Helical Magnetic Fields from Creation to Detection

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Motivation / Intro to Helical Magnetic Fields

Origin – Magnetogenesis

- AL, Sabancilar, Vachaspati [1309.1454] & [1509.XXXX]

Evolution – MHD and Exotica

- AL, Vachaspati [1504.03319]

Detection – CMB & Blazars

- AL, Vachaspati [1505.07846]

- Why Primordial Magnetic Fields?
  - Often a byproduct of our favorite phase transitions (inflation, EW, …)
  - A remnant of the PMF may persist today in the voids between galaxies and clusters, without significant processing by structure formation
  - Detecting the PMF would give a new handle on early universe cosmology … possibly discriminating between different models of inflation, baryogenesis, etc.
  - Empirical Motivation: Provide explanation of μG galactic fields via dynamo mechanism. Recent evidence for B ~ $10^{-17\ldots14}$ G from blazar observations. Lots of room for improvement in measurements.

- What is a helical magnetic field?

$$\langle B_i(t,\mathbf{k}) B_j(t,\mathbf{k}')^* \rangle = P_{ij}(t,\mathbf{k})\ (2\pi)^3\, \delta^{(3)}(\mathbf{k}-\mathbf{k}')$$

$$P_{ij}(t,\mathbf{k}) = \left(\delta_{ij} - \hat{k}_i \hat{k}_j\right) \underbrace{P_B(t,k)}_{\text{energy spectrum}} - i\epsilon_{ijm}\hat{k}_m \underbrace{P_{aB}(t,k)}_{\text{helicity spectrum}}$$

- A helical field has an excess of one circular polarization mode

$$\langle B^{(+)} B^{(+)*} \rangle + \langle B^{(-)} B^{(-)*} \rangle = 2\boxed{P_B(t,k)}\,(2\pi)^3 \delta^{(3)}(\mathbf{k}-\mathbf{k}')$$

$$\langle B^{(+)} B^{(+)*} \rangle - \langle B^{(-)} B^{(-)*} \rangle = 2\boxed{P_{aB}(t,k)}\,(2\pi)^3 \delta^{(3)}(\mathbf{k}-\mathbf{k}')$$

- Realizability condition & Maximally helical field

$$P_B(t,k) \geq |P_{aB}(t,k)|$$

- Magnetic helicity & Gauss linking number

$$H = \int \mathbf{A} \cdot \mathbf{B} \, d^3x$$

- Relationship to helicity spectrum (Coulomb gauge, div[A]=0)

$$\langle \mathbf{A}(t,\mathbf{k}) \cdot \mathbf{B}(t,\mathbf{k}')^* \rangle = \frac{2}{|\mathbf{k}|} P_{aB}(t,k) \; (2\pi)^3 \delta(\mathbf{k}-\mathbf{k}')$$

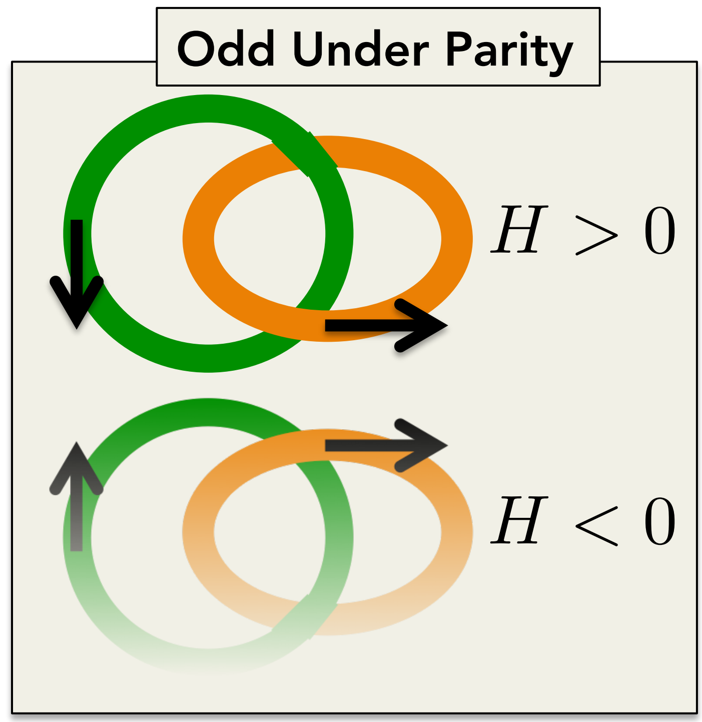
- Helicity quantifies the **linking of field lines**

flux tube: $\mathbf{B} \, d^3x = \Phi \, d\mathbf{l}$

linked tubes:

$$H = \Phi_1 \oint \mathbf{A} \cdot d\mathbf{l}_1 + \Phi_2 \oint \mathbf{A} \cdot d\mathbf{l}_2$$
$$= \pm 2 \Phi_1 \Phi_2$$

Moffatt (1969); Berger & Field (1984)

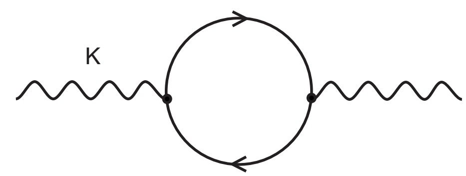


- Context – plasma of relativistic particles in early universe

$$T \gtrsim 100\,\mathrm{GeV} \qquad \mathrm{U}(1)_Y \to \text{hypermagnetic field}$$
$$T \lesssim 100\,\mathrm{GeV} \qquad \mathrm{U}(1)_{em} \to \text{(electro)magnetic field}$$

- There are always thermal fluctuations

$$P_B(\omega, k) = \frac{-8\pi\hbar\, k^2}{e^{\hbar\omega/T} - 1}\ \mathrm{Im}\Big[\frac{1}{\omega^2 - k^2 - \Pi_t(\omega, k)}\Big]$$



$$P_{aB}(\omega, k) = 0$$ (parity invariance of therm. ensemble)

$$B_{\mathrm{rms}} \approx \sqrt{\omega k^3 P_B(\omega, k)}$$

- Estimates from equipartition theorem

$$k^{-3}(B_k^2 + e^2T^2A_k^2) \sim T \quad \Rightarrow \quad \begin{cases} B_k \sim T^{1/2}k^{3/2} & , eT \lesssim k \lesssim T \\ B_k \sim k^{5/2}/eT^{1/2} & , k \lesssim eT \end{cases}$$

Magnetogenesis

(1) Inflation
(2) Electroweak Phase Transition
(3) Instability in Chiral Medium

….

