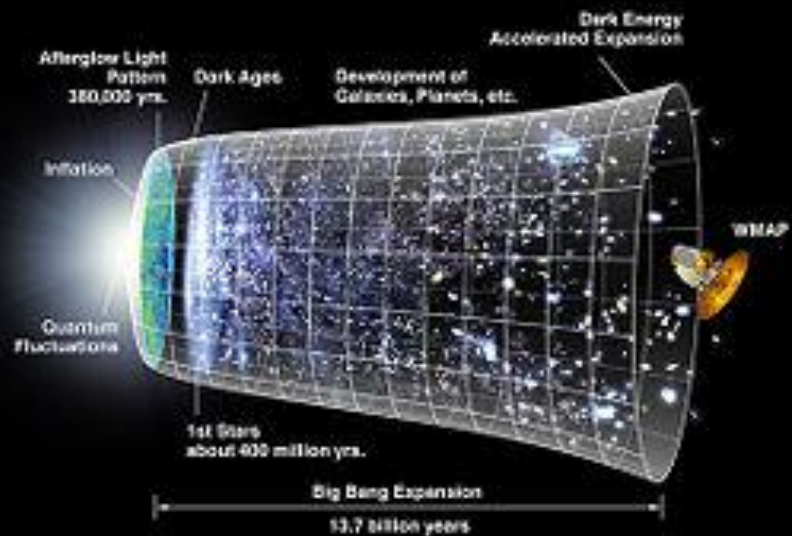
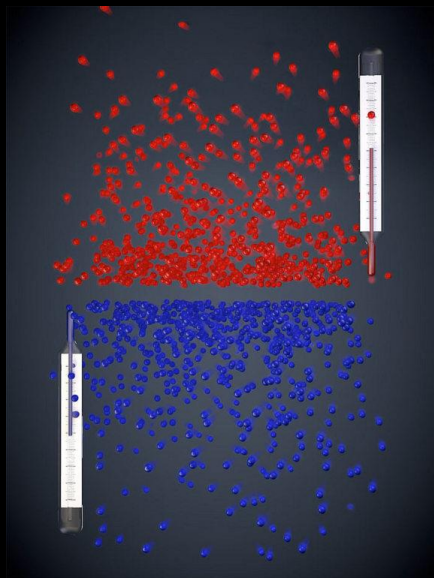


Cosmology With Negative Absolute Temperatures

José P. P. Vieira

COSMO 15

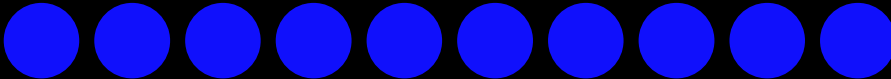


11th September 2015

What do you mean $T < 0$?

