COSMO15 @ Warsaw University 10/9/2015

Halo/Galaxy bispectrum with Equilateral-type Primordial Trispectrum

Shuntaro Mizuno (Waseda)

With Shuichiro Yokoyama

Phys. Rev. D 91, 123521 (arXiv: 1504.05505 [astro-ph.CO])

Primordial perturbations from inflation

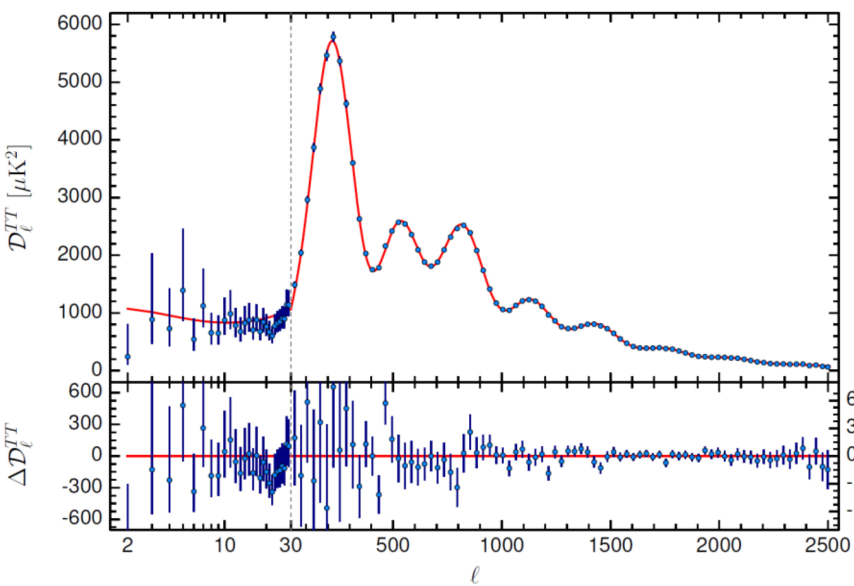
Inflation- extremely rapid expansion of the early universe

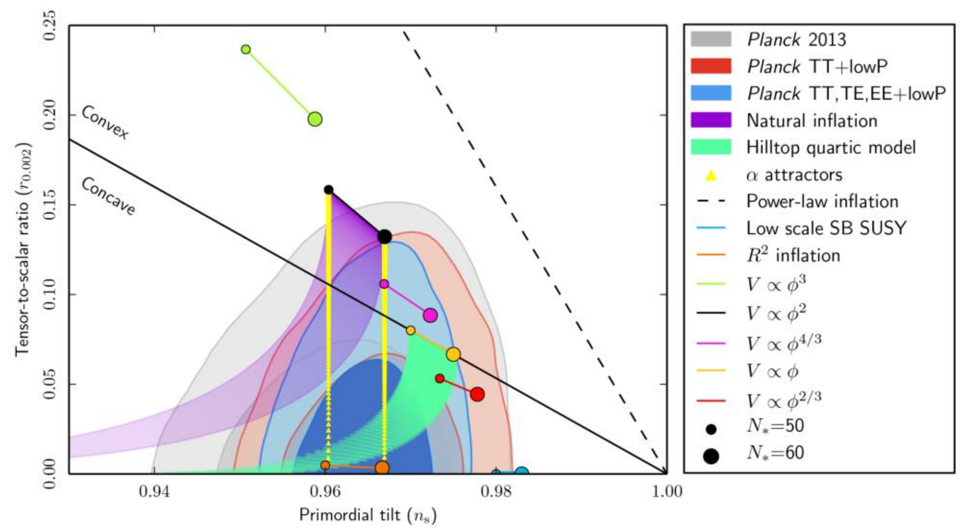
We can get information of high energy physics
by detailed observational results related with inflation

Predictions on primordial perturbations depend on inflation models

Ex.) Planck `15

$$n_s - 1 \equiv \frac{d\ln \mathcal{P}_\zeta}{d\ln k} \simeq -2\epsilon - 2\eta \quad \epsilon \equiv -\frac{\dot{H}}{H^2} \quad \eta \equiv \frac{\dot{\epsilon}}{H\epsilon} \quad r \equiv \frac{\mathcal{P}_h(k)}{\mathcal{P}_\zeta(k)} \simeq 16\epsilon$$





Primordial non-Gaussianity

$\mathcal{P}_\zeta^2 \, f_{\rm NL} \, \mathcal{S}(k_1, k_2, k_3)$

$$\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\rangle = (2\pi)^3 \delta^3(k_1 + k_2 + k_3) B_\zeta(k_1, k_2, k_3)$$