(4) Something else?
- Topological defects (see: Kouichirou Horiguchi's talk)
- Astrophysics (Biermann battery)
- Reionization epoch

(see: Ryo Namba's talk)

Magnetogenesis: Inflation

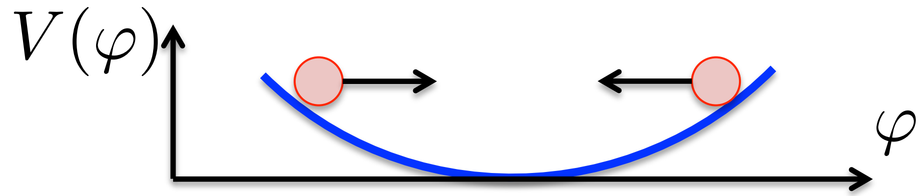
- Pseudoscalar axion field (spectator or inflaton):

Turner & Widrow (1987)
Garretson, Field, & Carroll (1992)
Anber & Sorbo (2006)
Durrer, Hollenstein, Jain (2010)
Barnaby, Namba, Peloso (2011)

$$-\mathcal{L}_{\mathrm{int}} = \frac{\varphi}{4f} F_{\mu\nu}\widetilde{F}^{\mu\nu} = \frac{d\varphi/dt}{2f}\mathbf{A}\cdot\mathbf{B} + \cdots$$

- P-violating background provided by rolling pseudoscalar

$$\xi \equiv \frac{d\varphi/dt}{2fH}$$



- This opens an instability toward the growth of one helicity mode:

$$\left(\frac{\partial^2}{\partial\eta^2} + k^2 \pm 2k\frac{\xi}{\eta}\right) A_\pm(\eta, k) = 0 \quad \Rightarrow \quad A_\pm \sim e^{\pm\pi\xi}$$

- Resulting spectra: (at end of inflation, super-horizon scales $k\eta << 1$)

$$P_B(k) = k\frac{\sinh(2\pi\xi)}{2\pi\xi} \quad , \quad P_{aB}(k) = k\frac{\cosh(2\pi\xi) - 1}{2\pi\xi}$$

(maximally helical for large ξ)

Magnetogenesis: EW Phase Transition

- First order phase transition proceeds through the nucleation of Higgs phase bubbles: <H> ~ v inside and <H> = 0 outside

1. Collisions of the shock fronts creates turbulence allowing for a dynamo amplification of thermal seed fields.

   Cheng & Olinto (1994); Baym, Bodecker, & McLerran (1995); Sigl, Olinto, & Jedamzik (1996)

2. Higgs phase equilibration follows bubble collisions. Higgs gradients induce a current & sources B-field. (Interpret as the formation and decay of electroweak Z-strings).

   Vachaspati (1991); Kibble & Vilenkin (1995); Ahonen & Enqvist (1997); Grasso & Riotto (1997)

EW Magnetogenesis <--> EW Baryogenesis

- Quantum anomalies reveal a profound connection between gauge field topology (~helicity) and fermion number violation

't Hooft (1976)

$$\partial_\mu j^\mu_{B+L} = 2N_f \left(\frac{g^2}{32\pi^2} W\widetilde{W} - \frac{g'^2}{32\pi^2} Y\widetilde{Y} \right)$$

$$\Delta N_{B+L} = 2N_f \left(\Delta N_{CS} - \frac{\alpha_Y}{4\pi} \Delta H_Y \right)$$

- **Question:** Can helical magnetic fields arise from baryogenesis via the anomaly?

Cornwall (1997); Vachaspati (2001)

- **Sometimes:** Bubble collisions may create linked Z-strings (~EW sphaleron). Their decay induces baryon number violation and creates magnetic helicity.

Krauss & Trodden (1999); Copi, Ferrer, Vachaspati, & Achucarro (2008); Diaz-Gil et. al. (2008)

- **Not Necessarily:** In non-local EW baryogenesis (charge transport in front of bubble wall), the anomaly is driven by CS diffusion in the SU(2) sector … not clear that there needs to be helical magnetic radiation

(see: Oleg Ruchayskiy's talk)

Magnetogenesis: Instability in Chiral Medium

Joyce & Shaposhnikov (1997); Giovannini & Shaposhnikov (1997)
see also Rubakov & Tavkhelidze (1985)

- Argument from free energy

$F_{\mathrm{initial}} \sim \mu^2 T^2 V$	(free energy of chiral medium)
$\Delta\mu \sim \dfrac{\alpha}{T^2 V}\Delta H$	(charge erasure accompanied by helicity generation)
$H \sim L\,(B^2 V)$	(best case scenario: maximally helical field)
$F_{\mathrm{final}} \sim B^2 V \sim \mu T^2 V/(\alpha L)$	(free energy of magnetic field)
$F_{\mathrm{final}} < F_{\mathrm{initial}} \;\Rightarrow\; (\alpha\mu)^{-1} < L$	(instability on large scales)

- Study co-evolution of helical B-field & asymmetry with chiral MHD

Boyarsky, Frolich, & Shaposhnikov (2015); also Frohlich & Pedrini (2000, 2002)

$$\text{instability time scale:} \qquad \tau \sim \sigma/\alpha^2\mu^2$$

- Implications for leptogenesis?
  - $\mu \sim 10^{-10}$ T ➔ instability too slow
  - $\mu \sim 10^{-4}$ T ➔ helical magnetogenesis

