Anisotropic Correlations in Fourier Phases

(MNRAS 453:797-809)

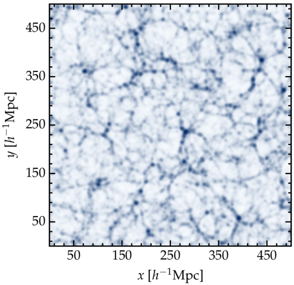
Alexander Eggemeier

University of Sussex

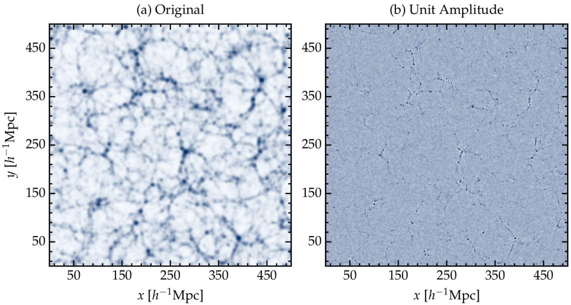
Collaborators:
T. Battefeld, R.E. Smith & J. Niemeyer

Why Phases Are Interesting!

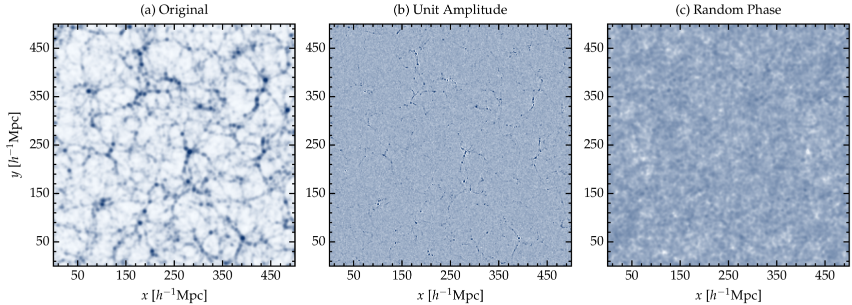
(a) Original



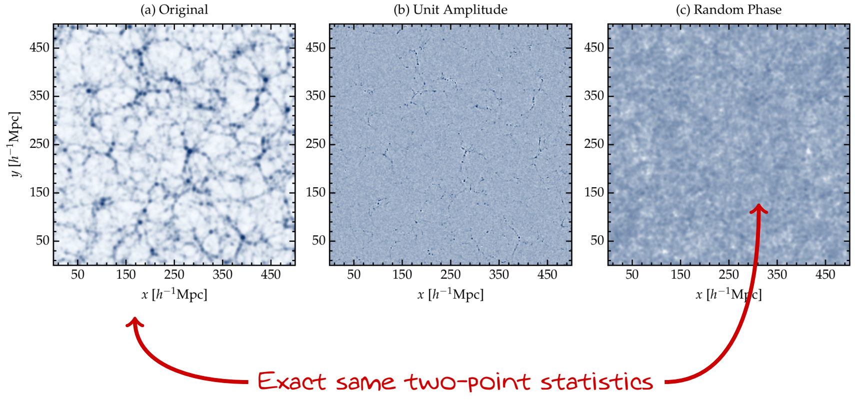
Why Phases Are Interesting!



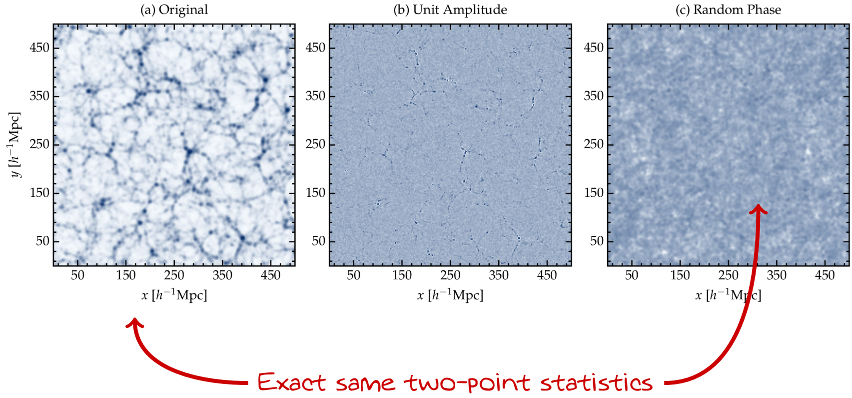
Why Phases Are Interesting!



