The CMB, String Inflation and UV inspired $f(R)$

Based on 1411.6010 and 1509.00024 (and 1507.02277)

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Dark Energy and String Cosmology Parallel Session

with D. Ciupke, M. Galante, F. Pedro, D. Roest, and A. Westphal

I will talk about . . .

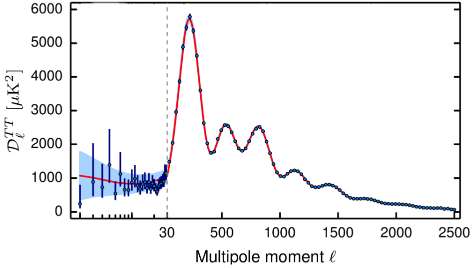
Inflation, the CMB and Power-loss

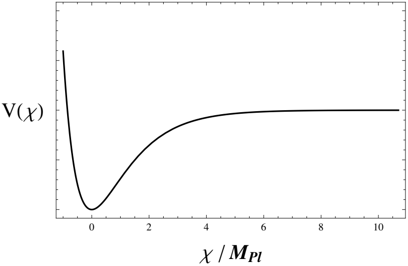
String Inflation from α'-corrections

$f(R)$ - Theory

$f(R)$ beyond R^2

Part I



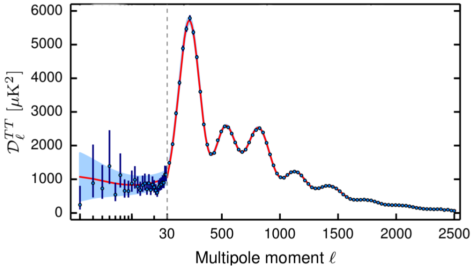


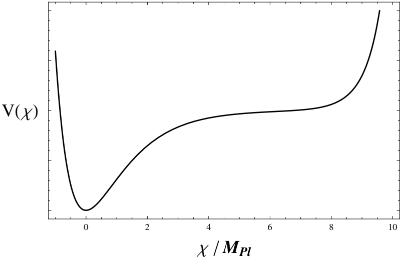
$$V_{inf} = V_0 \left(1 - e^{-\sqrt{\frac{2}{3}}\chi}\right)^2 \quad \Longleftrightarrow \quad f(R) = R + \alpha R^2, \quad \alpha = \frac{1}{8V_0}$$

[1502.02114]

Hints for power suppression at low ℓ

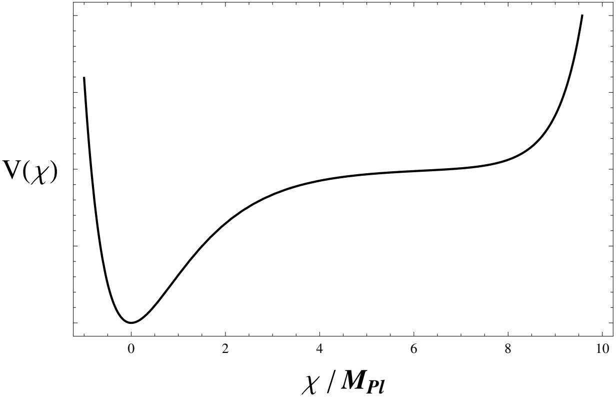
$$\Delta_s^2(k) \sim (k/k^*)^{n_s - 1}$$





$$V_{inf} = V_0 \left(1 - e^{-\sqrt{\frac{2}{3}}\chi}\right)^2 + \varepsilon\, e^{\sqrt{\frac{2}{3}}\chi} \quad \Longleftrightarrow \quad f(R) = R + \alpha R^2 + \ldots ?$$

"Take home message" - Part I [1509.00024]



Possible to obtain above potential from recently computed higher derivative $(\alpha')^3$-corrections in combination with string loop effects.
Caveat: Terms and Conditions may apply (ie. tuning)

Large Volume Scenario in a nutshell...

