

# Theoretical approaches to the Dark Energy paradigm

Cosmo-15 Conference, Warsaw, 7-11 Sept 2015

David Polarski

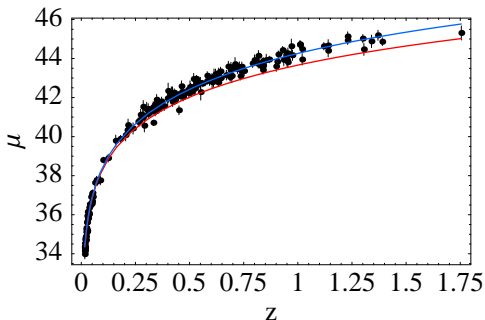
LCC, Université Montpellier

September 10 2015

# Dark Energy paradigm comes from **observations**: SNIa Luminosity-distances

$$\mathcal{F} = \frac{L}{4\pi d_L^2}$$

$$m - M = 5 \log d_L + 25$$



$\Lambda$ CDM     $\Omega_{m,0} = 0.3$      $\Omega_{k,0} = 0$     EdS

## Expansion different from standard cosmology

$$\ddot{a} < 0 \rightarrow \ddot{a} > 0 \quad @ \quad z \sim 0.5$$

### Dark Energy puzzle:

What is the origin of this accelerated expansion ?

We are not really unhappy...

$$\Omega_{m,0} \approx 0.3, \quad \Omega_{DE,0} \approx 0.7, \quad \Omega_{k,0} \approx 0$$

A consistent vision has emerged supported by many observations: SNIa, CMB, BAO,...

Dark Energy

Cosmological constant

Pending questions

Modified gravity

Scalar-tensor DE models

Chameleon models,  $f(R)$ 

Approaches and tools

Growth function, growth index

Outlook

“Old” simple solution: cosmological constant  $\Lambda$   
“...My greatest blunder...” A. Einstein

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_m + \frac{\Lambda}{3}$$

Conceptual problem :  $\Lambda \sim 10^{-122} l_{Pl}^{-2}$   
Consistent with *all* observations ?

Many contenders: Scalar field models, modified gravity,...

But  $\Lambda$ CDM remains the model to beat!

Dark Energy

Cosmological constant

Pending questions

Modified gravity

Scalar-tensor DE models

Chameleon models,  $f(R)$ 

Approaches and tools

Growth function, growth index

Outlook

Pending questions:

Is  $\rho_{DE}$  constant / Is  $w_{DE} = -1$  ?  $w_{DE}(z)$  ?

$\Omega_{DE} \rightarrow 0$  for  $z \gg 1$  ?

Is DE related to Inflation?

Coupling in the dark sector ?

Smooth component ?

Is DE connected to dark matter ?

Is DE a perfect fluid ?

Is gravity described by GR ?

Is our universe homogeneous ?

Higher dimensions ?

Theoretical  
approaches to the  
Dark Energy  
paradigm

David Polarski

Dark Energy

Cosmological  
constant

Pending questions

Modified gravity

Scalar-tensor DE  
models

Chameleon  
models,  $f(R)$

Approaches and  
tools

Growth function,  
growth index

Outlook

► **Modified gravity DE models**

Maybe gravity differs from GR on large scales ?  
Accelerated expansion without DE component ?

We keep the RW metric

$$ds^2 = dt^2 - a^2(t) d\ell^2$$

**We get modified Friedmann equations**

They can be often recast in an “Einsteinian” way

**The growth of perturbations gets modified as well!**

Crucial probe of modified gravity models

▶  $L = \frac{1}{16\pi G_*} \left( F(\Phi) R - Z \partial_\mu \Phi \partial^\mu \Phi - 2U(\Phi) \right) + L_m(g_{\mu\nu})$

▶ Brans-Dicke parametrization

$$F(\Phi) = \Phi \qquad Z(\Phi) = \frac{\omega_{BD}(\Phi)}{\Phi}$$

Another choice

$$F(\Phi) = \text{arbitrary} \qquad Z = 1 \Leftrightarrow \omega_{BD} > 0$$

$$\omega_{BD} = \frac{F}{(dF/d\Phi)^2} > -\frac{3}{2} \qquad \omega_{BD,0} > 4 \times 10^4$$