Why Phases Are Interesting!



Why Phases Are Interesting!



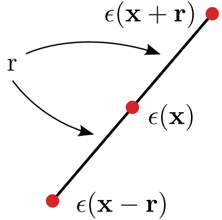
→ Loss of information by restriction to two-point function or power spectrum analyses

A Measure of Phase Information

Rough Definition of the Line Correlation Function

Line correlation function $\sim \langle \epsilon(\boldsymbol{x})\, \epsilon(\boldsymbol{x}+\boldsymbol{r})\, \epsilon(\boldsymbol{x}-\boldsymbol{r}) \rangle$

Obreschkow et al. 2013



$$\epsilon_{\boldsymbol{k}} \equiv \frac{\delta_{\boldsymbol{k}}}{|\delta_{\boldsymbol{k}}|}$$

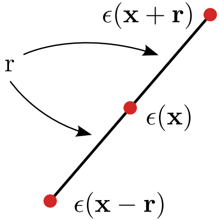
A Measure of Phase Information

Rough Definition of the Line Correlation Function

Line correlation function $\sim \langle \epsilon(\boldsymbol{x})\, \epsilon(\boldsymbol{x}+\boldsymbol{r})\, \epsilon(\boldsymbol{x}-\boldsymbol{r}) \rangle$

Obreschkow et al. 2013

$$\hat{\ell}(r) \backsimeq \iint \mathrm{d}^3k\, \mathrm{d}^3q\, j_0(|\boldsymbol{k}-\boldsymbol{q}|\, r)\, \epsilon_{\boldsymbol{k}}\epsilon_{\boldsymbol{q}}\epsilon_{-\boldsymbol{k}-\boldsymbol{q}}$$



$$\epsilon_{\boldsymbol{k}} \equiv \frac{\delta_{\boldsymbol{k}}}{|\delta_{\boldsymbol{k}}|}$$

A Measure of Phase Information

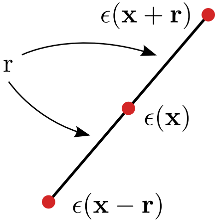
Rough Definition of the Line Correlation Function

Line correlation function $\sim \langle \epsilon(\boldsymbol{x})\, \epsilon(\boldsymbol{x}+\boldsymbol{r})\, \epsilon(\boldsymbol{x}-\boldsymbol{r}) \rangle$

Obreschkow et al. 2013

Scaling factor

$$\hat{\ell}(r) \backsimeq \left(\frac{r}{L}\right)^{9/2} \iint \mathrm{d}^3k\, \mathrm{d}^3q\, j_0(|\boldsymbol{k}-\boldsymbol{q}|\, r)\, \epsilon_{\boldsymbol{k}} \epsilon_{\boldsymbol{q}} \epsilon_{-\boldsymbol{k}-\boldsymbol{q}}$$



$$\epsilon_{\boldsymbol{k}} \equiv \frac{\delta_{\boldsymbol{k}}}{|\delta_{\boldsymbol{k}}|}$$

A Measure of Phase Information

Rough Definition of the Line Correlation Function

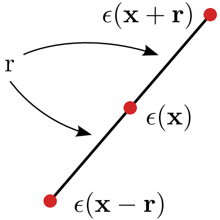
Line correlation function $\sim \langle \epsilon(\boldsymbol{x})\, \epsilon(\boldsymbol{x}+\boldsymbol{r})\, \epsilon(\boldsymbol{x}-\boldsymbol{r}) \rangle$

Scaling factor

Obreschkow et al. 2013

$$\hat{\ell}(r) \simeq \left(\frac{r}{L}\right)^{9/2} \iint \mathrm{d}^3k\, \mathrm{d}^3q\, j_0(|\boldsymbol{k}-\boldsymbol{q}|\, r)\, \epsilon_{\boldsymbol{k}}\epsilon_{\boldsymbol{q}}\epsilon_{-\boldsymbol{k}-\boldsymbol{q}} \quad \Theta(\boldsymbol{k})\Theta(\boldsymbol{q})\Theta(\boldsymbol{k}+\boldsymbol{q})$$

