Starobinsky, Higgs inflation and quantum gravity

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Motivation

- The **Starobinsky and Higgs** inflationary models are two of the most observationally successfull ones: Important to understand them deeper from a theoretical point of view.

- Both models correspond to leading-order terms in an **effective expansion** for the gravitational action

$$\mathcal{L} \supset \frac{1}{16\pi G} R + \frac{1}{b} R^2 + \frac{1}{c} \frac{R^3}{m_p^2} + \xi h^2 R + \ldots + \mathcal{L}_{\text{Standard Model}}$$

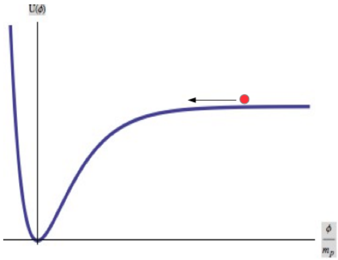
- Should we (and why) care about **quantum gravitational effects** beyond tree level during inflation?

- Is a **UV completion** for gravity necessary in this context?

Dynamics of Starobinsky and Higgs inflation

$$\frac{1}{16\pi G}R + \frac{1}{b}R^2 \quad \xleftrightarrow{\tilde{g}_{\alpha\beta} = g_{\alpha\beta}e^{\sqrt{2/3}\frac{\phi}{m_p}}} \quad \frac{\widetilde{R}}{16\pi G} + \frac{1}{2}(\partial\phi)^2 + U_{Star.}(\phi)$$

$$\left(\frac{1}{16\pi G} + \xi h^2\right)R - \frac{1}{2}(\partial h)^2 - \frac{\lambda}{4!}h^4 \quad \leftrightarrow \quad \frac{\widetilde{R}}{16\pi G} + \frac{1}{2}(\partial\phi)^2 + U_{Higgs}(\phi)$$



$$\frac{U(\phi)}{m_p^4} \sim \quad b \times \left(1 - e^{-\sqrt{2/3}\frac{\phi}{m_p}}\right)^2$$

$$\frac{U(\phi)}{m_p^4} \sim \quad \frac{\lambda}{\xi^2} \times \left(1 - e^{-\sqrt{2/3}\frac{\phi}{m_p}}\right)^2$$

- The scalar potentials for the two models share similar form
- The amplitude of tensor fluctuations is controlled by b or λ/ξ^2 .

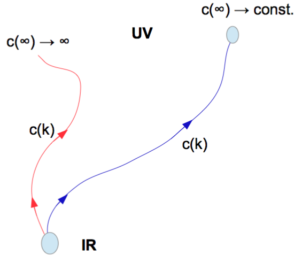
Quantum gravity is infinite, isn't it?

Gravity is a perturbatively non–renormalisable theory for cut–off scales larger than the Planck mass, $k \gtrsim m_p$:

order parameter: $\widetilde{G}(k) \equiv k^2 G(k)$, $\qquad \widetilde{G} \to \infty$ when $k \to \infty$

- **Renormalisable theories**: No divergences occur on the theory space, i.e couplings cannot diverge
  - $\rightsquigarrow$ UV completion of (metric) gravity?

- **S. Weinberg, 1979**[1]: " A theory is said to be Asymptotically Safe if the essential coupling parameters [of the theory] approach a fixed point as the momentum scale of their renormalisation point goes to infinity."

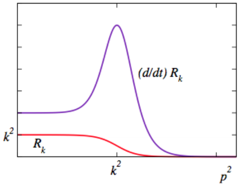


[1] S. Weinberg in General Relativity, an Einstein Centenary Survey, S.W. Hawking and W. Israel (Eds.), Cambridge University Press, (1979)

Going beyond perturbation theory

- An **Exact Renormalisation Group Equation** for the effective action Γ_k [2]

$$\frac{\partial}{\partial \ln k}\Gamma_k = \frac{1}{2}\mathrm{Trace}\left[\left(\Gamma_k^{(2)} + R_k \right)^{-1} \cdot k\partial_k R_k \right]$$



- **Assumptions**:
  - Background space-time is a Euclidean de Sitter
  - Landau gauge
  - Optimised (Litim's) regulator R_k [3]
  - $\Gamma_k = \int_{S^4} d^4x\sqrt{g}\left(-\frac{1}{16\pi G_k}R + \frac{1}{b_k}R^2\right) \equiv \int_{S^4} d^4x\sqrt{g}\, f(R,k)$

- Evaluation of the trace yields the flow of the effective action [4]

$$\partial_k\Gamma_k = \mathcal{F}_0[R;G,b] + \mathcal{F}_1[R;G,b]\cdot\partial_k\left(\frac{\partial f}{\partial R}\right) + \mathcal{F}[R;G,b]\cdot\partial_k\left(\frac{\partial^2 f}{\partial R^2}\right)$$

- The flow of the couplings under the RG

$$\frac{d}{d\ln k}\widetilde{G}(k) = \beta_{\widetilde{G}}(\widetilde{G},b), \qquad \frac{d}{d\ln k}b(k) = \beta_b(\widetilde{G},b)$$

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[2] C. Wetterich Phys. Lett. **B 301**, 90 (1993) |T. R. Morris, Int. J. Mod. Phys. A 9 (1994) 2411.

