Bound state formation in QFT

with Kalliopi Petraki and Michael Wiechers
1505.00109

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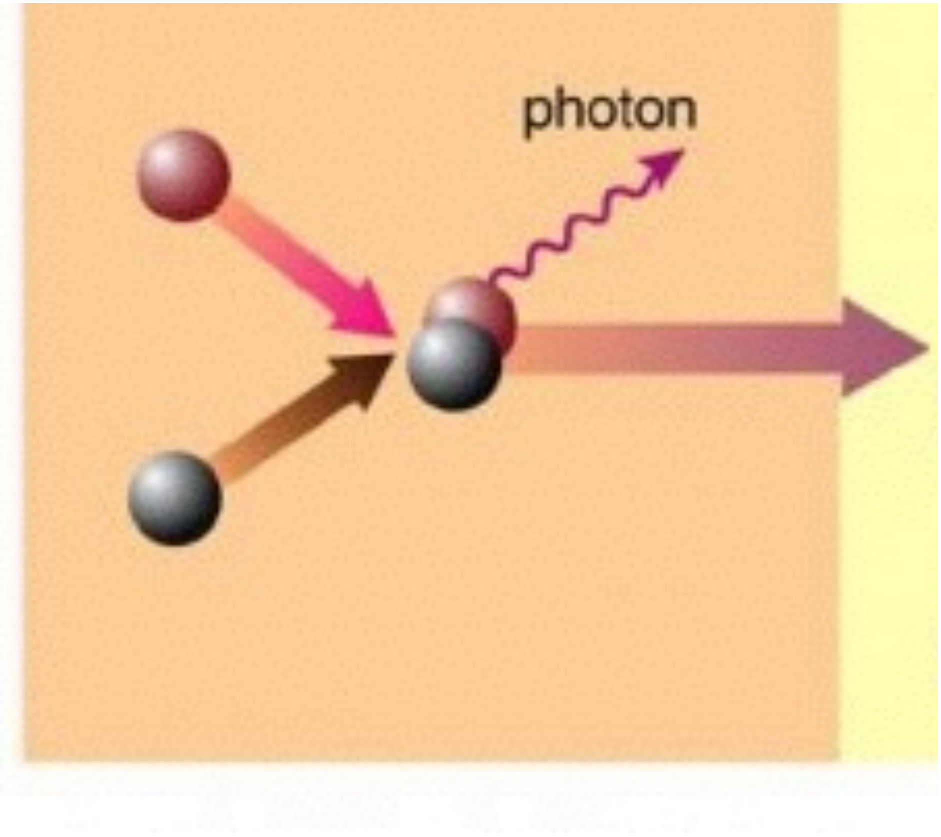
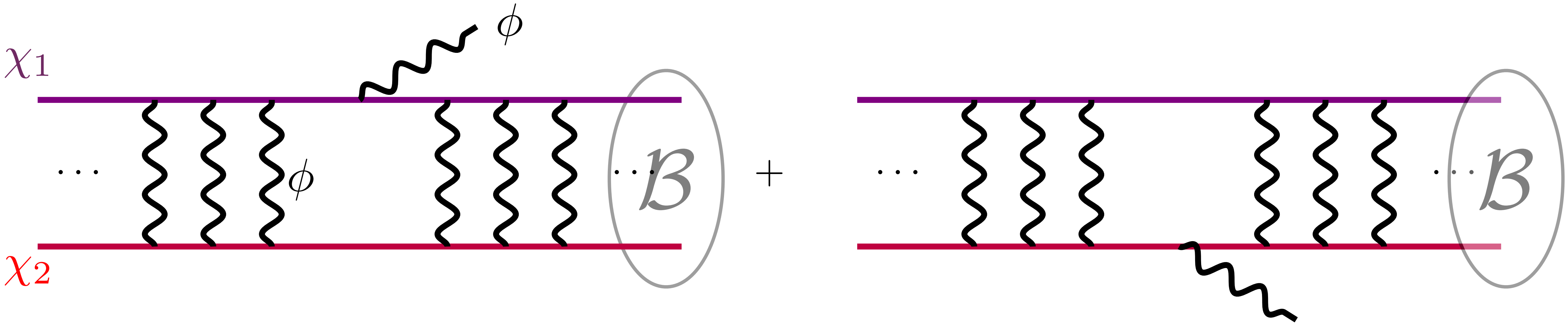


