Revisiting f(R) cosmology

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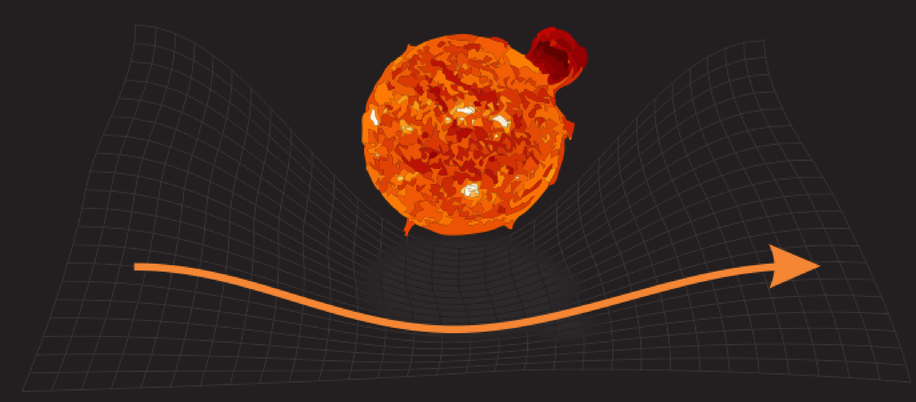

Abstract

In this poster we are presenting the results of several works where $f(R)$ gravity is revisited. The main goal of these papers has been the understanding of this kind of gravity as a geometric theory so we avoid the mapping to the scalar tensor representation which could present some countroversies, mostly regarding to the multivalued potentials.

Introduction

- Several modifications of gravity have been proposed in order to explain the acceleration of the Universe, however it is not clear if behind these generalizations of the Einstein-Hilbert action there is a modification or extension of some physical principle. Therefore, there is not a formal framework to guide the exploration of these models. $f(R)$ theories of gravity are perhaps the most straightforward modification of general relativity providing an extra geometric component which, in some cases, reproduces this acceleration.

f(R), field equations

The dependence of the Ricci scalar in $f(R)$ is a general function which will be defined in order to reproduce observations, the action is given by

$$S[g_{ab},\psi] = \int \frac{f(R)}{2\kappa}\sqrt{-g}\, d^4x + S_{\rm matt}[g_{ab},\psi] \;, \qquad (1)$$

where $G = 1$, $c = 1$ and $\kappa \equiv 8\pi$. Varying the action with respect to g^{ab},

$$f_R R_{ab} - \frac{1}{2} f g_{ab} - (\nabla_a \nabla_b - g_{ab}\Box)\, f_R = \kappa T_{ab} \;, \qquad (2)$$

where $f_R = \partial_R f$, $\Box = g^{ab}\nabla_a\nabla_b$ and T_{ab} is the EMT for matter.
The trace yields a second order equation for the Ricci scalar

$$\Box R = \frac{1}{3 f_{RR}}\left[\kappa T - 3 f_{RRR}(\nabla R)^2 + 2f - R f_R\right] \;, \qquad (3)$$

where $T := T^a{}_a$. Using (3) in (2) we find

$$G_{ab} = \frac{1}{f_R}\Big[f_{RR}\nabla_a\nabla_b R + f_{RRR}(\nabla_a R)(\nabla_b R) - \frac{g_{ab}}{6}\Big(R f_R + f + 2\kappa T\Big) + \kappa T_{ab}\Big] \;. \qquad (4)$$