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U} \right)_V$$

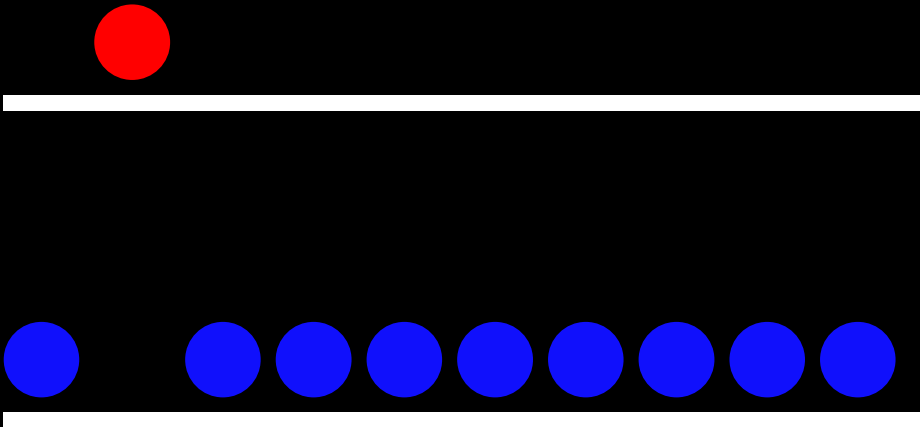
$$S = k_B \ln \Omega$$



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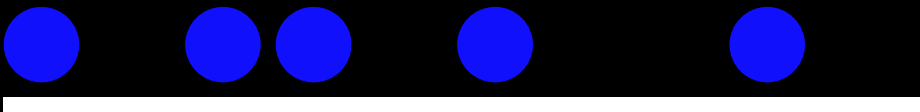
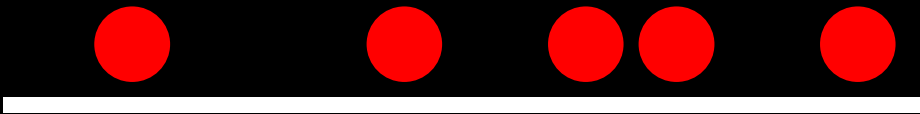
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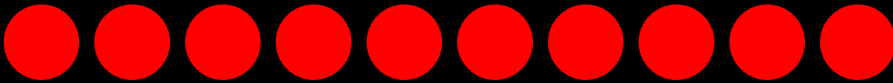
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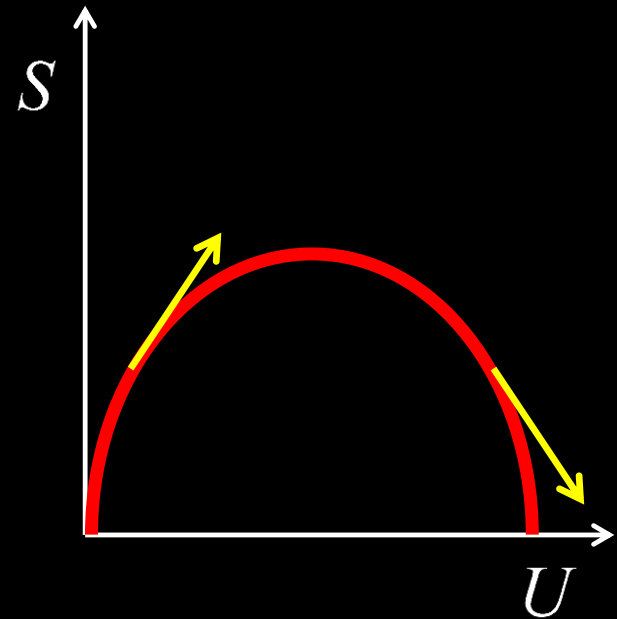
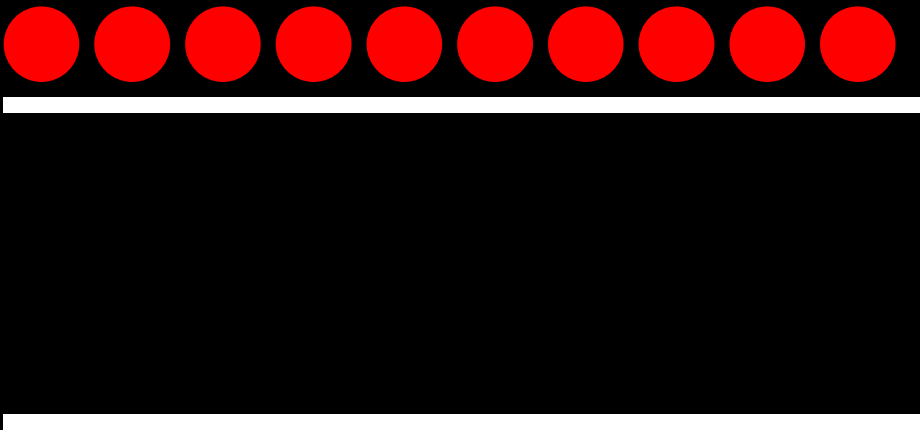
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Some landmarks in NAT

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- 1951 – LiF at -1K

Purcell & Pound, *Phys. Rev.*, 81:279-280

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Ramsay, *Phys. Rev.*, 103:20-28

Some landmarks in NAT

- 1951 – LiF at -1K
Purcell & Pound, *Phys. Rev.*, 81:279-280
- 1956 – Theoretical framework
Ramsay, *Phys. Rev.*, 103:20-28
- 2012 – $T < 0$ with motional degrees of freedom
Braun et al, *Science* 339(6115)

Why do we care?

