The scale-invariant-top-condensate model

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Motivation

- The stability of the EW scale against radiative corrections (the naturalness problem) demands a solution from BSM physics.
- The discovery of the Higgs-like 125 GeV resonance makes solving such a problem imperative.
- The measured Higgs mass is too high for the simplest SUSY models, and too low for composite Higgs models.

Motivation

- Guided by the naturalness principle, one considers additional symmetries that protect the Higgs mass:
  - Extra global symmetry – composite pseudo-Goldstone Higgs.

- We propose the simplest top-condensate model with classical scale invariance.

Top-quark condensation (TC)

- The minimal TC model generates the EWSB and the heavy top mass dynamically with very simple assumptions. [Miransky, Tanabashi Yamawaki (1989); Bardeen, Hill, Lindner (1990)]
- Four-fermion interaction at composite scale Λ:

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + G\left(\bar{\Psi}_L t_R\right)\left(\bar{t}_R \Psi_L\right) \qquad \Psi_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

$$H = \frac{\sqrt{G}}{m_0}\bar{t}_R\Psi_L$$

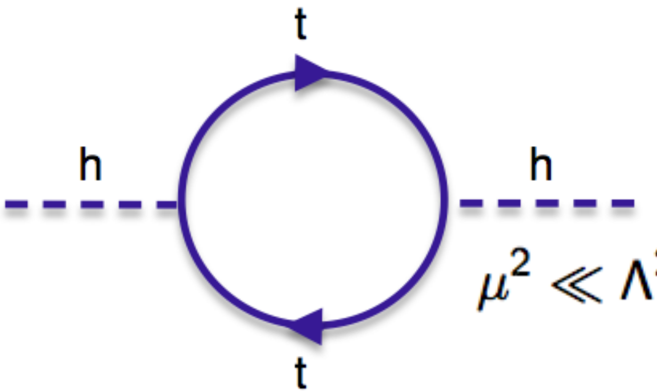
Auxiliary, non-dynamical Higgs doublet

- Equivalent tree-level Lagrangian after bosonisation:

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + y_t\left(\bar{\Psi}_L t_R H + \text{h.c.}\right) - m_0^2 H^\dagger H \qquad y_t = m_0\sqrt{G}$$

Top-quark condensation

- Quark-loop approximation



$\mu^2 \ll \Lambda^2$

$$\mathcal{L} = \ldots + Z(D_\nu H)^\dagger D^\nu H - m^2 H^\dagger H - \frac{1}{2}\lambda(H^\dagger H)^2$$

$$Z = \frac{N_c y_t^2}{(4\pi)^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$m^2 = m_0^2 - \frac{2N_c y_t^2}{(4\pi)^2}(\Lambda^2 - \mu^2)$$

$$\lambda = \frac{2N_c y_t^4}{(4\pi)^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

- But TC produces the wrong mass relation: $\boxed{m_H = 2m_t}$

- Beyond quark-loop approximation Higgs boson is still heavier than top-quark [Bardeen, Hill, Lindner]

Scale-invariance

- SI is the global symmetry under:

$$x^\mu \to tx^\mu,\ \phi(x) \to t^{-1}\phi(tx),\ A(x) \to t^{-1}A(tx),\ \psi(x) \to t^{-3/2}\psi(tx)$$

- It is typically an anomalous symmetry, ie., an exact symmetry of the classical action broken by quantum corrections.

- Example in nature: quantum dynamics in nearly SI strong interactions creates the mass of the visible matter.

- We assume: classical SI is the low-energy remnant of a fundamental theory, and is broken at low energies by quantum corrections through the Coleman-Weinberg mechanism, leading to mass generation.

