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Octopoles for Landau Damping

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FCC-hh Task 2.4

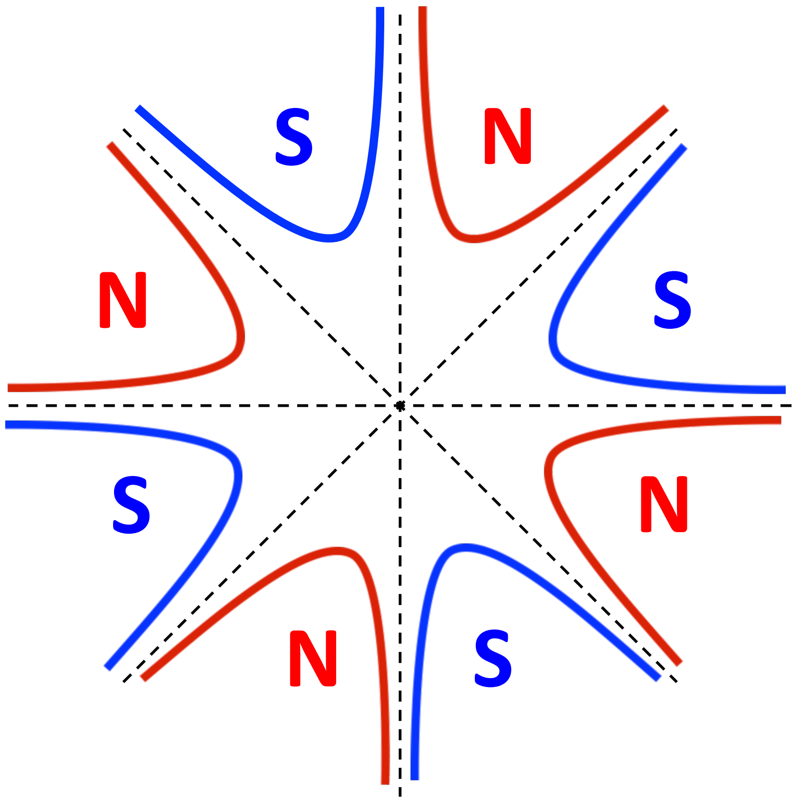
O.Boine-Frankenheim, X.Buffat, U.Niedermayer, et.al.

Octupole Field

$$B_x = O_3(3x^2y - y^3)$$
$$B_y = O_3(x^3 - 3xy^2)$$

First-order frequency shift of the anharmonic oscillations

$$\ddot{x} + \omega_0^2 x = \varepsilon x^3$$
$$\omega \approx \omega_0 - \frac{3}{8}\frac{\varepsilon A^2}{\omega_0}$$



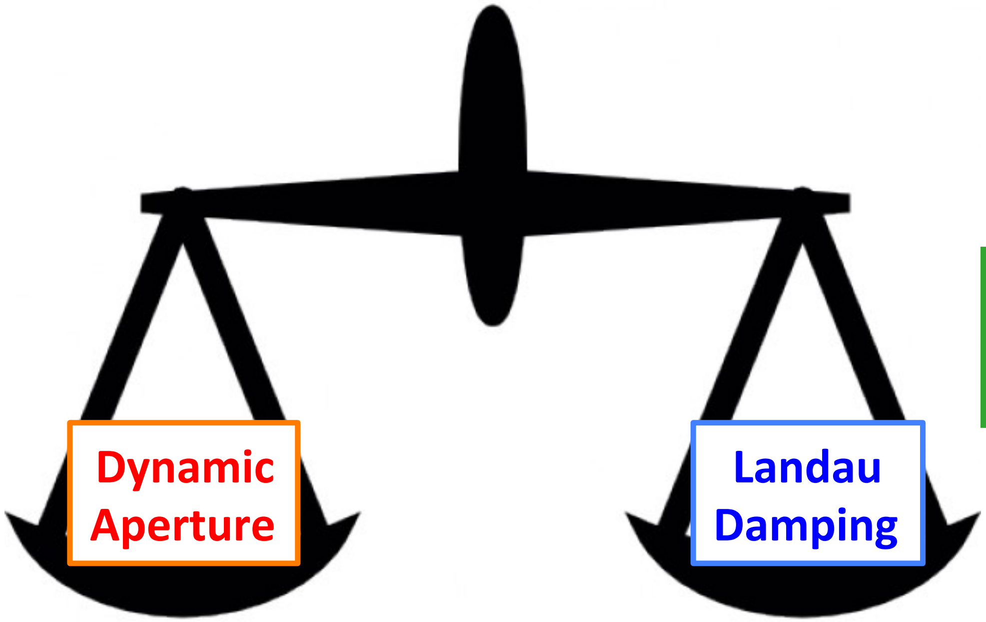
Schematic yoke profile of an octupole magnet