$$dU = TdS - PdV$$



$$\left(\frac{\partial S}{\partial V}\right)_U = \frac{P}{T} \geq 0$$

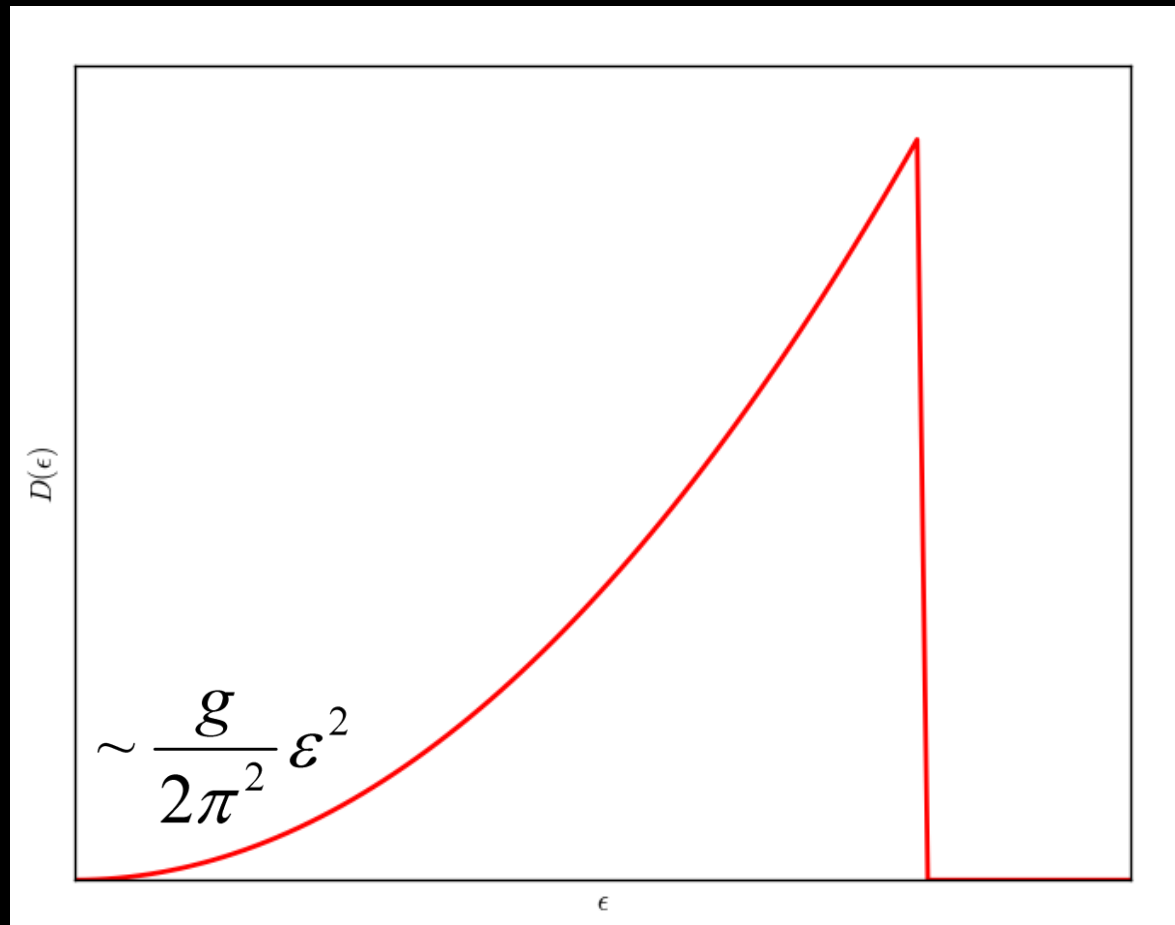


$$T < 0 \Rightarrow P < 0$$

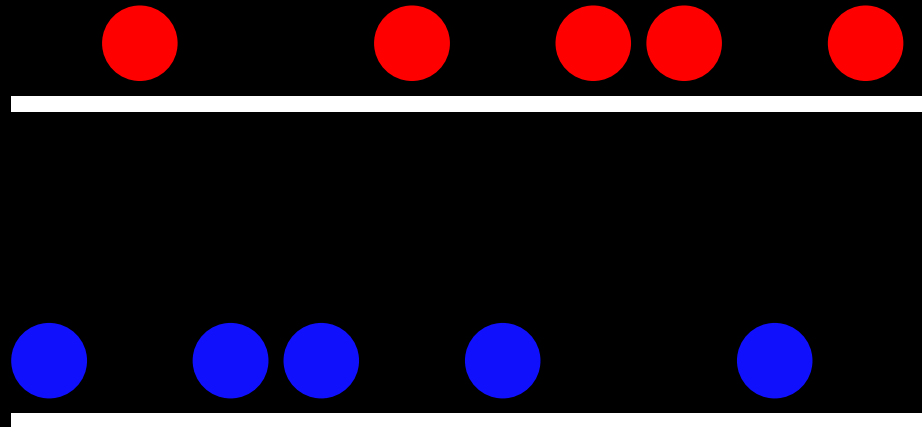


Inflation?

