Corrections from the quantum substructure of the background metric

Sebastian Zell
sebastian.zell@physik.uni-muenchen.de   Max-Planck-Institut für Physik, Munich
Work with Gia Dvali

1.) General relativity as mean field limit of gravitons

- Question: What is the microscopic nature of the cosmological constant Λ?
- Idea: Classical metric = collective effect of gravitons [1]
  - De Sitter as bound state of gravitons
  - Occupation number per Hubble patch (with Planck mass M_p):

$$N = M_p^2/\Lambda \tag{1}$$

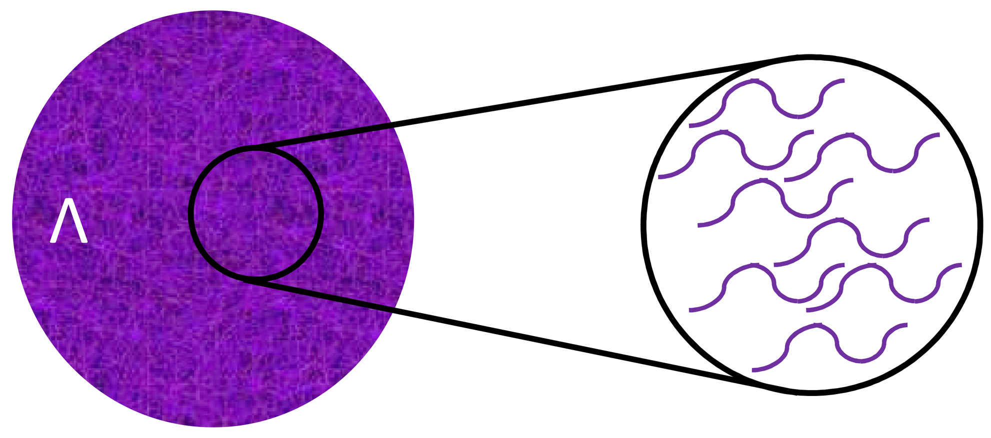

Figure 1: Resolution of the classical metric in constituent gravitons

2.) Weak gravity as collective graviton effect

- Newtonian potential Φ determines energy density in momentum space via the classical self-energy:

$$|E[\Phi]| \propto \int \mathrm{d}^3\vec{x}\, \left|\vec{\nabla}\Phi(\vec{x})\right|^2 \propto \int \mathrm{d}^3\vec{k}\, \underbrace{\vec{k}^2 \left|\mathcal{F}(\Phi)(\vec{k})\right|^2}_{\epsilon(\vec{k})} \tag{2}$$

- Conditions on quantum state:
  1. Energy is carried by gravitons $\Rightarrow$ Classical number density

$$n(\vec{k}) = \frac{\epsilon(\vec{k})}{|\vec{k}|} \propto \left|\mathcal{F}(\Phi)(\vec{k})\right|^2 \tag{3}$$

  2. Expectation value in fixed mode should be maximal.
     $\Rightarrow$ Coherent states
- This defines a unique graviton state $|n(\vec{k})\rangle$ on top of Minkowski vacuum, which reproduces the classical metric for all weak backgrounds:

$$\langle n(\vec{k})|\hat{\Phi}|n(\vec{k})\rangle = \Phi$$

3.) Application to de Sitter

- Coordinate system for small times $\Lambda t^2 \ll 1$ [2]

$$\mathrm{d}s^2 = (1+\Lambda t^2)\mathrm{d}t^2 + \ldots$$

- With a graviton of mass $m = \sqrt{\Lambda}$ as regularization, the canonically normalized Newtonian potential is

$$\Phi = M_p \cos(\sqrt{\Lambda} t).$$

- The free time evolution reproduces the classical metric.
- For $t = 0$, (3) yields a monochromatic graviton distribution:

$$n(\vec{k}) = \frac{\sqrt{\Lambda} M_p^2 V}{8\pi} \delta^{(3)}(\vec{k})$$

- In a Hubble patch, (1) implies that the well-known energy of de Sitter is naturally explained as sum of the graviton energies:

