Particle Production after Inflation with Non-minimal Derivative Coupling to Gravity

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based on arXiv:1504.07119 (accepted by JCAP)
with R. Jinno, K. Mukaida and K. Nakayama

Introduction

- The Higgs boson is discovered.
  $\rightarrow$ Are there any relations between Higgs and inflaton?

- There are a variety of Higgs inflation models:
  - Higgs inflation $\left(1 + \zeta\phi^2/M_P^2\right) R$, [Bezrukov, Shaposhnikov, '07]
  - ew Higgs inflation $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, [Germani, Kehagias, '10]
  - running kinetic inflation $K(\phi)\left(\partial\phi\right)^2$, [Nakayama, Takahashi, '10]

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- The reheating is also a key process of inflation models.

Here we consider the particle production due to the non-minimal derivative coupling to gravity (new Higgs-type): $\mathcal{L} \sim G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$.

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Outline

Background equations of motion

We consider the following action:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}\left(g^{\mu\nu} - \frac{G^{\mu\nu}}{M^2}\right)\partial_\mu\phi\partial_\nu\phi - V(\phi)\right],$$

where $g_{\mu\nu}$: metric, R: Ricci scalar, $G_{\mu\nu}$: Einstein tensor,

ϕ: inflaton (may not be Higgs), M: some mass parameter.

Background equations of motion are: $g_{\mu\nu} = \mathrm{diag}(-1, a^2, a^2, a^2), H \equiv \dot{a}/a,$

$$H^2 = \frac{\rho_\phi}{3M_P^2}, \ \dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0,$$

where

$$\rho_\phi = \left(1 + \frac{9H^2}{M^2}\right)\frac{\dot{\phi}^2}{2} + V, \ p_\phi = \left(1 - \frac{3H^2}{M^2}\right)\frac{\dot{\phi}^2}{2} - V - \underline{\underline{\frac{1}{M^2}\frac{d}{dt}\left(H\dot{\phi}^2\right)}}.$$

red: contributions from the non-minimal derivative coupling.

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Inflaton oscillation regime

Consider the inflaton oscillation regime:

$$H^2/M^2 \gg 1, \ m_{\rm eff} \equiv \frac{M}{H}\sqrt{\frac{V'}{\phi}} > H.$$

* Kinetic term is dominated by $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/M^2 \sim (H^2/M^2)\dot{\phi}^2$.

$\rightarrow$ inflaton oscillates with its frequency $\sim m_{\rm eff}$.

The following estimations hold:

$$\begin{cases} \rho_\phi \sim \frac{H^2}{M^2}\dot{\phi}^2 \sim V, \\ p_\phi \ni -\frac{1}{M^2}\frac{d}{dt}\left(H\dot{\phi}^2\right) \sim \frac{m_{\rm eff}H}{M^2}\dot{\phi}^2 \sim \frac{m_{\rm eff}}{H}\rho_\phi \gg \rho_\phi. \end{cases}$$

$\Rightarrow \dot{\rho}_\phi \sim \mathcal{O}(Hp_\phi) \sim \mathcal{O}(m_{\rm eff}\rho_\phi),$ and hence $\dot{H} \sim \mathcal{O}(m_{\rm eff}H)$.

The large contribution from p_ϕ plays a key role here.

Hubble parameter also oscillates with frequency $\sim m_{\rm eff}$!

$\rightarrow$ efficient particle production? (topic of next section)

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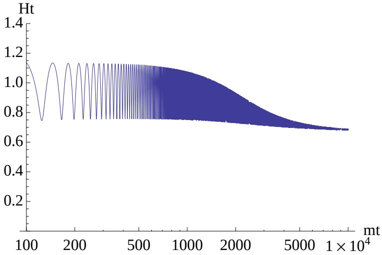
Numerical solution of the Hubble parameter

Right panel: plot of mt vs. Ht, where we take $V = m^2\phi^2/2$.

[YE, Jinno, Mukaida, Nakayama, '15]

We take the model parameters as $m/M = 10^3$ and solve EOM from the inflationary regime. Note that Ht is dimensionless, and hence depends only on the combination m/M.



Simple estimation of the sound speed squared

Now we consider the scalar perturbation around the background.
In particular, we pay attention to the sound speed squared.

Simple estimation:

$$G^{00}_{(\mathrm{bg})} = 3H^2,\ G^{ij}_{(\mathrm{bg})} = -\frac{\delta^{ij}}{a^2}\left(3H^2 + 2\dot{H}\right).$$

Therefore,

$$\frac{G^{\mu\nu}}{M^2}\partial_\mu\phi\partial_\nu\phi \sim \frac{3H^2}{M^2}\dot{\phi}^2 - \frac{\left(3H^2 + 2\dot{H}\right)}{M^2 a^2}\left(\partial_i\phi\right)^2$$

$$\Rightarrow c_s^2 \sim \frac{M^2}{3H^2}\frac{3H^2 + 2\dot{H}}{M^2} \sim \mathcal{O}\left(\frac{\dot{H}}{H^2}\right).$$

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Quadratic action for the scalar perturbation