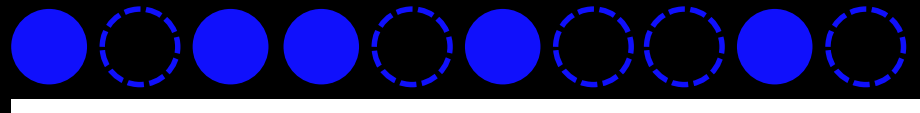
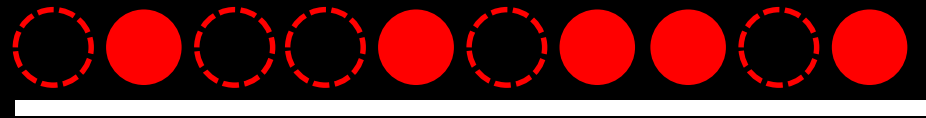
Cosmology with an energy cut-off?



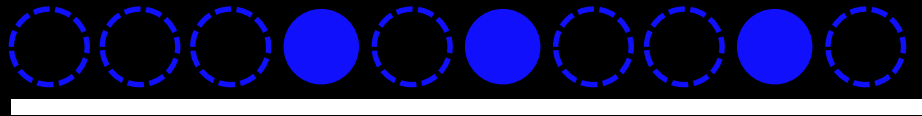
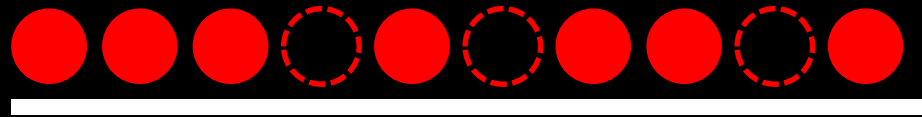
Making use of holes