Bound states

DM w/ long range (non-confining) interaction $\alpha \gtrsim m_\phi/\mu$

$$\boxed{\chi_1 + \chi_2 \to \mathcal{B} + \phi}$$



Bound states

DM w/ long range (non-confining) interaction $\alpha \gtrsim m_\phi/\mu$

This talk: calculation of non-relativistic cross section for bound state formation in QFT

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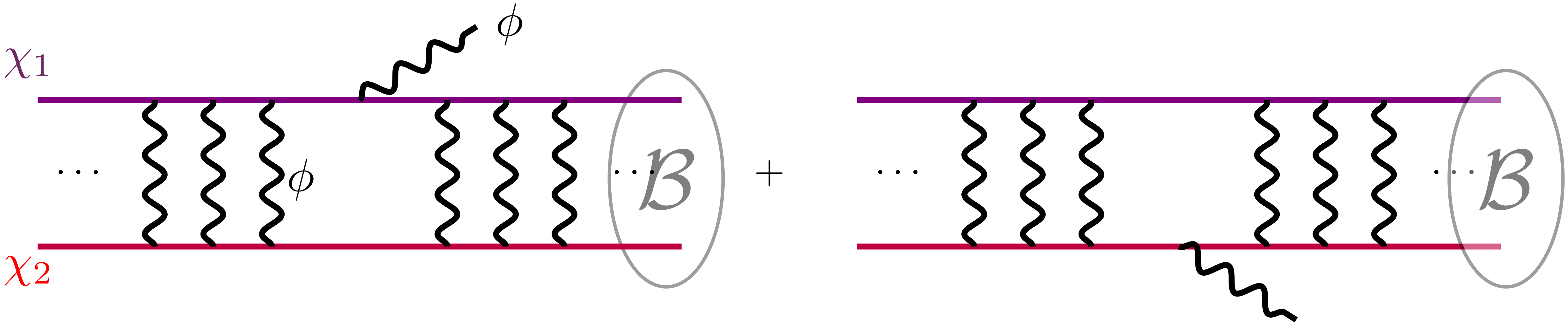


Motivation

Bound states — when important?

DM with long range interactions:

- self-interacting DM spergel & steinhardt ’00, etc.
- asymmetric DM Davoudiasl & Mohapatra ’12, etc.
- DM with Sommerfeld enhancement

  Hisano et al ’04, Cirelli et al ’07, etc.
- 10 TeV WIMP

  Hisano et al ’03, Cirelli et al ’07, etc.
- …

Bound states — why important?

Impact of bound states:

- relic density: unstable bound states extra annihilation channel

von Harling & Petraki ’14

- self-scattering in halos Cline et al ’12, Cyr-Racine & Sigurdson ’12, etc.

- indirect & direct detection experiments

Pearce & Kusenko ’13, Lah and Braaten ’13, etc.

