New approach to cosmological perturbation theory from a distribution function

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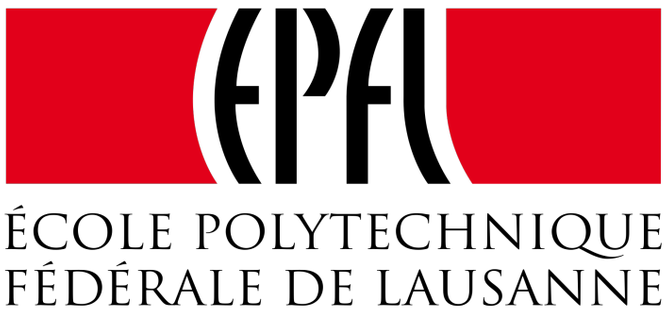
PhD at EPFL Lausanne



Supervisor: Sergey Sibiryakov (EPFL & CERN)
w/ D. Blas and M. Garny (CERN)

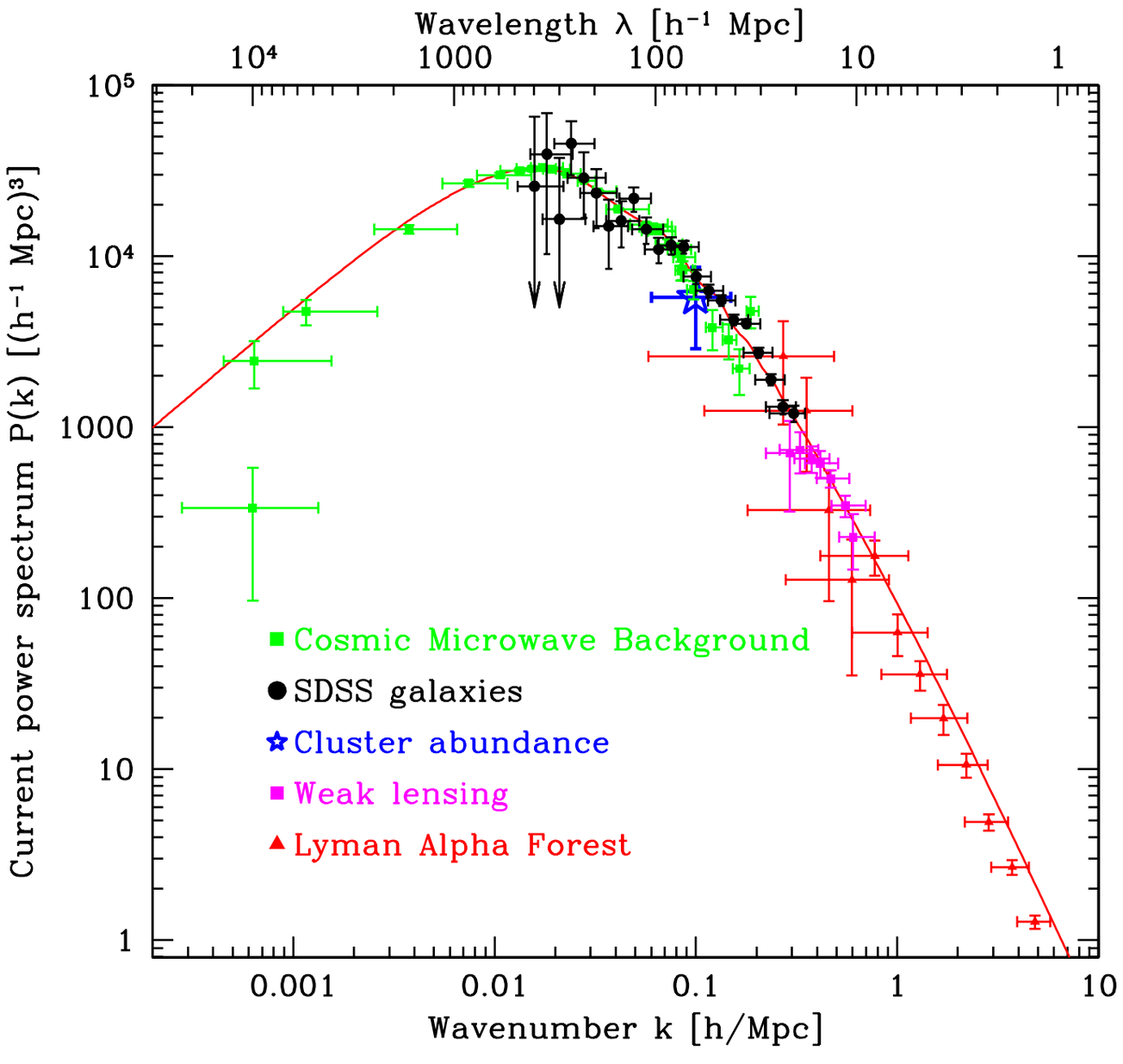
[arXiv: 1509.xxxxx]

COSMO-15, Warsaw, 7 - 11 Sept 2015

Large scale structure

Power spectrum of density contrast $\delta(\mathbf{x}, z) = \rho(\mathbf{x}, z)/\bar{\rho}(z) - 1$

$$\langle \delta(\mathbf{k}, z)\delta(\mathbf{k}', z)\rangle = \delta^{(3)}(\mathbf{k} + \mathbf{k}')P(k, z)$$



New checks of ΛCDM

Improve constraints on non-minimal extensions

Extract information on late-time effects (dark energy, neutrino masses)

SDSS 2003 (Tegmark et al) 0310725

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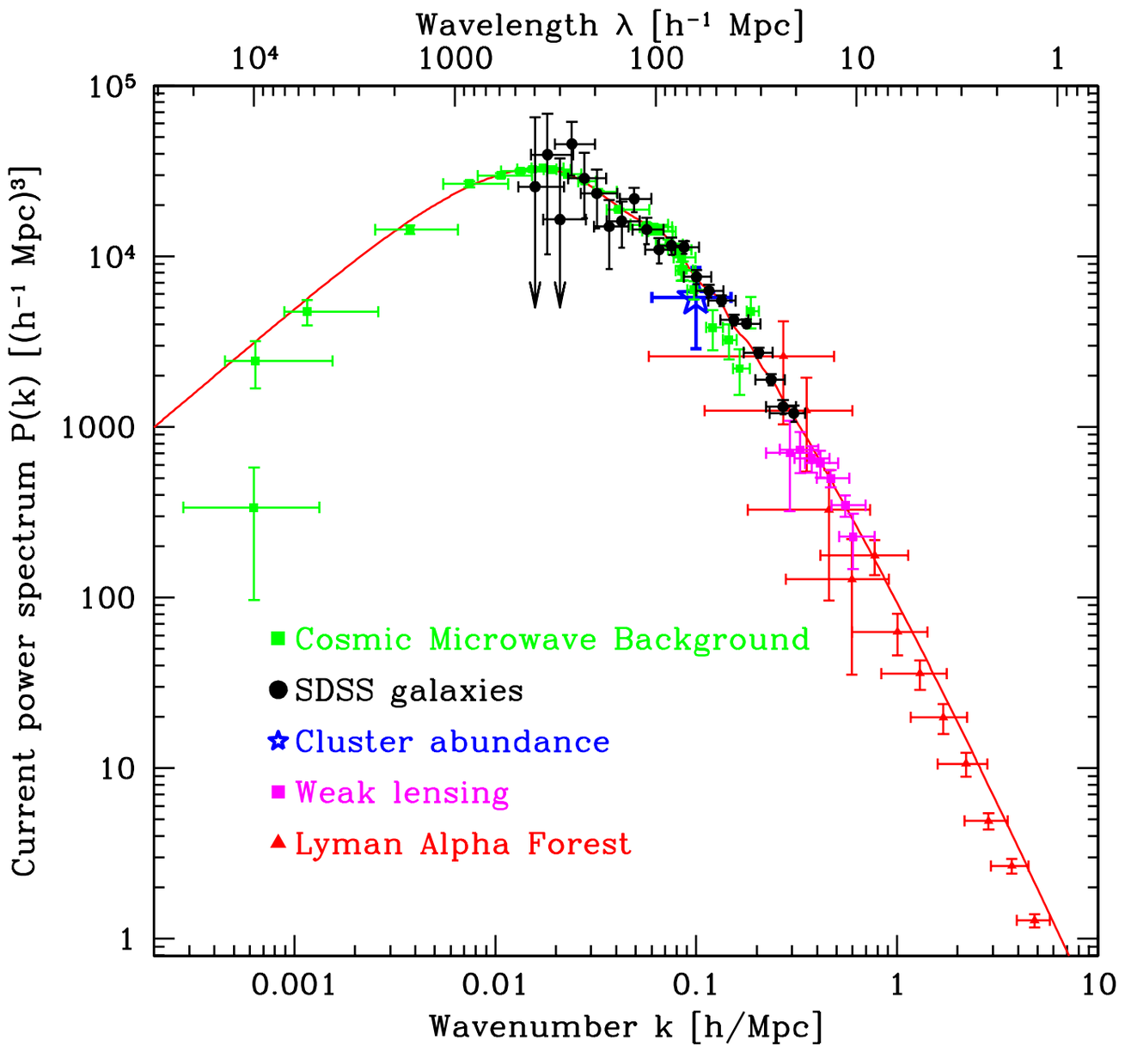