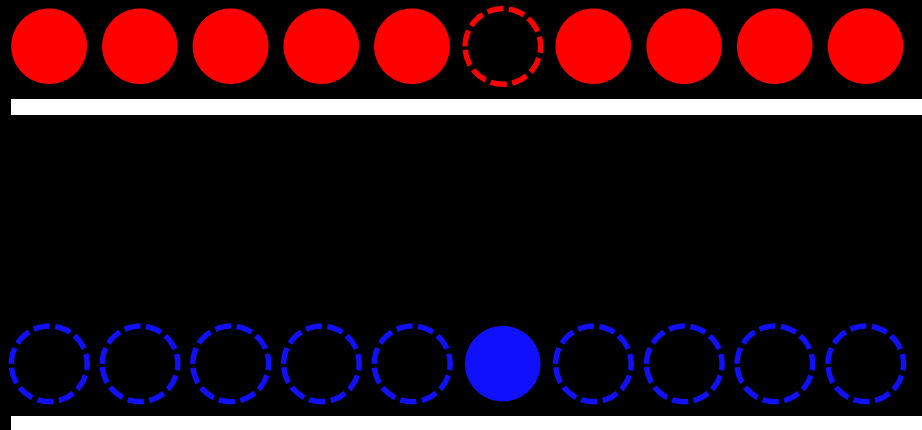
Making use of holes



Making use of holes

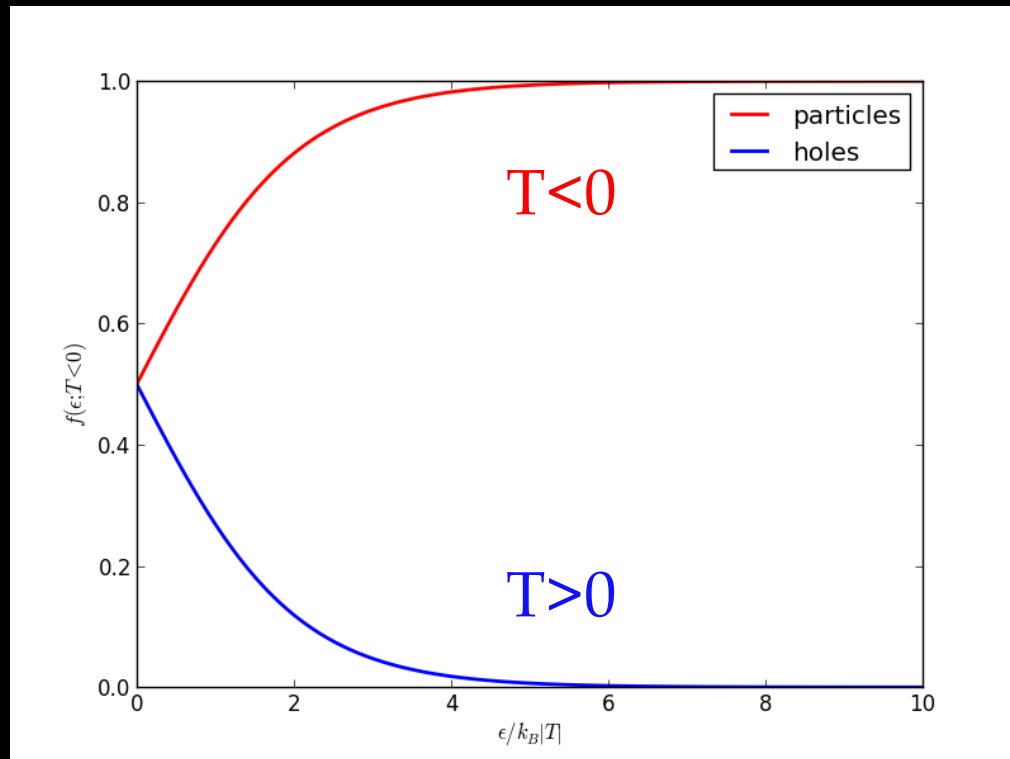


Making use of holes



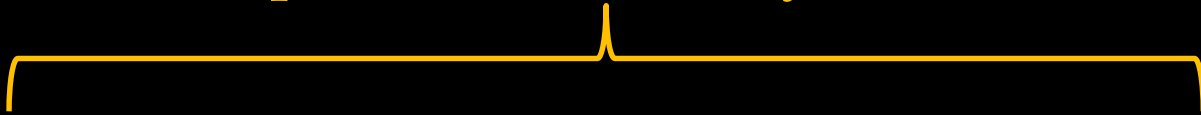
Making use of holes

$$f(\varepsilon; T) = 1 - f(\varepsilon; -T)$$
$$\rho(T) = \rho_{\max} - \rho(-T)$$
$$P(T) = -\rho_{\max} - P(-T)$$