Octupoles in Ring Machines

Octupole Magnets Usage:

- Landau Damping
- Correction of O_3–field errors (much weaker)

But: non-linearities can reduce DA



A balance is necessary

Octupoles are the essential part of the beam stability in LHC, in combination with the feedback system

Octupole Tune Shifts

Horizontal and vertical betatron tune shift ($\Delta Q = Q - Q_0$, $Q = f_\beta / f_0$) can be calculated using

$$\Delta Q_x = \left\{ \frac{3}{8\pi} \sum \hat{\beta}_x^2 \frac{O_3 L_\mathrm{m}}{B\rho} \right\} J_x - \left\{ \frac{3}{8\pi} \sum 2\hat{\beta}_x \hat{\beta}_y \frac{O_3 L_\mathrm{m}}{B\rho} \right\} J_y$$

$$\Delta Q_y = \left\{ \frac{3}{8\pi} \sum \hat{\beta}_y^2 \frac{O_3 L_\mathrm{m}}{B\rho} \right\} J_y - \left\{ \frac{3}{8\pi} \sum 2\hat{\beta}_x \hat{\beta}_y \frac{O_3 L_\mathrm{m}}{B\rho} \right\} J_x$$

Betatron Action J_x, J_y

Beta-function (The amplitude Courant-Snyder function)

$$x(s) = \sqrt{2 J_x \hat{\beta}_x(s)} \cos\left[\phi_x(s)\right]$$

$$y(s) = \sqrt{2 J_y \hat{\beta}_y(s)} \cos\left[\phi_y(s)\right]$$

$$\Delta Q_x = a_x J_x - b J_y$$

$$\Delta Q_y = a_y J_y - b J_x$$

The LHC Configuration

392 Main Arc Quadrupoles (MQ)
168 Landau Octupoles (MO):
84 F-Octupoles, 84 D-Octupoles

$$O_3 = 63100 \frac{I_{\rm oct}[{\rm A}]}{550}\ {\rm Tm}^{-3}$$
$$L_{\rm m} = 0.32\ {\rm m}$$
$$I_{\rm oct}^{\rm max} = 550\ {\rm A}$$

LHC Landau Octupoles

MO beta-function	MQ beta-function
$\beta_x^F = 30.1$ m	$\beta_x^F = 29.8$ m
$\beta_y^F = 178.8$ m	$\beta_y^F = 180.2$ m
$\beta_x^D = 175.5$ m	$\beta_x^D = 176.9$ m
$\beta_y^D = 33.6$ m	$\beta_y^D = 33.3$ m

Since the Landau Octupoles are close to the MQs, the beta-functions are very similar

The FCC Configuration

814 Arc Quadrupoles (MQ)

L_{FCC}/L_{LHC} = 3.7, but N_{MQ}/N_{MQ}= 2.1

(FCC L_{cell}=203m, and LHC L_{cell}=106.9m)

MQ LHC beta-function	**MQ FCC beta-function**
β_x^F = 29.8 m	β_x^F = 66.22 m
β_y^F = 180.2 m	β_y^F = 359.65 m
β_x^D = 176.9 m	β_x^D = 360.96 m
β_y^D = 33.3 m	β_y^D = 65.14 m

The FCC beta-functions are approx. twice of the LHC

We assume this scheme with the beta-functions at MQs

Ring Size Scaling

Coherent (real and imaginary) tune shift of the collective oscillations:

$$\Delta Q_{\mathrm{coh}} = \frac{\lambda_0 r_p}{\gamma} \frac{i\hat{\beta} Z_{\mathrm{eff}}^{\perp}}{Z_0}$$

$$\Delta Q_{\mathrm{coh}} \propto \frac{Z^{\perp}\hat{\beta}}{\gamma}$$

The players:

- impedance
- beta-function
- beam energy

Compare the LHC top energy with the FCC top energy for the same impedance per length:

$$\frac{\Delta Q_{\mathrm{coh}}^{\mathrm{FCC}}}{\Delta Q_{\mathrm{coh}}^{\mathrm{LHC}}} = \frac{L_{\mathrm{FCC}}}{L_{\mathrm{LHC}}} \times \frac{\hat{\beta}_{\mathrm{FCC}}}{\hat{\beta}_{\mathrm{LHC}}} \times \left(\frac{\gamma_{\mathrm{FCC}}}{\gamma_{\mathrm{LHC}}}\right)^{-1} \approx$$

$$\approx 3.7 \times 2 \times \left(\frac{50}{7}\right)^{-1} = 1$$

Ring Size Scaling

The tune shifts due to octupole magnets:

$$\Delta Q_{\mathrm{oct}} \propto (NI)_{\mathrm{oct}}\, \hat{\beta}^2 \frac{\epsilon_\perp}{\gamma^2}$$

$$(NI)_{\mathrm{oct}} \propto \frac{\gamma^2}{\hat{\beta}^2 \epsilon_\perp} \Delta Q_{\mathrm{oct}}$$

The players:
- beam energy
- beta-function
- transverse emittance

Compare the LHC top energy with the FCC top energy. The octupole power (number × current) needed to compensate a coherent mode:

$$\frac{(NI)_{\mathrm{oct}}^{\mathrm{FCC}}}{(NI)_{\mathrm{oct}}^{\mathrm{LHC}}} = \left(\frac{\gamma_{\mathrm{FCC}}}{\gamma_{\mathrm{LHC}}}\right)^2 \times \left(\frac{\hat{\beta}_{\mathrm{FCC}}}{\hat{\beta}_{\mathrm{LHC}}}\right)^{-2} \times \left(\frac{\epsilon_{\mathrm{FCC}}}{\epsilon_{\mathrm{LHC}}}\right)^{-1} \approx$$

$$\approx \left(\frac{50}{7}\right)^2 \times \frac{1}{4} \times \left(\frac{2.2}{3.75}\right)^{-1} = 21.7$$

Ring Size Scaling

In terms of the instability growth time:

$$a(t) = a_0 \; e^{t/\tau}$$

$$\frac{1}{\tau} = \mathrm{Im}(\Delta Q_{\mathrm{coh}}) \; 2\pi f_0$$

The same ΔQ_{coh} causes a slower instability in sec in FCC.

$$\frac{f_0^{\mathrm{FCC}}}{f_0^{\mathrm{LHC}}} = \frac{L_{\mathrm{LHC}}}{L_{\mathrm{FCC}}} = \frac{1}{3.73}$$

But, also a feedback system operates in kicks per turn (e.g. in LHC 2 μrad/turn). Stability means $\mathrm{Im}(\Delta Q_{coh})<0$.

Thus we consider ΔQ_{coh} .

FCC Landau Octupole Scheme

$$\Delta Q_{\mathrm{coh}} \propto \frac{Z^{\perp}\hat{\beta}}{\gamma} \qquad \Delta Q_{\mathrm{oct}} \propto (NI)_{\mathrm{oct}}\,\hat{\beta}^2\,\frac{\epsilon_{\perp}}{\gamma^2}$$

- The octupole requirements are more demanding at the top energy
- The expected ΔQ_{coh} in FCC may be similar to that in LHC
- The total octupole power in FCC should be ≈20 times stronger

Thus in our stability analysis we consider the FCC top energy 50 TeV, and we vary the number of octupole magnets

Landau octupole at each arc quadrupole: $N_{MO} = N_{MQ} = 814$
407 F-Octupoles, 407 D-Octupoles
FCC/LHC: $N_{MO}/N_{MO} = 814/168 = 4.8$