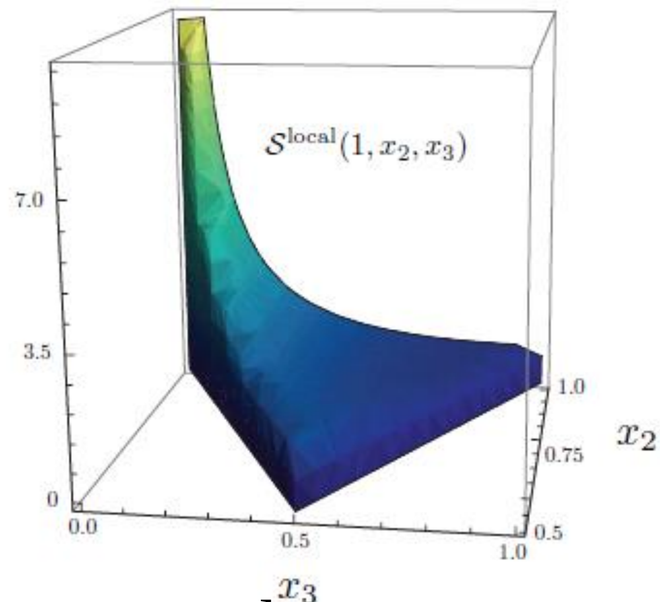
Local and equilateral shapes

$$x_2 \equiv k_2/k_1$$

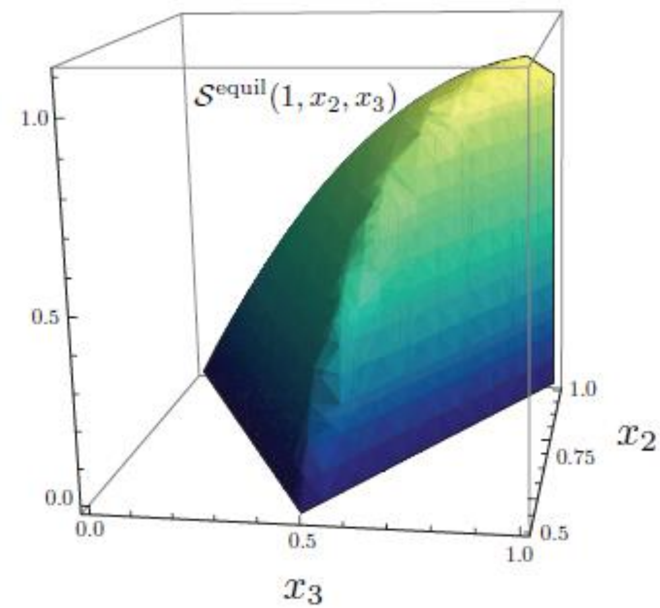
$$x_3 \equiv k_3/k_1$$

$k_3 < k_2 < k_1$

$k_2 + k_3 > k_1$



Local-type



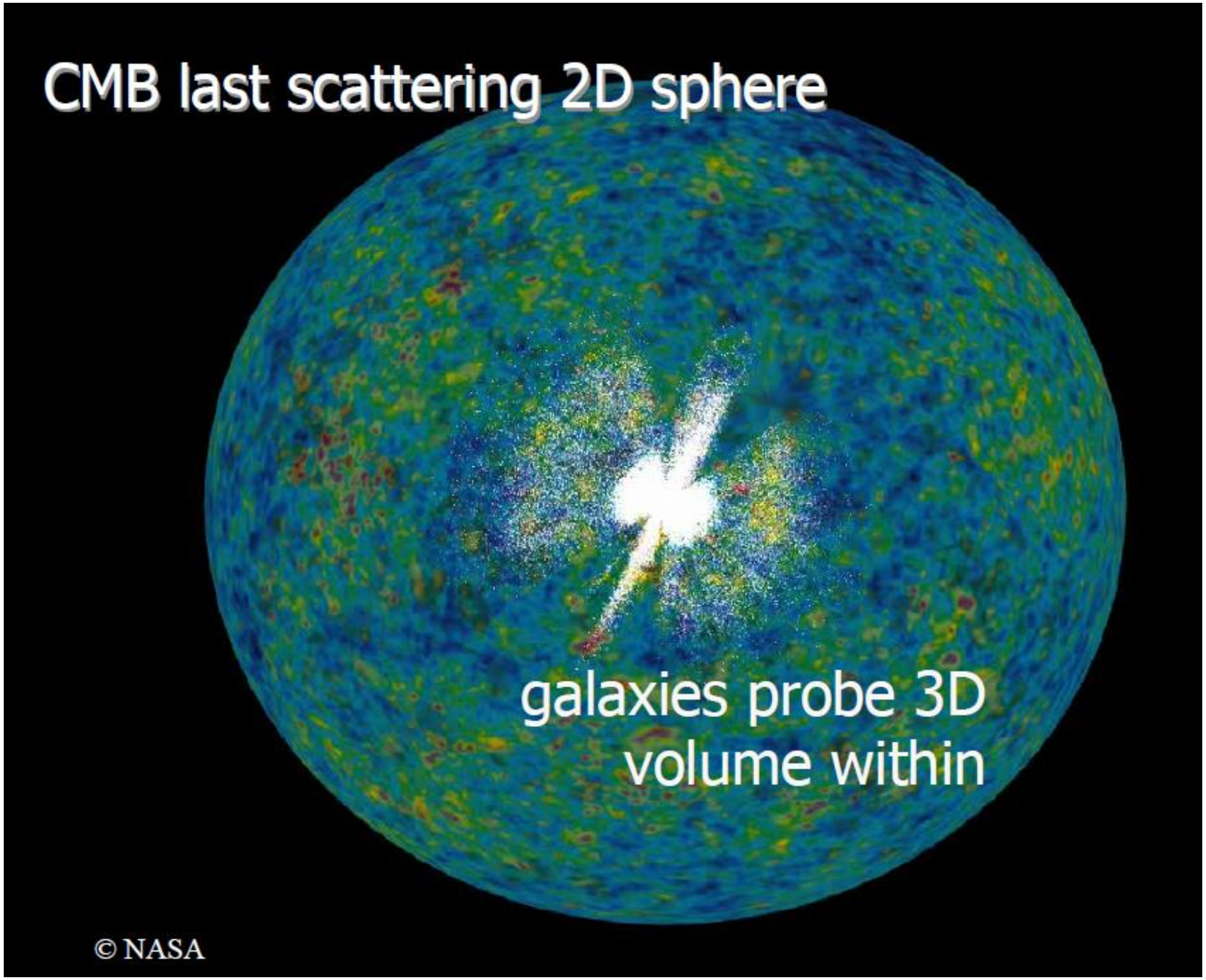
Equilateral-type

Planck constraint (68% CL)

$$f_{\rm NL}^{\rm local} = 0.8 \pm 5.0$$

$$f_{\rm NL}^{\rm equil} = -4 \pm 43$$

CMB vs LSS



LSS can give more stringent constraint !! (HSC, DES, LSST)

Integrated Perturbation Theory (iPT)

Matsubara `12, `13, Bernardeau et al `08

- Multi-point propagator of biased objects

$$\left\langle \frac{\delta^n \delta_X(\mathbf{k})}{\delta\delta_{\rm L}(\mathbf{k}_1)\delta\delta_{\rm L}(\mathbf{k}_2)\cdots\delta\delta_{\rm L}(\mathbf{k}_n)} \right\rangle = (2\pi)^{3-3n}\delta(\mathbf{k}_1+\mathbf{k}_2+\cdots+\mathbf{k}_n)\underline{\Gamma_X^{(n)}(\mathbf{k}_1,\mathbf{k}_2,\cdots,\mathbf{k}_n)}$$

Gravitational evolution

Lagrangian bias, ….