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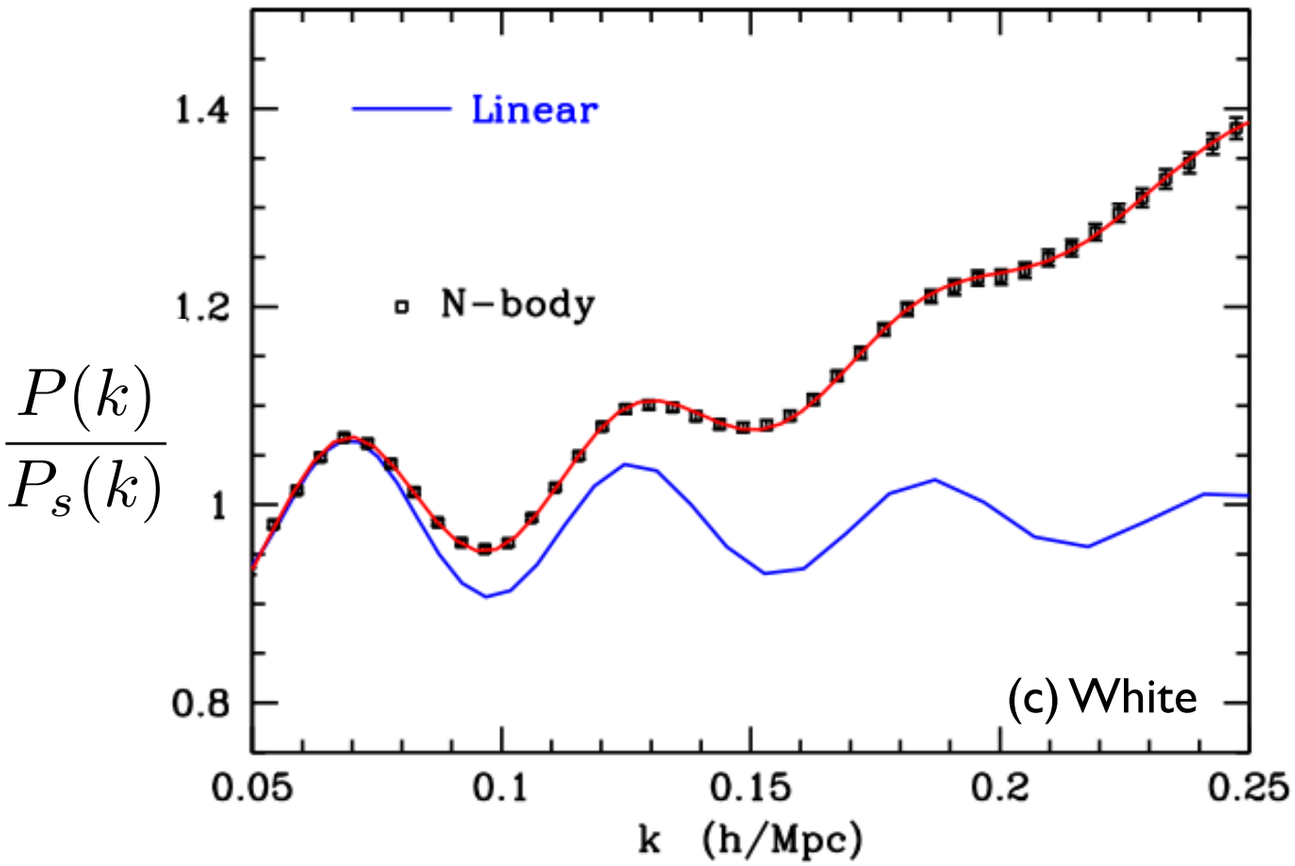
Elephant in the room: non-linearities

Non-linearities come into play



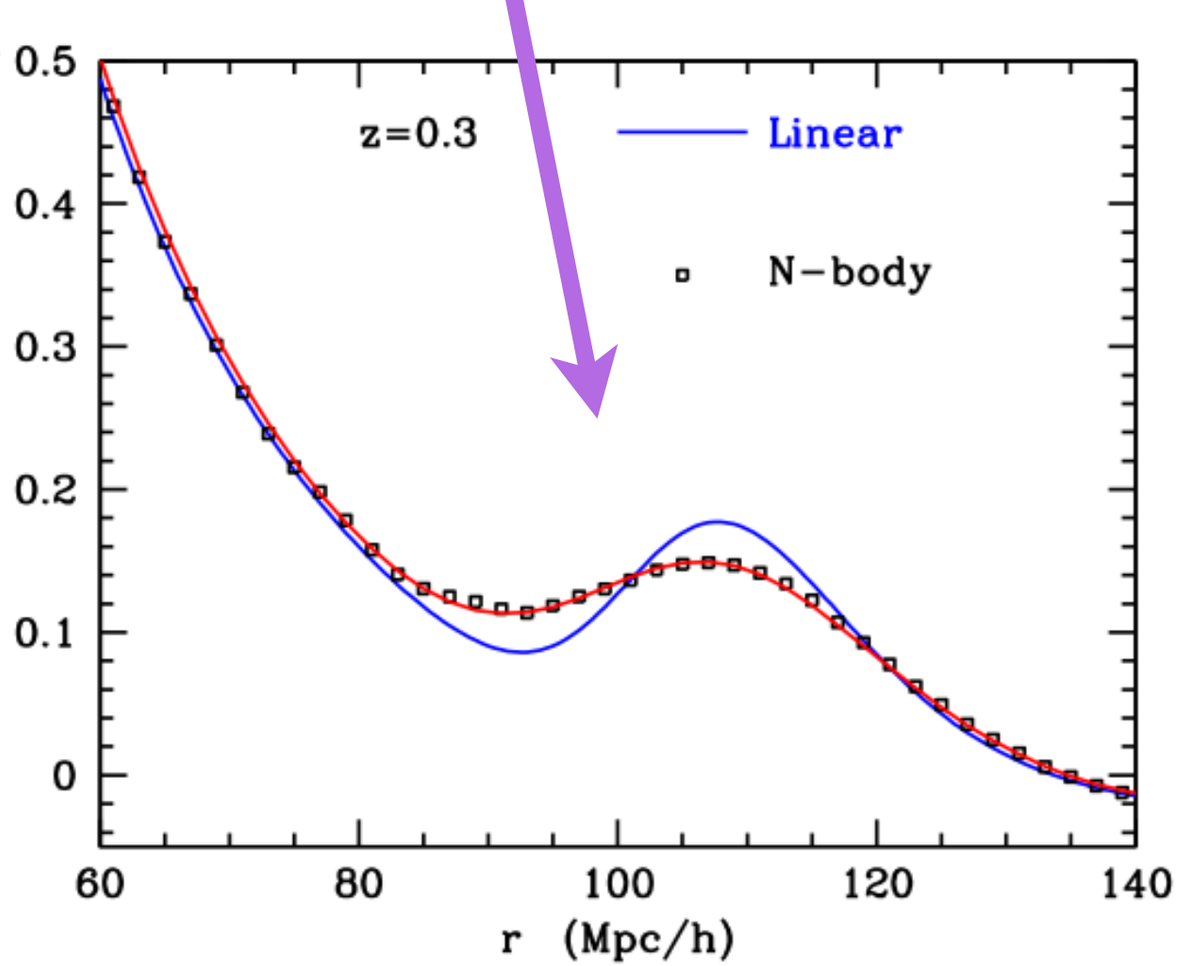
$$P(k) = P_s(k) + P_w(k)$$

BAO bump smeared

(length of the sound waves at recombination)

Galaxies correlation function

$$\xi(r) = \int d^3k e^{i\vec{k}\cdot\vec{r}} P(k)$$



How to get around?