FCC Relevant Parameters

Circumference	100 km
Beam kinetic energy	Injection 3.3 TeV Collisions 50 TeV
rms Bunch Length	80 mm
rms normalized transverse emittance	2.2 μm-rad
Particle Number	10600 bunches 10^{11} ppb
Tunes	Q_x=107.32 Q_y=108.31

D.Schulte, FCC Week 2015, FCC Week 2016
M.Schaumann, PRSTAB **18**, 091002 (2015)

Dispersion Relation

L.Laslett, V.Neil, A.Sessler, 1965
D.Möhl, H.Schönauer, 1974
J.Berg, F.Ruggiero, CERN SL-96-71 AP 1996

$$\Delta Q_{\mathrm{coh}} \int \frac{1}{\Delta Q_{\mathrm{oct}} - \Omega/\omega_0} J_x \frac{\partial \psi_\perp}{\partial J_x} dJ_x dJ_y = 1$$

complex coherent tune shift for the beam without damping

The solution: collective mode frequency Ω for the given impedance and beam

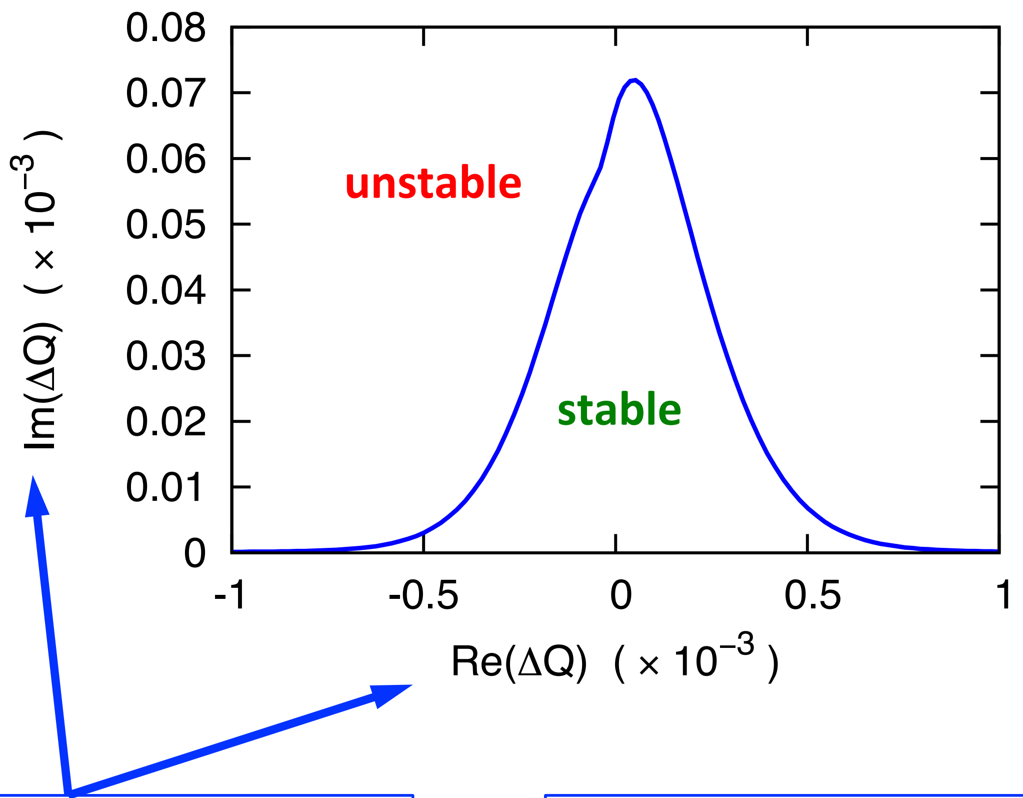
Gaussian distribution

$$\psi_\perp(J_x, J_y) = \frac{1}{\sigma^4} \exp\left(- \frac{J_x + J_y}{\sigma^2} \right)$$

The resulting damping is a complicated 2D convolution of the distribution {$d\psi/dJ_x$, $\psi(J_y)$} and tune shifts $\Delta Q_{\mathrm{oct}}(J_x, J_y)$