δ_X : number density field of the biased objects

$\delta_{\rm L}$: linear density field which is related with

primordial curvature perturbation ζ through

$$\delta_{\rm L}(k) = \mathcal{M}(k)\zeta(k);\ \mathcal{M}(k) = \frac{2}{3}\frac{D(z)}{D(z_*)(1+z_*)}\frac{k^2T(k)}{H_0^2\Omega_{\rm m0}}$$

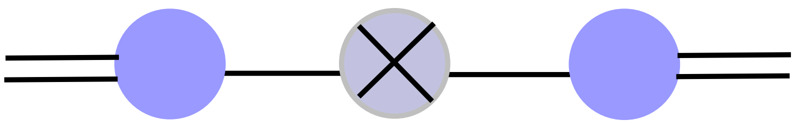
$D(a)$: growth factor $T(k)$: transfer function

⇒ spectra of biased objects (Halo/Galaxy) systematically !!

Effects on Halo/galaxy power spectrum

- Diagrams for the power spectrum of the biased objects

P_0



$\Gamma_X^{(1)}(\mathbf{k})\,P_{\rm L}(k)\,\Gamma_X^{(1)}(-\mathbf{k})$

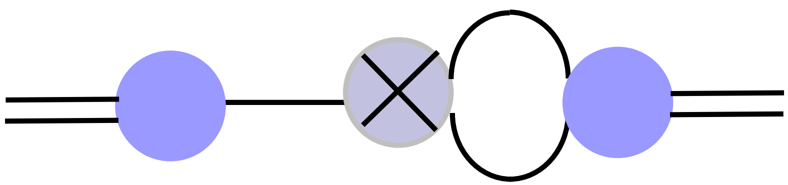
$\Rightarrow \propto \mathcal{M}(k)^2 P_\zeta(k) \propto k$

large scale limit

$k \ll p$

typical scale of the biased objects

$P_{\rm bis}$



$\Gamma_X^{(1)}(\mathbf{k})\,B_{\rm L}(k, p, |\mathbf{p}+\mathbf{k}|)\,\Gamma_X^{(2)}(\mathbf{p}, -\mathbf{p}-\mathbf{k})$

large scale limit $\Rightarrow$

$\propto \mathcal{M}(k)k^{-3} \propto k^{-1}$ for $B_\zeta^{\rm local}$ — enhancement

$\propto \mathcal{M}(k)k^{-1} \propto k$ for $B_\zeta^{\rm equil}$ — no enhancement

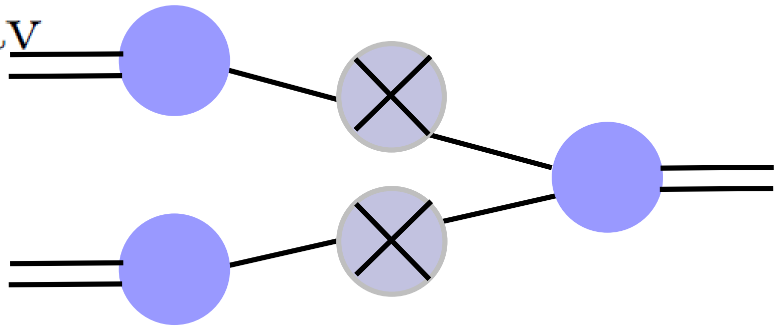
Dalal et al `08

Effects on Halo/galaxy bispectrum

Yokoyama, Matsubara, Taruya `13

- Diagrams for the bispectrum of the biased objects

$B_{\rm grav}$



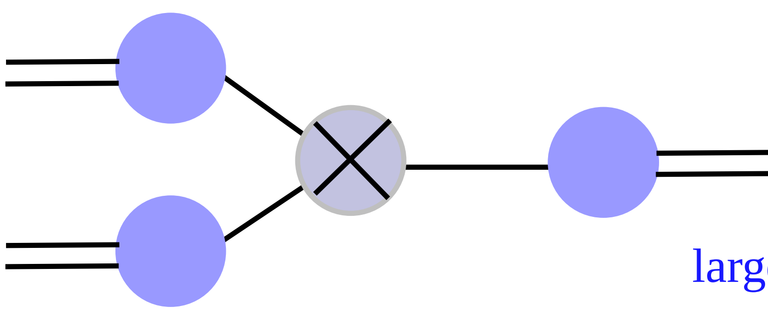
large scale limit

$\propto \mathcal{M}(k)^4 P_\zeta(k)^2 \propto k^2$

$k \equiv k_1 = k_2 = k_3 \ll p$

$$\Gamma_X^{(1)}(\mathbf{k}_1) P_L(k_1) \Gamma_X^{(1)}(\mathbf{k}_2) P_L(k_2) \Gamma_X^{(2)}(-\mathbf{k}_1, -\mathbf{k}_2)$$

$B_{\rm bis}$



large scale limit

$\propto \mathcal{M}(k)^3 P_\zeta(k)^2 \propto k^0$

for $B_\zeta^{\rm equil}$

$$\Gamma_X^{(1)}(\mathbf{k}_1) \Gamma_X^{(1)}(\mathbf{k}_2) B_{\rm L}(k_1, k_2, k_3) \Gamma_X^{(1)}(\mathbf{k}_3)$$

Halo/galaxy bispectrum with $f_{\rm NL}^{\rm equil}$

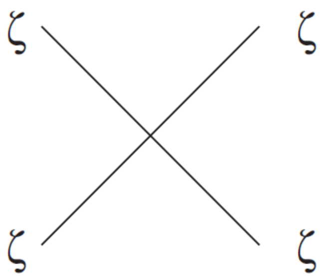
Making use of halo/galaxy bispectrum,
we can obtain $\Delta f_{\rm NL}^{\rm equil} < 20$
by future surveys !!