Mode cutoff

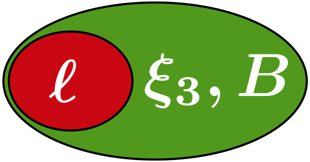


$$\epsilon_{\boldsymbol{k}} \equiv \frac{\delta_{\boldsymbol{k}}}{|\delta_{\boldsymbol{k}}|}$$

$$\Theta(\boldsymbol{k}) = \begin{cases} 1 & |\boldsymbol{k}| < 2\pi/r \\ 0 & \text{else} \end{cases}$$

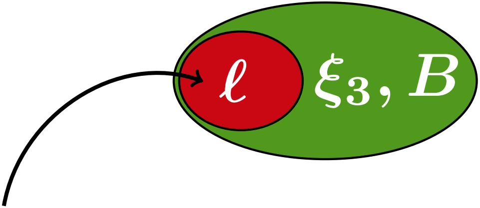
Information Content

Just a subset of information contained in 3p-statistics?



Information Content

Just a subset of information contained in 3p-statistics?

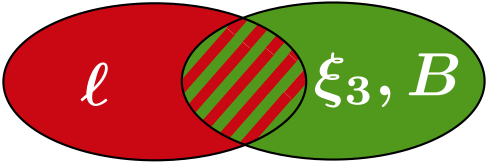


- independency of modulus $|\delta_{\boldsymbol{k}}|$
- independency of linear bias

$\rightarrow$ probes information complementary to $P(k)$ and with less contamination from $P(k)$-variance / bias

Information Content

Just a subset of information contained in 3p-statistics?



- independency of modulus $|\delta_{\boldsymbol{k}}|$
- independency of linear bias

$\rightarrow$ probes information complementary to $P(k)$ and with less contamination from $P(k)$-variance / bias

$$\text{PT:} \quad \langle \epsilon_{\boldsymbol{k}} \epsilon_{\boldsymbol{q}} \epsilon_{-\boldsymbol{k}-\boldsymbol{q}} \rangle \sim \frac{B(\boldsymbol{k}, \boldsymbol{q})}{\sqrt{P(k)P(q)P(|\boldsymbol{k}+\boldsymbol{q}|)}} + \sum_{\text{all higher order spectra}}$$

Matsubara 2003, Wolstenhulme et al. 2015

Why Anisotropies Matter

Alcock-Paczyński Effect

Converting redshifts and angles to physical distances requires the assumption of a prior cosmology

$$r_{||} \sim H(z)^{-1}\,\Delta z$$
$$r_{\perp} \sim D_A(z)\,\Delta\theta$$

$\rightarrow$ choosing a wrong cosmology leads to distorted scales

Why Anisotropies Matter

Alcock-Paczyński Effect

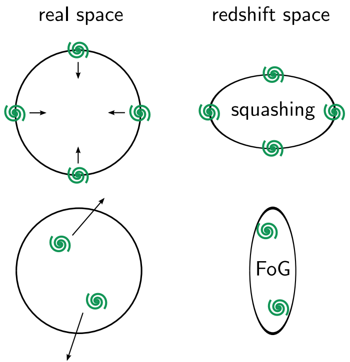
Converting redshifts and angles to physical distances requires the assumption of a prior cosmology

$$r_{||} \sim H(z)^{-1}\,\Delta z$$

$$r_{\perp} \sim D_A(z)\,\Delta\theta$$

→ choosing a wrong cosmology leads to distorted scales

Redshift Space Distortions



Separating Radial and Transverse Scales

Average over transverse scales only: Eggemeier et al. 2015

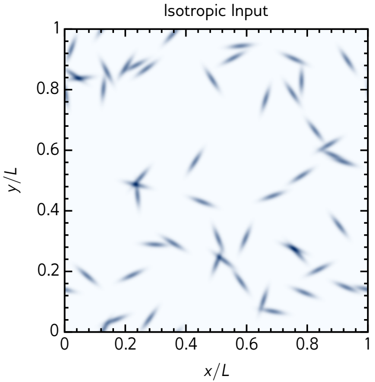
$$\hat{\ell}(r_\perp, r_\parallel) \backsimeq \left(\frac{r}{L}\right)^{9/2} \iint \mathrm{d}^3k\, \mathrm{d}^3q \cos\left[\left(k_\parallel - q_\parallel\right) r_\parallel\right] J_0\left[|k_\perp - q_\perp| r_\perp\right]$$