Cicoli, Conlon, Burgess, Quevedo

Consider IIB Flux compactifications with K3-fibred $\mathcal{V}(\tau_1, \tau_2, \tau_3)$ where $\tau_1, \tau_2 \gg \tau_3$

$$K = -2\log\left(\mathcal{V} + \frac{\hat{\xi}}{2}\right) \qquad \text{and} \qquad W = W_0 + Ae^{-a\tau_3},$$

F-term scalar potential gets generated for the Kähler moduli:

$$V^{LVS}(\mathcal{V}, \tau_3) = g_s\left[\frac{8a_3^2A_3^2}{3\alpha\gamma}\frac{\sqrt{\tau_3}}{\mathcal{V}}\mathrm{e}^{-2a_3\tau_3} - 4W_0a_3A_3\frac{\tau_3}{\mathcal{V}^2}\mathrm{e}^{-a_3\tau_3} + \frac{3\hat{\xi}W_0^2}{4\mathcal{V}^3}\right]$$

The above potential does not depend on τ_1, which is, hence, left as a flat direction. V^{LVS} has a minimum at exponentially large volume given by

$$\langle\tau_3\rangle = \left(\frac{\hat{\xi}}{2\alpha\gamma}\right)^{2/3} \quad , \quad \langle\mathcal{V}\rangle = \frac{3\alpha\gamma}{4a_3A_3}W_0\sqrt{\langle\tau_3\rangle}\mathrm{e}^{a_3\langle\tau_3\rangle}$$

Four-Superspace-Derivative Correction [1505.03092]

Ciupke, Louis, Westphal

$$V_{(1)} = -g_s^2 \hat{\lambda} \frac{|W_0|^4}{\mathcal{V}^4} \Pi_i t^i$$

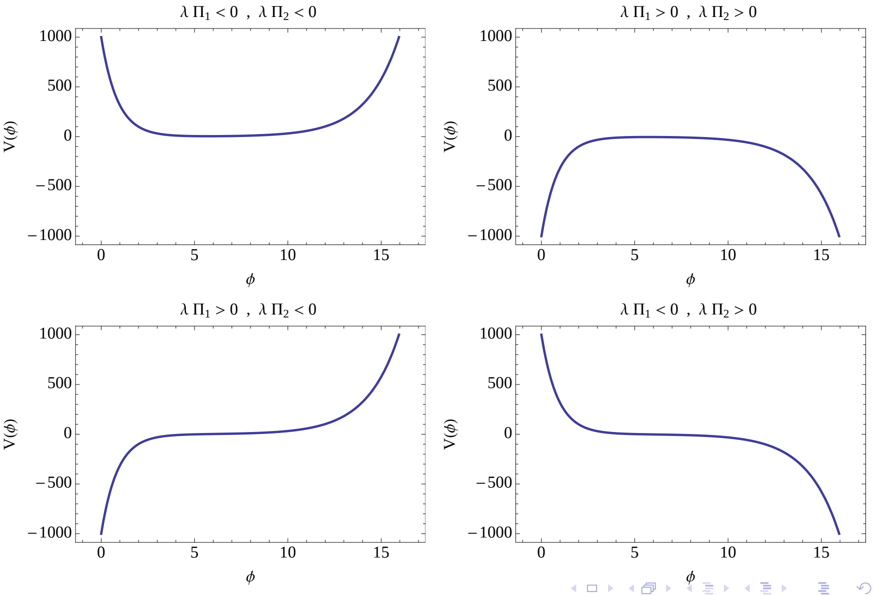
Choose a geometry

$$\mathcal{V} = \alpha \left(\sqrt{\tau_1}\tau_2 - \gamma \tau_3^{3/2} \right)$$

$$V_{(1)} \simeq -g_s^2 \hat{\lambda} \frac{|W_0|^4}{\mathcal{V}^4} \left(\Pi_1 \frac{\mathcal{V}}{\tau_1} + \Pi_2 \lambda_1^{-1/2} \sqrt{\tau_1} \right)$$

$$V(\varphi) = V^{LVS}(\langle \tau_3 \rangle, \langle \mathcal{V} \rangle) - g_s^2 \hat{\lambda} \frac{|W_0|^4}{\langle \mathcal{V} \rangle^4} \left(\Pi_1 \langle \mathcal{V} \rangle e^{-2/\sqrt{3}\varphi} + \Pi_2 \lambda_1^{-1/2} e^{\varphi/\sqrt{3}} \right)$$

Possible Inflationary Potentials



String Loop Corrections

$$\delta V_{(g_s)} \simeq \frac{g_s|W_0|^2}{\mathcal{V}^2}\left(g_s^2\frac{(C_1^{KK})^2}{\tau_1^2} + 2g_s^2(\alpha C_2^{KK})^2\frac{\tau_1}{\mathcal{V}^2}\right)$$