Joyce & Shaposhnikov (1997);
Semikoz & Valle (2008); Semikoz, Smirnov, & Sokoloff (2013)
AL, Sabancilar, Vachaspati [1309.1454] & [1509.XXXX]

Evolution in a Turbulent Medium

- MHD is used to study the evolution of the field
- Helicity is approximately conserved in highly conductive early universe

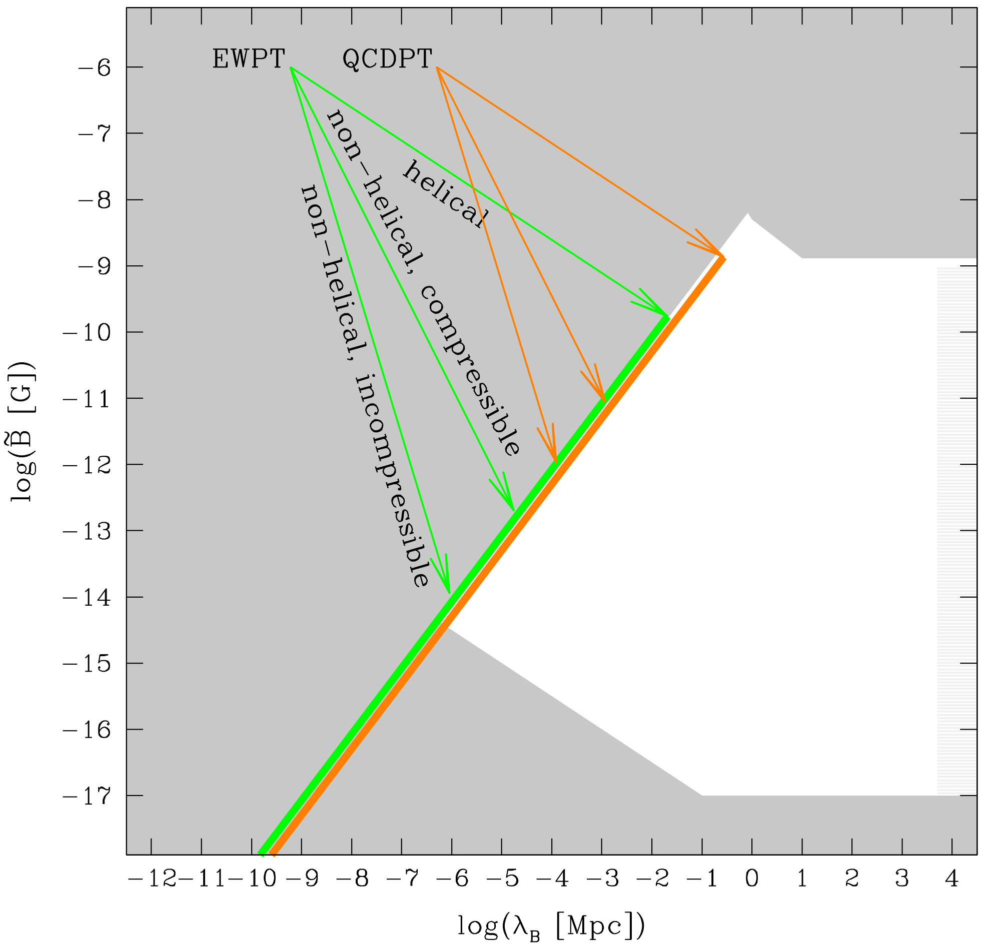
$$H = \mathcal{E} L$$

$$-\dot{\mathcal{E}} = \mathcal{E}^{3/2}/L$$

$$\Rightarrow \begin{cases} \mathcal{E} \sim t^{-2/3} \\ L \sim t^{2/3} \\ H \sim t^{0} \end{cases}$$

- "Inverse Cascade" – energy is transferred from small to large scales

Poquet, Frisch, & Leorat (1976)



Kahniashvilli, Tevzadze, Brandenburg, Neronov (2013)
see also: Christensson, Hindmarsh, & Brandenburg (2000)

AL, Vachaspati [1504.03319]

Evolution with Exotica

- **Magnetic Monopoles**
  - The B-field does work on a gas of monopoles.
  - Persistence of the PMF leads to an upper bound on the monopole density … analogous to the Parker Bound on galactic fields

Parker (1970); Turner, Parker, Bogdan (1982); Freese, et. al. (1993)

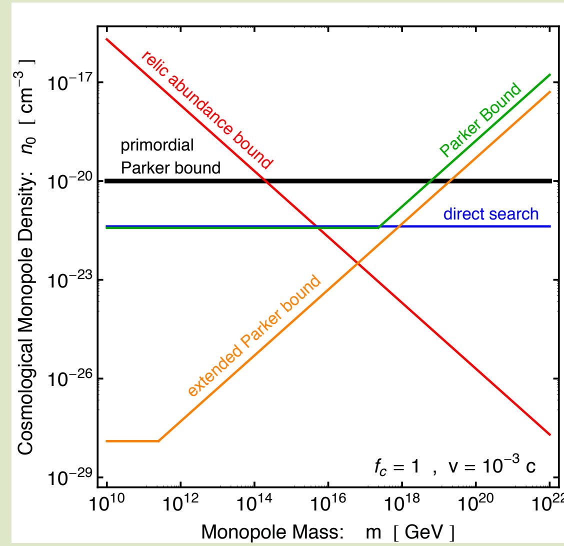
- **Axions**
  - B-field amplification (recall inflation model)
  - Axion gets sourced by helical field