- kinetic decoupling of DM from radiation

Cyr-Racine et al ‘14

- …

Bound state formation

Quantum mechanics vs. Quantum field theory

Sommerfeld ‘31

Bethe & Salpeter ‘57

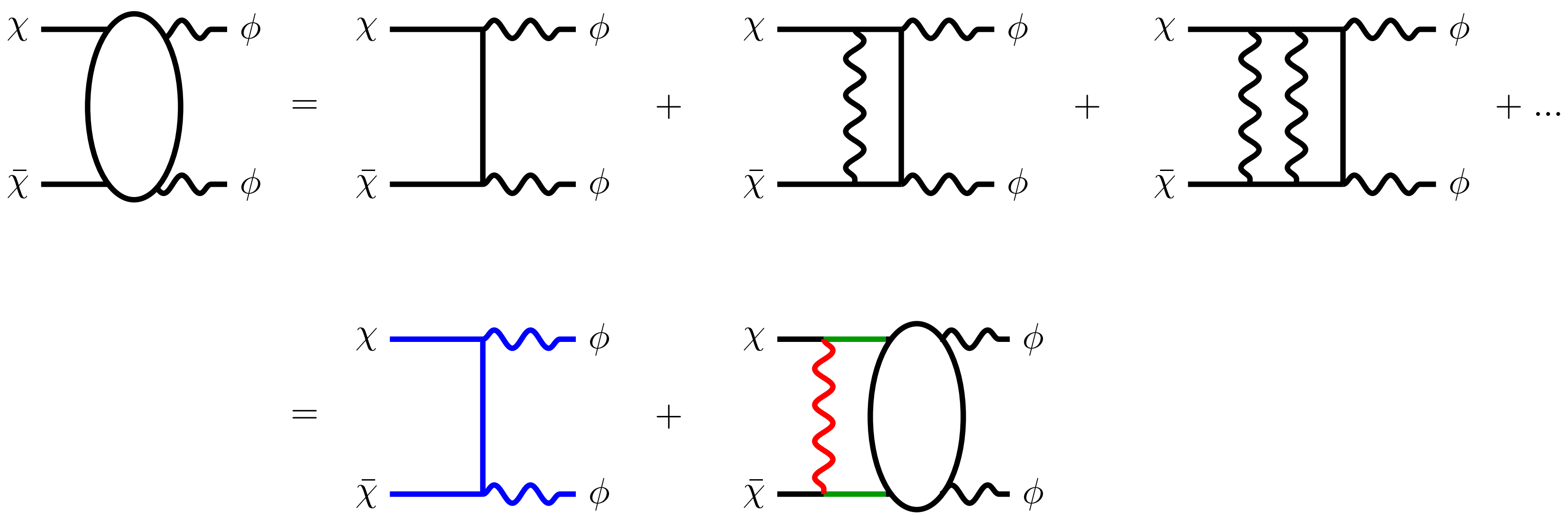
Akhiezer & Merenkov ‘96

- modern approach
- systematic relativistic and radiative corrections
- generalization to non-abelian interactions

Bound state formation

Bound state formation — Challenge

Sommerfeld enhancement in QFT in $\chi\bar{\chi} \to \phi\phi$ Iengo ’09, Cassel ‘09