$$\times\ \epsilon_{\boldsymbol{k}}\epsilon_{\boldsymbol{q}}\epsilon_{-\boldsymbol{k}-\boldsymbol{q}}\, \Theta(\boldsymbol{k})\Theta(\boldsymbol{q})\Theta(\boldsymbol{k}+\boldsymbol{q})$$

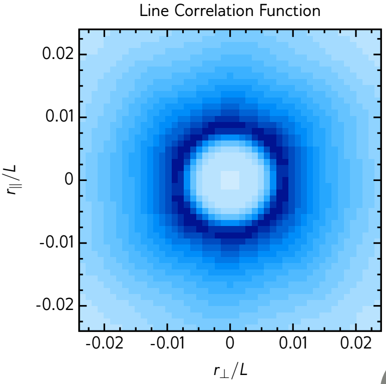
Separating Radial and Transverse Scales

Average over transverse scales only: Eggemeier et al. 2015

$$\hat{\ell}(r_\perp, r_\parallel) \backsimeq \left(\frac{r}{L}\right)^{9/2} \iint \mathrm{d}^3k\, \mathrm{d}^3q \cos\left[\left(k_\parallel - q_\parallel\right) r_\parallel\right] J_0\left[|\boldsymbol{k}_\perp - \boldsymbol{q}_\perp| r_\perp\right] \times \epsilon_{\boldsymbol{k}}\epsilon_{\boldsymbol{q}}\epsilon_{-\boldsymbol{k}-\boldsymbol{q}}\, \Theta(\boldsymbol{k})\Theta(\boldsymbol{q})\Theta(\boldsymbol{k}+\boldsymbol{q})$$