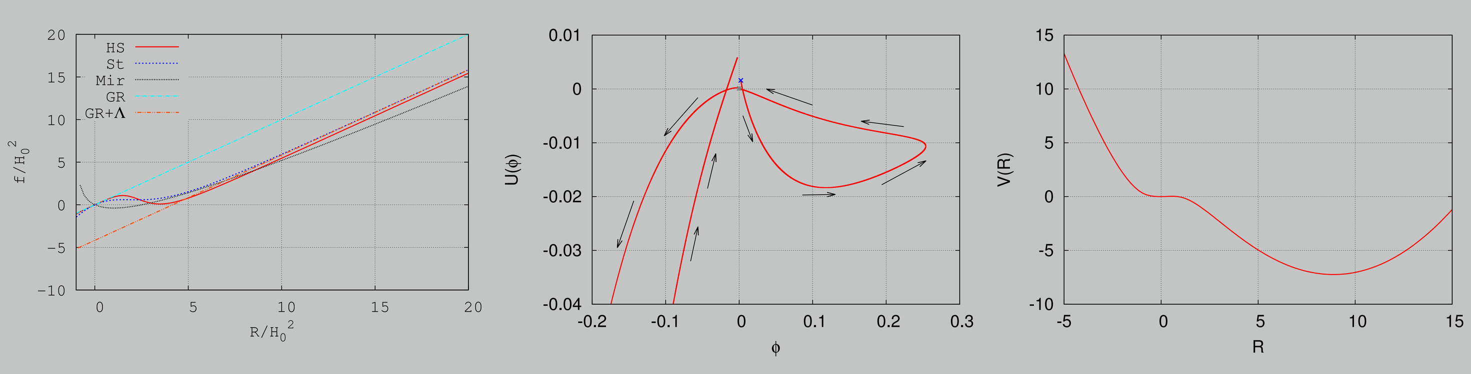

Figure 1: $f(R)$ Models, $U(\Phi)$ in the Einstein frame and $V(R)$

F(R) Cosmology

- We focus now on homogeneous and isotropic space-times described by the FRW metric,

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right] \;, \qquad (5)$$

From Eq. (3) we find

$$\ddot R = -3H\dot R - \frac{1}{3f_{RR}}\left[3 f_{RRR}\dot R^2 + 2f - f_R R + \kappa T\right] \;, \qquad (6)$$

And the field equations are

$$H^2 + \frac{k}{a^2} + \frac{1}{f_R}\left[f_{RR}H\dot R - \frac{1}{6}(f_R R - f)\right] = \frac{-\kappa T^t{}_t}{3 f_R} \;, \qquad (7)$$

$$\dot H = -H^2 + \frac{1}{f_R}\left(f_{RR}H\dot R + \frac{f}{6} + \frac{\kappa T^t{}_t}{3}\right) \qquad (8)$$

References

T. P. Sotiriou and V. Faraoni, RMP Vol.82, 451, 2010 / L. G. Jaime, L. Patiño, and M. Salgado, PRD 83, 024039, 2011 / L. G. Jaime, L. Patiño, and M. Salgado, arXiv: 1206.1642 / L. G. Jaime, L. Patiño, and M. Salgado, PRD 89, 084010, 2014 / V. Sahni, A. Shafieloo, and A. Starobinsky, PRD 78, 103502, 2008 / L. G. Jaime, PRD 91, 124070, 2015

The EOS for the Geometric Dark Energy (GDE)

EMT_X	Energy-density and presure of GDE
$AT^{\rm tot}_{ab} - BT_{ab}$	$\tilde\rho_X = \frac{A}{\kappa f_R}\left[\frac{1}{2}(f_R R - f) - 3 f_{RR} H\dot R + \kappa\rho\left(1 - \frac{Bf_R}{A}\right)\right]$ $\tilde p_X = -\frac{A}{3\kappa f_R}\left[\frac{1}{2}(f_R R + f) + 3 f_{RR} H\dot R - \kappa\left(\rho - 3p_{\rm rad}\frac{Bf_R}{A}\right)\right]$

Table 1: Geometric dark energy (GDE) variables in terms of the scalar functions A and B. The different definitions of the EMT, energy-density, pressure and EOS of GDE are obtained from the quantities A with $(A: 1, f_R^0, f_R, 1)$ and B $(B: 1, 1, 1, f_R^{-1})$ in that order. Definitions III and IV produce the same EOS since $Bf_R/A = 1$ in both cases while the factor A outside the brackets cancels out when taking the ratio of the pressure and the energy-density and they are not conserved. Definition I gives the relation: $\omega_X = \frac{3H^2 - 3\kappa\, p_{\rm rad} - R}{3(3H^2 - \kappa\rho)}$.

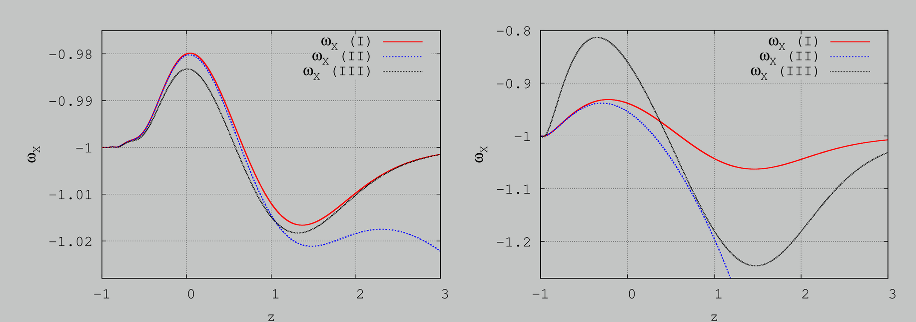

Figure 2: Evolution of ω_X for Hu-Sawicki and Starobinsky $f(R)$ models for the three definitions.