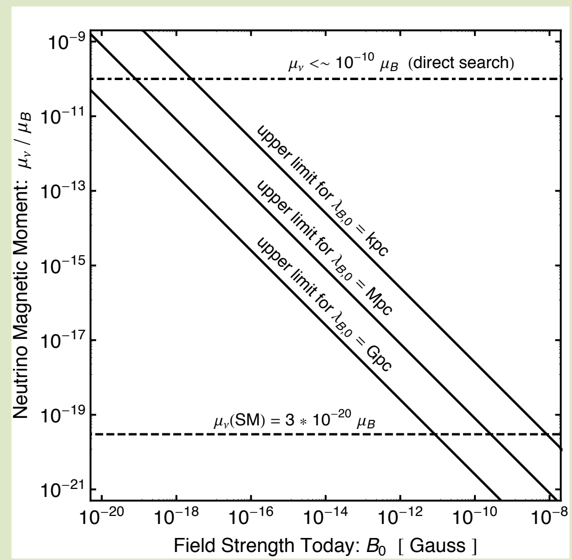
$$\ddot{a} + \frac{g_{a\gamma}^2 \eta_{\mathrm{d}} \langle |\mathbf{B}|^2 \rangle}{4\pi} \dot{a} + m_a^2 a^2 = \frac{g_{a\gamma} \eta_{\mathrm{d}}}{4\pi} \langle \mathbf{B} \cdot \boldsymbol{\nabla} \times \mathbf{B} \rangle$$

Ahonen, Enqvist, & Raffelt (1996)

- **Dirac Neutrinos**
  - B-field couples to ν magnetic moment … induces spin flip $\nu_L \rightarrow \nu_R$ … populates sterile states … Neff = 6 inconsistent with BBN

Enqvist, Olesen, & Semikoz (1992)





Detection Strategies using the CMB

- A primordial magnetic has many potential effects on the CMB
  - B-field energy density induces curvature perturbations
  - BAO in the presence of B-field shifts acoustic peaks
  - Faraday rotation of E → B polarization modes (Jacques Wagstaff's talk)
  - Dissipation of magnetic energy induces spectral distortion
  - Non-Gaussianities: <δρ δρ> ~ <B^2 B^2>

- Combined limit is easy to estimate:

$$\frac{\Omega_B}{\Omega_\gamma} = \frac{B^2/8\pi}{2\pi^2 T^4/30} \simeq 10^{-5}\left(\frac{B}{10^{-8}\,\mathrm{G}}\right)^2 \quad \Rightarrow \quad B_{1\,\mathrm{Mpc}} \lesssim (1-5)\,\mathrm{nG}$$

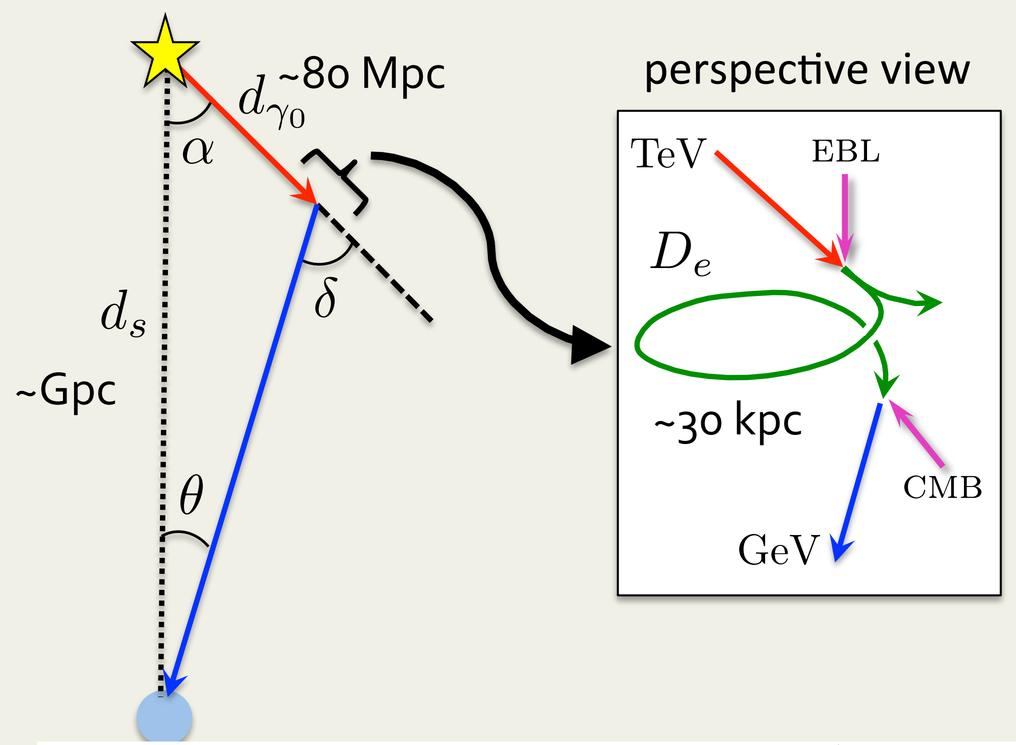
Planck (2015) 95%CL limits

- Unique effect of helical fields
  - Parity-odd observables like <TB>, and <EB> cross correlation

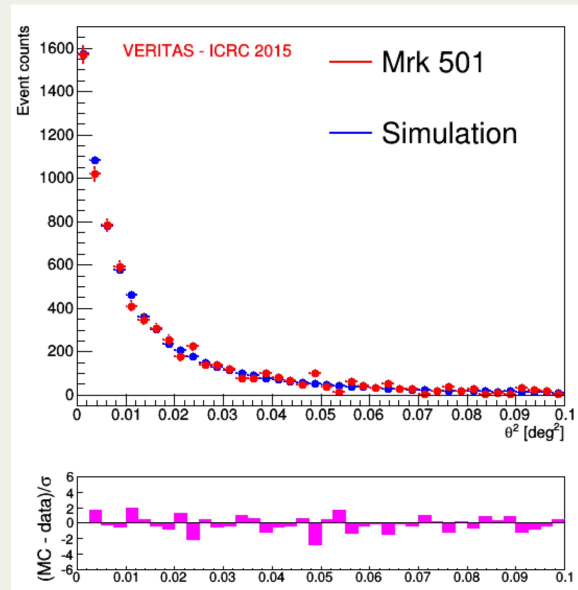
Detection with TeV Blazar Halos

- Gamma rays from TeV blazars develop an electromagnetic cascade, which is deflected back toward the line of sight by B-field, inducing a halo