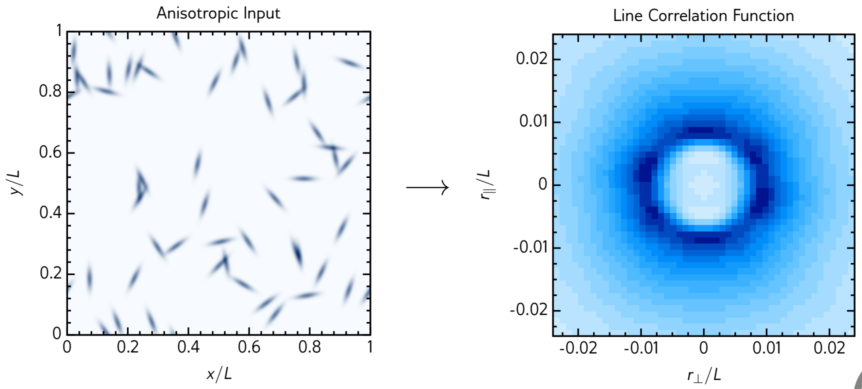
⟶



Separating Radial and Transverse Scales

Average over transverse scales only: Eggemeier et al. 2015

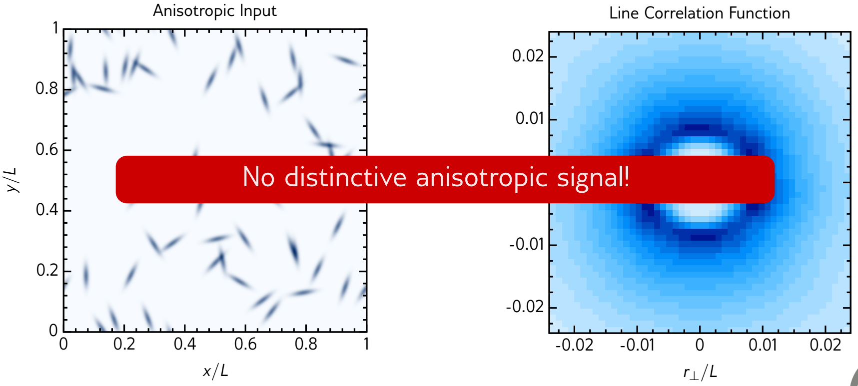
$$\hat{\ell}(r_\perp, r_\parallel) \backsimeq \left(\frac{r}{L}\right)^{9/2} \iint \mathrm{d}^3k\, \mathrm{d}^3q \cos\left[\left(k_\parallel - q_\parallel\right) r_\parallel\right] J_0\left[|k_\perp - q_\perp| r_\perp\right]$$
$$\times\ \epsilon_{\boldsymbol{k}}\epsilon_{\boldsymbol{q}}\epsilon_{-\boldsymbol{k}-\boldsymbol{q}}\, \Theta(\boldsymbol{k})\Theta(\boldsymbol{q})\Theta(\boldsymbol{k}+\boldsymbol{q})$$



Separating Radial and Transverse Scales

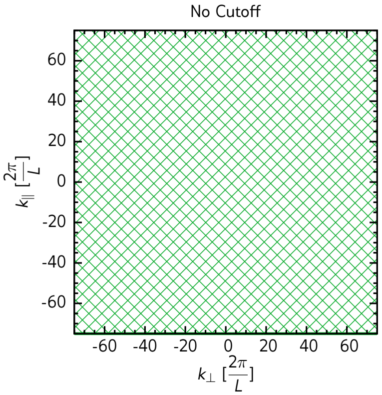
Average over transverse scales only: Eggemeier et al. 2015

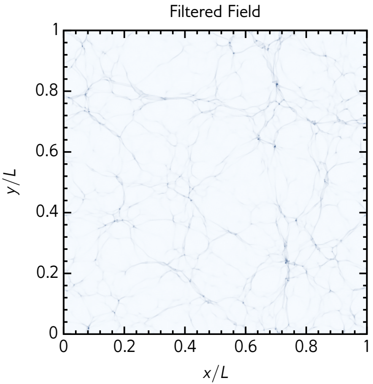
$$\hat{\ell}(r_\perp, r_\parallel) \simeq \left(\frac{r}{L}\right)^{9/2} \iint \mathrm{d}^3k\, \mathrm{d}^3q \cos\left[\left(k_\parallel - q_\parallel\right) r_\parallel\right] J_0\left[|k_\perp - q_\perp| r_\perp\right] \times \epsilon_{\boldsymbol{k}}\epsilon_{\boldsymbol{q}}\epsilon_{-\boldsymbol{k}-\boldsymbol{q}}\, \Theta(\boldsymbol{k})\Theta(\boldsymbol{q})\Theta(\boldsymbol{k}+\boldsymbol{q})$$



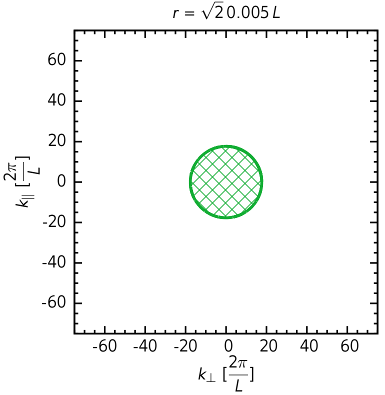
No distinctive anisotropic signal!