Matter domination era in R^n gravity.

We integrate the differential equations starting at some redshift, by assuming matter domination for R^n models. We find that varying them in several ways it turns out impossible to recover the actual abundances of the different components at present time while having an adequate accelerating phase. This means that compared to the ΛCDM model, the Universe expands faster or slower depending on n but it never reproduces the correct accelerated expansion and matter domination eras within the same model; it reproduces one or the other in the best of scenarios but not both.

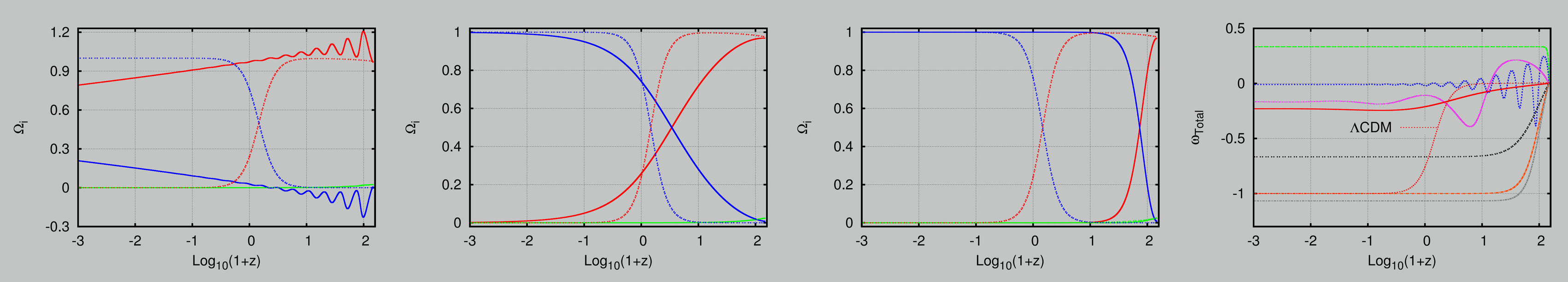

Figure 3: Evolution of $\Omega_{\rm matt}$, Ω_X and $\Omega_{\rm rad}$ for $n = 1.01$, $n = 1.3$ and $n = 2$ and the total EOS $\omega_{\rm tot}$ in R^n gravity. For reference the ΛCDM model are included.

The Om(z) function as a test.

Shafieloo *et al.* proposed a diagnostic by using the $Om(z)$ function at two different points. This way we can take observations about the determination of $H(z)$ at several redshifts

$$Omh^2(z_i; z_j) = \frac{h^2(z_i) - h^2(z_j)}{(1+z_i)^3 - (1+z_j)^3}. \qquad (9)$$

Taking the observed values at $z_1 = 0$, $z_2 = 0.57$ and $z_3 = 2.34$ with $H = 70.6 \pm 3.2$, 92.4 ± 4.5 and $222 \pm 7 km/sec/Mpc$ respectively. The two-point relation gives

$$\begin{aligned} Omh^2(z_1; z_2) &= 0.124 \pm 0.045, \\ Omh^2(z_1; z_3) &= 0.122 \pm 0.01, \\ Omh^2(z_2; z_3) &= 0.122 \pm 0.012, \end{aligned} \qquad (10)$$

while for ΛCDM the value is $Omh^2 = 0.1426$ (constant) with $P = 0.98$

Ω_m^0	(z_i, z_j)	$Omh^2(z_i, z_j)$	χ^2_{St}	$P(f(R))$
	(z_1, z_2)	0.131		
0.24	(z_1, z_3)	0.123	0.041	0.16
	(z_2, z_3)	0.123		
	(z_1, z_2)	0.126		
0.25	(z_1, z_3)	0.123	0.013	0.09
	(z_2, z_3)	0.123		

Table 2: Values for the two points two-point relation $Omh^2(z_1, z_2)$ for the Starobinsky (first) and Hu-Sawicki (second) $f(R)$ models. Column 3 is the value of $Omh^2(z_1, z_2)$. In column 4 is the value of $\chi^2_{f(R)}$. In column 5 is the cumulative probability.

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