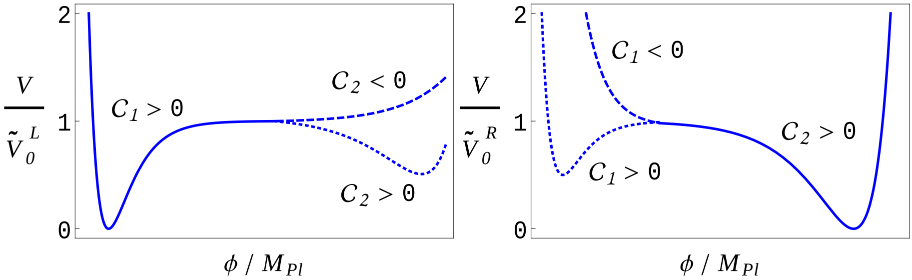
$$V = V_{\delta_{up}}^{LVS} + V_0\left(-\mathcal{C}_1 e^{-2/\sqrt{3}\varphi} - \mathcal{C}_2 e^{\varphi/\sqrt{3}} + \mathcal{C}_1^{loop} e^{-4/\sqrt{3}\varphi} + \mathcal{C}_2^{loop} e^{2\sqrt{3}\varphi}\right)$$

where we have defined

$$V_0 = g_s^2\frac{|W_0|^4}{\mathcal{V}^4}, \quad \mathcal{C}_1 = \hat{\lambda}\,\Pi_1\mathcal{V}, \quad \mathcal{C}_2 = \hat{\lambda}\,\Pi_2\lambda_1^{-1/2},$$

$$\mathcal{C}_1^{loop} = \frac{\mathcal{V}^2}{|W_0|^2}g_s(C_1^{KK})^2 > 0, \quad \mathcal{C}_2^{loop} = \frac{2g_s}{|W_0|^2}(\alpha C_2^{KK})^2 > 0$$

Viable Inflationary Potentials



$$V_{inf}^{L} \sim V_0 \left(-\frac{\mathcal{C}_1}{\tau_1} + \frac{\mathcal{C}_1^{loop}}{\tau_1^2} \right) \qquad V_{inf}^{R} \sim V_0 \left(-\mathcal{C}_2 \sqrt{\tau_1} + \mathcal{C}_2^{loop} \tau_1 \right)$$

$$V_{inf}^{L} = \tilde{V}_0^{L} \left(1 - e^{-\kappa\phi} \right)^2 \qquad V_{inf}^{R} = \tilde{V}_0^{R} \left(1 - e^{\frac{\kappa}{2}\phi} \right)^2$$

What you have to ensure. . .

To the Left	*resulting bound*
minimum at $\tau_1 \gtrsim 1$	$g_s^{5/2}\,\mathcal{V}(C_1^{KK})^2 \gtrsim 1$
$\tau_1^{min} < \tau_1^c$	$\mathcal{C}_1^{loop} < \frac{1}{2}\left(\frac{2}{\mathcal{C}_2}\right)^{2/3}\mathcal{C}_1^{5/3}$
PLANCK	$\lambda^2\,\lvert W_0\rvert^6\,\mathcal{V}^{-4}g_s^{-2}(C_1^{KK})^{-2} \sim 10^{-9}$

To the Right	*resulting bound*
plateau at $\tau_1 \gtrsim 1$	$2\,\frac{g_s^{5/2}(C_2^{KK})^2}{\lambda\,\lvert W_0\rvert^2\Pi_2} \ll 1$
$\tau_1^{min} > \tau_1^c$	$\mathcal{C}_2^{loop} < \left(\frac{\mathcal{C}_2^4}{\lvert\mathcal{C}_1\rvert}\right)^{1/3}$
PLANCK	$\lambda^2\,\lvert W_0\rvert^6\,\mathcal{V}^{-4}g_s^{-2}(C_2^{KK})^{-2} \sim 5\times 10^{-9}$

It's not a free lunch :(. . . What about masses?

Mass Hierarchy I

Remaining scalars (complex structure moduli, axio-dilaton, volume) must play no role during inflation, they must be heavier than Hubble scale during inflation H

$$m_{cs}^2\,,\, m_S^2\,,\, m_{\tau_3}^2 \sim g_s \frac{|W_0|^2}{\mathcal{V}^2}$$

Overall volume $\mathcal{V}$:

$$m_{\mathcal{V}}^2 \sim g_s \frac{|W_0|^2}{\mathcal{V}^3}$$

One must therefore make sure that

$$m_{\mathcal{V}}^2 \gg H^2 \sim V$$

such that no other field besides the fibre modulus τ_1 plays a role in inflation.