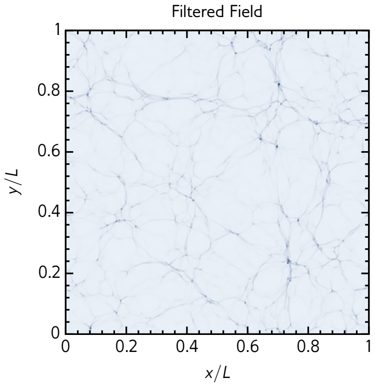
An Anisotropic Cutoff





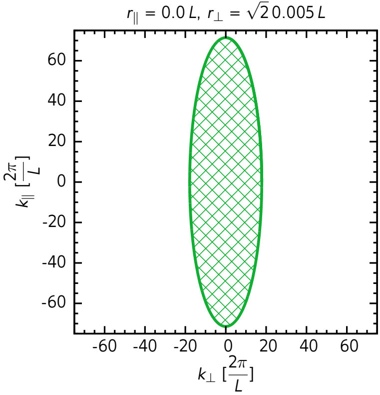
An Anisotropic Cutoff

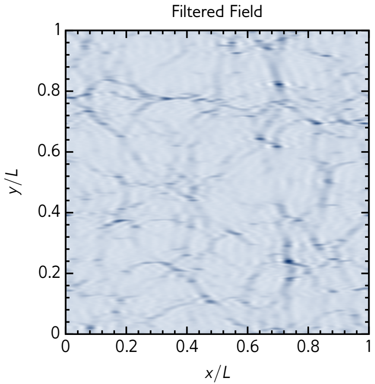




$$\Theta(k):\quad k^2\,r^2 \qquad \leq 4\pi^2$$

An Anisotropic Cutoff



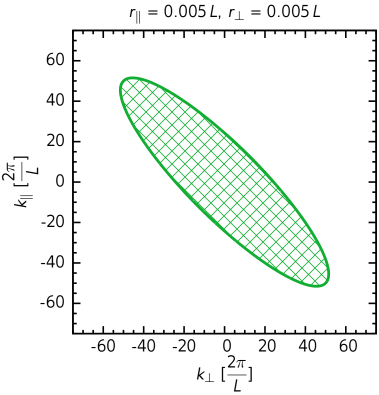


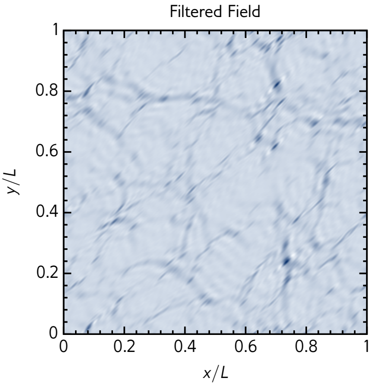
$$\Theta(k):\quad k^2\, r^2 + \left(\eta^2 - 1\right)\,(\boldsymbol{k}\cdot\boldsymbol{r})^2 \;\leq 4\pi^2$$

Anisotropic modification, depending on η

Eggemeier et al. 2015

An Anisotropic Cutoff



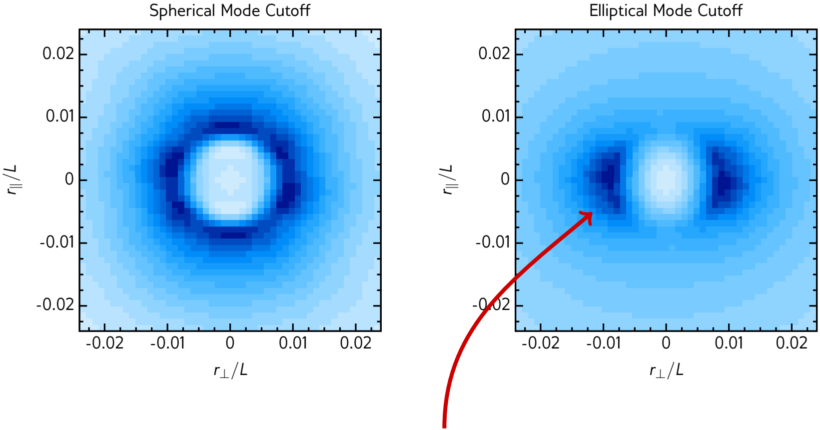


$$\Theta(k):\quad k^2\, r^2 + \left(\eta^2 - 1\right)\, (\boldsymbol{k}\cdot\boldsymbol{r})^2 \;\leq 4\pi^2$$

Anisotropic modification, depending on η

Eggemeier et al. 2015

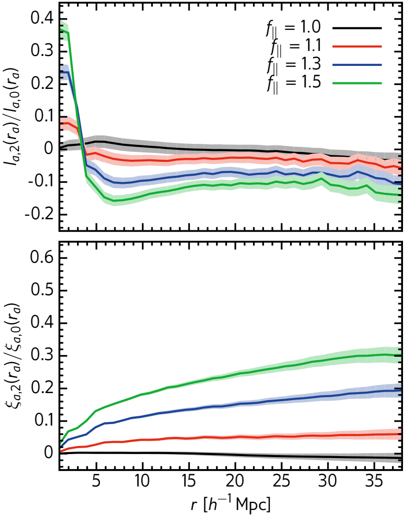
An Anisotropic Cutoff



Clear Anisotropy:
Enhanced signal in transverse direction!