Run N-body

1) Time consuming,
2) Expensive,
3) Hard to adopt for modified gravity

Develop analytical tools
for being able to
test theoretical models
against data quickly

Large scale structure: basics

Two cornerstones:

I) Initial probability distribution

$$\langle \delta(\vec{k})\delta(\vec{k}')\rangle|_{\tau=0} = \delta(\vec{k}+\vec{k}')P_0(k)$$

δ is a random gaussian-distributed variable

II) Time evolution

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot (1+\delta)\vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial \tau} + H\vec{v} + \vec{v}\cdot\nabla\vec{v} = -\nabla\Phi$$

$\delta = \rho/\bar{\rho} - 1$ density contrast

$\vec{v}$ fluid velocity

H Hubble parameter

Φ grav. potential

Poisson equation:

$$\nabla^2\Phi = 4\pi G a^2 \bar{\rho}\delta$$

The case study: Zel'dovich approximation

Zel'dovich'70

Hydrodynamical equations:

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot (1 + \delta)\vec{v} = 0$$

$$\frac{\partial \vec{v}}{\partial \tau} + H\vec{v} + \vec{v} \cdot \nabla \vec{v} = -\nabla \Phi$$

Linear solution:

$$\delta_L = -\frac{\nabla \vec{v}}{H}$$

Poisson equation:

$$\nabla^2 \Phi = 4\pi G a^2 \bar{\rho} \delta$$

$$\nabla \Phi = -\frac{4\pi G a^2 \bar{\rho}}{H} \vec{v} = -\frac{3}{2} H \vec{v}$$

$$\frac{\partial \vec{v}}{\partial \tau} - \frac{1}{2} H \vec{v} + \vec{v} \cdot \nabla \vec{v} = 0$$

A bit of reshuffling

$$\frac{\partial \vec{v}}{\partial \tau} - \frac{1}{2} H \vec{v} + \vec{v} \cdot \nabla \vec{v} = 0$$

New time $\quad t \equiv \log a(\tau)$

New variable: velocity div. $\quad \psi \equiv -\frac{\nabla \vec{v}}{aH}$

$$\frac{\partial \psi(\vec{k},t)}{\partial t} - \psi(\vec{k},t) = \int d^3k_1 d^3k_2 \delta_D^{(3)}(\vec{k} - \vec{k}_1 - \vec{k}_2) \frac{(\vec{k}_1 \cdot \vec{k}_2)(\vec{k}_1 + \vec{k}_2)^2}{2k_1^2 k_2^2} \psi(\vec{k}_1,t)\psi(\vec{k}_2,t)$$

$$\beta(\vec{k}_1,\vec{k}_2) \equiv \frac{(\vec{k}_1 \cdot \vec{k}_2)(\vec{k}_1 + \vec{k}_2)^2}{2k_1^2 k_2^2}$$

$$\frac{\partial \psi}{\partial t} - \psi = \int \beta \psi^2$$

Bernardeau, Scoccimarro, Crocce…

Ways to proceed:

Standard perturbation theory

I) Time evolution: solve for fields

$$\psi = \sum_n \psi^{(n)}$$

$$= \sum_n a(t)^n \int G_n \cdot (\psi_0)^n$$

II) Statistics: average over

$$P_{mn}^{loop} = \langle \psi^{(n)} \psi^{(m)} \rangle \sim \langle \psi_0^{n+m} \rangle$$

using initial distr. function for ψ_0

$$\langle \psi_0(k) \psi_0(-k) \rangle = P_0(k)$$

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Time - sliced perturbation theory

I) Statistics: introduce a distribution function

II) Time evolution: instead of time-evol. for fields consider time-evol. for a distribution function

Fokker - Planck -like equation

Time - sliced perturbation theory (TSPT)

Initial distribution - Gaussian Euclidian partition function

$$Z_0[J] = \mathcal{N}^{-1} \int \mathcal{D}\psi_0 \exp\left\{ \int \left[-\frac{\Gamma_2^{(0)}}{2}\psi_0^2 + J\psi_0 \right] \right\}$$

$$\langle \psi_0(k)\psi_0(-k) \rangle = \frac{\delta^2 Z_0[J]}{\delta J \delta J} = \frac{1}{\Gamma_2^{(0)}} = P_0(k)$$

Time - sliced perturbation theory (TSPT)

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Finite - time distribution

$$Z[J] = \mathcal{N}^{-1} \int \mathcal{D}\psi_0 \ \exp\left\{ \int \left[-\frac{\Gamma_2^0}{2}\psi_0^2 + J\psi_t \right] \right\}$$

Change variable: $\psi_0 \to \psi_t(\psi_0)$

$$Z[J] = \mathcal{N}^{-1} \int \mathcal{D}\psi_t \ \exp\left\{ \ln\det\frac{\partial\psi_0}{\partial\psi_t} + \int \left[-\frac{\Gamma_2^0}{2}\psi_0^2 + J\psi_t \right] \right\}$$

Time - sliced perturbation theory (TSPT)

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Cancelled

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$$\equiv -\sum_{n=2}^{\infty} \int \frac{\Gamma_n(t)}{n!}\psi_t^n$$

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$$\frac{d}{dt}\int \sum_{n=2}^{\infty} \frac{\Gamma_n(t)}{n!}\psi_t^n = \int \sum_{n=2}^{\infty} \frac{1}{n!}\left(\frac{\partial\Gamma_n}{\partial t}\psi_t^n + n\Gamma_n\psi_t^{n-1}\frac{\partial\psi_t}{\partial t} \right)$$

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$$\equiv \frac{d}{dt}\int \frac{\Gamma_2^0}{2}\psi_0^2 = 0$$

$$\frac{\partial\psi_t}{\partial t} = \psi_t + \int \beta\psi_t^2$$

$$\frac{\partial\Gamma_n}{\partial t} + n\Gamma_n + \beta\Gamma_{n-1} = 0$$

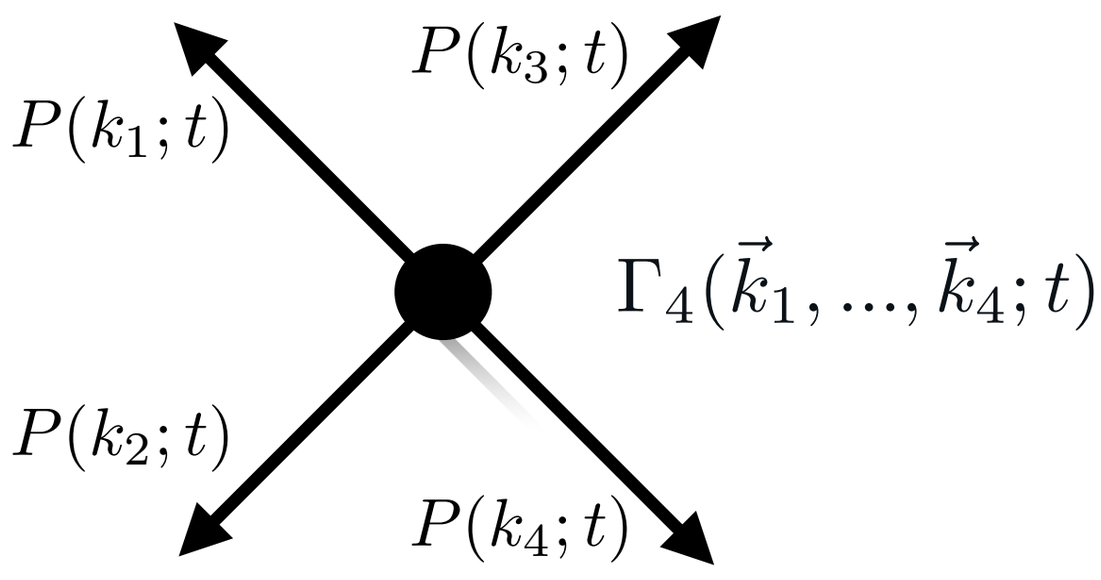
Fokker - Planck eq.