Sefusatti and Komatsu `07

Primordial trispectrum in general k-inflation

Arroja, SM, Koyama, Tanaka `09, Chen et al `09, (Smith, Senatore, Zaldarriaga `15)

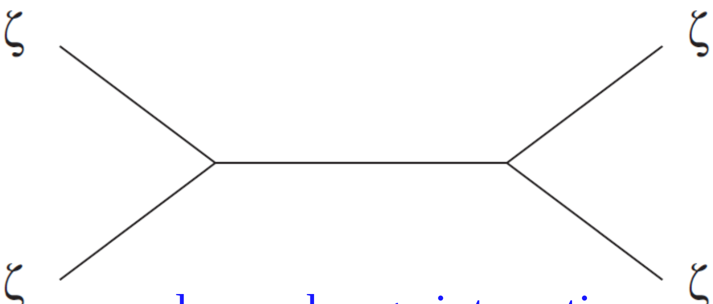
$$S = \frac{1}{2}\int d^4x\sqrt{-g}[M_{\rm Pl}^2 R + 2P(X,\phi)] \qquad X \equiv -(1/2)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$



contact interaction

$$\langle\Omega|\delta\phi(0,\mathbf{k}_1)\delta\phi(0,\mathbf{k}_2)\delta\phi(0,\mathbf{k}_3)\delta\phi(0,\mathbf{k}_4)|\Omega\rangle^{\rm CI}$$
$$= -i\int_{-\infty}^{0} d\eta\langle 0|[\delta\phi_I(0,\mathbf{k}_1)\delta\phi_I(0,\mathbf{k}_2)\delta\phi_I(0,\mathbf{k}_3)$$
$$\times\,\delta\phi_I(0,\mathbf{k}_4), \underline{H_I^{(4)}(\eta)}]|0\rangle,$$

$$H_I^{(4)}(\eta) = \int d^3x[\underline{\beta_1\delta\phi_I'^4} + \underline{\beta_2\delta\phi_I'^2(\partial\delta\phi_I)^2} + \underline{\beta_3(\partial\delta\phi_I)^4}],$$



scalar-exchange interaction

$$\langle\Omega|\delta\phi(0,\mathbf{k}_1)\delta\phi(0,\mathbf{k}_2)\delta\phi(0,\mathbf{k}_3)\delta\phi(0,\mathbf{k}_4)|\Omega\rangle^{\rm SE}$$
$$= -\int_{-\infty}^{0} d\eta\int_{-\infty}^{\eta} d\tilde{\eta}\langle 0|[[\delta\phi_I(0,\mathbf{k}_1)\delta\phi_I(0,\mathbf{k}_2)$$
$$\times\,\delta\phi_I(0,\mathbf{k}_3)\delta\phi_I(0,\mathbf{k}_4), \underline{H_I^{(3)}(\eta)}], \underline{H_I^{(3)}(\tilde{\eta})}]|0\rangle,$$

$$H_I^{(3)}(\eta) = \int d^3x[\underline{Aa\delta\phi_I'^3} + \underline{Ba\delta\phi_I'(\partial\delta\phi_I)^2}],$$

$$T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = T_\zeta^{c_1} + T_\zeta^{c_2} + T_\zeta^{c_3} + T_\zeta^{s_1} + T_\zeta^{s_2} + T_\zeta^{s_3}$$

Trispectra from contact interactions

- Concrete expressions

$$\frac{T_\zeta^{c_1}}{(2\pi^2\mathcal{P}_\zeta)^3} = \frac{221184}{25}\frac{g_{\rm NL}^{\dot\sigma^4}}{(\sum k_i)^5 k_1k_2k_3k_4}$$

$$\frac{T_\zeta^{c_2}}{(2\pi^2\mathcal{P}_\zeta)^3} = -\frac{27648}{325}g_{\rm NL}^{\dot\sigma^2(\partial\sigma)^2}\left[\frac{k_1^2k_2^2(\mathbf{k}_3\cdot\mathbf{k}_4)}{(\sum k_i)^3\Pi k_i^3}\left(1+3\frac{(k_3+k_4)}{\sum k_i}+12\frac{k_3k_4}{(\sum k_i)^2}\right)+\text{perms.}\right]$$

$$\frac{T_\zeta^{c_3}}{(2\pi^2\mathcal{P}_\zeta)^3} = \frac{165888}{2575}g_{\rm NL}^{(\partial\sigma)^4}\frac{[(\mathbf{k}_1\cdot\mathbf{k}_2)(\mathbf{k}_3\cdot\mathbf{k}_4)+\text{perms.}]}{\sum k_i\Pi k_i} \times\left(1+\frac{\sum_{i<j}k_ik_j}{(\sum k_i)^2}+3\frac{\Pi k_i}{(\sum k_i)^3}\sum\frac{1}{k_i}+12\frac{\Pi k_i}{(\sum k_i)^4}\right)$$

These trispectra also appear in effective field theory of inflation !!