Stability Diagram

The contour line of Im(Ω)=0
for FCC nominal parameters, 814 LHC–magnets, I–max



complex coherent tune shift for the beam without damping

The solution: collective mode frequency Ω for the given impedance and beam

Stability Diagram

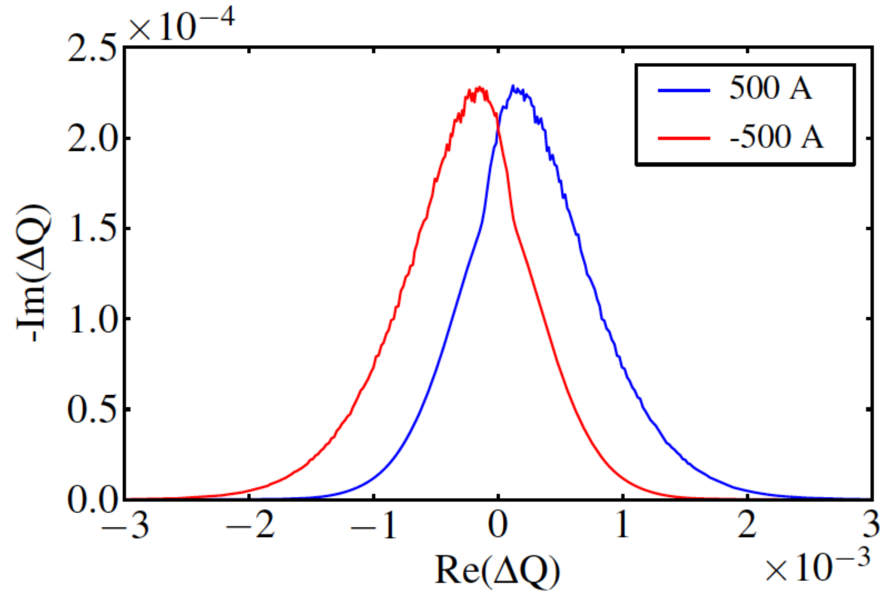
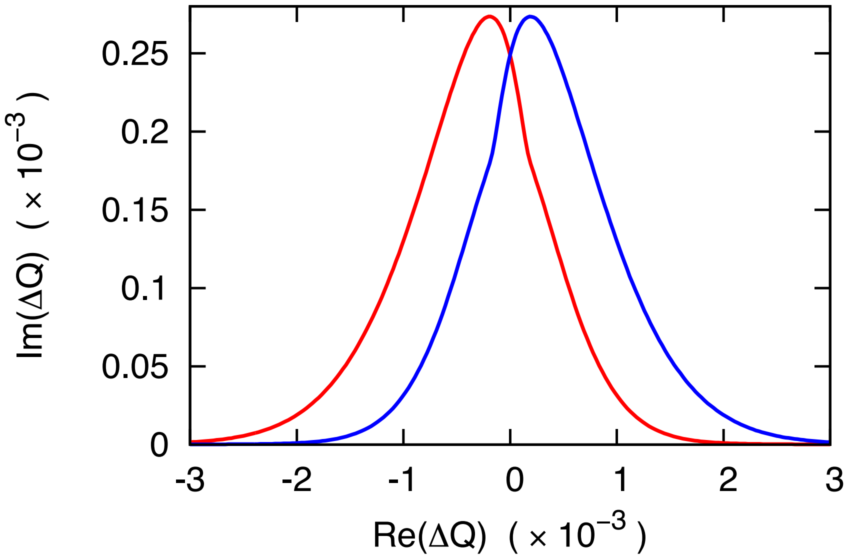


Figure from X.Buffat,et.el, PRSTAB **17**, 111002 (2014)



My calculations

LHC nominal beam parameters at 7 TeV,
maximum octupole magnet current

FCC Landau Octupole Scheme

The general tune shifts due to octupoles:

$$\Delta Q_x = \left\{\frac{3}{8\pi}\sum \hat{\beta}_x^2 \frac{O_3 L_\mathrm{m}}{B\rho}\right\} J_x - \left\{\frac{3}{8\pi}\sum 2\hat{\beta}_x\hat{\beta}_y \frac{O_3 L_\mathrm{m}}{B\rho}\right\} J_y$$

$$\Delta Q_y = \left\{\frac{3}{8\pi}\sum \hat{\beta}_y^2 \frac{O_3 L_\mathrm{m}}{B\rho}\right\} J_y - \left\{\frac{3}{8\pi}\sum 2\hat{\beta}_x\hat{\beta}_y \frac{O_3 L_\mathrm{m}}{B\rho}\right\} J_x$$