▶  $V = -G_{\text{eff}} \frac{M_1 M_2}{r}$  massless  $\Phi$  field

$$G_{\text{eff}} = G_N \left( 1 + \frac{1}{2\omega_{BD} + 3} \right) \qquad G_N = \frac{G_*}{F}$$

▶  $G_{\text{eff},0} \simeq G_{N,0} \simeq G$

Dark Energy

Cosmological constant

Pending questions

Modified gravity

Scalar-tensor DE models

Chameleon models,  $f(R)$

Approaches and tools

Growth function, growth index

Outlook

▶  $L = \frac{1}{16\pi G_*} \left( F(\Phi) R - Z \partial_\mu \Phi \partial^\mu \Phi - 2U(\Phi) \right) + L_m(g_{\mu\nu})$

▶ Brans-Dicke parametrization

$$F(\Phi) = \Phi$$

$$Z(\Phi) = \frac{\omega_{BD}(\Phi)}{\Phi}$$

Another choice

$$F(\Phi) = \text{arbitrary}$$

$$Z = 1 \Leftrightarrow \omega_{BD} > 0$$

$$\omega_{BD} = \frac{F}{(dF/d\Phi)^2} > -\frac{3}{2}$$

$$\omega_{BD,0} > 4 \times 10^4$$



$$V = -G_{\text{eff}} \frac{M_1 M_2}{r}$$

massless  $\Phi$  field

$$G_{\text{eff}} = G_N \left( 1 + \frac{1}{2\omega_{BD} + 3} \right)$$

$$G_N = \frac{G_*}{F}$$



$$G_{\text{eff},0} \simeq G_{N,0} \simeq G$$



$$\blacktriangleright L = \frac{1}{16\pi G_*} \left( F(\Phi) R - Z \partial_\mu \Phi \partial^\mu \Phi - 2U(\Phi) \right) + L_m(g_{\mu\nu})$$

$\blacktriangleright$  Brans-Dicke parametrization

$$F(\Phi) = \Phi \qquad Z(\Phi) = \frac{\omega_{BD}(\Phi)}{\Phi}$$

Another choice

$$F(\Phi) = \text{arbitrary} \qquad Z = 1 \Leftrightarrow \omega_{BD} > 0$$

$$\omega_{BD} = \frac{F}{(dF/d\Phi)^2} > -\frac{3}{2} \qquad \omega_{BD,0} > 4 \times 10^4$$

$\blacktriangleright$

$$V = -G_{\text{eff}} \frac{M_1 M_2}{r} \qquad \text{massless } \Phi \text{ field}$$

$$G_{\text{eff}} = G_N \left( 1 + \frac{1}{2\omega_{BD} + 3} \right) \qquad G_N = \frac{G_*}{F}$$

$\blacktriangleright$   $G_{\text{eff},0} \simeq G_{N,0} \simeq G$

$$\blacktriangleright L = \frac{1}{16\pi G_*} \left( F(\Phi) R - Z \partial_\mu \Phi \partial^\mu \Phi - 2U(\Phi) \right) + L_m(g_{\mu\nu})$$

$\blacktriangleright$  Brans-Dicke parametrization

$$F(\Phi) = \Phi \qquad Z(\Phi) = \frac{\omega_{BD}(\Phi)}{\Phi}$$

Another choice

$$F(\Phi) = \text{arbitrary} \qquad Z = 1 \Leftrightarrow \omega_{BD} > 0$$

$$\omega_{BD} = \frac{F}{(dF/d\Phi)^2} > -\frac{3}{2} \qquad \omega_{BD,0} > 4 \times 10^4$$

$$\blacktriangleright V = -G_{\text{eff}} \frac{M_1 M_2}{r} \qquad \text{massless } \Phi \text{ field}$$

$$G_{\text{eff}} = G_N \left( 1 + \frac{1}{2\omega_{BD} + 3} \right) \qquad G_N = \frac{G_*}{F}$$