Time - sliced perturbation theory (TSPT)

Physical meaning

$$\frac{\partial \Gamma_n}{\partial t} + n\Gamma_n + \beta\Gamma_{n-1} = 0$$

$$\langle \psi(\vec{k}_1;t)...\psi(\vec{k}_n;t)\rangle^{tree,1PI} = \Gamma_n(\vec{k}_1,...,\vec{k}_n;t)\prod_{i=1}^{n} P_L(k_i;t)$$



$1PI$ - Trispectrum

IR - safety theorem

$$\Gamma_{n+1}(\vec{k}_1,...,\vec{k}_{n+1}) = 2\delta^{(3)}\left(\sum_{i=1}^{n+1}\vec{k}_i\right)\sum_{i\neq j=1}^{n+1}\beta(\vec{k}_i,\vec{k}_j)\Gamma_n(\vec{k}_1,...,\vec{k}_i+\vec{k}_j,...\vec{k}_{n+1})$$

Take one momentum soft $\quad k_{n+1} = q \ll k_i$

$$\Gamma_{n+1}(\vec{k}_1,...,\vec{k}_n,\vec{q})|_{q\to 0} \simeq \delta^{(3)}\left(\sum_{i=1}^{n}\vec{k}_i\right)\sum_{i=1}^{n}\frac{(\vec{k}_i\cdot\vec{q})}{q^2}\Gamma_n(\vec{k}_1,...,\vec{k}_n) + \text{IR OK}$$

(cf. Consistency conditions - *Criminelli et al'13*
Valages'13, Kehagias et al.'13 also *L. Sagunski's* talk)

NB. Consequence of the Galilean invariance, according to which all the equal time correlators are IR - safe

Frieman, Scoccimarro 95; Pietroni, Peloso 13

Time - sliced perturbation theory (TSPT)

1) Compute all the statistical weights

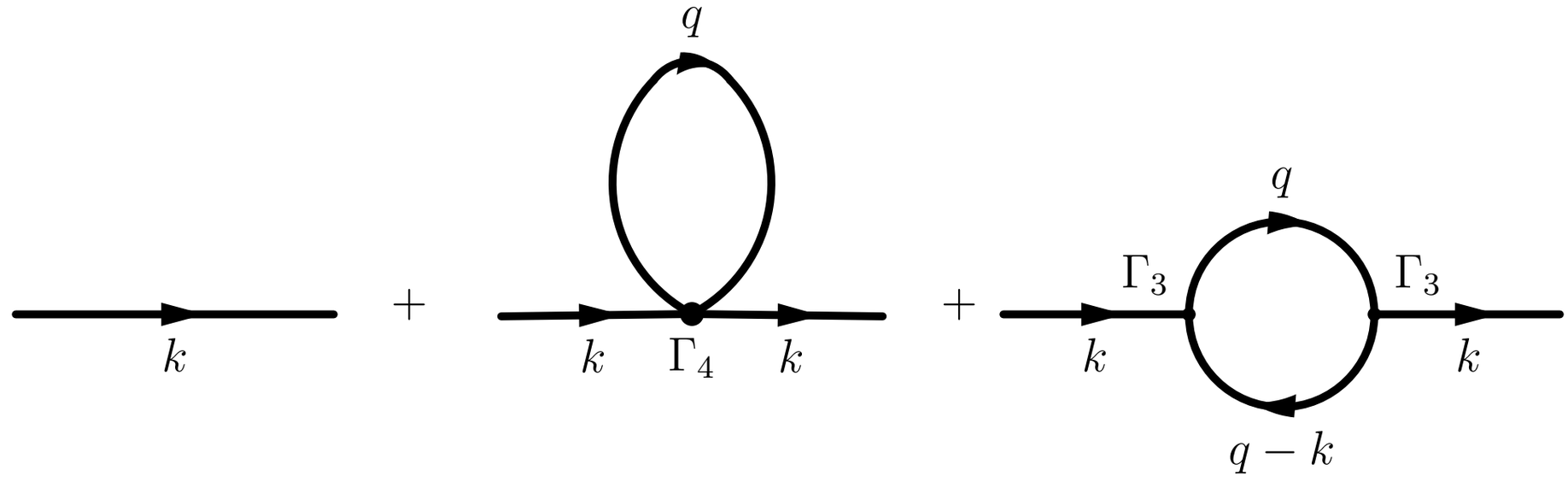
$$\frac{\partial \Gamma_n}{\partial t} + n\Gamma_n + \beta\Gamma_{n-1} = 0$$

2) Insert them into the partition function

$$Z[J] = \mathcal{N}^{-1} \int \mathcal{D}\psi \left(1 - \frac{1}{3!}\int \Gamma_3 \psi^3 - \frac{1}{4!}\int \Gamma_4 \psi^4 + \frac{1}{2}\left(\frac{1}{3!}\int \Gamma_3 \psi^3\right)^2 + ...\right) e^{-\frac{\Gamma_2 \psi^2}{2} + J\psi}$$

3) Compute correlation functions like in QFT

$P(k;t) + P^{1-loop}(k;t) =$



BAO IR - resummation

1) BAO smearing comes from interaction with long-wavelength perturbations
(= Bulk motion)

2) Because of the spurious IR - enhancements, studying Bulk motion
is complicated in SPT

3) In TSPT one can compute the leading behaviour relevant for BAO - Γ_n

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'BAO consistency condition' (cf. *Zaldarriaga et al'15*)

$$\Gamma_n(\vec{k},-\vec{k},\vec{q}_1,...,\vec{q}_{n-2}) = \prod_{j=1}^{n-2} \frac{(\vec{k}\cdot\vec{q}_j)}{q_j^2}(1-e^{-q_j\cdot\nabla_k})\frac{1}{P(k)}$$

$$\frac{q_j}{k} \to 0$$

$$\approx \prod_{j=1}^{n-2} \frac{(\vec{k}\cdot q_j)}{q_j^2}(q_j\cdot\nabla_k)\frac{1}{P(k)} \sim (k\cdot\nabla_k)^{n-2}\frac{1}{P(k)}$$

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$$P(k) = P_s(k) + P_w(k) \longrightarrow k\cdot\nabla_k P_s(k) \sim P_s(k)$$

$$k\cdot\nabla_k P_w(k) \sim (kr_{BAO})P_w(k) \gg P_w(k)$$

$$r_{BAO} \sim 150Mpc \quad for \quad k \sim 0.1Mpc^{-1}$$

$$k\cdot r_{BAO} \sim 15$$

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Must be resummed!

BAO IR - resummation

$$\Gamma_n^w(\vec{k}, -\vec{k}, \vec{q}_1, ..., \vec{q}_{n-2}) = \prod_{j=1}^{n-2} \frac{(\vec{k} \cdot \vec{q}_j)}{q_j^2} (1 - e^{-iq_j r_s \cdot \cos\theta}) \frac{P_w(k)}{(P_s(k))^2}$$