$$E = N\sqrt{\Lambda}$$

4.) Decay rate of particle production

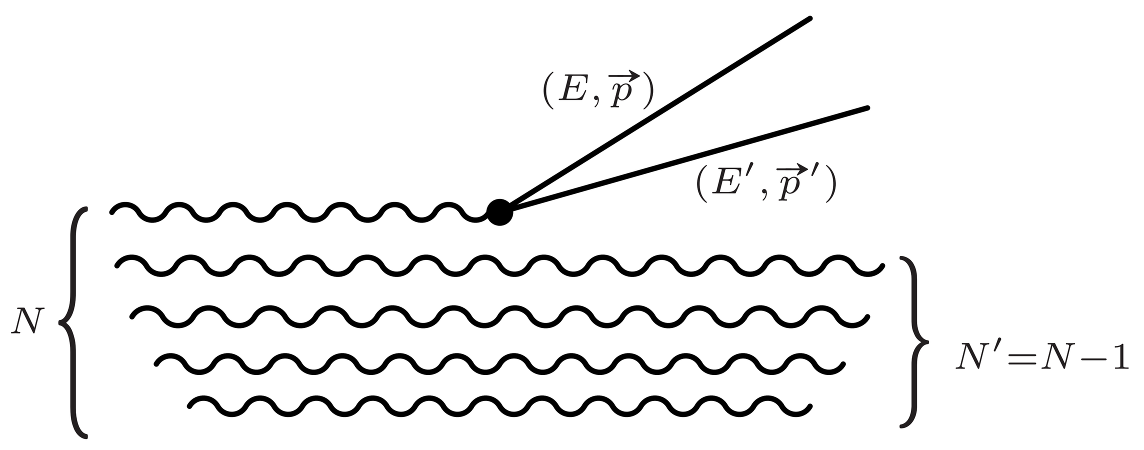

Figure 2: Production of two particles as decay of a background graviton

- Well-known semi-classical decay constant for a general weak background

$$\frac{\mathrm{d}\Gamma_{sc}}{\mathrm{d}\Omega} \propto |\mathcal{F}(\Phi)|^2(E+E', \vec{p}+\vec{p}\,')$$

- Fully quantum formula using (3) (with final background state $n'(\vec{k})$)

$$\frac{\mathrm{d}\Gamma_q}{\mathrm{d}\Omega} \propto n(E+E', \vec{p}+\vec{p}\,') \exp\left\{-\int \mathrm{d}^3\vec{k}\left(\sqrt{n(\vec{k})} - \sqrt{n'(\vec{k})}\right)^2\right\}$$

$\Rightarrow$ Reinterpretation (already in the semi-classical limit $n' = n$): Graviton 4-momentum = Sum of 4-momenta of produced particles

$\Rightarrow$ Quantum correction because of the back-reaction ($n' \neq n$)

- Concrete result for de Sitter (with $N' = N - 1$):

$$\Gamma_q = \Gamma_{sc}\left(1 - \frac{5}{4N}\right) = \frac{\Lambda^2 V}{16\pi}\left(1 - \frac{5}{4N}\right)$$

5.) Final state of the background

- Source-free representation of the background: On quantum level, the gravitons suffice to account for the classical metric.

$\Rightarrow$ States which do not correspond to a classical source are also possible.

- Two corrections: Not only the rate, but also the final state of the background field is changed.

$\Rightarrow$ Evolution of the cosmological constant?

$\Rightarrow$ Question of decoherence [1]:
Why should the final state admit an effective description by general relativity?
Breakdown of the classical metric-description?

6.) Summary

- **Weak gravitational background as mean field limit of gravitons**
- **Particle production because of graviton decay**
- **1/N-correction of the rate caused by back-reaction**
- **Quantum change of the background state**

7.) Outlook

- Go beyond first order, i.e. include interaction
- Model final state of de Sitter-background
- Consider other processes such as redshift
- Investigate analogous description for e.g. black holes

References

[1] G. Dvali and C. Gomez, *Quantum Compositeness of Gravity: Black Holes, AdS and Inflation*, J. Cosmol. Astropart. Phys. **1401** (2014) no. 01, , arXiv:1312.4795.

[2] G. Dvali, S. Hofmann, and J. Khoury, *Degravitation of the cosmological constant and graviton width*, Phys.Rev. **D76** (2007) 084006, arXiv:hep-th/0703027.