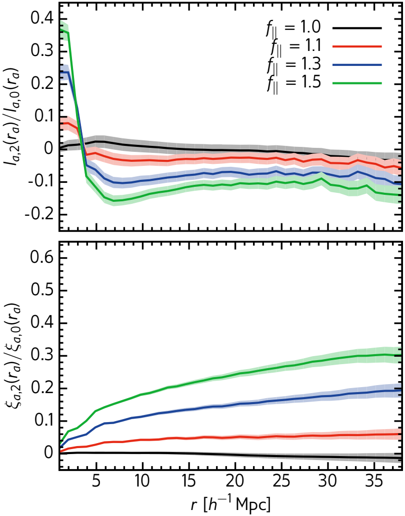
Testing Anisotropies with Zel'dovich Fields

Alcock-Paczyński Effect

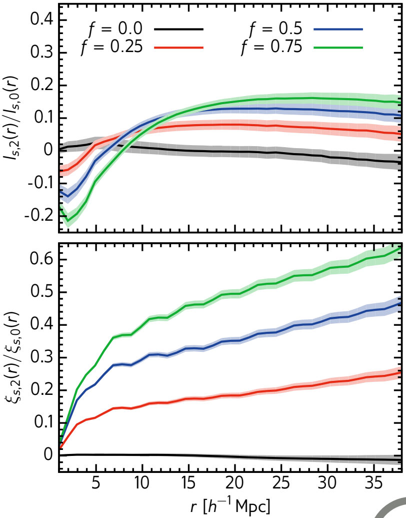


Testing Anisotropies with Zel'dovich Fields

Alcock-Paczyński Effect



Kaiser Effect



Summary and Outlook

Two take home messages:

- LCF is a complementary statistical tool, probing the non-gaussian regime of structure formation
- for making best use of the data, an extension to 2D LCF is inevitable; change of formalism (mode cutoff) enables detection of AP effect and kinematical RSD

Summary and Outlook

Two take home messages:

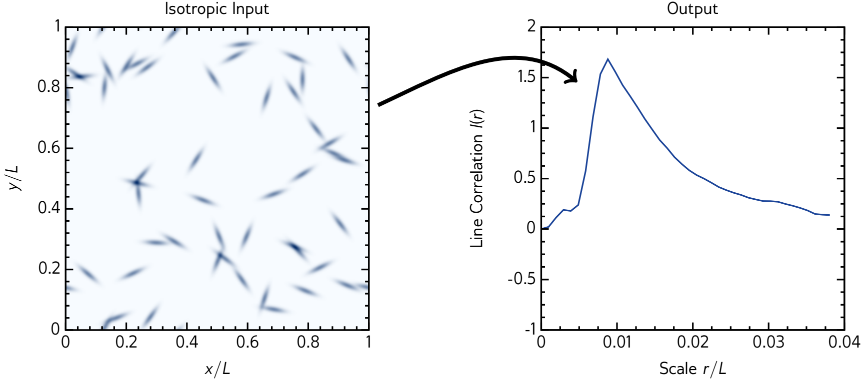
- LCF is a complementary statistical tool, probing the non-gaussian regime of structure formation
- for making best use of the data, an extension to 2D LCF is inevitable; change of formalism (mode cutoff) enables detection of AP effect and kinematical RSD

What's up next?