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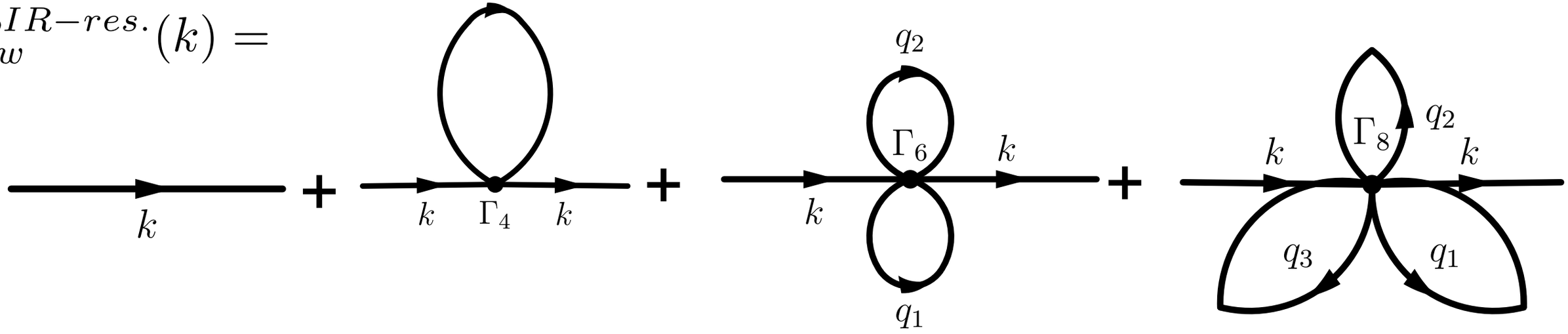
Daisy graphs

$$P_w^{IR-res.}(k) =$$



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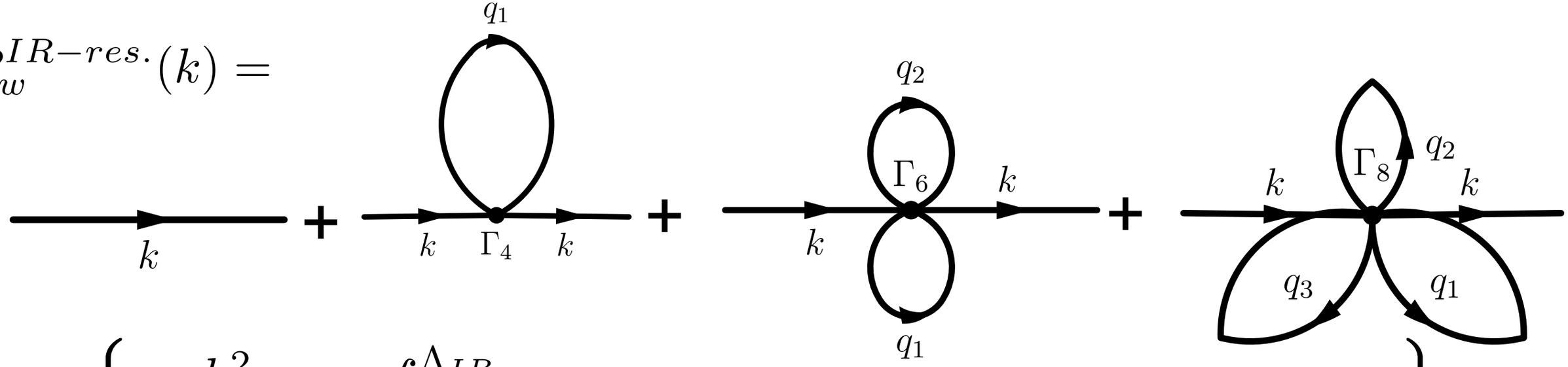


$$= \exp\left\{ -\frac{k^2}{3} \cdot 4\pi \int_0^{\Lambda_{IR}} dq P_s^L(q)(1 - j_0(qr_{BAO}) + 2j_2(qr_{BAO})) \right\} P_w(k)$$

less power in CF

BAO IR - resummation

$$P_w^{IR-res.}(k) = \exp\left\{ -\frac{k^2}{3}\cdot 4\pi \int_0^{\Lambda_{IR}} dq P_s^L(q)(1-j_0(qr_{BAO})+2j_2(qr_{BAO}))\right\} P_w(k)$$

$$\equiv e^{-\Sigma^2(\Lambda_{IR})k^2} P_w(k)$$

The same resumm (daisy graphs) goes for all the c.f. , e.g.

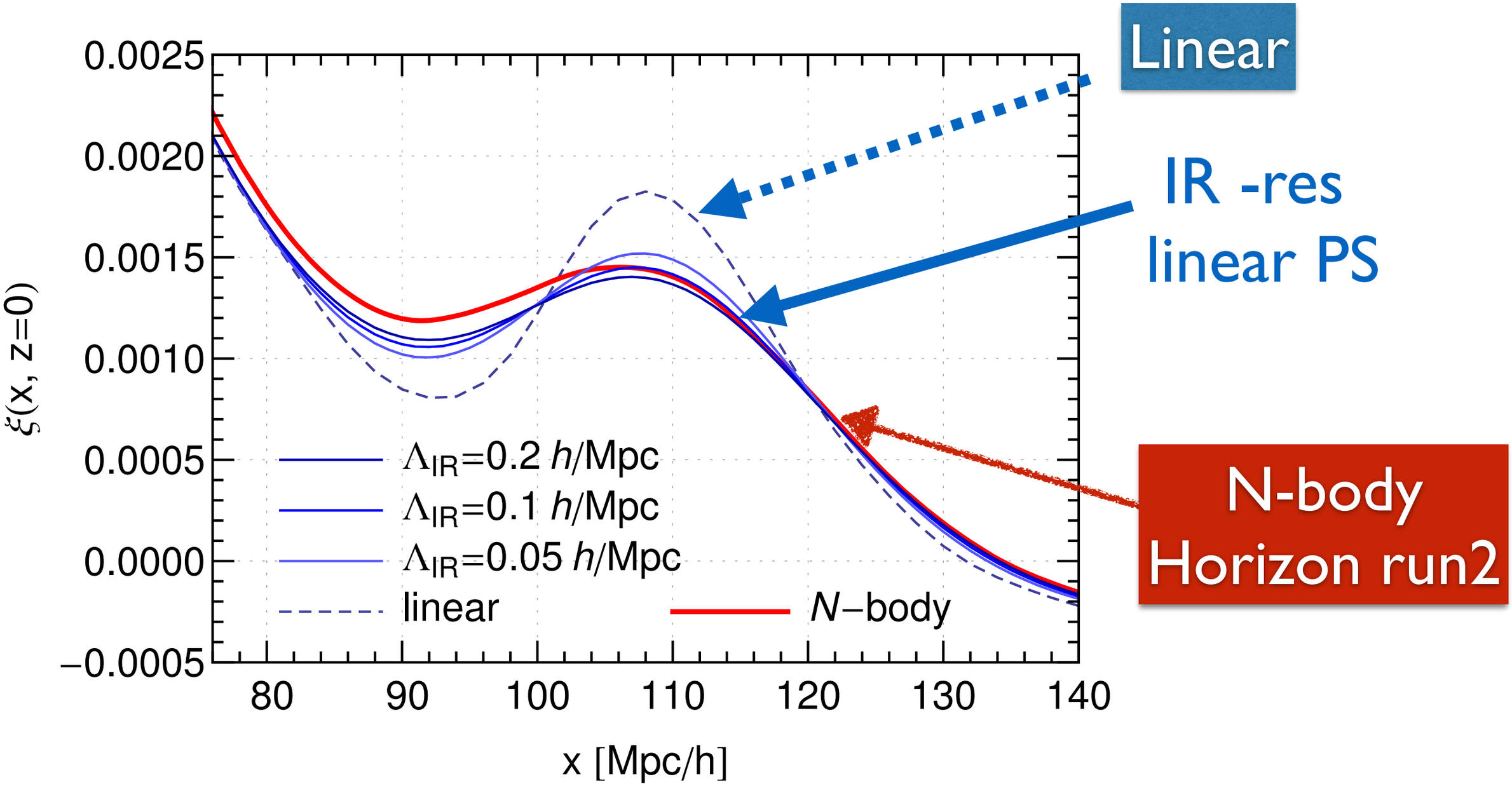
$$B(k_1,k_2,k_3)^{IR-res.} = 2F_2(k_1,k_2)\left(e^{-\Sigma^2(\Lambda_{IR})k_1^2} P_w(k_1)P_s(k_2) + e^{-\Sigma^2(\Lambda_{IR})k_2^2} P_w(k_2)P_s(k_1)\right)$$
$$+ (1\to 2\to 3)$$