[3] D. Litim (2000) arXiv: 0005245 [hep–th]

[4] for an explicit calculation see P.F. Machado & F. Saueressig arXiv:0712.0445 [hep-th] | K. Falls, D. Litim, K. Nikolakopoulos & C. Rahmede (2014) arXiv:1410.4815 [hep-th]

The (not so inspiring) form of the beta functions

The explicit form of the beta functions is quite complex ...

$$\frac{d}{d\ln k}\widetilde{G} = \beta_{\widetilde{G}}(\widetilde{G}, b) \equiv \frac{A_0(1-B_2)+A_2B_0}{1-A_1-B_2+A_1B_2-A_2B_1}$$

$$\frac{d}{d\ln k}b = \beta_{\widetilde{G}}(\widetilde{G}, b) \equiv \frac{B_0(1-A_1)+A_0B_1}{1-A_1-B_2+A_1B_2-A_2B_1}$$

$$A_0 \equiv \frac{\widetilde{G}\left(b^3(144\pi - 301\widetilde{G}) + 3456\pi b^2(17\widetilde{G}-8\pi)\widetilde{G} + 9216\pi^2 b(144\pi - 323\widetilde{G})\widetilde{G}^2 + 17694720\pi^3\widetilde{G}^4\right)}{72\pi b(b-96\pi\widetilde{G})^2}$$

$$A_1 \equiv \frac{4\widetilde{G}\left(b^3 - 225\pi b^2\widetilde{G} + 15840\pi^2 b\widetilde{G}^2 - 276480\pi^3\widetilde{G}^3\right)}{9\pi b(b-96\pi\widetilde{G})^2}$$

$$A_2 \equiv \frac{16\widetilde{G}^3\left(b^2 - 200\pi b\widetilde{G} + 7680\pi^2\widetilde{G}^2\right)}{b^2(b-96\pi\widetilde{G})^2}$$

$$B_0 \equiv -\frac{491b^5 - 157088\pi b^4\widetilde{G} + 18275328\pi^2 b^3\widetilde{G}^2 - 916586496\pi^3 b^2\widetilde{G}^3 + 17694720000\pi^4 b\widetilde{G}^4 - 135895449600\pi^5\widetilde{G}^5}{2880\pi^2(b-96\pi\widetilde{G})^3}$$

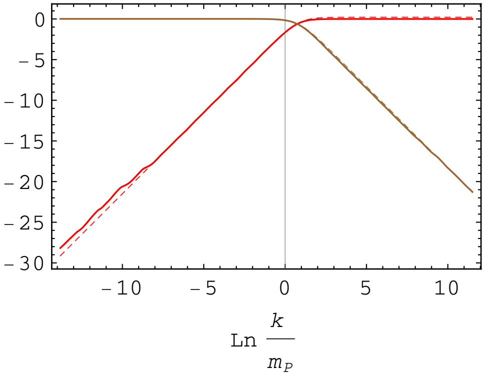
$$B_1 \equiv -\frac{-89b^5 + 31818\pi b^4\widetilde{G} - 4328064\pi^2 b^3\widetilde{G}^2 + 276203520\pi^3 b^2\widetilde{G}^3 - 8493465600\pi^4 b\widetilde{G}^4 + 101921587200\pi^5\widetilde{G}^5}{4320\pi^2\widetilde{G}(b-96\pi\widetilde{G})^3}$$

$$B_2 \equiv \frac{\widetilde{G}\left(731b^4 - 222912\pi b^3\widetilde{G} + 24247296\pi^2 b^2\widetilde{G}^2 - 1150156800\pi^3 b\widetilde{G}^3 + 16986931200\pi^4\widetilde{G}^4\right)}{720\pi b(b-96\pi\widetilde{G})^3}$$

A new non–trivial UV fixed point [5]

A non–trivial UV fixed point: $\widetilde{G}(k \to \infty) = 24\pi/17, \qquad b(k \to \infty) = 0 \qquad \left[\widetilde{G} \equiv k^2 G(k)\right]$

Approximately for $b \ll 1$: $\dfrac{d}{d \ln k}\widetilde{G} \simeq 2\widetilde{G} - \dfrac{61 \cdot \widetilde{G}^2}{36\pi - 5\widetilde{G}} + \mathcal{O}(\widetilde{G}^3, b), \qquad \dfrac{d}{d \ln k} b \simeq -\dfrac{41\widetilde{G} \cdot b}{36\pi - 5\widetilde{G}} + \mathcal{O}(\widetilde{G}^2, b^2)$



Red $= Log\left(\dfrac{k^2 \cdot G(k)}{G(k \to \infty)}\right)$

Brown $= Log\left(b(k) \cdot 10^9\right)$

- Persistence under higher-order curvature corrections?