The Scale-invariant-top-condensate (SITC) model

- Introduce a dynamical dilaton field χ
- In the original tree-level Lagrangian modify all the dimensionful parameters:

$$m_0^2 \to m_0^2 \frac{\chi^2}{f^2}, \quad \Lambda^2 \to \Lambda^2 \frac{\chi^2}{f^2}, \quad V_0 \to V_0 \frac{\chi^4}{f^4}.$$

- The tree-level Lagrangian becomes:

$$\mathcal{L}_{SITC} = \mathcal{L}_{\text{kin}} + y_t \left(\bar{\Psi}_L t_R H + \text{h.c.} \right) - V_0 \frac{\chi^4}{f^4} - m_0^2 \frac{\chi^2}{f^2} H^\dagger H$$

$$V_{tree} = \frac{\lambda_\chi}{4} \chi^4 + \sigma \chi^2 H^\dagger H \qquad \lambda_\chi = 4V_0/f^4, \; \sigma = m_0^2/f^2$$

- The tree-level couplings now depend on the rescaled cut-off

$$\lambda_\chi = \lambda_\chi \left(\frac{\Lambda^2 \chi^2}{f^2} \right), \quad \sigma = \sigma \left(\frac{\Lambda^2 \chi^2}{f^2} \right)$$

The SITC model: tree-level analysis

$$V_{tree} = \frac{\lambda_\chi}{4}\chi^4 + \frac{\sigma}{2}\chi^2 h^2 \qquad H = \left(0,\ \frac{h}{\sqrt{2}}\right)^T$$

- Minimisation:

$$\left.\frac{\partial V_{tree}}{\partial h}\right|_{VEV} = 0 \quad \Longrightarrow \quad \sigma\chi^2 h = 0$$

$$\left.\frac{\partial V_{tree}}{\partial \chi}\right|_{VEV} = 0 \quad \Longrightarrow \quad \chi\left[\left(\lambda_\chi + \frac{\beta_{\lambda_\chi}}{4}\right)\chi^2 + \left(\sigma + \frac{\beta_\sigma}{2}\right)h^2\right] = 0$$

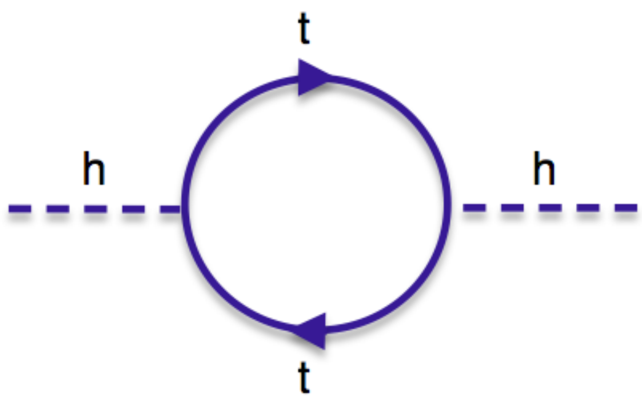
$$h_0 \neq 0 \text{ and } \chi_0 \neq 0 \quad \Longrightarrow \quad \boxed{\sigma\left(\frac{\Lambda^2\chi^2}{f^2}\right) = 0 \text{ and } \frac{h_0^2}{\chi_0^2} = -2\frac{\lambda_\chi + \beta_{\lambda_\chi}/4}{\beta_\sigma}}$$

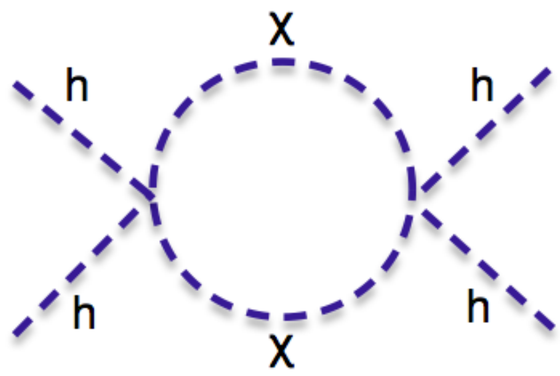
$$V_{tree}|_{VEV} = 0 \quad \Longrightarrow \quad \lambda_\chi\left(\frac{\Lambda^2\chi^2}{f^2}\right) = 0$$