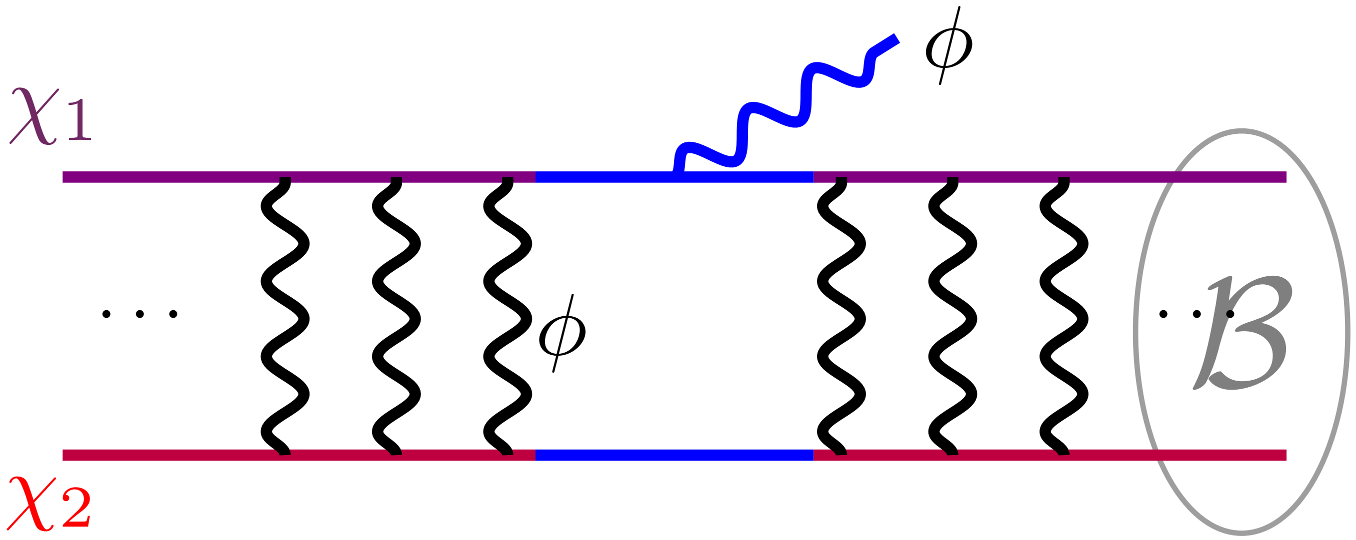


Bethe-Salpeter equation

$$\mathcal{A}(p,p') = \mathcal{A}_0(p,p') + \int \frac{\mathrm{d}^4 q}{(2\pi)^4} D_\phi(p-q) D_\chi(q) D_\chi(-q)\, \mathcal{A}(q,p')$$

End result in non-relativistic limit $|\mathcal{U}(p)\rangle = \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \phi_{\vec{p}}(q) |q,-q\rangle$

