Relativistic Corrections In N-Body Simulations

in collaboration with C Rampf, T Tram, R Crittenden, D Wands and K Koyama

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Cosmology, Particle Physics and Phenomenology - CP3

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Today

Motivation

- The large scale structure of the Universe is expected to be one of the main cosmological probes in the near future
- The precision of large-scale structure data is increasing and will soon surpass the predictive power of the CMB data

Are Theoretical Predictions Or Simulations Ready?

- Analytical computations typically rely on perturbation theory
- N-body simulations solve Newtonian equations of motion

In this talk we present a method for including relativistic corrections in N-body simulations

Relativistic N-Body Simulations

GR Is A Theory Of Space-Time

Gauge Freedom: Choose any coordinate system

In an N-body simulation it is necessary to fix a gauge

- The gauge defines the meaning of the N-body coordinates
- The gauge specifies which equations need to be solved

If a gauge exists in which the equations of motion are Newtonian, then a conventional N-body includes all GR effects if its coordinates are understood in that Gauge

What Are Current N-Body Simulations Doing?

- Initial conditions are generated via Zel'dovich approximation using **synchronous** gauge power spectra
- The **Newtonian** equations of motion are solved in the N-body simulation
- The resulting matter power spectrum is interpreted in **synchronous** gauge
- Velocities however are to be understood in **longitudinal** gauge

While this prescription does work in practice, it does look unsatisfactory from a theoretical point of view

What Does A Simulation Compute?

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1. Compute the density contrast: $\rho_{\text{sim}} = \frac{1}{a^3} \sum_{\text{particles}} m\, \delta_{\mathrm{D}}^{(3)}(\boldsymbol{x} - \boldsymbol{x}_p)$
2. Compute the Bardeen potential: $\boldsymbol{\nabla}^2 \Phi_{\text{sim}} = -4\pi G \bar{a}^2 \rho_{\text{sim}}$
3. Move the particles: $\left(\frac{\partial}{\partial \eta} + \frac{\dot{a}}{a}\right) \boldsymbol{v}_{\text{sim}} = \boldsymbol{\nabla} \Phi_{\text{sim}}$

The Metric Including Linear Perturbations

$$\mathrm{d}s^2 = a^2 \Big(-(1+2A)\mathrm{d}\eta^2 - 2\partial^i B\, \mathrm{d}x_i \mathrm{d}\eta + \left[\delta^{ij}(1+2H_{\mathrm{L}}) + 2D^{ij} H_{\mathrm{T}}\right] \mathrm{d}x_i \mathrm{d}x_j \Big)$$

The full density $\rho = (1 - 3H_{\mathrm{L}})\rho_{\text{sim}}$ takes deformation of space into account

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The N-Body Gauge ($B = v$, $H_{\mathrm{L}} = 0$)

Find a gauge with equations of motion matching the N-body simulation:

We fix the temporal Gauge by enforcing $B = v$:

$$\boldsymbol{\nabla}^2 \Phi = -4\pi G \bar{a}^2 \bar{\rho} \delta$$

When using the spatial Gauge to set $H_{\mathrm{L}} = 0$, the Bardeen potential is computed according to GR in the N-body simulation

Relativistic Corrections

$$\left(\frac{\partial}{\partial \eta} + \frac{\dot{a}}{a} \right) \boldsymbol{v} = \boldsymbol{\nabla} \Phi + \boldsymbol{\nabla} \gamma$$

$$\gamma = \ddot{H}_{\mathrm{T}} + \frac{\dot{a}}{a} \dot{H}_{\mathrm{T}} - 8\pi G a^2 p \Pi$$

with the total anisotropic stress Π
In ΛCDM the correction vanishes as H_{T} is related to the comoving curvature and therefore conserved

Why Do Current N-Body Simulations Work?