- Absence of Tree-level masses

Quantum corrections at one loop

- From considering loop diagrams such as:





$$V = V_{tree} + \Delta V_{1-loop}$$

$$\Delta V_{1-loop} = \frac{1}{16\pi^2}\left[\left(N_c y_t^2 - \frac{\sigma}{2}\right)\mu^2 h^2 + \frac{N_c y_t^4 - \sigma^2}{4}\ln\left(\frac{\Lambda^2\chi^2}{\mu^2 f^2}\right)h^4 \right.$$
$$- \left(\frac{3}{2}\lambda_\chi \sigma \ln\left(\frac{\Lambda^2\chi^2}{\mu^2 f^2}\right) + \left(N_c y_t^2 - \frac{\sigma}{2}\right)\frac{\Lambda^2}{f^2}\right)\chi^2 h^2$$
$$\left. -\frac{3}{2}\lambda_\chi \mu^2 \chi^2 + \left(\frac{3}{2}\lambda_\chi \frac{\Lambda^2}{f^2} - \frac{9}{4}\lambda_\chi^2 \ln\left(\frac{\Lambda^2\chi^2}{\mu^2 f^2}\right)\right)\chi^4\right]$$

$$\mu^2 \ll \Lambda^2$$

SITC: one-loop analysis

- Minimising the full one-loop potential sets the VEVs

$$\frac{\partial \tilde{V}}{\partial \chi} = 0 = \chi \left[\lambda_\chi(\mu)\chi^2 + \left[\sigma(\mu) + \frac{1}{16\pi^2}\left(\frac{\sigma}{2} - N_c y_t^2\right)\frac{\Lambda^2}{f^2}\right] h^2 - \frac{3}{16\pi^2}\lambda_\chi\left(2\frac{\Lambda^2}{f^2}\chi^2 - \mu^2\right)\right] + \text{2-loop effects}$$

$$\frac{\partial \tilde{V}}{\partial h} = 0 = h\left[\sigma(\mu)\chi^2 + \frac{1}{8\pi^2}\left(N_c y_t^2 - \frac{\sigma}{2}\right)\left(\mu^2 - \frac{\Lambda^2}{f^2}\chi^2\right) + \lambda_h(\mu)h^2\right]$$

$$\lambda_\chi(\mu) = \lambda_\chi - \frac{9}{16\pi^2}\lambda_\chi^2 \ln\left(\frac{\Lambda^2\chi^2}{\mu^2 f^2}\right)$$

$$\lambda_h(\mu) = \frac{1}{16\pi^2}\left(N_c y_t^4 - \sigma^2\right)\ln\left(\frac{\Lambda^2\chi^2}{\mu^2 f^2}\right)$$

$$\sigma(\mu) = \sigma - \frac{3}{16\pi^2}\lambda_\chi\sigma \ln\left(\frac{\Lambda^2\chi^2}{\mu^2 f^2}\right)$$

SITC: Mass predictions at one loop

- In the limit $\lambda\chi = 0$, the one-loop masses are:

$$m_h^2 = \frac{1}{4\pi^2}\left(N_c - \frac{\sigma}{2y_t^2}\right)\frac{m_t^2}{v^2}\mu^2 + 6\left(1 - \frac{\sigma^2}{N_c y_t^4}\right)m_t^2$$

$$m_\chi^2 = 0$$

- We can adjust parameters to obtain the measured values of Higgs and top quark masses.
- There is no mixing between dilaton and Higgs induced at 1-loop level.
- A dilaton mass and dilaton-Higgs mixing emerge at 2-loop level and are expected to be small.

Conclusions

- Scale invariance: all mass scales arise quantum-mechanically.
- The simplest top-condensate model with classical SI can resolve the hierarchy problem (in the technical sense) and accommodate measured masses for Higgs boson and top-quark. Some of the predictions are testable at LHC and/or future linear colliders.
- In the limit of a vanishing cosmological constant, this model predict a dilaton that is massless at one-loop.
- Studies of phenomenological and cosmological implications of the model are underway.