Aharonian, Coppi, & Voelk (1994); Neronov & Semikoz (2006)



$$\Theta_{\rm ext} \simeq (0.68^\circ)\left(\frac{B_0}{10^{-15}\,{\rm G}}\right)\left(\frac{E_\gamma}{10\,{\rm GeV}}\right)^{-3/2}\left(\frac{d_s}{1\,{\rm Gpc}}\right)$$

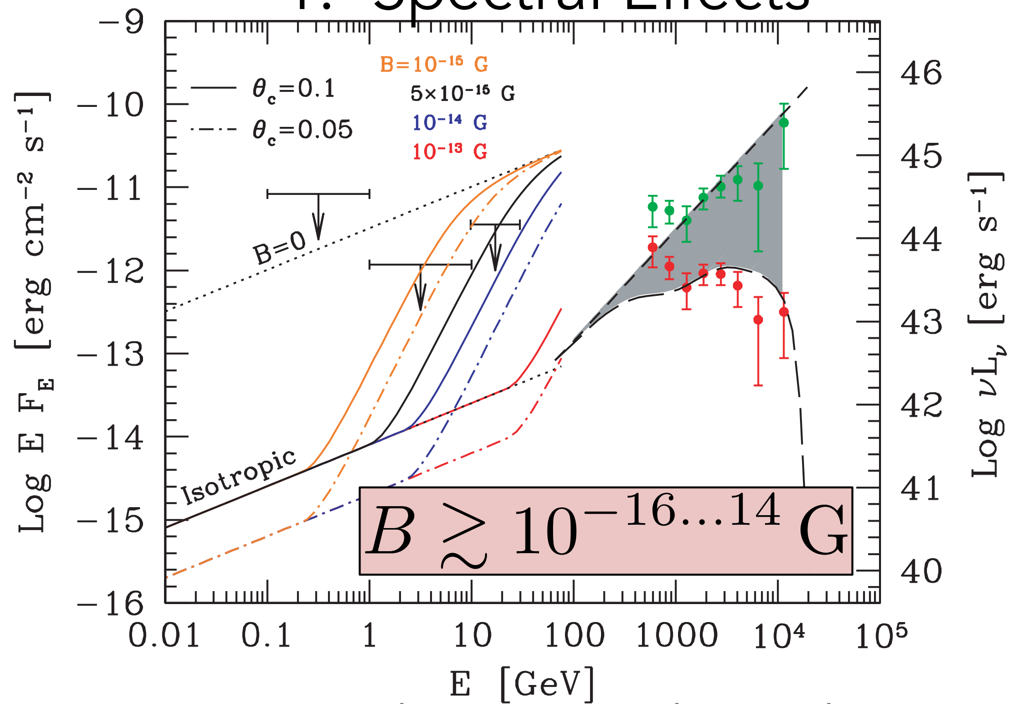


VERITAS (Pueschel, 2015) – no halo seen … leads to model dependent exclusion @ 95% CL

$$(5-10)\times 10^{-15}\ {\rm G}\ \ ({\rm excluded})$$

Recent Hints in Blazar Data

1. Spectral Effects



Neronov & Vovk (2010); Tavecchio, et. al. (2010)

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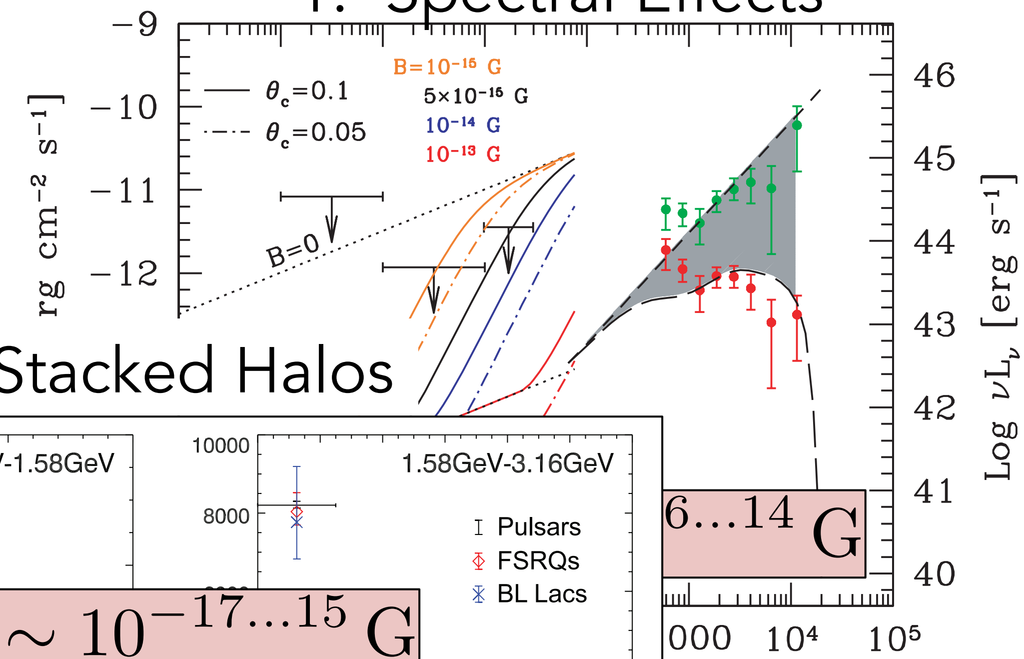
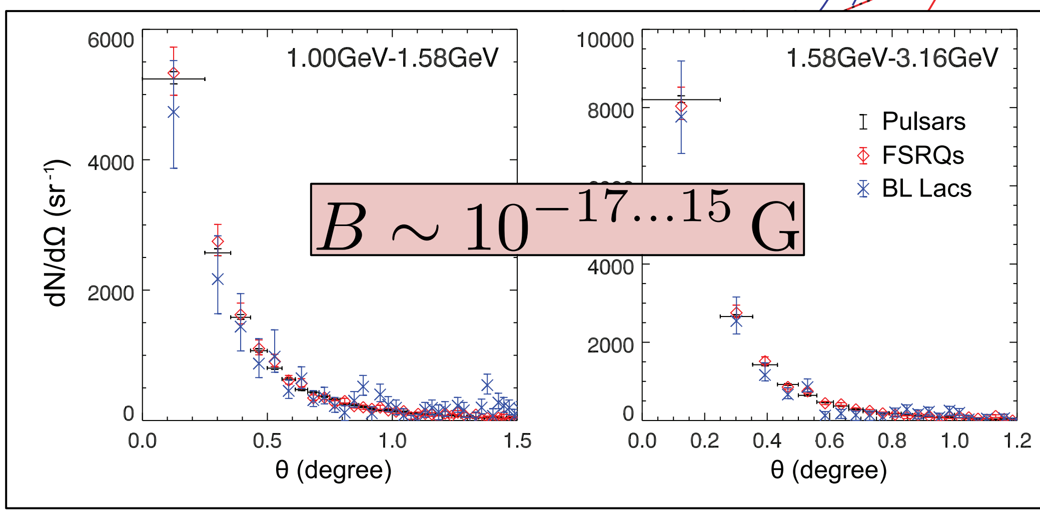