Hole dynamics

Low-temperature holes may behave like



radiation

$$w = -\frac{1}{3} \left(4 \frac{\rho_{\max}}{\rho} - 1 \right)$$

matter

$$w = -\frac{\rho_{\max}}{\rho}$$

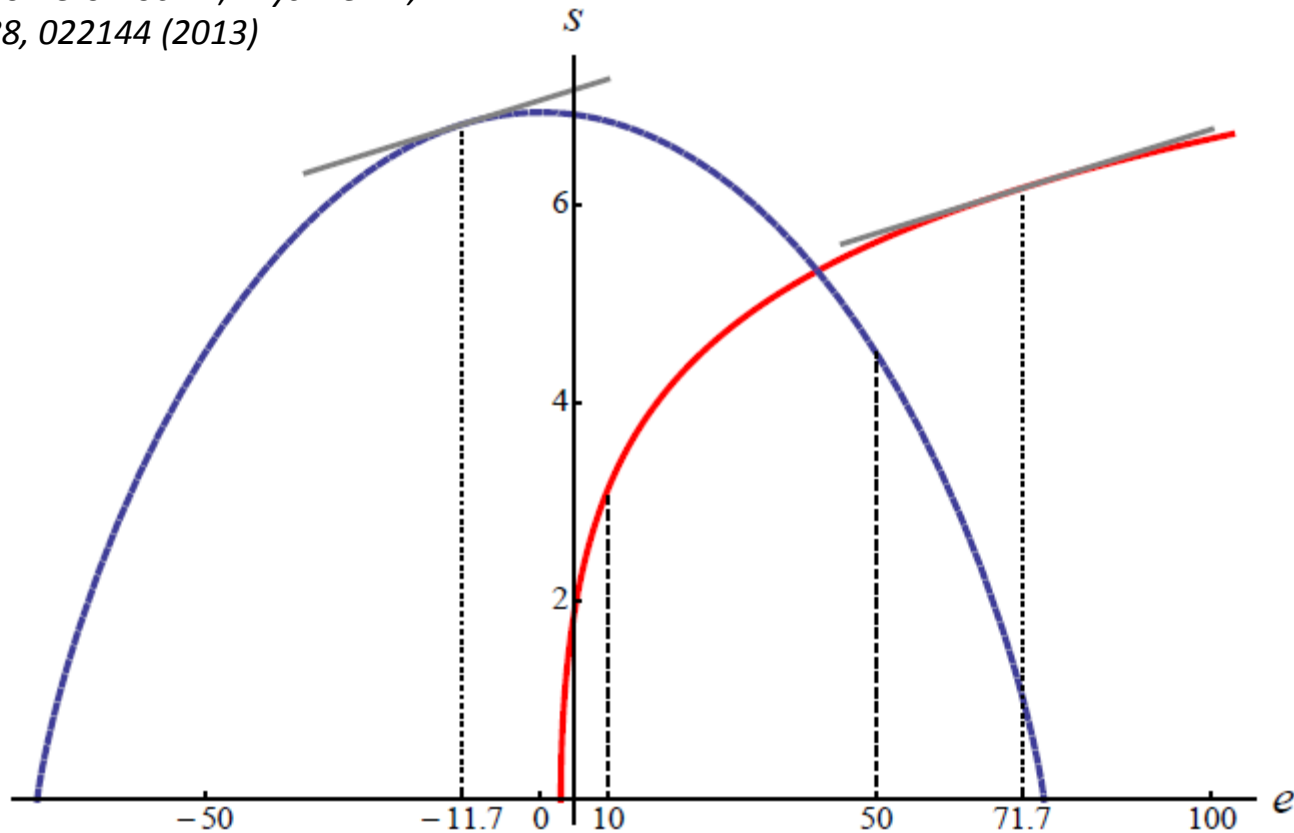
$$\frac{d \ln \rho}{dt} = -3H(1+w) \geq 0$$

Full of holes?

$$\#e - \text{foldings} = -\frac{1}{3} \int_{\rho_i}^{\rho_f} \frac{d\rho / \rho}{1+w} = \begin{cases} \frac{1}{4} \ln \left[\frac{1 - \rho_i / \rho_{\max}}{1 - \rho_f / \rho_{\max}} \right] \\ \frac{1}{3} \ln \left[\frac{1 - \rho_i / \rho_{\max}}{1 - \rho_f / \rho_{\max}} \right] \end{cases}$$

Full of holes?

Romero-Rochín, *Phys. Rev. E*,
88, 022144 (2013)



In conclusion

- If a component of the Universe is allowed to be at $T < 0$:

accelerated expansion with $w \approx -1$ naturally arises

(inflation? dark energy? Big Crunch averted?)

BUT...

In Conclusion

- Where could the cut-off come from?
Quantum gravity (discrete spacetime)?

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In Conclusion

- Where could the cut-off come from?
Quantum gravity (discrete spacetime)?
- What is the microscopic origin of $P < 0$?
Modified Heisenberg uncertainty?
Could thermalisation at high temperatures be forbidden?
- How would inflation end?
"Temperon" decay?
Thermalisation with bosons?