Theorem. *IR - resummation of an arbitrary 'wiggly' n-point correlation function results in dressing of all the 'wiggly' power spectrum with the exp's:*

$$P_w(k) \to \ e^{-\Sigma^2(\Lambda_{IR})k^2} P_w(k)$$

NB. accuracy $\mathcal{O}(\Lambda_{IR}/k)$

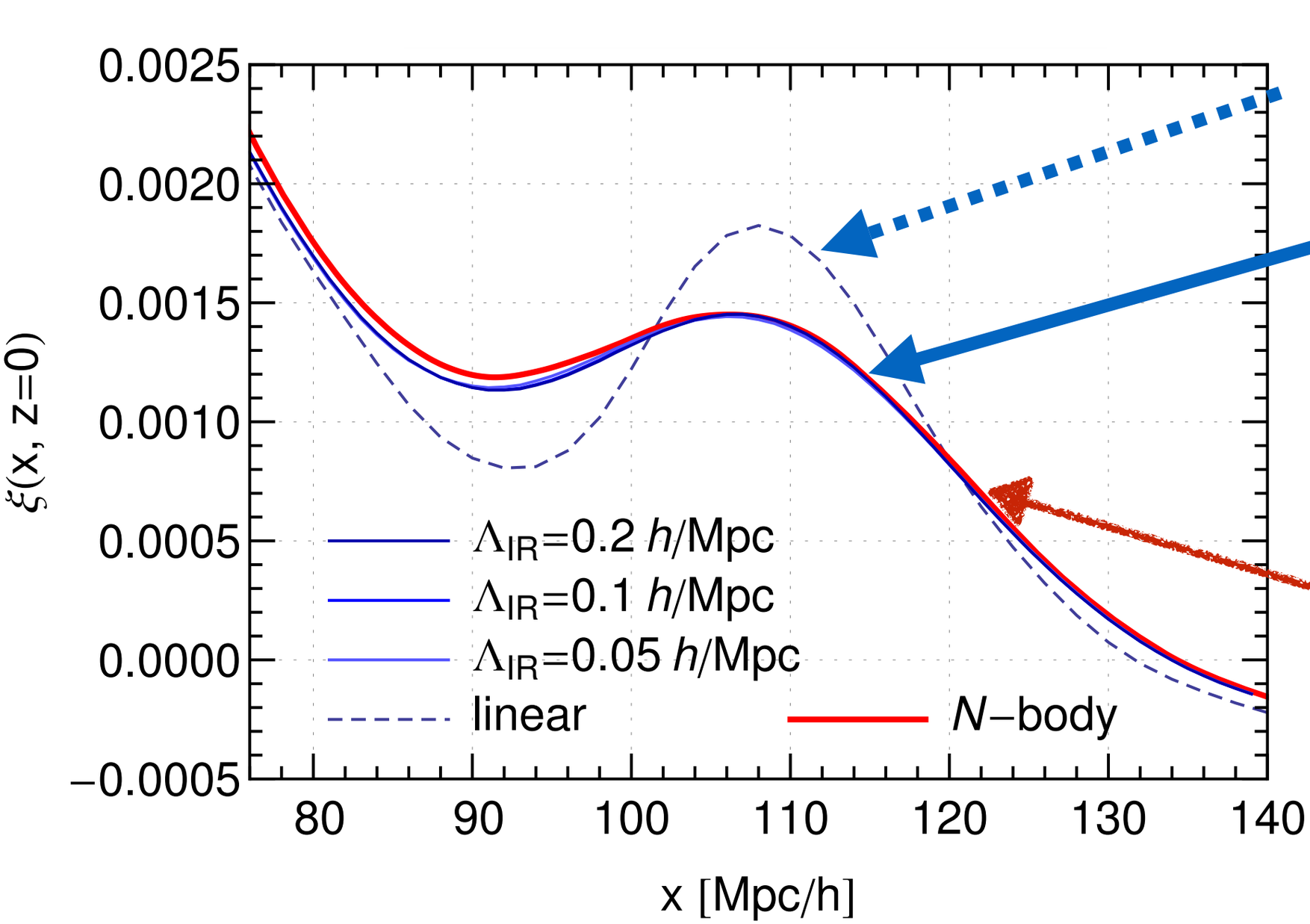
Galaxy correlation function in TSPT



Tree-level IR - resummed

$$P_w^{tree,res.}(k) = e^{-\Sigma^2 k^2} P_w(k)$$

Galaxy correlation function in TSPT



Linear

IR -res
1-loop PS

N-body
Horizon run2

1-loop IR - resummed

$$P_w^{1loop,res.}(k) = e^{-\Sigma^2 k^2} P_w(k)(1+\Sigma^2 k^2) + e^{-\Sigma^2 k^2} P_w^{1loop}(k)$$

Conclusions:

TSPT - Fokker Planck approach to LSS

IR - safe (Galilean symm. - !)

Accounts for bulk motion to all orders in Eulerian Pert. Theory (IR - resumm of BAO)

Systematic IR - resumm. of all the cfs (Power spectrum, Bispectrum, Trispectrum etc.)

Resummation of UV - effects -? EFT of LSS - ?

Subleading IR - resumm: shift of BAO peak

Galaxy bias

Modified gravity …

Thank you for your attention !

BAO IR - resummation

$$\Gamma_n^w(\vec{k},-\vec{k},\vec{q}_1,...,\vec{q}_{n-2}) = \prod_{j=1}^{n-2} \frac{(\vec{k}\cdot\vec{q}_j)}{q_j^2}(1-e^{-iq_j r_s\cdot\cos\theta})\frac{P_w(k)}{(P_s(k))^2}$$

Daisy graphs

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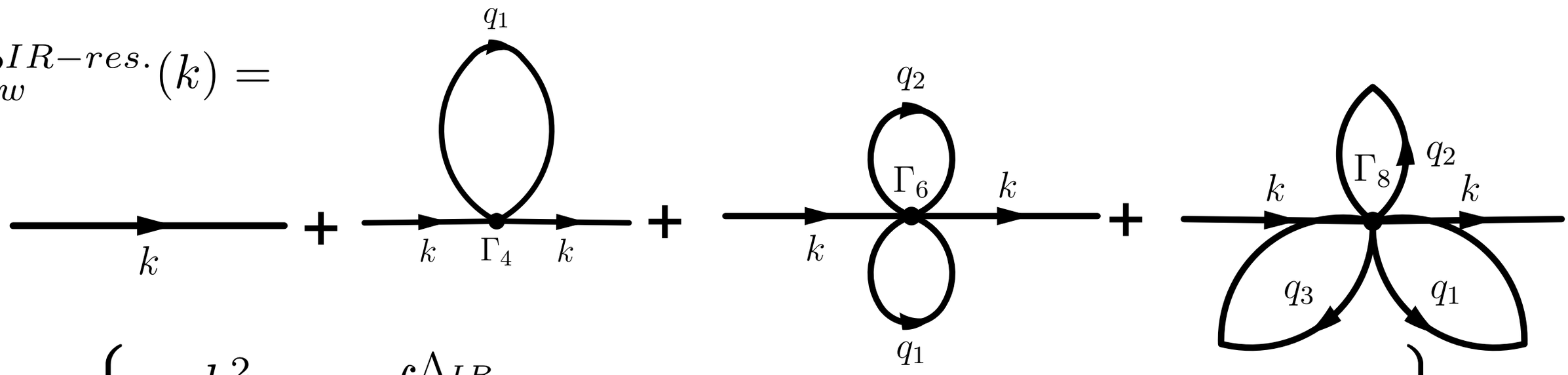