chio, et. al. (2010)

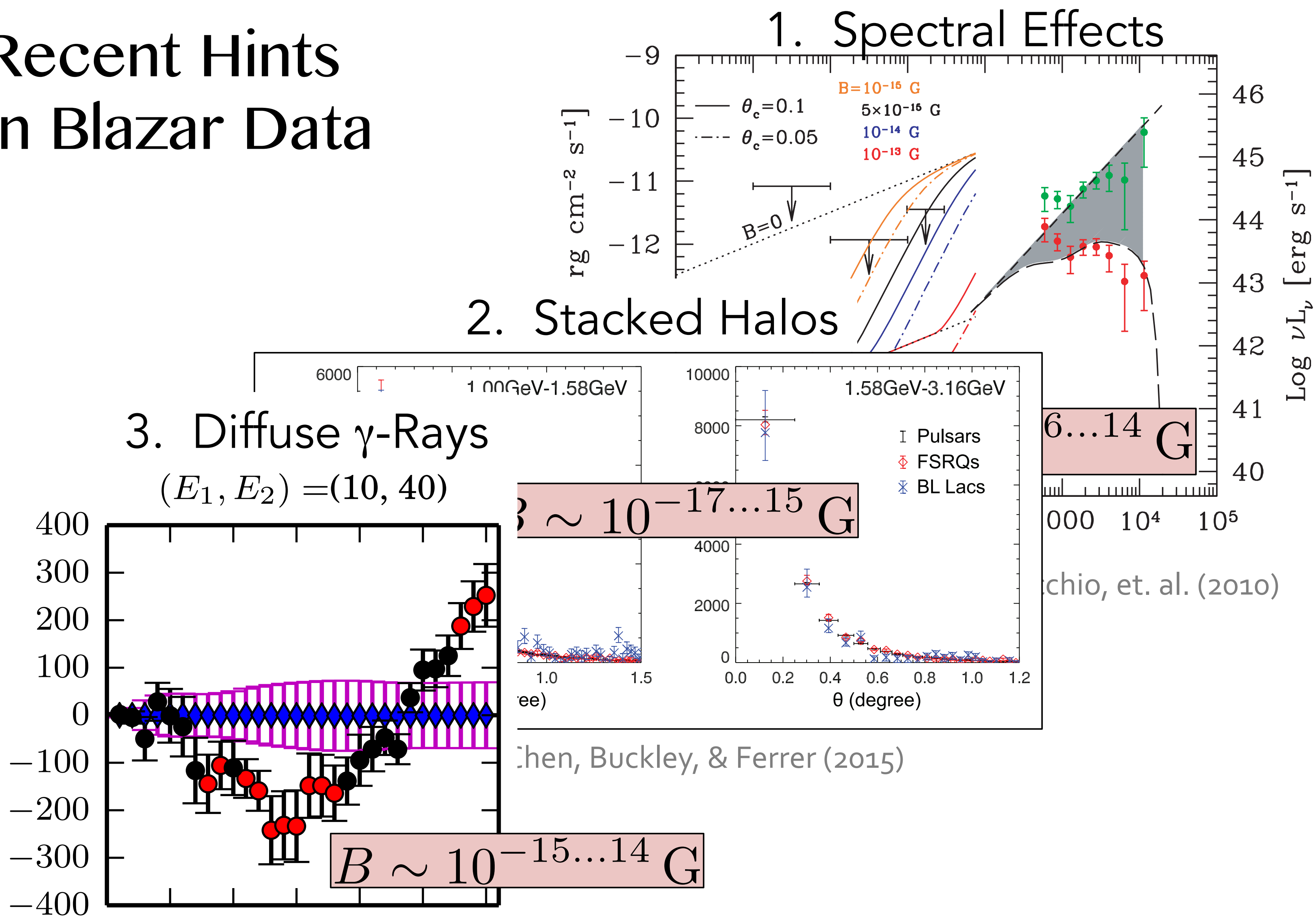
2. Stacked Halos



$$B \sim 10^{-17 \ldots 15}\ \mathrm{G}$$

Chen, Buckley, & Ferrer (2015)

Recent Hints in Blazar Data



…chio, et. al. (2010)

…hen, Buckley, & Ferrer (2015)

Chen, Chowdhury, Ferrer, Tashiro, & Vachaspati (2015)

AL, Vachaspati [1505.07846]