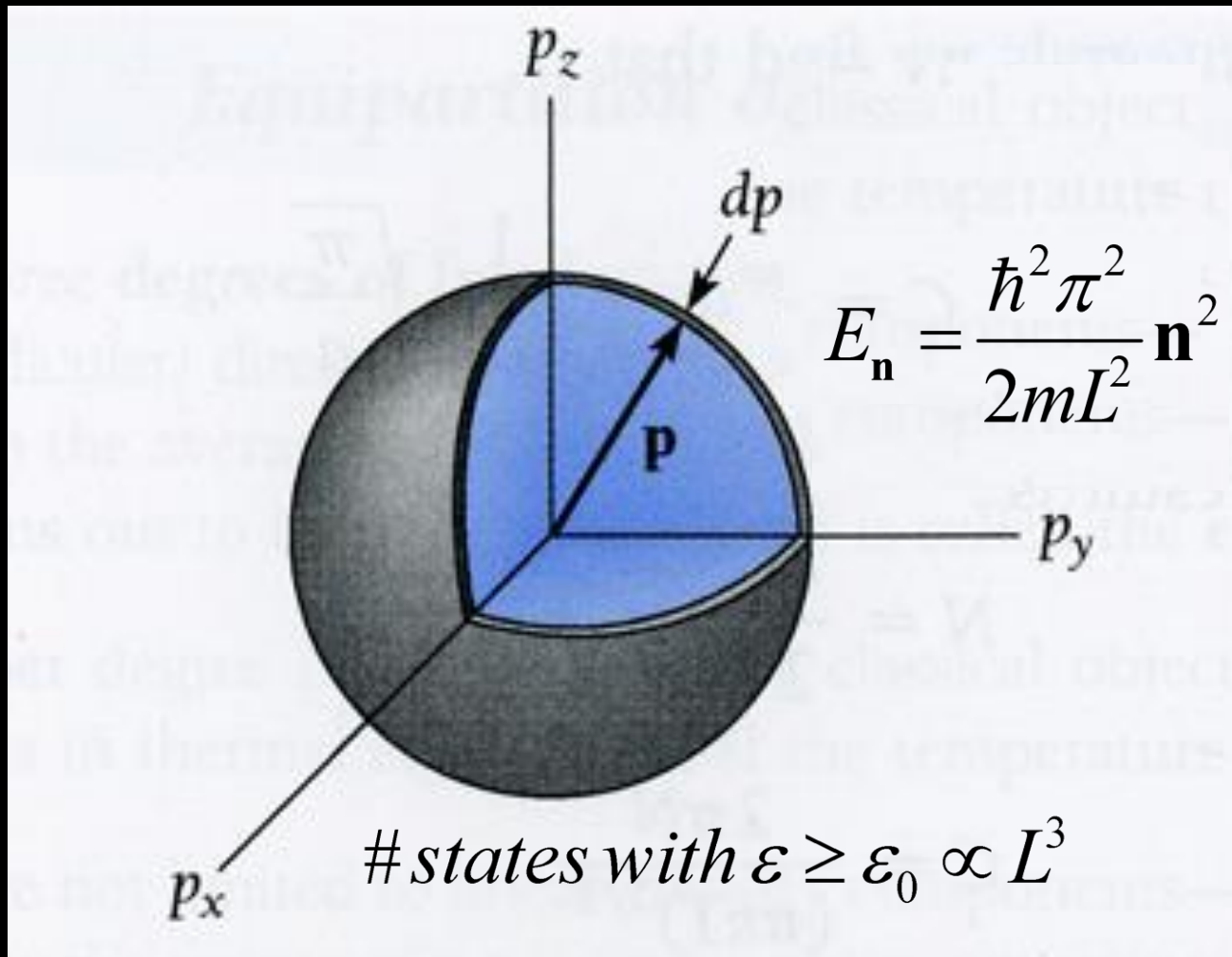
The End

COSMO-15

7-11 SEPTEMBER 2015
WARSAW, POLAND



Cosmology with an energy cut-off?