Bound state formation — Challenge



- double summation
- ‘missing’ propagator
- amplitude vanishes on-shell

Bound state formation — Solution

LSZ-reduction

n-point function $\Longleftrightarrow$ *n*-point amputated amplitude

pole & branche cut structure

bound state formation

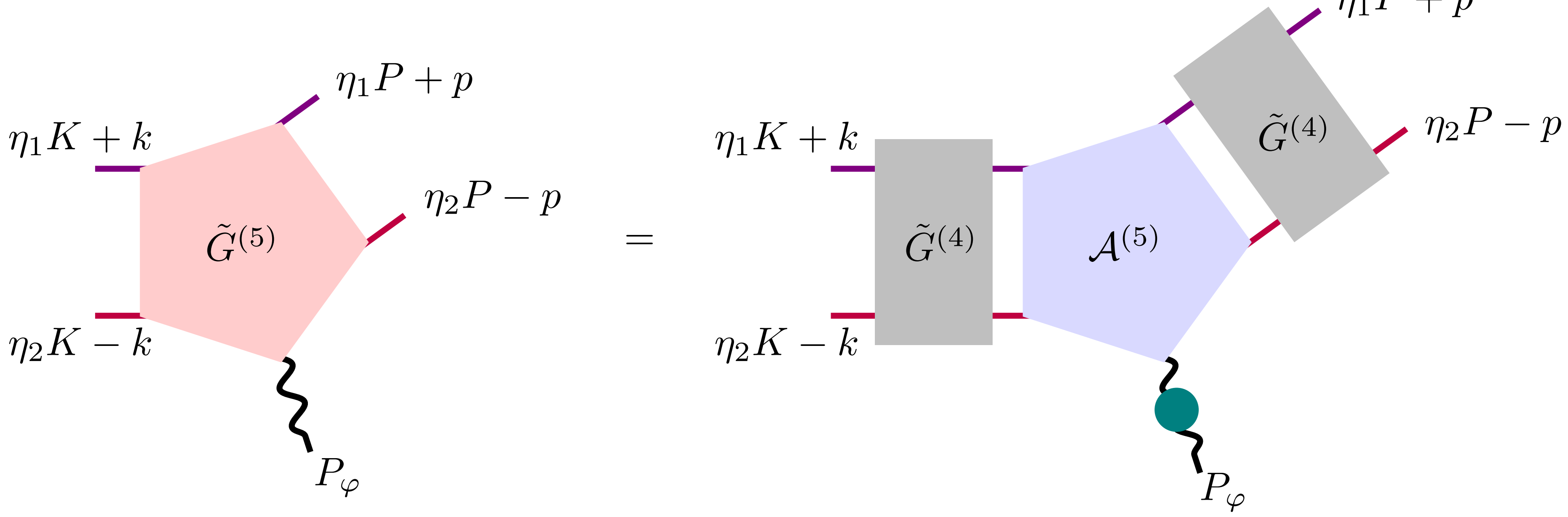
Sommerfeld enhancement

Bound state formation — Solution

LSZ-reduction

n-point function $\Longleftrightarrow$ *n*-point amputated amplitude

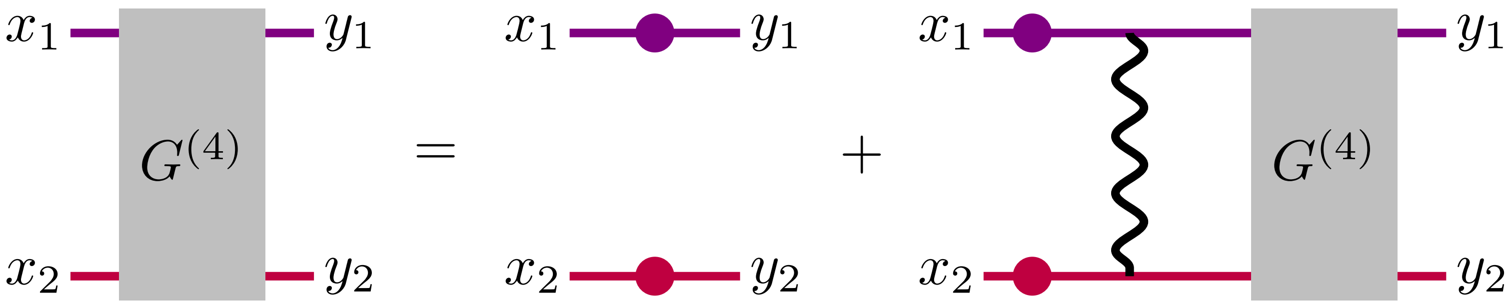
pole & branche cut
structure



$$\boxed{\chi_1 + \chi_2 \to \mathcal{B} + \phi}$$

Bound state formation — Solution

Bethe-Salpeter equation

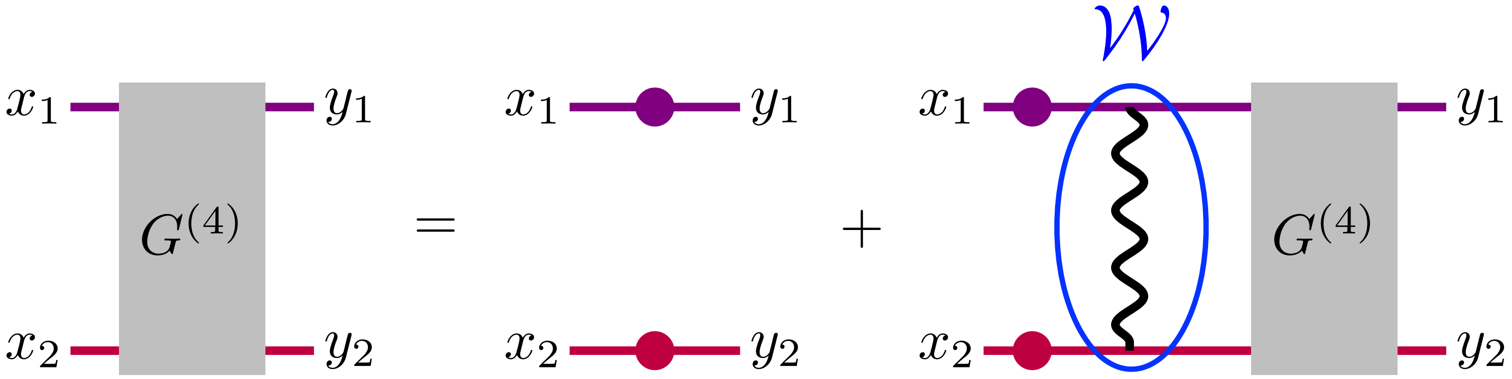


$$G^{(4)} = \langle \Omega | T \chi_1(x_1) \chi_2(x_1) \chi_1^\dagger(y_1) \chi_2^\dagger(y_2) | \Omega \rangle$$

insert completeness relation