Halo Morphology

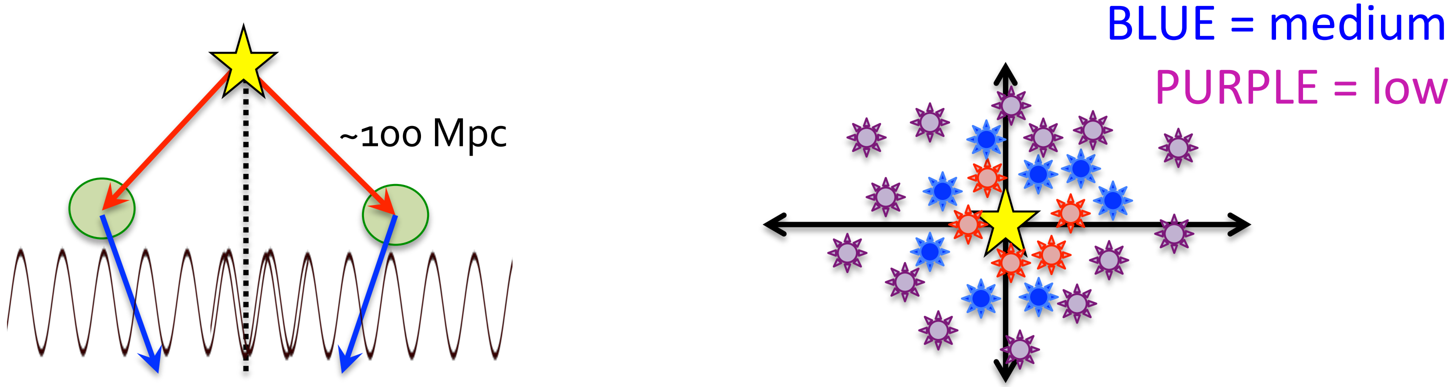
- How does magnetic helicity affect the halo?

  Halo Size ➔ Field Strength
  Halo Shape ➔ Magnetic Helicity

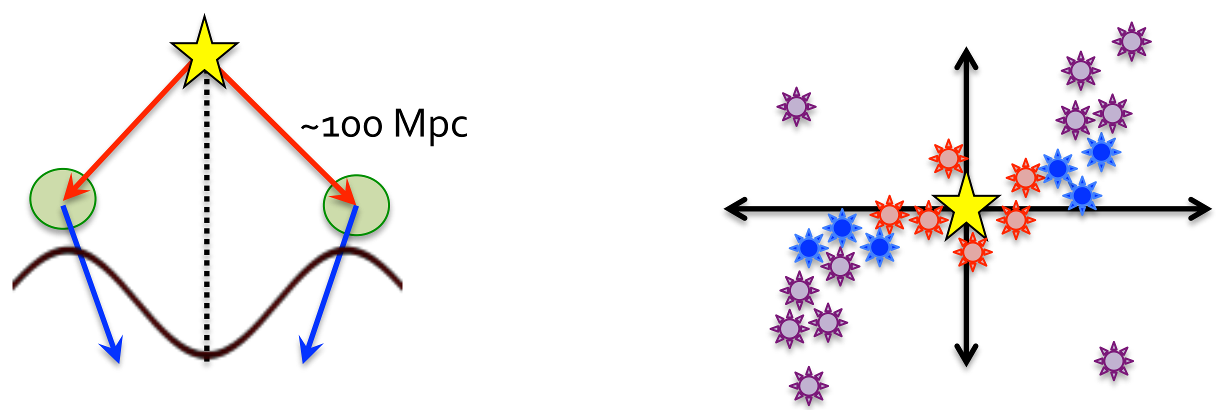
  Tashiro & Vachaspati (2013)

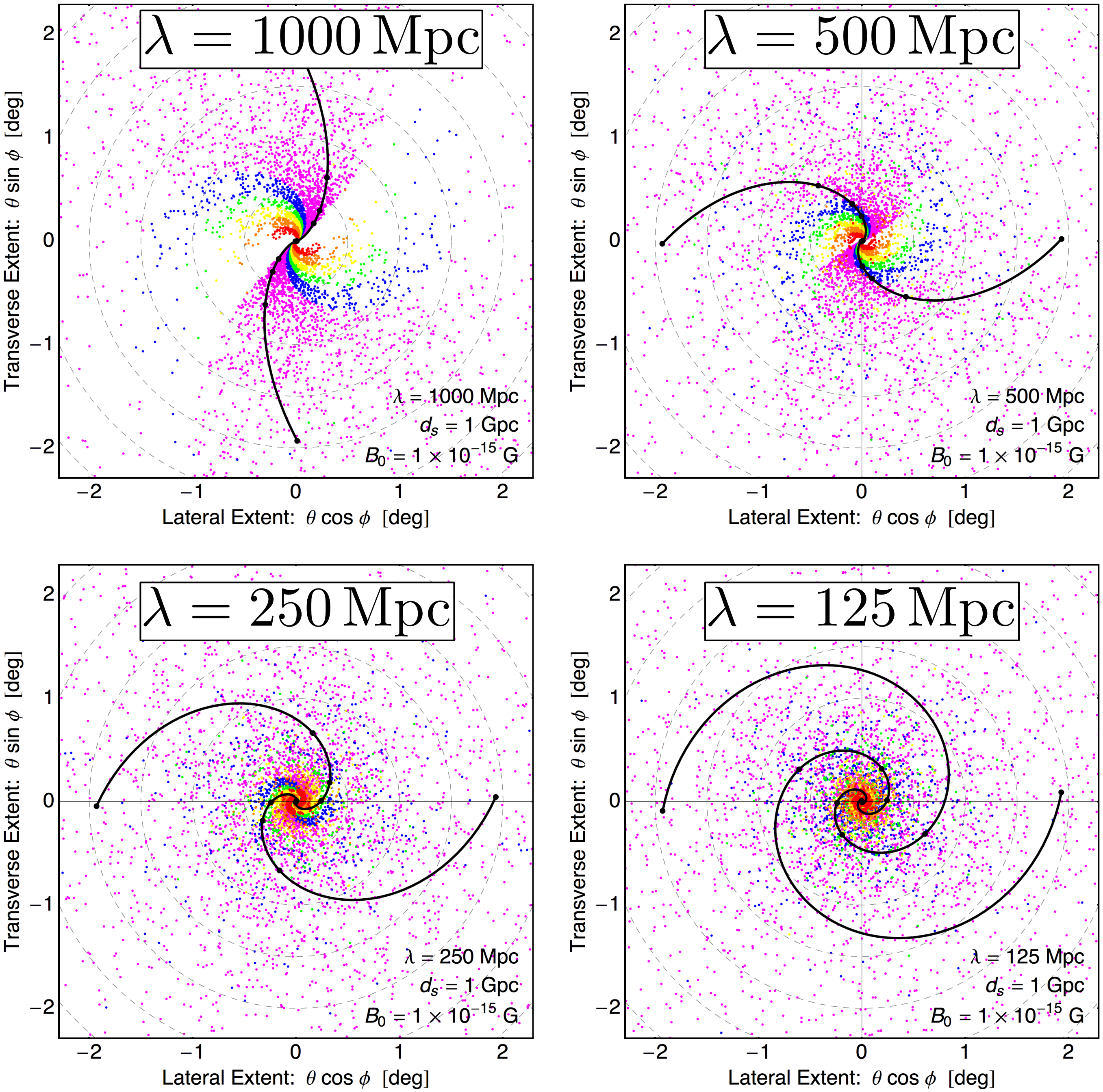
- Endows it with parity-violating property: skew