Thermodynamic Formulas

$$f(\varepsilon; T, \mu) = 1 - f(\varepsilon; -T, \mu)$$

$$\rho(T, \mu) = \rho_{\max} - \rho(-T, \mu)$$

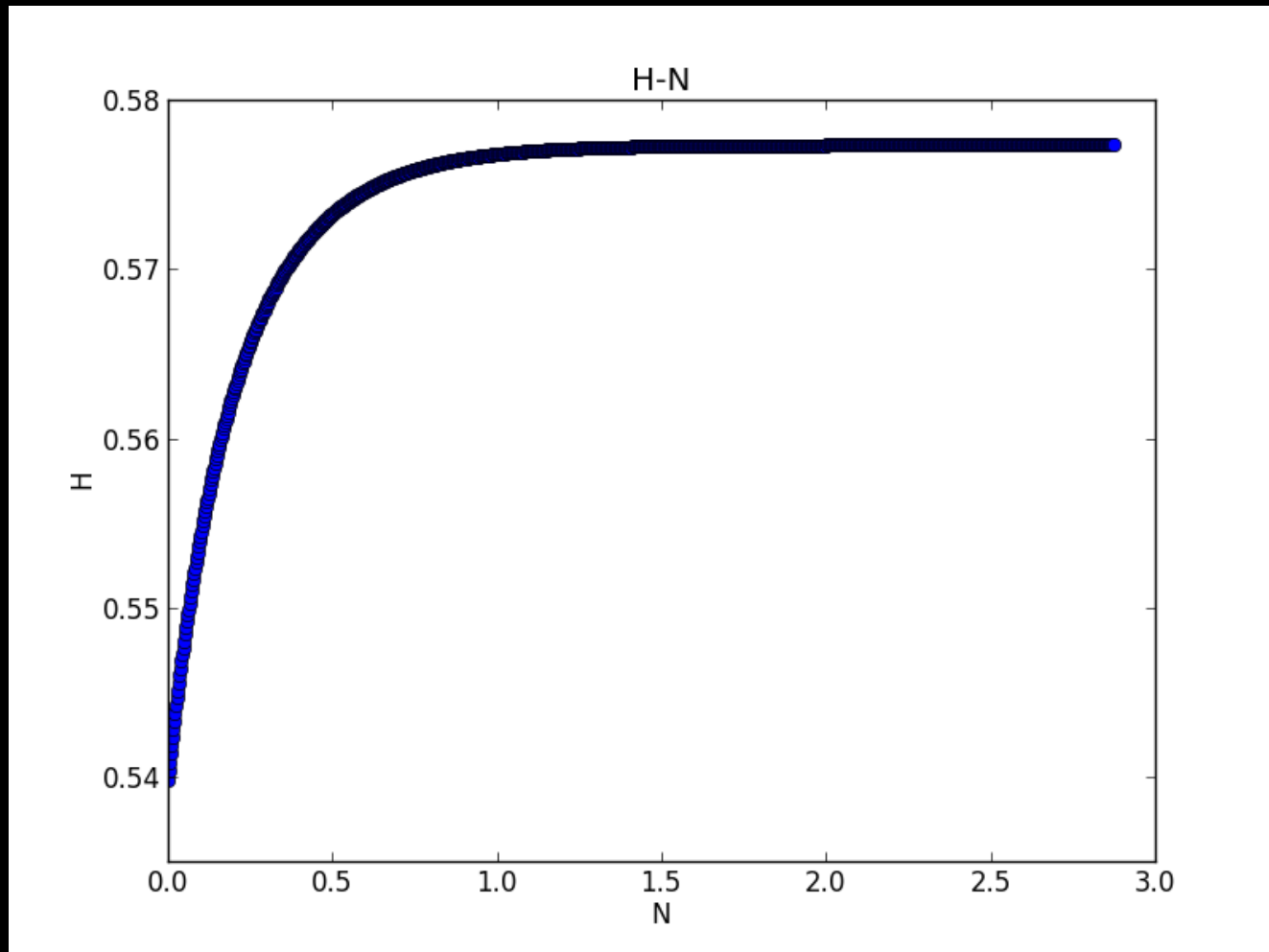
$$n(T, \mu) = n_{\max} - n(-T, \mu)$$

$$P(T, \mu) = 2\mu n_{\max} - \rho_{\max} - P(-T, \mu)$$

$$P(T, \mu) = \mu n(T, \mu) - \left(\frac{\partial F}{\partial V} \right)_{T, n}$$

$$\frac{F}{V} = -k_B T \int f(\varepsilon; T, \mu) \ln \left(1 + e^{-\frac{(\varepsilon - \mu)}{k_B T}} \right) d\varepsilon$$

Thermodynamic Formulas



Gibbs-Hertz Entropy

Loris Ferrari (2015)
arXiv:1501.04566v3

$$S_B(E) = \ln \left[\sum_u \delta_{H(u), E} \right]$$

$$S_G(E) = \ln \left[\sum_u \Theta(E - H\{u\}) \right]$$