- Constraints from CMB (95 % CL) Smith, Senatore, Zaldarriaga \`15

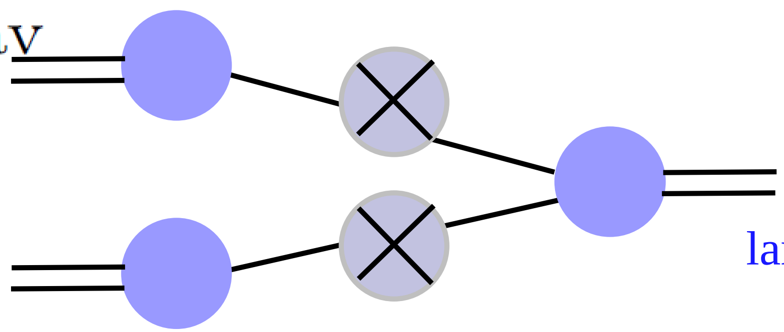
$$(-9.38\times10^6) < g_{NL}^{\dot\sigma^4} < (2.98\times10^6)$$

$$(-2.34\times10^6) < g_{NL}^{(\partial\sigma)^4} < (0.19\times10^6)$$

Effects on Halo/galaxy bispectrum (cont'd)

- Diagrams for the bispectrum of the biased objects

$B_{\rm grav}$

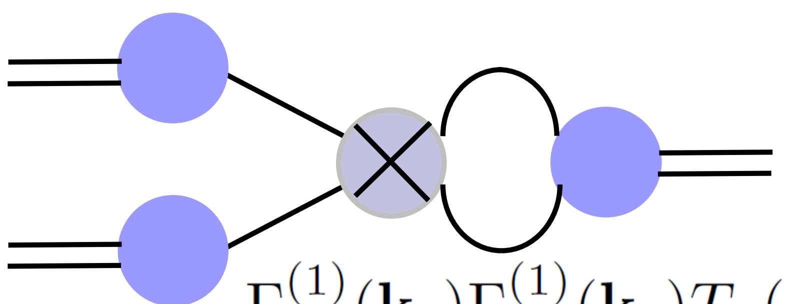


large scale limit

$$\propto \mathcal{M}(k)^4 P_\zeta(k)^2 \propto k^2$$

$$k \equiv k_1 = k_2 = k_3 \ll p$$

$B_{\rm tri}$



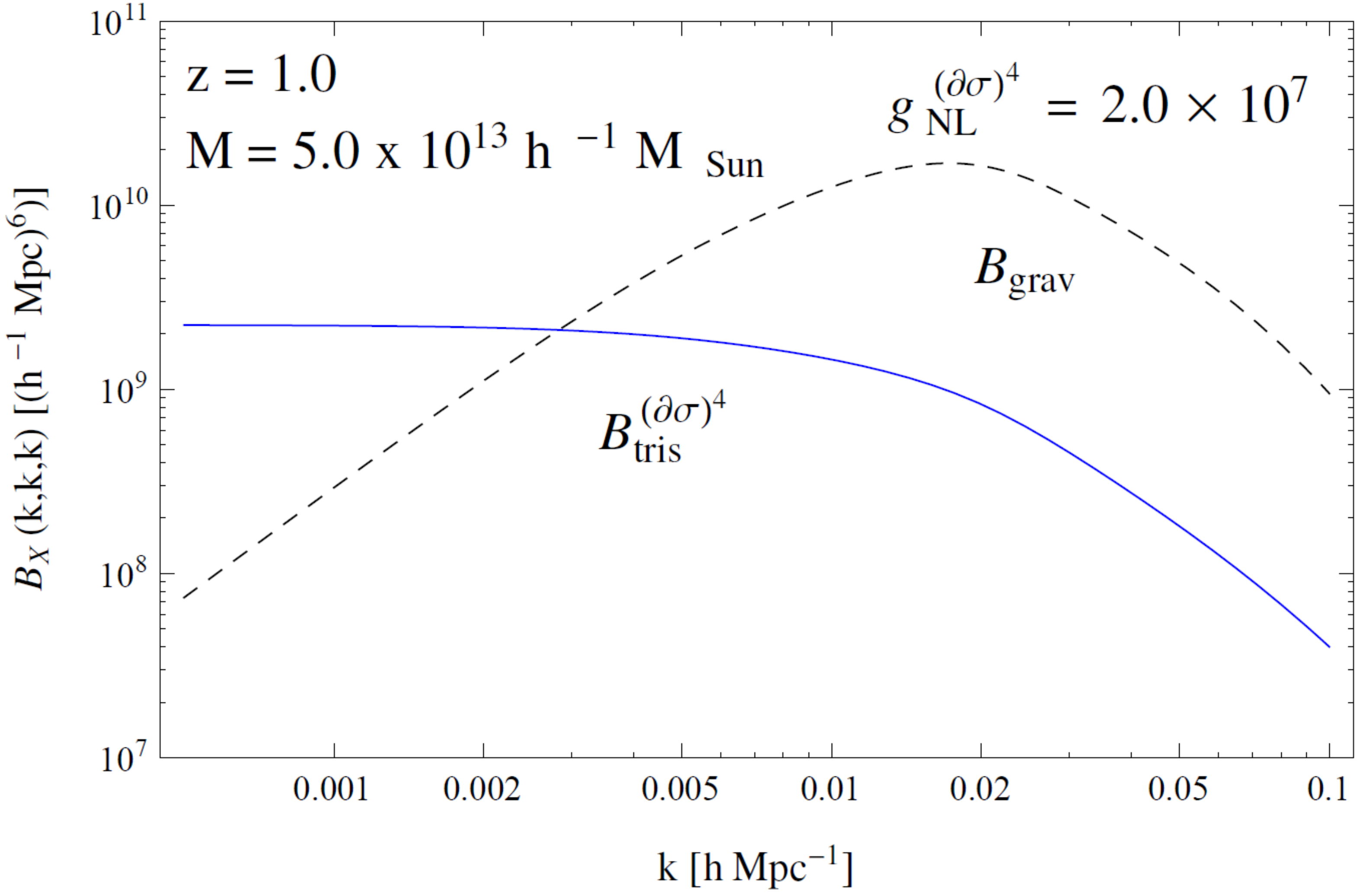
$$\Gamma_X^{(1)}(\mathbf{k}_1)\Gamma_X^{(1)}(\mathbf{k}_2)T_{\rm L}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{p}, \mathbf{k}_3 - \mathbf{p})\Gamma_X^{(2)}(\mathbf{p}, \mathbf{k}_3 - \mathbf{p})$$