The estimation of the previous slide is in fact true. The quadratic action for the curvature perturbation ζ is given by

$$S = M_P^2 \int d^4x \, a^3 \frac{F^2 G}{H^2} \left[\dot{\zeta}^2 - \frac{c_s^2}{a^2} \left(\partial_i \zeta\right)^2 \right],$$

$F \equiv \frac{1-\frac{1}{2}\epsilon}{1-\frac{3}{2}\epsilon}$, $G \equiv \frac{\dot{\phi}^2}{2M_P^2}\left(1 + \frac{3H^2}{M^2}\frac{1+\frac{3}{2}\epsilon}{1-\frac{1}{2}\epsilon}\right)$, $\epsilon \equiv \frac{\dot{\phi}^2}{M_P^2 M^2}$, Freedman eq. $\Rightarrow 0 < \epsilon < \frac{2}{3}$.

Here, the sound speed squared is given by

$$c_s^2 = \frac{1}{K}\left[\left(1 + \frac{3}{2}\epsilon\right) + \frac{3H^2}{M^2}\left[1 + \left(\frac{3}{2} + \frac{2}{3F}\right)\epsilon\right] + \frac{6\dot{H}}{M^2}\left(1 - \frac{1}{2}\epsilon\right)\right]$$

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c_s^2 oscillates between positive and negative values.

$\Rightarrow$ A gradient instability exists in this model!

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Gradient instability

Sound speed squared becomes negative during some period
= We call this situation as a gradient instability.

- Higher momentum modes are more likely to be enhanced (exponential enhancement):

$$\omega \sim c_s k \sim \pm i|c_s|k \;\Rightarrow\; e^{i\omega t} \sim e^{\pm|c_s|kt}.$$

- The dynamics soon becomes highly non-linear, and hence some tough numerical simulation may be needed.
- Some UV completion is also needed to treat the situation.
- Back-reactions of such an enhancement should be taken into account, and the reheating process may be much affected by this process.

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Quadratic action for the tensor perturbation

Next, we consider the quadratic action for the graviton:

$$S_{\rm grav} = \frac{M_P^2}{8}\int d^4x\, a^3 \left[\left(1-\frac{1}{2}\epsilon\right)\left(\dot{h}_{ij}\right)^2 - \frac{1}{a^2}\left(1+\frac{1}{2}\epsilon\right)(\partial_l h_{ij})^2\right],$$

where we take $ds^2 = -dt^2 + a^2\left(e^h\right)_{ij} dx^i dx^j,\ \partial_i h_{ij} = h_{ii} = 0.$

- The tensor modes do not have the ghost/gradient instability.
- Direct coupling between the inflaton and the graviton.
  $\rightarrow$ Graviton production also occurs, but that of the scalar perturbation is much more drastic.

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Summary

- We consider the inflaton oscillation regime of the model with the non-minimal derivative coupling to gravity:

$$\mathcal{L} \ni \frac{G^{\mu\nu}}{2M^2}\partial_\mu\phi\partial_\nu\phi.$$

- We solve the background dynamics of this model.
  The Hubble parameter oscillates violently due to the large contribution from the pressure p_ϕ:

$$\dot{H} \sim \mathcal{O}\left(m_{\rm eff} H\right).$$

- The sound speed squared of the scalar perturbation becomes negative during the inflaton oscillation regime,
  and hence there is a gradient instability in this model.

$$c_s^2 \sim \mathcal{O}(\dot{H}/H^2).$$

Back up

Cut-off scale of this model

The first non-renormalizable operator is [Germani, Kehagias, '10]

$$\mathcal{L}_{\rm NR} \sim \frac{\partial^2 h^{ij}}{M^2} \partial_i \phi \partial_j \phi.$$

- In the Minkowski background, the canonically normalized graviton is $\tilde{h} \equiv M_P h/2$, and hence the operator is

$$\mathcal{L}_{\rm NR} \sim \frac{\partial^2 \tilde{h}^{ij}}{M_P M^2} \partial_i \phi \partial_j \phi.$$

  Therefore, the cut-off scale is given by $\Lambda_{(\rm Min)} \simeq \left(M_P M^2\right)^{1/3}$.

- In the FLRW background with $H^2/M^2 \gg 1$, the canonically normalized scalar is $\tilde{\phi} \sim H\phi/M$, and hence the operator is

$$\mathcal{L}_{\rm NR} \sim \frac{\partial^2 \tilde{h}^{ij}}{M_P H^2} \partial_i \tilde{\phi} \partial_j \tilde{\phi}.$$

  Therefore, the cut-off scale is given by $\Lambda_{(\rm FRW)}(H) \simeq \left(M_P H^2\right)^{1/3}$.

*** At the onset of the oscillation, $m_{\rm eff} \simeq H$ and hence $\Lambda_{(\rm FRW)} > m_{\rm eff}$ as long as $M_P > H$.