[5] E. Copeland, C. Rahmede, IDS, (2014), arXiv: 1311.0881 [gr-qc], and work in progress.

The role of quantum gravitational corrections for Higgs inflation

$$\Gamma[g_{\mu\nu}, \phi] = -\int \sqrt{\bar{g}} \left(\frac{1}{16\pi G_k} + \xi_k \phi^2 \right) R - \frac{1}{2}(\partial\phi)^2 - \frac{\lambda_k}{4} \left(\phi^2 - v^2 \right)^2$$

- Inflation is realised for large ξ and field expectation value: [6]

$$\xi \sim 10^4, \ \langle\phi\rangle \gg m_p/\sqrt{\xi} \quad \left[\lambda \sim \mathcal{O}(10^{-2} - 10^{-1}) \text{ at inflationary scales} \right]$$

- Quantum fluctuations of the inflaton occur on the top of its v.e.v, $\phi = \langle\phi\rangle + \delta\phi$. Under the choice of a Landau-type gauge, in a Eucledian de Sitter background, the running of the couplings under the Wilsonian cut–off k are of the form

$$\frac{d}{d \ln k} c_i(k) = \beta_{c_i}(c_j), \quad c_i = \{\widetilde{G}, b, \lambda\}$$

- An attractive UV fixed point ($\beta_{c_i} = 0$) of the non–perturbative RG flow exists [7]

$$\widetilde{G}(k \to \infty) = 0.559118, \qquad \xi(k \to \infty) = 0, \qquad \lambda(k \to \infty) = 0$$

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[6] For an alternative scenario see: M. Shaposhnikov and F. Bezrukov, 1403.6078 [hep-th]

[7] See for example also: R. Percacci, G. Narain, arXiv: 0911.0386 [hep-th]
G. Narain, C. Rahmede, arXiv: 0911.0394 [hep-th]

The role of quantum gravitational corrections for Higgs inflation

- We can understand the leading-order gravitational effects considering an expansion in Newton's G, assuming $\widetilde{G} \ll 1$

- To leading order in Newton's G the running of the couplings is controlled by

$$k\partial_k \widetilde{G} \simeq 2\widetilde{G} + \frac{1}{24\pi}\left(\frac{14\xi}{\gamma^2} - 55\right) \cdot \widetilde{G}^2 + \mathcal{O}\left(\frac{\widetilde{G}^n}{\gamma^m}\right)$$

$$k\partial_k \xi \simeq \frac{1}{64\pi^2}\frac{\lambda \cdot (28\xi + 5)}{\gamma^3} + \mathcal{O}\left(\frac{\widetilde{G}^n}{\gamma^m}\right)$$

$$k\partial_k \lambda \simeq \frac{21}{64\pi^2}\frac{\lambda^2}{\gamma^3} + \mathcal{O}\left(\frac{\widetilde{G}^n}{\gamma^m}\right)$$

$$\left[\widetilde{G} \equiv k^2 \cdot G(k)\right]$$

$$\gamma \sim \lambda \cdot \frac{\langle\phi\rangle}{k} + 1$$

Large v.e.v regime: $\gamma \gg 1$

Small v.e.v regime: $\gamma \ll 1$

(1)

- Can the initial conditions set during inflation be connect smoothly with the UV?

Summary

- Starobinsky and Higgs inflation require **particularly large values for their respective free parameters (couplings) during inflation**, and it is important to understand whether these can be reconciled with low/higher energy regimes. Embedding the models within more fundamental contexts can provide us with stronger motivations and possible explanations for the required initial conditions.

- For the Starobinsky action, an **attractive, UV fixed point** exists under the RG, where Newton's coupling is asymptotically safe, and **the R^2 coupling vanishes**. The fixed point ensures the existence of RG trajectories connecting the UV with the IR, with primordial fluctuations small. We find that its existence persists under the inclusion of up to fourth-order terms in curvature.

- During Higgs inflation **quantum corrections from gravitons and Higgs loops are sufficiently suppressed**, however, it appears they could potentially grow dangerously when extrapolating the theory to higher energies due to the large value of the non–minimal coupling ξ. A closer investigation of the quantum dynamics of the theory is for this purpose required.