$$\blacktriangleright G_{\text{eff},0} \simeq G_{N,0} \simeq G$$

► Modified background equations

$$3FH^2 = 8\pi G_* \rho_m + \frac{\dot{\phi}^2}{2} + U - 3H\dot{F}$$

$$-2F\dot{H} = 8\pi G_* \rho_m + \dot{\phi}^2 + \ddot{F} - H\dot{F}$$

*Define*  $\rho_{DE}$  and  $p_{DE}$ :

$$3H^2 = 8\pi G_{N,0} (\rho_m + \rho_{DE})$$

$$-2\dot{H} = 8\pi G_{N,0} (\rho_m + \rho_{DE} + p_{DE})$$

►  $\frac{d\ln^2}{dz} < 3 \Omega_{m,0} (1+z)^2 + 2 \Omega_{k,0} (1+z) \iff$  phantom

Possible in scalar-tensor models

$$8\pi G_* (\rho_{DE} + p_{DE}) = \dot{\phi}^2 + \ddot{F} - H\dot{F} + 2(F - F_0) \dot{H}$$

► Modified background equations

$$3FH^2 = 8\pi G_* \rho_m + \frac{\dot{\phi}^2}{2} + U - 3H\dot{F}$$

$$-2F\dot{H} = 8\pi G_* \rho_m + \dot{\phi}^2 + \ddot{F} - H\dot{F}$$

*Define*  $\rho_{DE}$  and  $p_{DE}$ :

$$3H^2 = 8\pi G_{N,0} (\rho_m + \rho_{DE})$$

$$-2\dot{H} = 8\pi G_{N,0} (\rho_m + \rho_{DE} + p_{DE})$$

►  $\frac{d^2 r^2}{dz^2} < 3 \Omega_{m,0} (1+z)^2 + 2 \Omega_{k,0} (1+z) \iff$  phantom

Possible in scalar-tensor models

$$8\pi G_* (\rho_{DE} + p_{DE}) = \dot{\phi}^2 + \ddot{F} - H\dot{F} + 2(F - F_0) \dot{H}$$

$$L = \frac{1}{2} \left( F(\Phi) R - Z \partial_\mu \Phi \partial^\mu \Phi - 2U(\Phi) \right)$$

$$ZF = -\frac{1}{6} \Phi^2 + \kappa^{-2}$$

$$ZU = \frac{\Lambda}{\kappa^2} - c\Phi^4 \quad \Lambda, c > 0$$

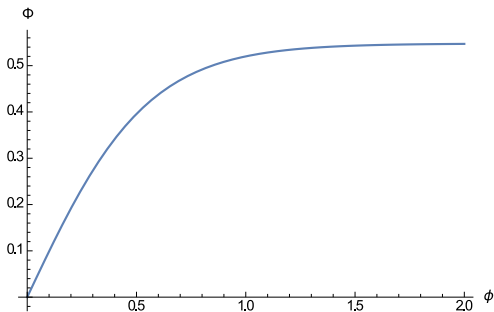
$$3H^2 = \Lambda + \kappa^2 \frac{A}{a^4},$$

$$\frac{1}{2} \left( \frac{d\chi}{d\eta} \right)^2 - c\chi^4 = A \quad \chi \equiv a\Phi$$

$$a_B = \left( \frac{-A\kappa^2}{\Lambda} \right)^{\frac{1}{4}} \quad A < 0$$

Family of non-degenerate spatially flat integrable bouncing solutions

## In EF (Quintessence!): integrable solutions with inverted double-well potential



**Figure:** It is seen that  $\Phi$  is a monotonically growing function of  $\phi$ . The limit  $\Phi \rightarrow \frac{\sqrt{6}}{\kappa} \equiv \Phi_{max}$  corresponds to  $\phi \rightarrow \infty$ . The interval  $0 < \phi < \infty$  covers the physically viable interval  $0 < \Phi < \Phi_{max}$  for which  $F > 0$  in the JF .