Mass Hierarchy II

For the case when inflation takes place as the fibre modulus rolls to the left, the Hubble scale during the observable inflation is approximately

$$H^2 \sim \tilde{V}_0^L \sim g_s^{-2} \frac{W_0^6}{\mathcal{V}^4} \frac{\lambda^2 \Pi_1^2}{(C_1^{KK})^2} \quad \Rightarrow \quad W_0^2 \lambda \Pi_1 \ll \sqrt{g_s}$$

Noting that for inflation towards the right, the Hubble scale is

$$H^2 \sim \tilde{V}_0^R \sim g_s^{-2} \frac{W_0^6}{\mathcal{V}^4} \frac{\lambda^2 \Pi_2^2}{(C_2^{KK})^2} \quad \Rightarrow \quad \frac{W_0^2}{\mathcal{V}} \lambda \Pi_2 \ll \sqrt{g_s}$$

First Order Observables

$$V_{inf} = V_0 \left(1 - e^{\pm\nu\phi}\right)^2 \quad \Rightarrow \quad n_s = 1 - \frac{2}{N}\,, \quad r = \frac{1}{\nu^2}\frac{8}{N^2}$$

	$n_s(50)$	$n_s(60)$	$r(50)$	$r(60)$
right	0.960	0.967	0.0096	0.0067
left	0.960	0.967	0.0043	0.0016

Second Order Observables

Inflation to the Left

$$V_{inf}^{L} \sim V_0 \left(-\frac{\mathcal{C}_1}{\tau_1} + \frac{\mathcal{C}_1^{loop}}{\tau_1^2} + \mathcal{C}_2 \sqrt{\tau_1} \right) \quad \Rightarrow \quad \tilde{V}_0^L \left(1 - 2e^{-\kappa\phi} + \varepsilon^2 e^{\frac{\kappa}{2}\phi} \right)$$

$$n_s = 1 - \frac{2}{N} - \frac{3\sqrt{2}\,\varepsilon^2\kappa}{\sqrt{N}} + \frac{\varepsilon^2\kappa^3}{\sqrt{2}}\sqrt{N} - \frac{3}{2}\varepsilon^4\kappa^4 N + \dots$$

Considering the 2-σ bounds by PLANCK, i.e. requiring $\delta n_s \lesssim 0.008$ at $N = 60$, we obtain the upper bound

$$\varepsilon^2 \sim \lambda^{-3/2}\,\mathcal{V}^{-1} \left(g_s^{5/2}\mathcal{V} \right)^{3/2} \Pi_2\,\Pi_1^{-5/2} (C_1^{KK})^{3/2} \lesssim 10^{-3}$$

Second Order Observables

Inflation to the Right

$$V_{inf}^{R} \sim V_0 \left(-\frac{\mathcal{C}_1}{\tau_1} - \mathcal{C}_2 \sqrt{\tau_1} + \mathcal{C}_2^{loop} \tau_1 \right) \quad \Rightarrow \quad V_0^R \left(1 - 2 e^{\frac{\kappa}{2}\phi} + \varepsilon^2 e^{-\kappa\phi} \right)$$

$$n_s = 1 - \frac{2}{N} - 3\,\varepsilon^2 \kappa^4 N + \frac{\varepsilon^2 \kappa^6}{2} N^2 + \dots$$

$$\lambda^{-3}\, \mathcal{V}\, g_s^{15/2}\, \Pi_1\, \Pi_2^{-4} (C_2^{KK})^6 \lesssim 2.4 \times 10^{-6}$$

Aside: $n_s = d\ln P / d\ln k = P^{-1} dP/dN$ and hence

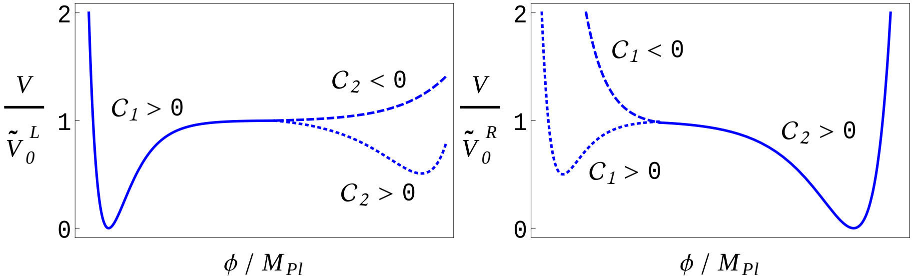
$$\left. \frac{\Delta P(\delta n_s)}{P} \right|_{N+\Delta N}^{N} = \int_{N+\Delta N}^{N} \delta n_s \sim \delta n_s\, \Delta N \quad \rightarrow \quad 4\%$$