The scheme with the F-Octupoles and D-Octupoles:

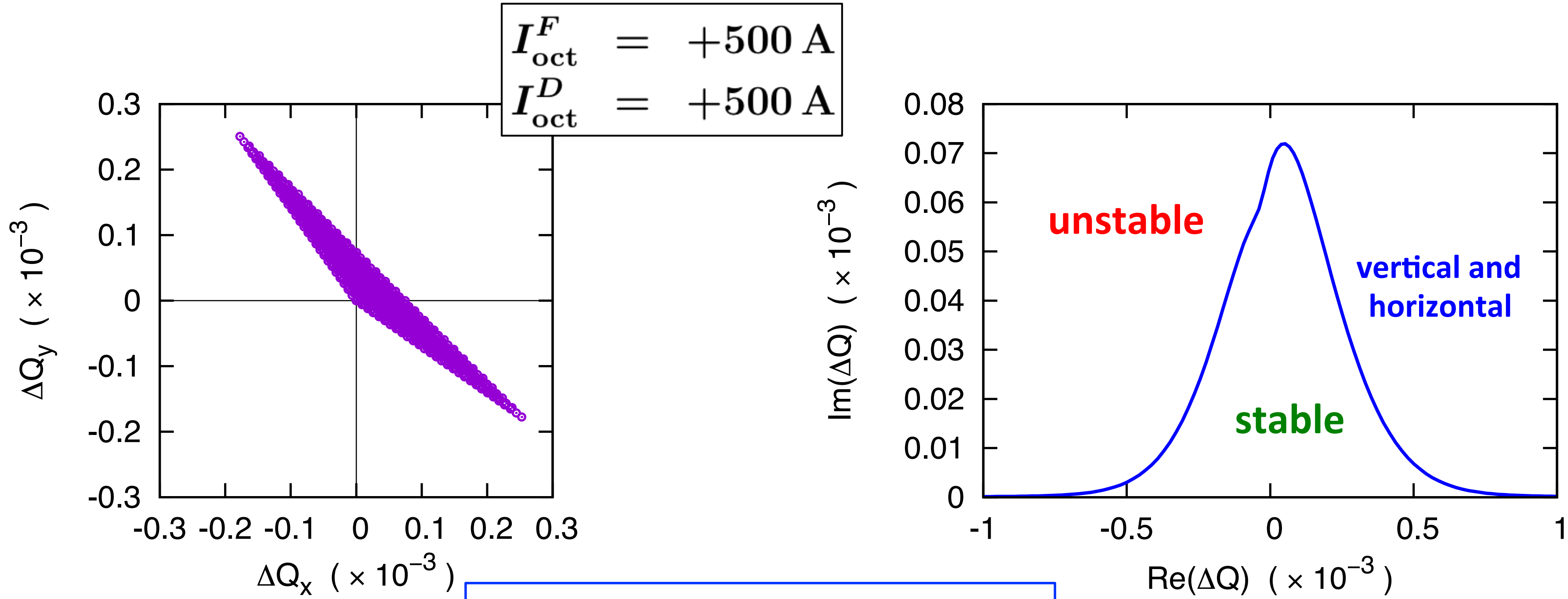
$$\Delta Q_x = (\alpha_x^F I_\mathrm{oct}^F + \alpha_x^D I_\mathrm{oct}^D) J_x - (\alpha_{xy}^F I_\mathrm{oct}^F + \alpha_{xy}^D I_\mathrm{oct}^D) J_y$$

$$\Delta Q_y = (\alpha_y^F I_\mathrm{oct}^F + \alpha_y^D I_\mathrm{oct}^D) J_y - (\alpha_{xy}^F I_\mathrm{oct}^F + \alpha_{xy}^D I_\mathrm{oct}^D) J_x$$

the α−coefficients depend on the beta-functions and on the magnet parameters.