Dark Energy

Cosmological constant

Pending questions

Modified gravity

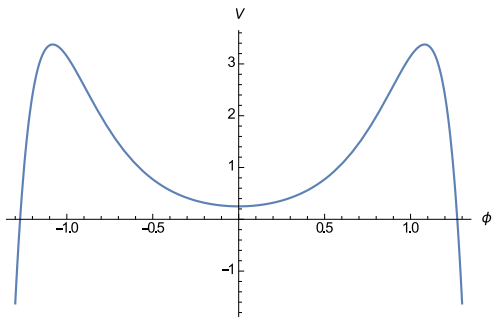
Scalar-tensor DE models

Chameleon models,  $f(R)$ 

Approaches and tools

Growth function, growth index

Outlook



**Figure:** The EF potential  $V$  is shown in the case  $Z = 1$  for the same parameters. The value  $\phi = \infty$  corresponds to  $\Phi = \frac{\sqrt{6}}{\kappa} \equiv \Phi_{max}$ , the unphysical limit where  $F$  vanishes and for which either a Big Bang or a Big Crunch takes place in the EF.

## Technical details

$$ds^2 = (1 + 2\phi)dt^2 - a^2 (1 - 2\psi)d\mathbf{x}^2$$

$$\phi = \psi - \frac{\delta F}{F}$$

### In quasi static limit

Perturbed dilaton equation of motion:

$$\delta\Phi = (\phi - 2\psi) \frac{dF}{d\Phi} = -\phi \sqrt{F} \frac{\sqrt{\omega_{BD}}}{2 + \omega_{BD}}$$

Combination of the perturbed Einstein equations:

$$\frac{k^2}{a^2} \phi = 4\pi G_{\text{eff}} \rho_m \delta_m$$

$$\Rightarrow \ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\text{eff}} \rho_m \delta_m = 0$$



$$L = \frac{R}{16\pi G_*} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) + L_m [\Psi_m; A^2(\phi) g_{\mu\nu}]$$

## Interacting dark sector

$f(R)$  modified gravity DE models:  $R \rightarrow f(R)$

e.g.  $R - \lambda R_c \left( 1 - \left( 1 + \frac{R^2}{R_c^2} \right)^{-n} \right)$ ,  $n, \lambda > 0$  ( $n \geq 2$ )

$$G_{\text{eff}}(\mathbf{z}, k) = \frac{G_*}{F} \left( 1 + \frac{1}{3} \frac{\frac{k^2}{a^2 m^2}}{1 + \frac{k^2}{a^2 m^2}} \right) \quad \frac{F}{F'} \equiv 3 m^2$$

$$\Leftrightarrow V(r) = -\frac{G_*}{F} \frac{M_1 M_2}{r} \left( 1 + \frac{1}{3} e^{-mr} \right) \quad (1)$$

## Chameleon mechanism

Dark Energy

Cosmological constant

Pending questions

Modified gravity

Scalar-tensor DE models

Chameleon models,  $f(R)$ 

Approaches and tools

Growth function, growth index

Outlook

## Horndeski model

## Galileon model

modified Friedmann eqs with “effective”  $\rho_{DE}(\phi, \dot{\phi})$

$w_{DE,0} = -1$ ,  $w_{DE}$  can be  $< -1$

$G_{\text{eff}}(z) \Rightarrow$  signature in the perturbation growth

Laboratory and solar system constraints: Vainshtein mechanism

## Massive gravity

Dark Energy

Cosmological constant

Pending questions

Modified gravity

Scalar-tensor DE models

Chameleon models,  $f(R)$

Approaches and tools

Growth function, growth index

Outlook

Various theoretical frameworks and approaches:

PPF (Parametrized Post-Friedmannian)

EFT (Effective Field Theory)

Cosmographic approach, Reconstruction

Phenomenological tools:

EoS parametrizations, null-tests, growth index,...

Dark Energy

Cosmological  
constant

Pending questions

Modified gravity

Scalar-tensor DE  
models

Chameleon  
models,  $f(R)$

Approaches and  
tools

Growth function,  
growth index

Outlook

Matter perturbations can be characterized by the “growth function”  $f = \frac{d \ln \delta}{d \ln a} \equiv \frac{d \ln \delta}{dx}$

$$\frac{df}{dx} + f^2 + \frac{1}{2} (1 - 3 w_{\text{eff}}) f = \frac{3}{2} \frac{G_{\text{eff}}}{G} \Omega_m$$

A convenient “parameterization”  $f = \Omega_m^\gamma$ .