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No ΛCDM:

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- Velocities are to be understood in **N-body** gauge
- And all linear relativistic corrections are automatically included

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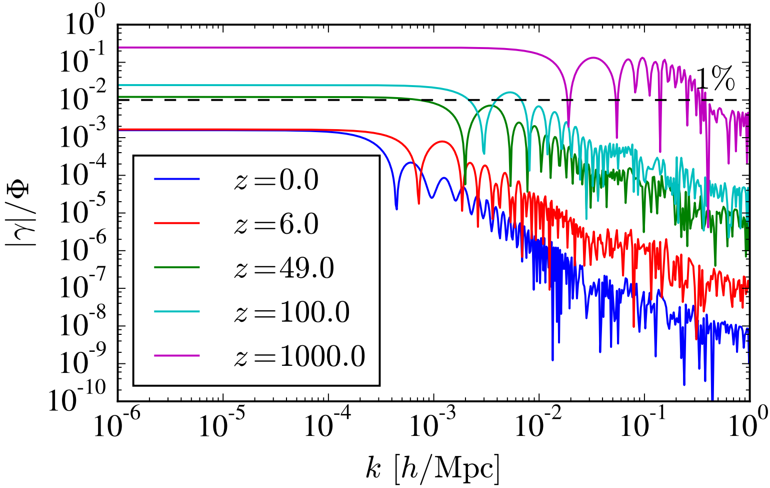
An Example For γ

Including residual radiation and neutrinos:



Conclusions

- Defined the N-body gauge, uniquely suited for N-body simulations
- Provide a clean and simple framework, relevant especially out of ΛCDM
- N-body gauge power spectra for generating initial conditions can be generated with `CLASS`

`Thank You For Your Attention`

Setting Initial Conditions

Newtonian Zel'dovich Approximation

$$\begin{aligned}\dot{\boldsymbol{\psi}} &= \boldsymbol{v} \\ \dot{\delta} &= -\boldsymbol{\nabla} \cdot \boldsymbol{v}\end{aligned}$$

Integrate starting from a homogenous distribution ($\psi(0) = 0$):

$$-\boldsymbol{\nabla} \cdot \boldsymbol{\psi} = \delta$$

Setting Initial Conditions

Relativistic Zel'dovich Approximation

$$
\begin{aligned}
\dot{\boldsymbol{\psi}} &= \boldsymbol{v} \\
\dot{\delta} &= -\boldsymbol{\nabla} \cdot \boldsymbol{v} - 3\dot{H}_{\mathrm{L}}
\end{aligned}
$$

Integrate starting from a homogenous distribution ($\psi(0) = 0$):

$$-\boldsymbol{\nabla} \cdot \boldsymbol{\psi} = \delta + 3H_{\mathrm{L}}$$

- This correction is absent in the N-body gauge ($H_{\mathrm{L}} = 0$)

Other Popular Gauge Choices

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- Total matter gauge ($H_{\mathrm{T}} = 0$, $B = v$) has formally Newtonian equations of motion
  - ➔ well suited for analytical Newtonian computations
  - ➔ not suited for simulations as the simulation density ignores volume deformation
  - ➔ modified initial conditions
- Longitudinal gauge ($B = H_{\mathrm{T}} = 0$) has modified Poisson equation
- Synchronous gauge ($A = B = 0$) has modified Poisson and Euler equation
  - ➔ often used to generate initial conditions (`CAMB` uses synchronous gauge) as in a pure CDM Universe the density coincides with the comoving density
  - ➔ we have updated `CLASS` to compute the actual N-body gauge density to generate initial conditions

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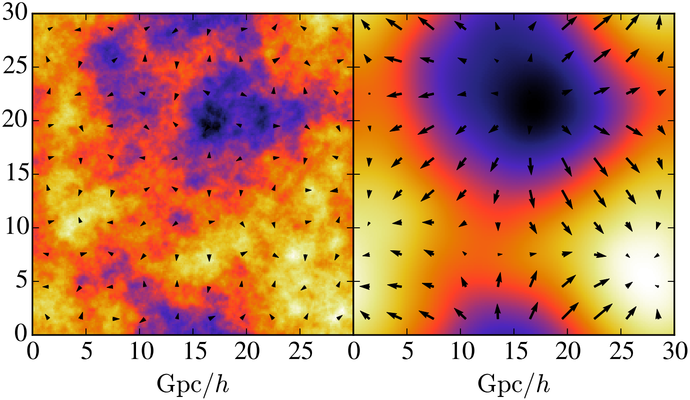

The potential of the displacement field from the classical Zel'dovich approximation on the left and from GR corrections in total matter gauge on the right

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