$$\mathbf{1} = \sum_n \int \frac{\mathrm{d}^3 Q}{(2\pi)^3 2\omega_{\vec{Q},n}} |\mathcal{B}_{\vec{Q},n}\rangle\langle\mathcal{B}_{\vec{Q},n}| + \int \frac{\mathrm{d}^3 q}{(2\pi)^3 2\omega_q} \frac{\mathrm{d}^3 Q}{(2\pi)^3 2\omega_Q} |\mathcal{U}_{\vec{Q},\vec{q}}\rangle\langle\mathcal{U}_{\vec{Q},\vec{q}}|$$

Bound state formation — Solution



Bethe-Salpeter equation

1. non-relativistic schroedinger equation for bound and scattering state wave function

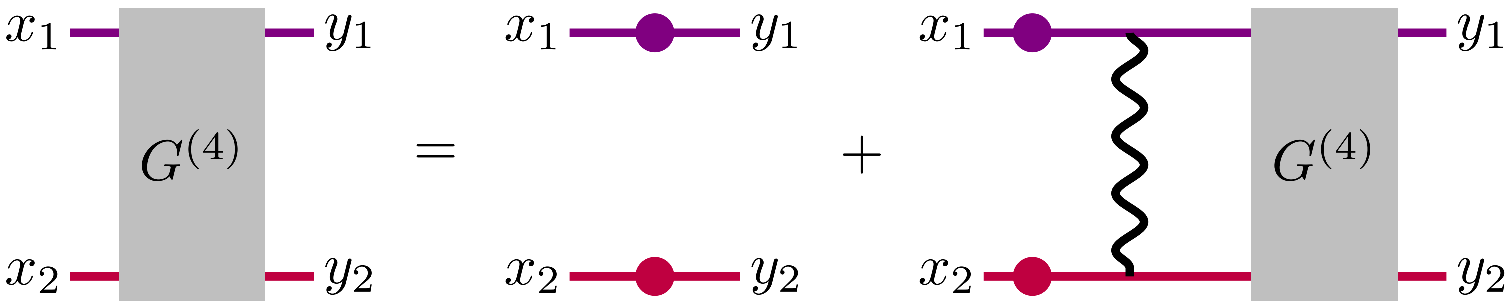
$$\left[-\frac{\nabla^2}{2\mu}+V(\vec{r})\right]\psi_n(\vec{r})=\mathcal{E}_n\psi_n(\vec{r}) \qquad \psi(\vec{x})\propto\langle\Omega|T(\chi_1\chi_2)|\mathcal{B}_{Q,n}\rangle\Big|_{x^0=0}$$

$$\left[-\frac{\nabla^2}{2\mu}+V(\vec{r})\right]\phi_q(\vec{r})=\mathcal{E}_{\vec{q}}\phi_q(\vec{r}) \qquad \phi(\vec{x})=\langle\Omega|T(\chi_1\chi_2)|\mathcal{U}_{Q,\vec{q}}\rangle\Big|_{x_0=0}$$

$$V(\vec{r})=-\frac{1}{i4m\mu}\int\frac{\mathrm{d}^3k}{(2\pi)^3}\mathcal{W}(\vec{k})\mathrm{e}^{i\vec{k}\cdot\vec{r}}$$