large scale limit

$$\begin{cases} \propto \mathcal{M}(k)^2 k^{-2} \propto k^2 & \text{for } g_{\rm NL}^{\dot\sigma^4} \\ \propto \mathcal{M}(k)^2 k^{-4} \propto k^0 & \text{for } g_{\rm NL}^{\dot\sigma^2(\partial\sigma)^2},\ g_{\rm NL}^{(\partial\sigma)^4} \end{cases}$$

Halo/galaxy bispectrum with g_{NL}^{equil}

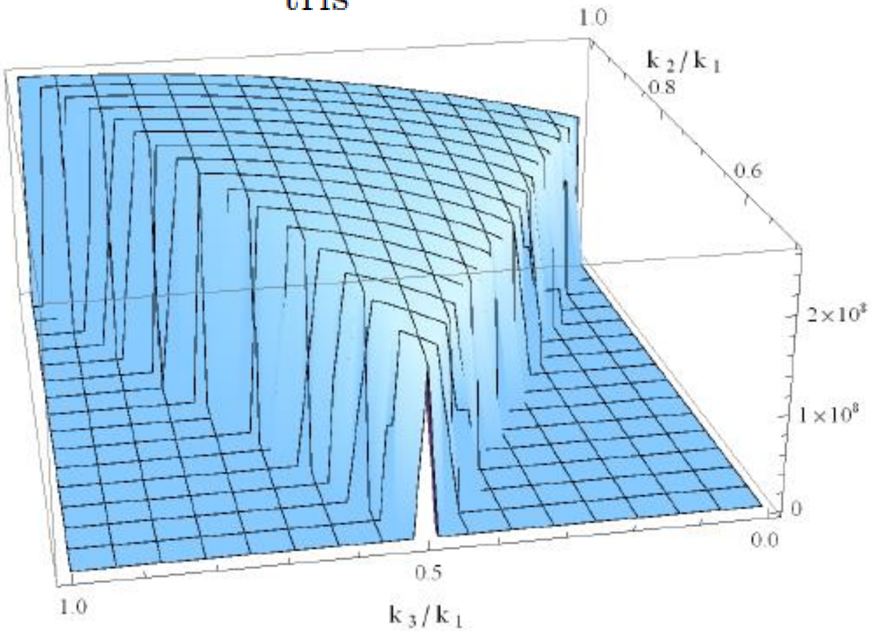
Adopting maximum allowed values by CMB observations



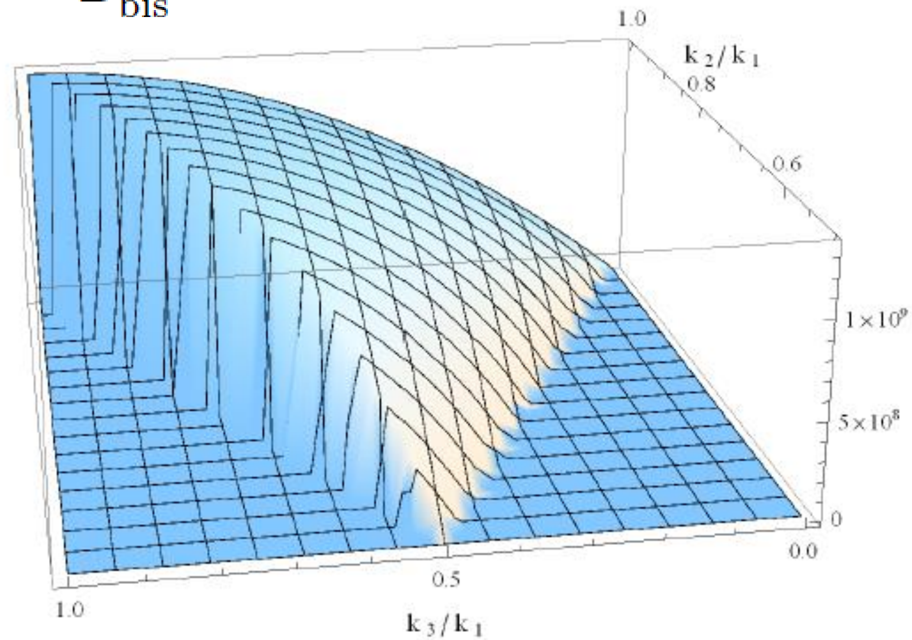
Contributions from $g_{\rm NL}^{(\partial\sigma)^4}$ and $f_{\rm NL}^{\rm equil}$

- Shape-dependence of Halo/galaxy bispectrum

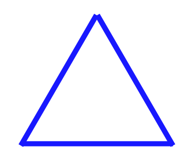
$B_{\rm tris}^{(\partial\sigma)^4}$



$B_{\rm bis}^{\rm equil}$



So far, we have limited the equilateral configuration ($k_1 = k_2 = k_3 = k$)



But the folded configuration ($k_1/2 = k_2 = k_3 = k$)
is also helpful to distinguish $B_{\rm tris}^{(\partial\sigma)^4}$ with $B_{\rm bis}^{\rm equil}$

Conclusions and Discussions

• Halo/galaxy bispectrum was shown to be useful tool to distinguish equilateral-type NG from gravitational nonlinearity.

$$B_{\rm grav} \propto k^2, \quad B_{\rm bis}^{\rm equil} \propto k^0 \Longrightarrow \Delta f_{\rm NL}^{\rm equil} < 20$$

• We can also constrain primordial trispectrum generated by general k-inflation based on halo/galaxy bispectrum.

$$B_{\rm tris}^{\dot\sigma^4} \propto k^2, \quad B_{\rm tris}^{\dot\sigma^2(\partial\sigma)^2,\,(\partial\sigma)^4} \propto k^0 \Longrightarrow \Delta g_{\rm NL}^{\rm equil} = ??$$

• Constraints on more general class of inflation models which give equilateral-type bispectrum

k-inflation (scalar-exchange interaction)

Ghost inflation, Lifshitz scalar, Galileon inflation,….