Numerical Examples

	W_0	g_s	$\mathcal{V}$	τ_1^{min}	Π_1	Π_2	C_1^{KK}	C_2^{KK}	n_s
$\mathcal{R}_1$	5	0.2	625.5	3000	0	100	0.00242	0.799	0.968
$\mathcal{R}_2$	25	0.3	1886.2	3500	0	10	0.000859	0.732	0.967
$\mathcal{L}_1$	2	0.3	460	3	100	1	0.163	0.0288	0.966
$\mathcal{L}_2$	5	0.4	1031.6	6	50	0	0.189	0.0266	0.969

Table: Examples of compactifications parameters and inflationary observables for inflation to the left ($\mathcal{L}_1$ and $\mathcal{L}_2$) and to the right ($\mathcal{R}_1$ and $\mathcal{R}_2$). The spectral index is computed at $N_e = 55$.

Conclusions I



Possible to obtain above potentials from recently computed higher derivative $(\alpha')^3$-corrections in combination with string loop effects.

Part II

$f(R)$ in a Nutshell

Consider

$$R \to f(R)$$

Weyl-transform via

$$\tilde{g}_{\mu\nu} = \Omega \; g_{\mu\nu} \,, \quad \Omega = \frac{\partial f}{\partial R}$$

and obtain

$$\frac{\mathcal{L}}{\sqrt{-\tilde{g}}} = \frac{\tilde{R}}{2} - \frac{1}{2} (\partial\chi)^2 - V(\chi),$$

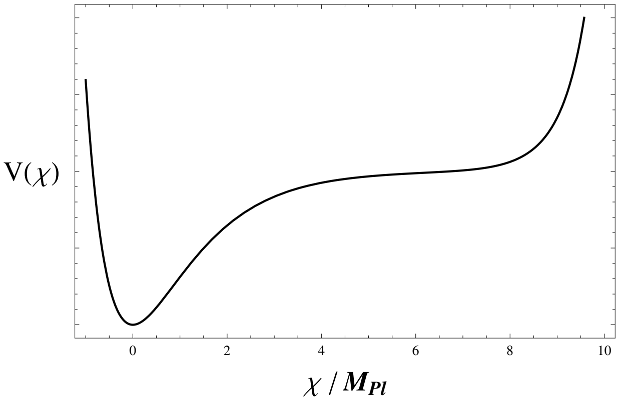
for $\chi \equiv \sqrt{3/2} \ln f'$ with potential $V(\chi) = (f'R - f)/(2f'^2)$.

Note: To obtain closed form expressions when going from $V(\chi) \to f(R)$:

$$V(\chi) = f\left(e^{\kappa\chi}\right) , \quad \kappa = \sqrt{2/3}$$

"Take home message" - Part II

[1411.6010]
JCAP03 (2015) 029



For broken shift-symmetry at large fields, corresponding $f(R)$ - dual is to leading order R^n with $1 < n < 2$.

The Potential at Large Fields

If potential at large field values is

$$V(\chi) \sim V_0 e^{n\,\kappa\,\chi},$$

with $n \geq 1$, have to solve differential equation

$$V_0 f'^{\,n} = \frac{f'R - f}{2f'^{\,2}}.$$

Obtain asymptotic solution

$$f(R) \sim R^{(n+2)/(n+1)} + \dots$$

An exact $f(R)$ - Toy Model

Consider

$$V(\chi) = V_0 \left[\left(1 - e^{-\sqrt{2/3}\chi} \right)^2 + \varepsilon \, e^{\sqrt{2/3}\chi} \right] - \varepsilon \, V_0$$

to obtain exact

$$f(R) = \frac{\varepsilon - 1}{3\varepsilon} R + 4\varepsilon V_0 \left[\frac{(1-\varepsilon)^2}{9\varepsilon^2} + \frac{2}{3\varepsilon} + \frac{R}{6\varepsilon V_0} \right]^{3/2} + K$$

Taylor expanding for $\varepsilon \to 0$ recovers Starobinsky coefficients, e.g.

$$\lim_{\varepsilon \to 0} c_2 = \frac{1}{8 \, V_0}$$

Need $\varepsilon \lesssim \mathcal{O}\left(10^{-4}\right)$ for $n_s \sim 0.97$

A non-zero Λ for free?