Generalized Galileon theories

The most general Lagrangian whose equations of motion contain up to 2nd order time derivatives = generalized Galileon theories

[Horndeski, '74; Deffayet, Gao, Steer, Zahariade, '11]

$$S_G = \int d^4x\sqrt{-g}\,\sum_{i=2}^{5}\mathcal{L}_i,$$

where $(X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi)$

$$\begin{aligned}
\mathcal{L}_2 &= G_2(\phi, X),\\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi,\\
\mathcal{L}_4 &= G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],\\
\mathcal{L}_5 &= G_5(\phi, X)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi\\
&\qquad - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right].
\end{aligned}$$

Our case corresponds to

$$G_2 = X - V,\ G_4 = M_P^2/2,\ G_5 = -\phi/2M^2.$$

Adiabatic invariant

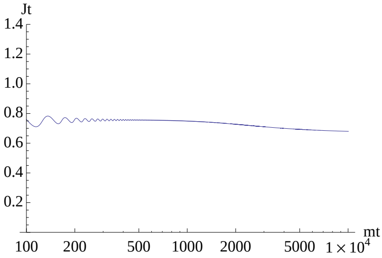
The Hubble parameter oscillates violently in this model.

Instead, $J \equiv H\left(1 - \frac{1}{2}\frac{\dot{\phi}^2}{M_P^2 M^2}\right)$ satisfies $\dot{J} \sim \mathcal{O}(HJ)$, and can be used to estimate the expansion rate. (cf. 1504.07119, 1505.04670.)

[YE, Jinno, Mukaida, Nakayama, '15]

Right panel: plot of J.

The model parameters and the initial conditions are the same as the figure of the Hubble parameter in the main text.



Details of the calculation

1. We use the ADM metric,

$$ds^2 = -N^2 dt^2 + \gamma_{ij}\left(dx^i + \beta^i dt\right)\left(dx^j + \beta^j dt\right),$$

   and take the gauge condition as

$$\phi = \phi(t),\ N = 1 + \alpha,\ \beta_i = \partial_i \psi,\ \gamma_{ij} = a^2 e^{2\zeta}\delta_{ij}$$

2. N and β^i do not have time derivatives in the action, and hence their EOMs give constraints:

$$\alpha = \frac{F}{H}\dot{\zeta},\ \psi = -\frac{F}{H}\zeta + \chi,\ \partial_i^2\chi = a^2\frac{M^2}{H^2}\frac{F^2 G}{1-\frac{1}{2}\epsilon}\dot{\zeta}.$$

3. Substitute them into the original action, and expand to 2nd order in the perturbations.

Calculation in another gauge condition

We also calculate the action for the scalar perturbation in the following gauge:

$$N = 1 + \alpha,\ \beta_i = \partial_i \psi,\ \gamma_{ij} = a(t)^2 \delta_{ij},\ \phi = \bar{\phi}(t) + \delta\phi.$$

The solution of the constraint equations is

(the explicit form ψ is irrelevant for the quadratic action)

$$\alpha = \frac{\dot{\bar{\phi}}}{2HM_P^2\left(1 - \frac{3}{2}\epsilon\right)}\left[\left(1 + \frac{3H^2}{M^2}\right)\delta\phi - \frac{2H}{M^2}\delta\dot{\phi}\right].$$

Then, the kinetic term for $\delta\phi$ is

$$S_{\mathrm{kin}} = M_P^2 \int d^4x\, a^3 \frac{F^2 G}{\dot{\bar{\phi}}^2}\left[\delta\dot{\phi}^2 - \frac{c_s^2}{a^2}\left(\partial_i \delta\phi\right)^2\right].$$

and hence the sound speed squared is the same as before.

Inflation with non-minimal derivative coupling

During inflationary regime, $G^{00} \simeq 3H^2$ and $G^{ij} \simeq -3H^2/a^2$, and hence for $V = \frac{\lambda}{n}\phi^n$,

$$\mathcal{L} \simeq \frac{3H^2}{2M^2}\left(\partial\phi\right)^2 - V \simeq -\frac{\lambda\phi^n}{2nM_P^2M^2}\left(\partial\phi\right)^2 - V$$
$$\simeq -\frac{1}{2}\left(\partial\tilde{\phi}\right)^2 - V(\tilde{\phi}),$$

where $\tilde{\phi} \equiv \frac{\sqrt{\lambda}}{\sqrt{n}M_P M}\phi^{(n+2)/2}$, $V(\tilde{\phi}) = \left(\frac{\lambda}{n}\right)^{2/(n+2)}\left(M_P M\tilde{\phi}\right)^{2/(n+2)}$.

Thus, the spectral index and the tensor-to-scalar ratio are given by

$$n_s = 1 - \frac{2(n+1)}{(n+2)N}, \; r = \frac{8n}{(n+2)N},$$

which are inside the 2σ constraint region for $n \leq 4$. [Planck collaboration, '15]
From the CMB normalization, ($\sqrt{\lambda} = m$ for $n = 2$)

$$\frac{mM}{M_P^2} \simeq 2\times 10^{-10}\left(\frac{50}{N}\right)^{3/2} \; (n=2), \; \frac{\lambda M^4}{M_P^4} \simeq 10^{-31}\left(\frac{50}{N}\right)^5 \; (n=4).$$