$\lambda \ll 100\,\mathrm{Mpc}$



$\lambda \gtrsim 100\,\mathrm{Mpc}$





$$B_0 = 10^{-15}\,\mathrm{G}$$
$$d_s = 1\,\mathrm{Gpc}$$

γ-Ray Energy:

RED = high

BLUE = medium

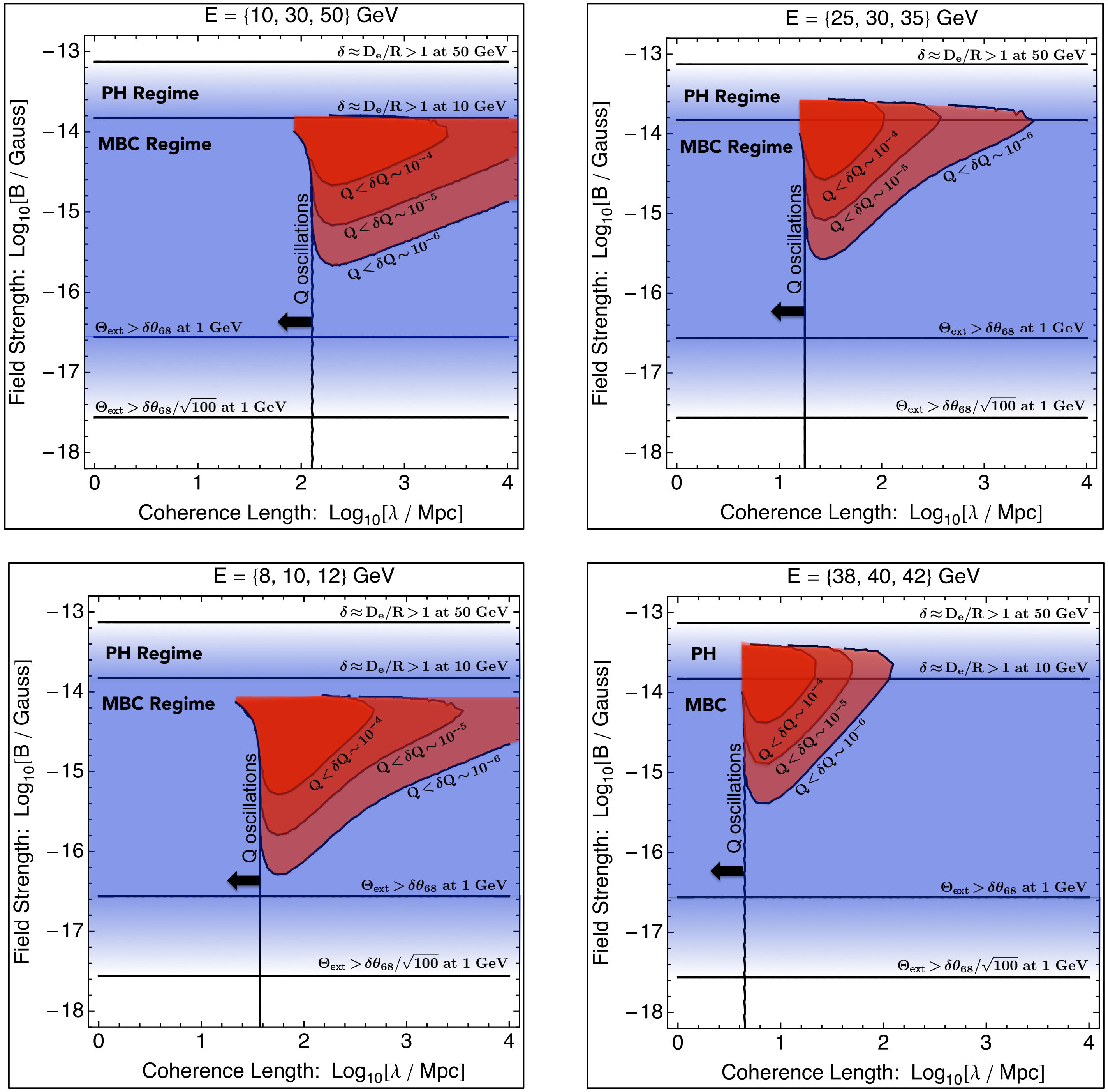
PURPLE = low

AL, Vachaspati (2015)

look out for: Batista, Saveliev, Sigl, Vachaspati (in prep.)

Blue Region
-- best suited to probe field strength via halo size measurement

Red Region
-- best suited to probe coherence length (helicity) via halo shape measurement (Q-statistic)



Conclusions

- Primordial magnetic fields represent an untapped probe of the early universe: inflation, phase transitions, baryogenesis, …
- If future blazar halo observations confirm the presence of an intergalactic magnetic field, measurements of halo morphology may be used to probe magnetic strength and helicity