Actually

$$\delta_m(\mathbf{z}, \mathbf{k}) \Leftrightarrow \gamma = \gamma(\mathbf{z}, \mathbf{k})$$

In  $\Lambda$ CDM:  $\gamma \simeq 0.55$

It can be very different in modified gravity models!

Dark Energy

Cosmological constant

Pending questions

Modified gravity

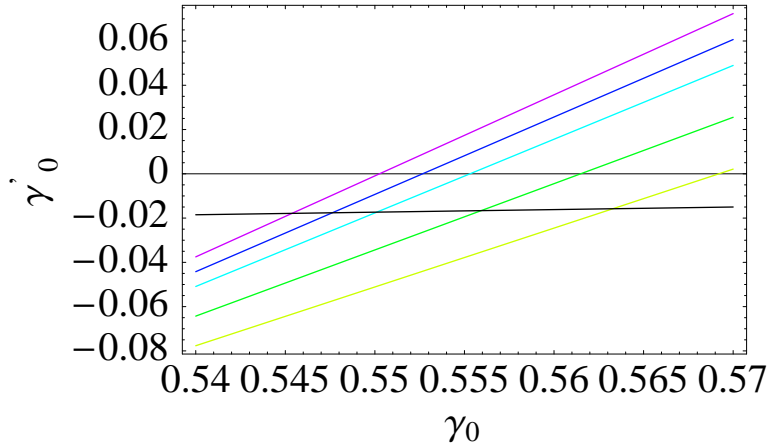
Scalar-tensor DE models

Chameleon models,  $f(R)$ 

Approaches and tools

Growth function, growth index

Outlook



$\Omega_{m,0} = 0.3$  and various values of  $w_{DE,0}$

We have from top to bottom:

$w_{DE,0} = -1.4, -1.3, -1.2, -1, -0.8.$

The black line gives the **true** value of  $\gamma_0$  realized:

same non vanishing  $\gamma_0' \approx -0.02.$

## BAO

Acoustic scale at various  $z \Rightarrow D_A(z)$  and  $H(z)$ 

BAO forecasts for a Full-Sky survey

| $z_{\min}$ | $z_{\max}$ | Vol  | % Err $D_A(z)$ | % Err $H(z)$ | $\Omega_\Lambda$ | $\sigma_w$ |
|------------|------------|------|----------------|--------------|------------------|------------|
| 0.00       | 0.15       | 0.33 | 2.8            | 4.9          | 0.708            | 0.64       |
| 0.15       | 0.32       | 2.62 | 0.95           | 1.7          | 0.616            | 0.088      |
| 0.32       | 0.51       | 7.89 | 0.53           | 0.96         | 0.515            | 0.036      |
| 0.51       | 0.73       | 16.5 | 0.35           | 0.63         | 0.413            | 0.021      |
| 0.73       | 0.99       | 28.4 | 0.26           | 0.46         | 0.318            | 0.015      |
| 0.99       | 1.28       | 42.9 | 0.21           | 0.36         | 0.236            | 0.013      |
| 1.28       | 1.62       | 59.0 | 0.17           | 0.28         | 0.170            | 0.012      |
| 1.62       | 2.00       | 75.8 | 0.14           | 0.24         | 0.119            | 0.013      |
| 2.00       | 2.44       | 92.3 | 0.13           | 0.21         | 0.082            | 0.014      |
| 2.44       | 2.95       | 108  | 0.12           | 0.18         | 0.056            | 0.016      |
| 2.95       | 3.53       | 121  | 0.11           | 0.17         | 0.038            | 0.020      |
| 3.53       | 4.20       | 133  | 0.10           | 0.15         | 0.025            | 0.025      |
| 4.20       | 4.96       | 142  | 0.10           | 0.15         | 0.017            | 0.033      |

Theoretical approaches to the Dark Energy paradigm

David Polarski

Dark Energy

Cosmological constant

Pending questions

Modified gravity

Scalar-tensor DE

Models

Chameleon

Models,  $f(R)$ 

Approaches and

tools

Growth function, growth index

Book

The art of inducing accelerated expansion:  
Large variety of models and approaches

A new vision with challenging questions!

Accurate observations will single out viable models

$\Lambda$  or not  $\Lambda$  ?

To be continued...