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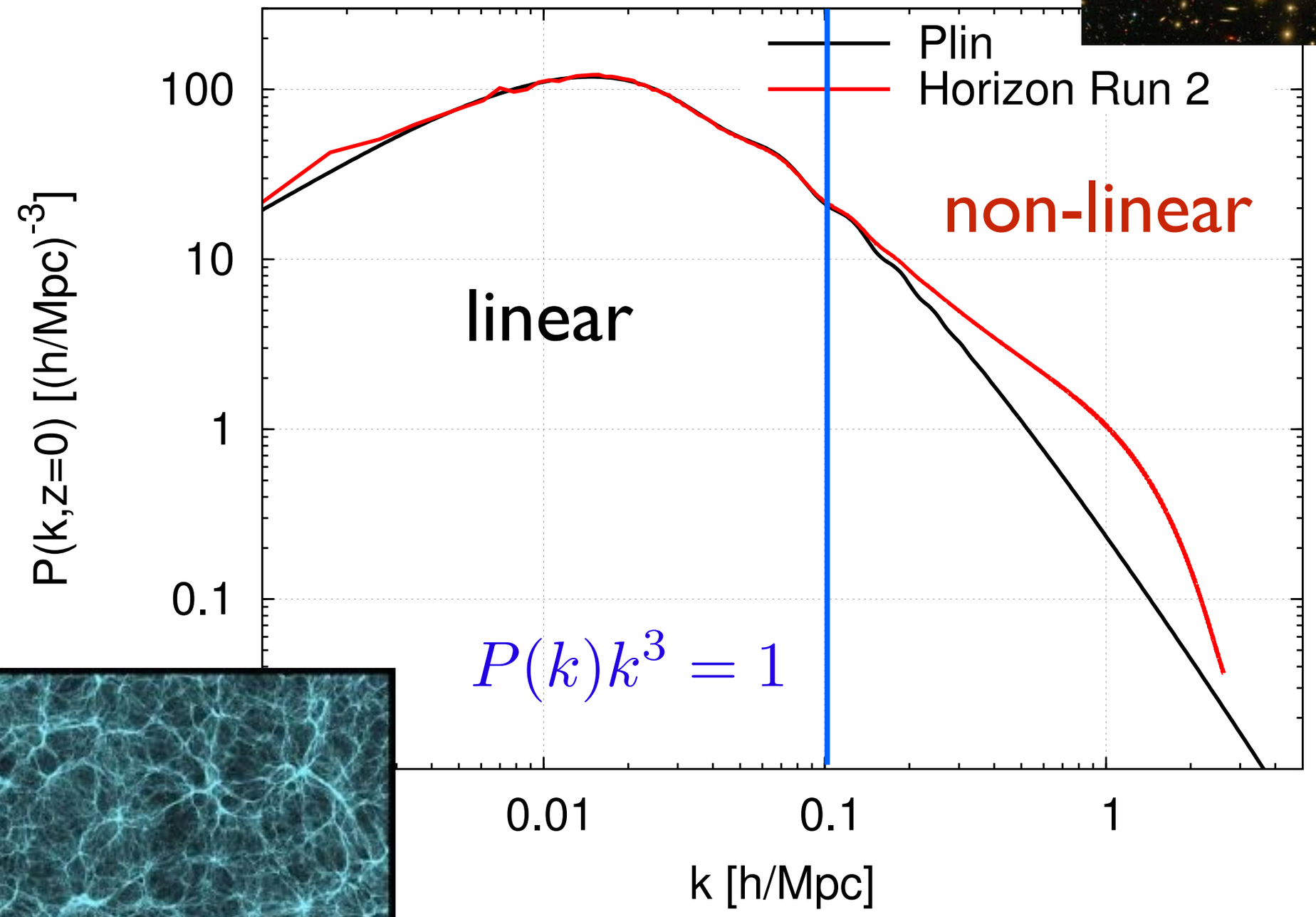
less power in CF

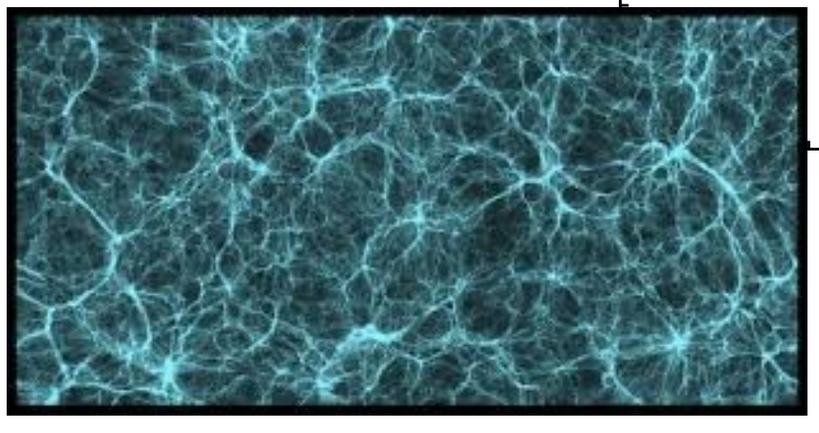
$$P_w^{RPT}(k) = \exp\left\{ -\frac{k^2}{3}\cdot 4\pi \int_0^{\Lambda_{IR}} dq P_s^L(q)\right\} P_w(k)$$

Crocce, Scoccimarro'06

Non-linearities come into play

$\delta \sim 1$ A lot of data at mildly nonlinear scales!