- application to N-body and real data
- using RSD to break degeneracy between f and b
- finding the optimal cutoff $\Theta(\boldsymbol{k})$

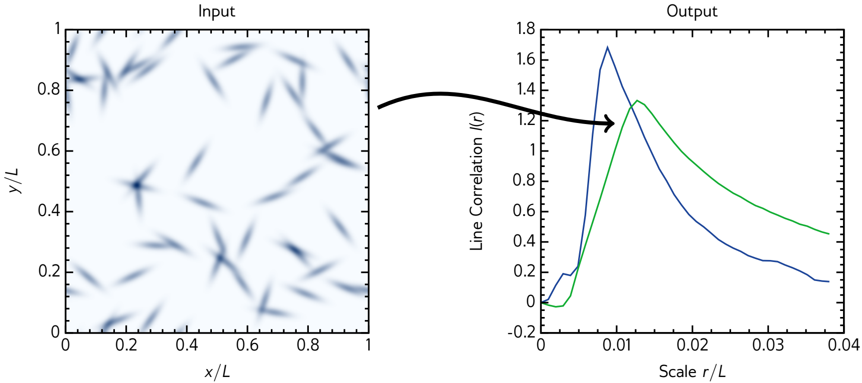
Backup: LCF - A Mesaure of Filamentary Structure

The line correlation function is sensitive on filamentary structures:



Backup: LCF - A Mesaure of Filamentary Structure

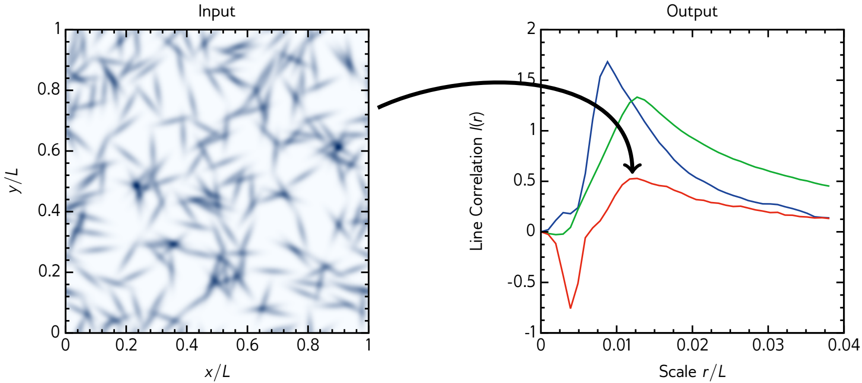
The line correlation function is sensitive on filamentary structures:



→ increasing size of structures shifts the peak

Backup: LCF - A Mesaure of Filamentary Structure

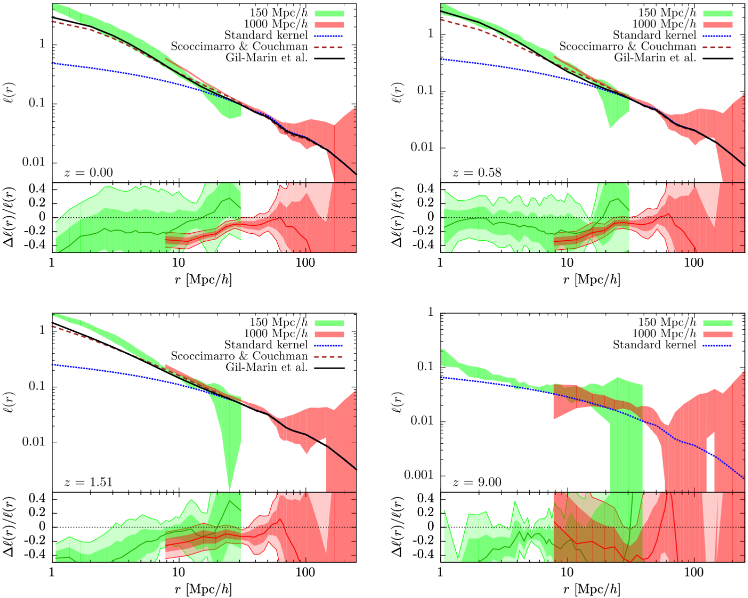
The line correlation function is sensitive on filamentary structures:



→ increasing number of objects decreases amplitude

Backup: LCF vs N-Body

Wolstenhulme et al. 2015



Backup: Anisotropic Cutoff

Eggemeier et al. 2015