Bound state formation — Solution

Bethe-Salpeter equation



1. non-relativistic schroedinger equation for bound and scattering state wave function

2. pole & branch cut structure needed for LSZ reduction

$$\tilde{G}^{(4)}_n(p, p'; Q) \to \frac{i\tilde{\Psi}_{Q,n}(p)\tilde{\Psi}^*_{Q,n}(p')}{2\omega_{Q,n}[Q^0 - \omega_{Q,n} + i\epsilon]}$$

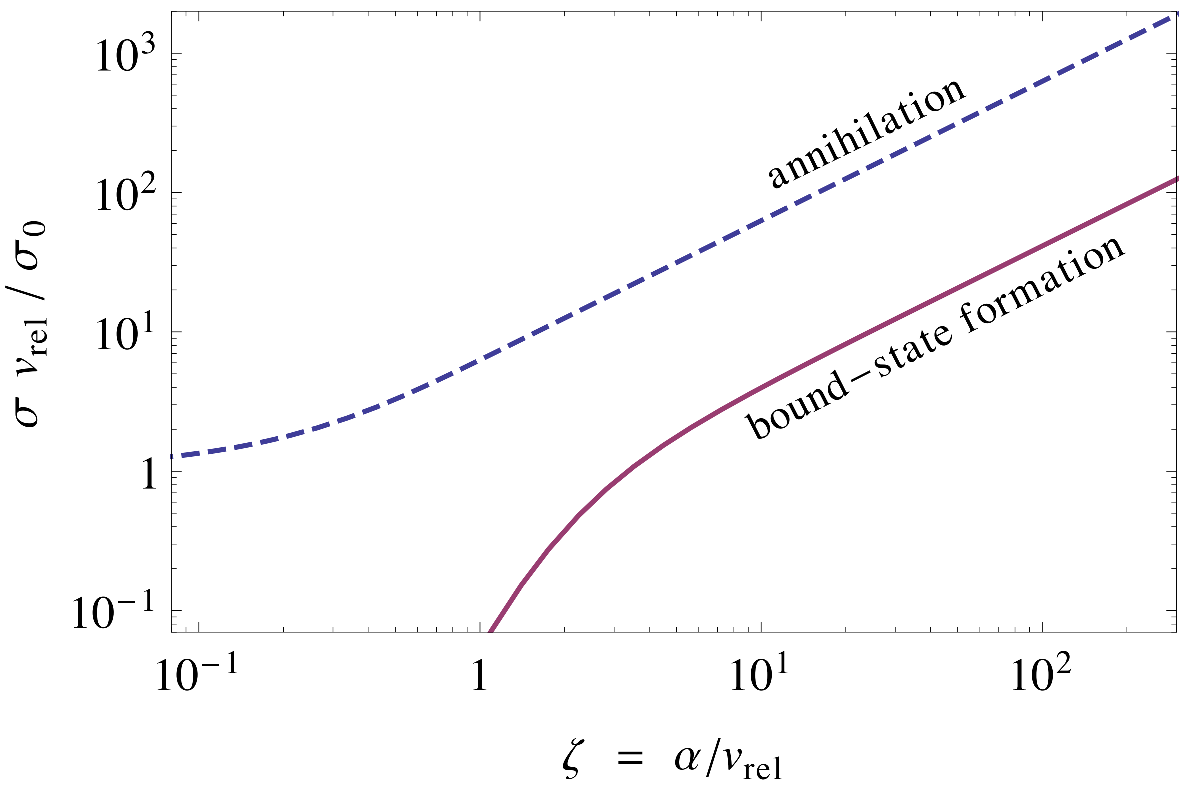
Bound state formation — Result

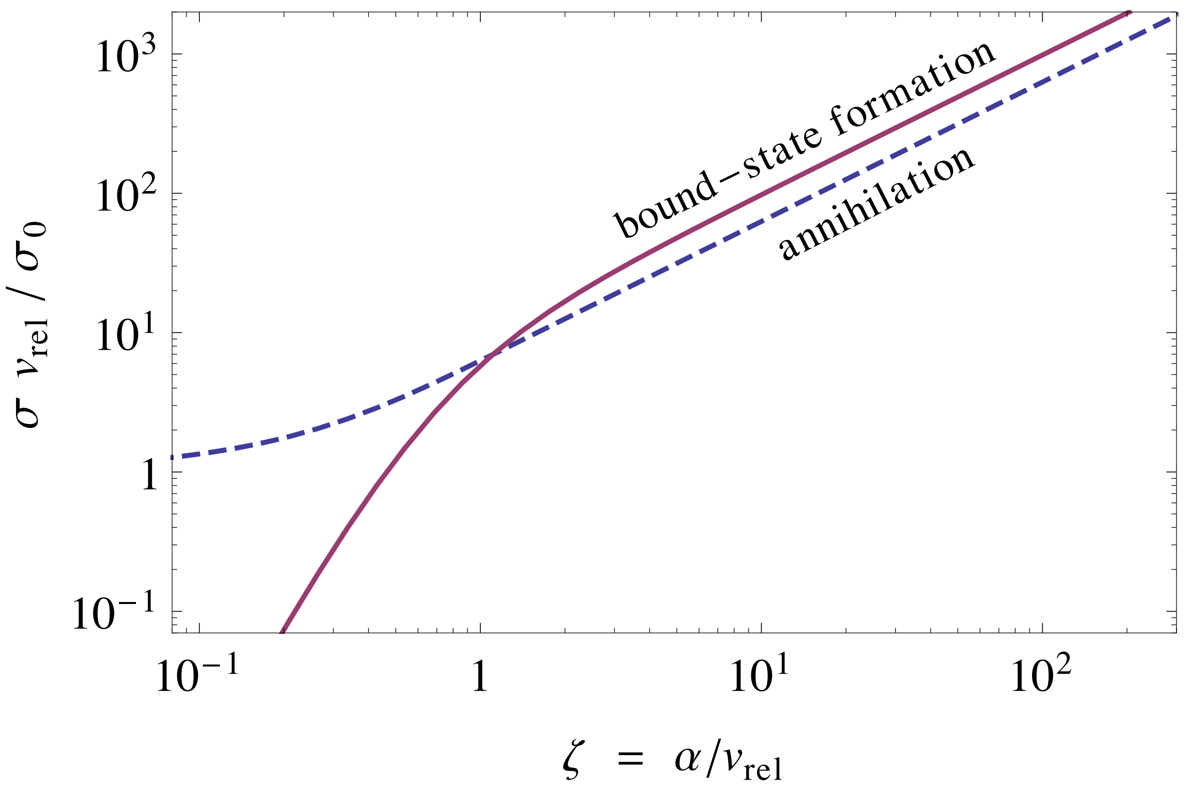
-scalar mediator and real scalar DM $\mathcal{L}_{\text{int}} = -gm\phi\chi^2$

$$\mathcal{M}_{k\to n} \simeq -2gm^{3/2} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left(1 + \frac{\vec{p}^2}{2m^2}\right) \psi_n^*(\vec{p}) \left[\phi_k(\vec{p} + \frac{1}{2}\vec{P}_\phi) + \phi_k(\vec{p} - \frac{1}{2}\vec{P}_\phi)\right]$$

Bound state formation — Result

This talk: calculation of non-relativistic cross section for bound state formation in QFT


Scalar mediator


Vector mediator