Appendix

Cosmological parameters

- Planck (2015)

$$h = 0.678\,, \quad \Omega_{m0} = 0.308\,,$$

$$\Omega_{b0}h^2 = 0.02226\,, \quad \Omega_{r0}h^2 = 4.15 \times 10^{-5}$$

- Growth factor

Linder `05

$$D(a) = \left(e^{\int_0^a d\ln a [\Omega_m(a)^{6/11} - 1]} \right) a$$

Transfer function

Chongchitnan and Silk `10

$$T(x) = \frac{\ln[1+(0.124x)^2]}{(0.124x)^2}\left[\frac{1+(1.257x)^2+(0.4452x)^4+(0.2197x)^6}{1+(1.606x)^2+(0.8568x)^4+(0.3927x)^6}\right]^{1/2}$$

$$x_{\rm EH} = \frac{k\Omega_{r0}^{1/2}}{H_0\Omega_{m0}}\left[\alpha+\frac{1-\alpha}{1+(0.43ks)^4}\right]^{-1}$$

$$\alpha = 1-0.328\ln(431\Omega_{m0}h^2)\frac{\Omega_{b0}}{\Omega_{m0}}+0.38\ln(22.3\Omega_{m0}h^2)\left(\frac{\Omega_{b0}}{\Omega_{m0}}\right)^2$$

$$s = \frac{44.5\ln(9.83/\Omega_{m0}h^2)}{\sqrt{1+10(\Omega_{b0}h^2)^{3/4}}}\,\mathrm{Mpc}$$

Multi-point propagators on large scales

Matsubara `12

$$\Gamma_X^{(1)}(\mathbf{k}) \approx 1 + \underline{c_1^{\rm L}(k)}$$

$$\Gamma_X^{(2)}(\mathbf{k}_1,\mathbf{k}_2) \approx F_2(\mathbf{k}_1,\mathbf{k}_2) + \left(1+\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_2^2}\right)\underline{c_1^{\rm L}(\mathbf{k}_1)} + \left(1+\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1^2}\right)\underline{c_1^{\rm L}(\mathbf{k}_2)} + \underline{c_2^{\rm L}(\mathbf{k}_1,\mathbf{k}_2)}$$

$$F_2(\mathbf{k}_1,\mathbf{k}_2) = \frac{10}{7} + \left(\frac{k_2}{k_1}+\frac{k_1}{k_2}\right)\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1 k_2} + \frac{4}{7}\left(\frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1 k_2}\right)^2$$

$c_n^{\rm L}$: renormalized bias function defined in Lagrangian space

$$\left\langle \frac{\delta^n \delta_X^{\rm L}(\mathbf{k})}{\delta\delta_{\rm L}(\mathbf{k}_1)\delta\delta_{\rm L}(\mathbf{k}_2)\cdots\delta\delta_{\rm L}((\mathbf{k}_n)} \right\rangle = (2\pi)^{3-3n}\delta(\mathbf{k}_1+\mathbf{k}_2+\cdots+\mathbf{k}_n)c_n^{\rm L}(\mathbf{k}_1,\mathbf{k}_2,\cdots,\mathbf{k}_n)$$

The other parts include the information of displacement field in Lagrangian perturbation theory

Renormalized bias function

Matsubara `12

Cf. Press-Schechter formalism `74

$$c_n^{\mathrm{L}}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n) = \frac{(-1)^n \frac{\partial}{\partial M}\left[\frac{\partial^n P(M,\delta_{\mathrm{c}})}{\partial \delta_{\mathrm{c}}{}^n} W(k_1 R) \cdots W(k_n R)\right]}{\frac{\partial P(M,\delta_{\mathrm{c}})}{\partial M}}$$

$\delta_M(\boldsymbol{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} W(kR)\delta_{\mathrm{L}}(\boldsymbol{k})$: linear density field smoothed over mass scale M

$W(kR) = 3\left[\frac{\sin(kR)}{(kR)^3} - \frac{\cos(kR)}{(kR)^2}\right]$: top-hat window function

$$M \equiv \frac{4}{3}\pi R^3 \rho_{m0} \simeq 1.16 \times 10^{12} \Omega_{m0} \left(\frac{R}{h^{-1}\mathrm{Mpc}}\right)^3 h^{-1} M_\odot$$

$n(\boldsymbol{x}, M) = -\frac{2\bar{\rho}_0}{M}\frac{\partial}{\partial M}\Theta[\delta_M(\boldsymbol{x}) - \delta_{\mathrm{c}}]$ $\delta_c(\simeq 1.686)$: critical density for collapse

step functional behavior

averaging ⇒ $n(M) = -\frac{2\bar{\rho}_0}{M}\frac{\partial}{\partial M}P(M, \delta_{\mathrm{c}})$ probability that $\delta_M > \delta_c$

$$\delta_{\mathrm{h}}^{\mathrm{L}}(\boldsymbol{x}) = n(\boldsymbol{x}, M)/n(M) - 1$$

Renormalized bias function (cont'd)