It is easy to show that

$$R|_{\phi=0} = 2\,\varepsilon\,V_0$$

and

$$f(R|_{\phi=0}) = 2\,\varepsilon\,V_0$$

when $V(0) = 0$. $f(R)$ also satisfies

$$f(0) = 0\,, \quad f'|_{R=0} < 1\,, \quad f'|_{R=2\delta V_0} = 1$$

At first attempt, **not possible** to have both

$$f(R|_{\phi=0})\,,\; f(0) = 0$$

at the same time (e. g. shifting $\chi \to \chi + \chi_0$ or adjusting K)

Consider your standard SUGRA/String derived set-up with kinetic term

$$K_{\Phi\bar{\Phi}}\partial\Phi\partial\bar{\Phi}$$

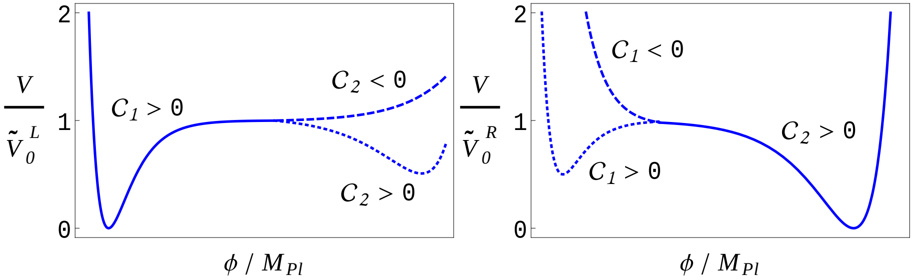
and e.g. monomial effective potential such as $V \sim \Phi\bar{\Phi}$.

Pole structure of $K_{\Phi\bar{\Phi}}$ can break inflationary plateau at large fields with complex pole, i.e.

$$K_{\Phi\bar{\Phi}} \sim \frac{1}{\Phi\bar{\Phi} + \varepsilon^2}$$

Complex pole determines where shift symmetry is broken.

Recap: Conclusions I



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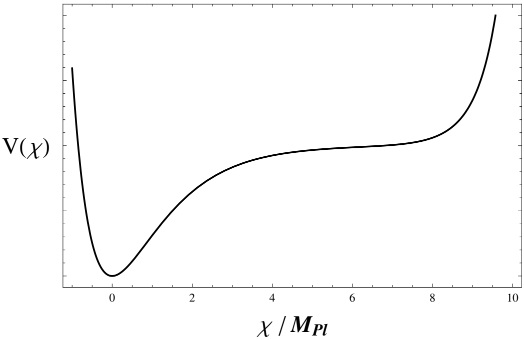
Conclusions II



For broken shift-symmetry at large fields, corresponding $f(R)$ - dual is to leading order R^n with $1 < n < 2$.

Thank you very much for your attention!

Exemplary $f(R)$ for Fibre Inflation

(Cicoli, Burgess, Quevedo [0808.0691])

Inflaton is Kähler modulus and has string loop generated potential

$$V(\phi) = V_0' \left(\mathcal{C}_0 e^{\kappa' \phi} - \mathcal{C}_1 e^{-\kappa' \phi/2} + \mathcal{C}_2 e^{-2\kappa' \phi} + \mathcal{C}_{up} \right),$$

with $\kappa' = 2/\sqrt{3}$. Recast in terms of $\kappa = \sqrt{2/3}$ and consider plateau

$$V(\phi) \sim V_0 \left(1 - \mathcal{C}_1 e^{-\frac{\kappa}{\sqrt{2}}\phi} + \mathcal{C}_2 e^{-2\sqrt{2}\kappa\phi} \right).$$

One obtains

$$f(R) = \frac{1}{8V_0} R^2 + \alpha' R^{2-1/\sqrt{2}} + \beta' R^{2-2\sqrt{2}} + \ldots$$

Consider large field regime

$$V(\phi) \sim V_0 \; \mathcal{C}_0 e^{\sqrt{2}\kappa\phi}$$

to obtain

$$f(R) \sim R^{\sqrt{2}}.$$