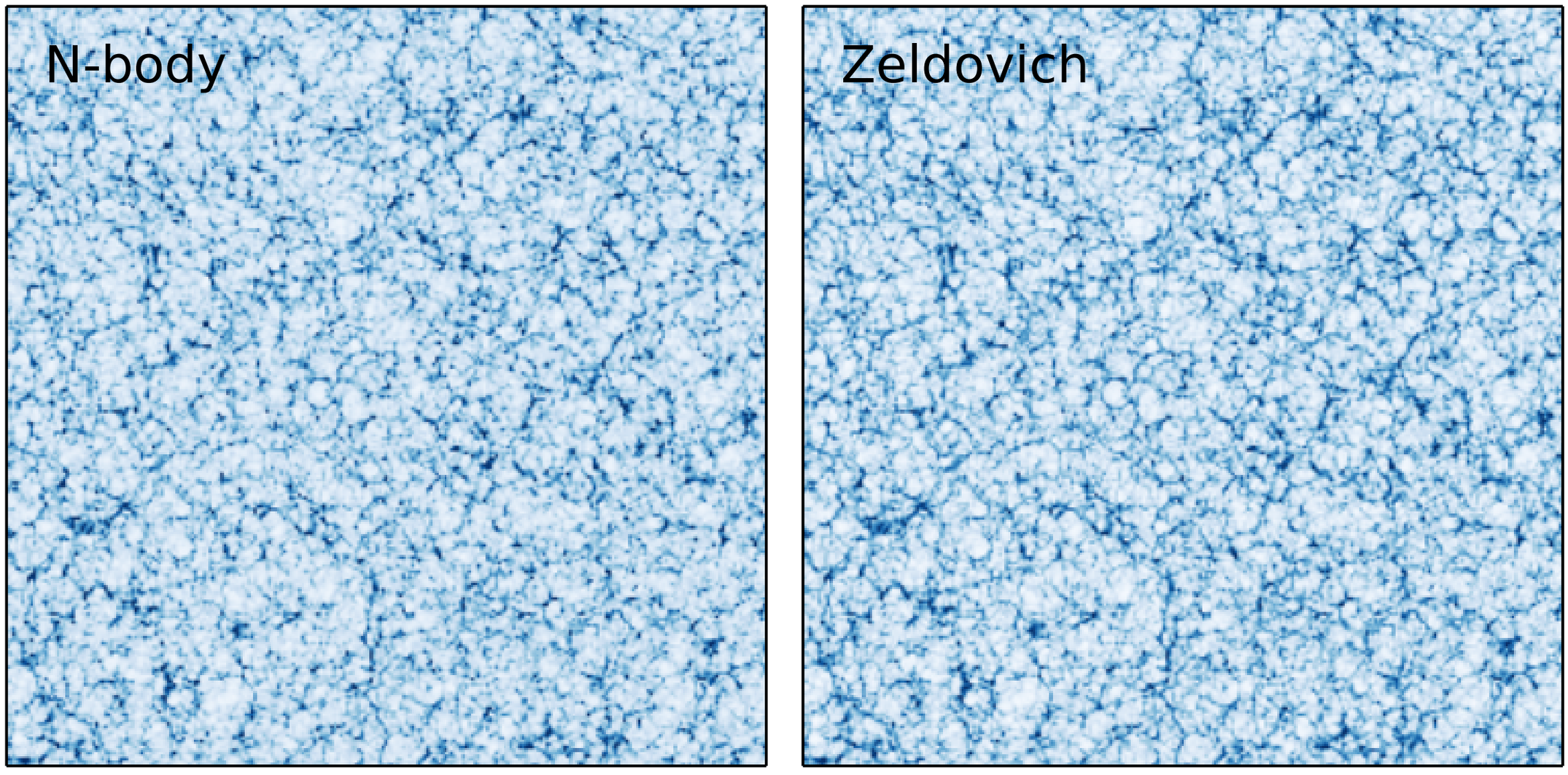




Zel'dovich approximation

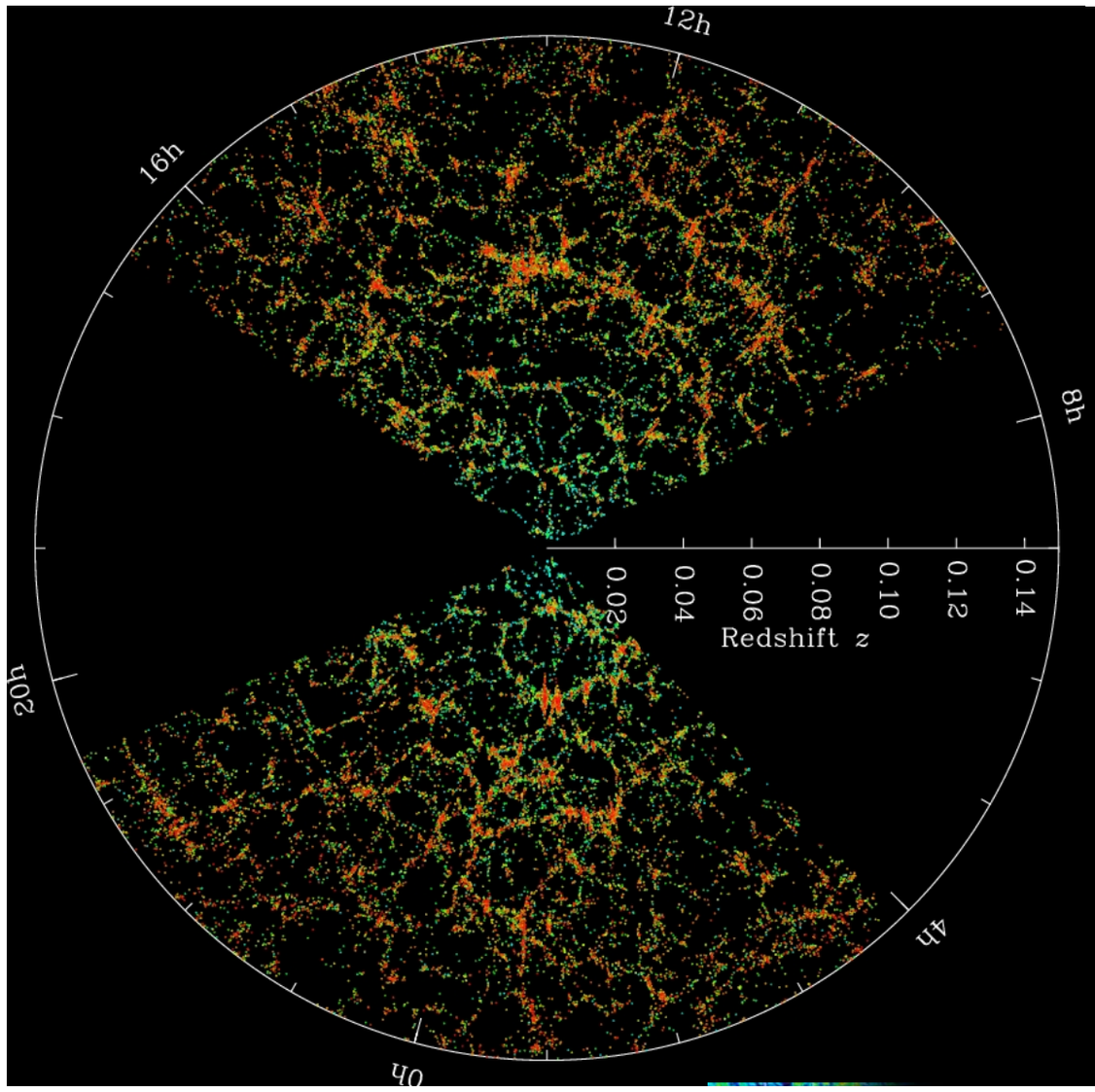
$$\frac{\partial \vec{v}}{\partial \tau} - \frac{1}{2} H \vec{v} + \vec{v} \cdot \nabla \vec{v} = 0$$

N.B. ZA is accurate up to shell-crossing (Zel'dovich pancakes)

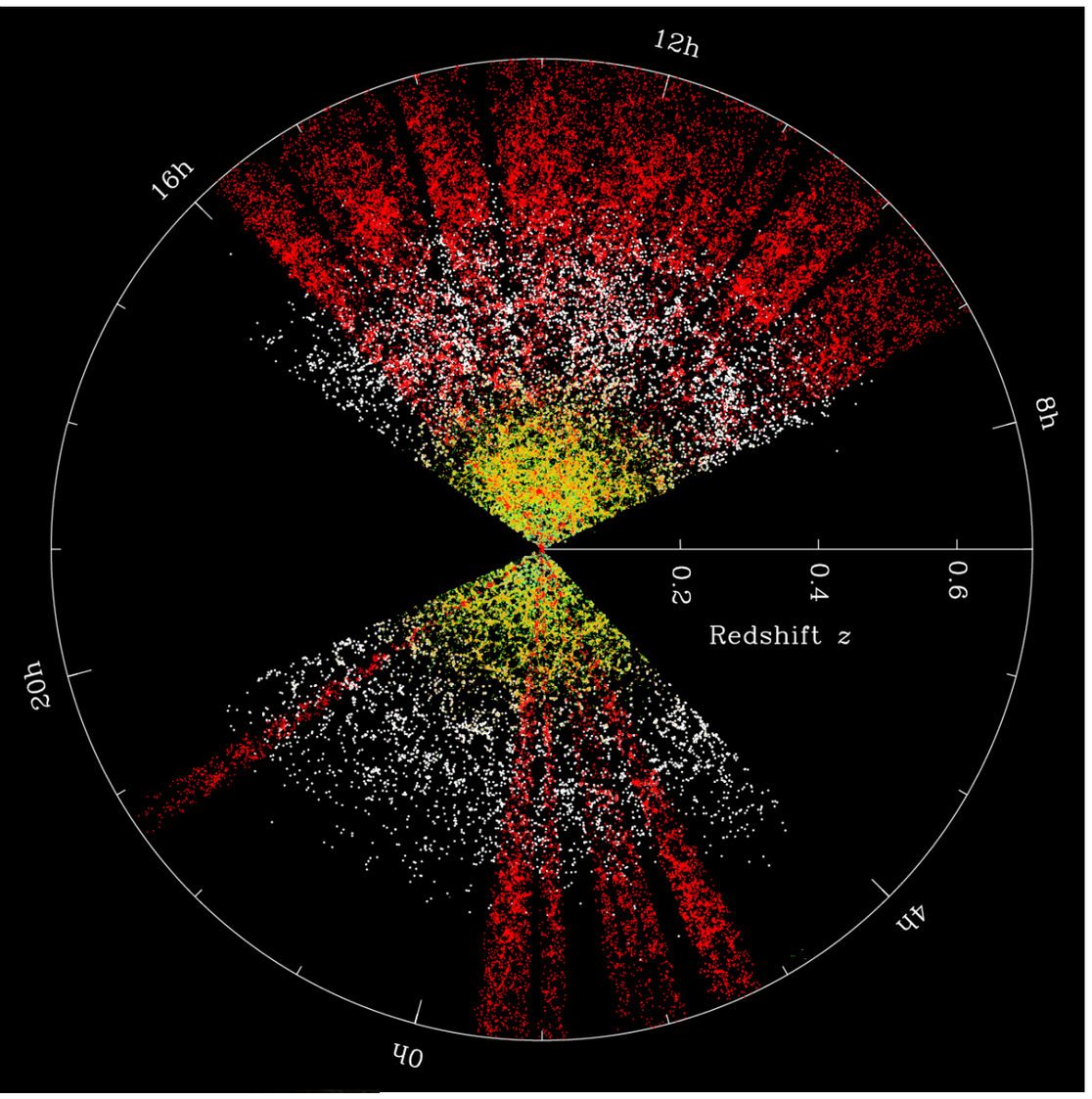


(c) M.White

Large scale structure



SDSS-III (sdss3.org)



BOSS (red), SDSS (white/yellow)

LSST (constr. started 2014)
DESI (starting in 2018)
Euclid (launch in 2020)

Plenty of data coming!