Matsubara `12

For universal mass functions $f_{\rm MF}(\nu)$ satisfying

$$n(M)dM = \frac{\bar{\rho}_0}{M} f_{\rm MF}(\nu) \frac{d\nu}{\nu} \quad \text{and} \quad \int_0^\infty f_{\rm MF}(\nu)\frac{d\nu}{\nu} = 1 \qquad \text{with} \quad \nu = \delta_c/\sigma_M$$

$$\sigma_M{}^2 = \int \frac{d^3k}{(2\pi)^3} W^2(kR) P_{\rm L}(k)$$: density variance of mass scale M

$$c_n^{\rm L}(\mathbf{k}_1,\cdots,\mathbf{k}_n) = \frac{A_n(M)}{\delta_c^n} W(k_1;M)\cdots W(k_n;M) + \frac{A_{n-1}(M)\sigma_M^n}{\delta_c^n} \frac{d}{d\ln\sigma_M}\left[\frac{W(k_1;M)\cdots W(k_n;M)}{\sigma_M^n}\right]$$

$$A_n(M) \equiv \sum_{j=0}^{n} \frac{n!}{j!} \delta_c^j \underline{b_j^{\rm L}(M)}$$

j-th order Lagrangian bias parameter

$$b_n^{\rm L}(M) = (-\sigma_M)^{-n} f_{\rm MF}^{-1} \frac{d^n}{d\nu^n}(f_{\rm MF}(\nu))$$

Mass function

For the mass function, we adopted Sheth-Tormen model given by

$$f_{\mathrm{ST}}(\nu) = A(p)\sqrt{\frac{2}{\pi}}\left[1+(q\nu^2)^{-p}\right]\sqrt{q}\nu e^{-q\nu^2/2}$$

$$p = 0.3,\ q = 0.707$$

$$A(p) = \left[1+\Gamma(1/2-p)/(\sqrt{\pi}2^p)\right]^{-1}$$

$$\Rightarrow \quad c_n^{\mathrm{L}}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) \approx b_n^{\mathrm{L}}(M) \qquad (|\boldsymbol{k}_i| \to 0)$$

$$b_1^{\mathrm{L}}(M) = \frac{1}{\delta_{\mathrm{c}}}\left[q\nu^2 - 1 + \frac{2p}{1+(q\nu^2)^p}\right]$$

$$b_2^{\mathrm{L}}(M) = \frac{1}{{\delta_{\mathrm{c}}}^2}\left[q^2\nu^4 - 3q\nu^2 + \frac{2p(2q\nu^2+2p-1)}{1+(q\nu^2)^p}\right]$$

no scale-dependence on large scales

Primordial trispectrum $T_\zeta^{s_1}$

Arroja, SM, Koyama, Tanaka `09

$$T_\zeta^{s_1} \propto \mathcal{F}_1(k_1, k_2, -k_{12}, k_3, k_4, k_{12}) - \mathcal{F}_1(-k_1, -k_2, -k_{12}, k_3, k_4, k_{12}))$$
$$+ \text{perms.}$$

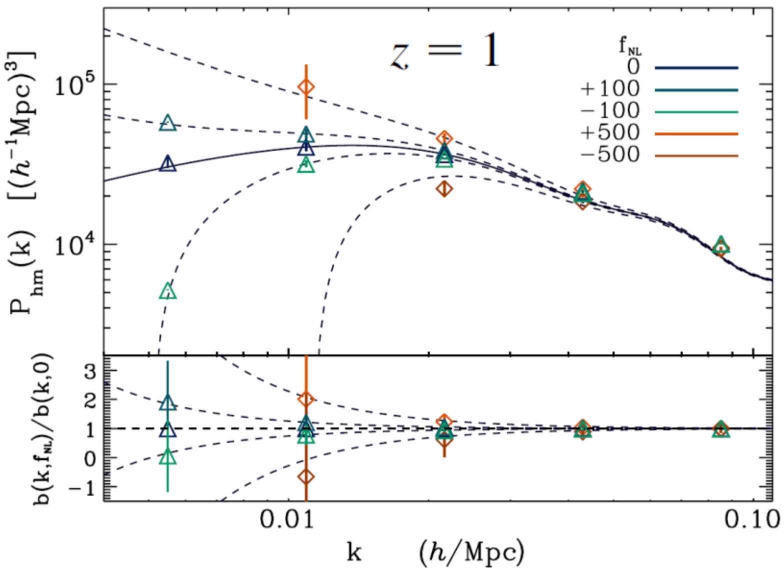
$$\mathcal{F}_1(k_1, k_2, k_3, k_4, k_5, k_6) = \frac{1}{\mathcal{A}^3\mathcal{C}^3}\left(1 + 3\frac{\mathcal{A}}{\mathcal{C}} + 6\frac{\mathcal{A}^2}{\mathcal{C}^2}\right)$$

$$\mathcal{A} = k_4 + k_5 + k_6$$

$$\mathcal{C} = k_1 + k_2 + k_3 + k_4 + k_5 + k_6$$

Constraints on local-type NG from LSS

Dalal et al `08



$$1.6 \times 10^{13} M_\odot < M < 3.2 \times 10^{13} M_\odot$$

Constraints on bispectrum

Giannatonio et al `13

$$-36 < f_{\rm NL}^{\rm local} < 45 \quad \text{(95\% CL)}$$

Future constraints:

Yamauchi et al `14

SKA (Square Km Array)

$|f_{NL}| < 0.1$?

Constraints on trispectrum

Desjacques and Seljak `10

$$\zeta(x) = \zeta_G(x) + \frac{9}{25} g_{\rm NL}^{\rm local} \zeta_G^3(x)$$

$$-3.5 \times 10^5 < g_{\rm NL}^{\rm local} < 8.2 \times 10^5 \quad \text{(95\% CL)}$$

How about equilateral-type NG ?

Halo/galaxy bispectrum with $f_{\rm NL}^{\rm equil}$