Two knobs: I_{oct}^F, I_{oct}^D (not really F and D, just the beta functions)

Tune Footprint and Stability Diagram



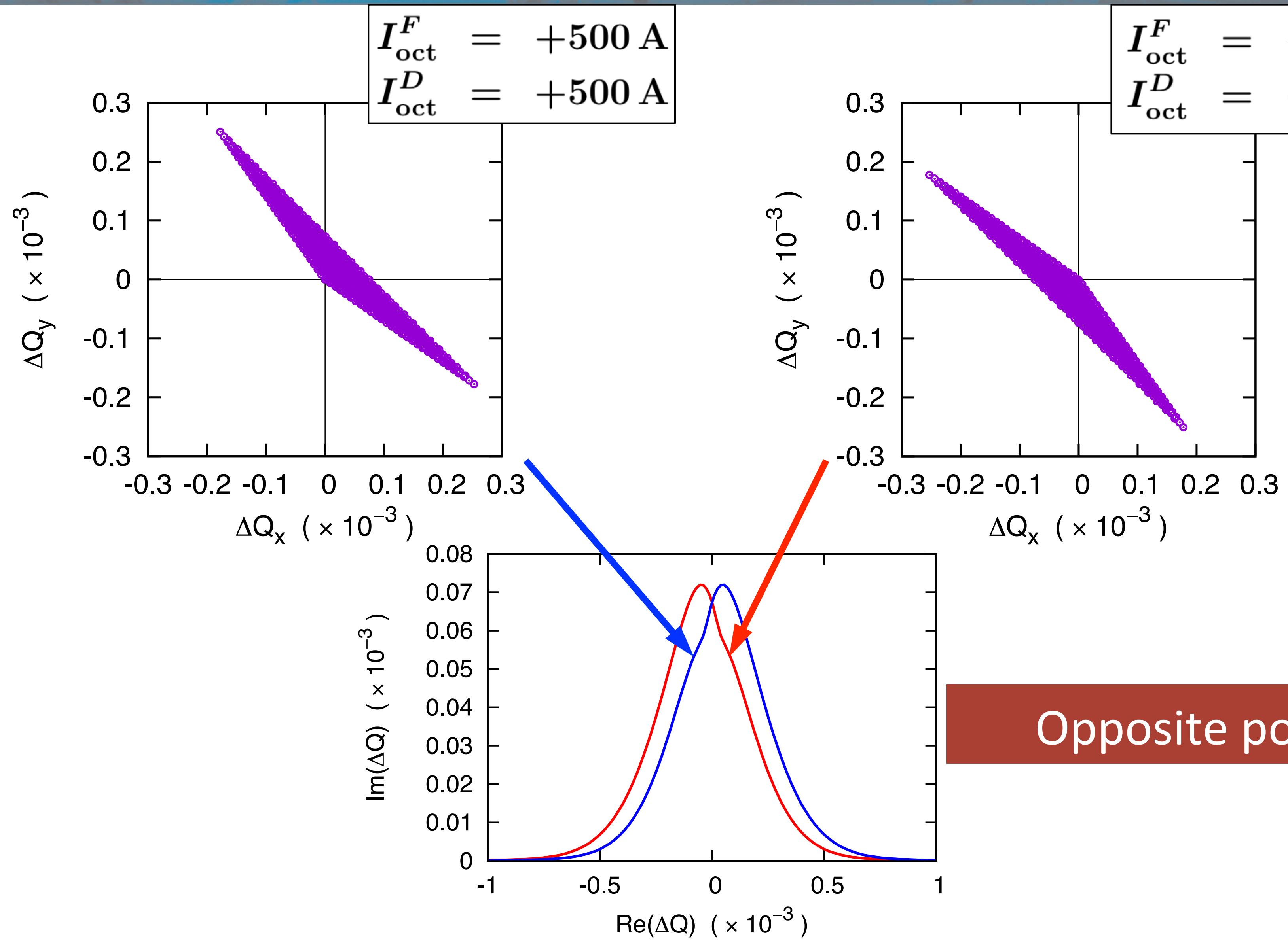
footprint pictures

$$J_x + J_y < 8\sigma^2$$

8σ in action (2.8σ in ampl)
gives ≈ one half of damping

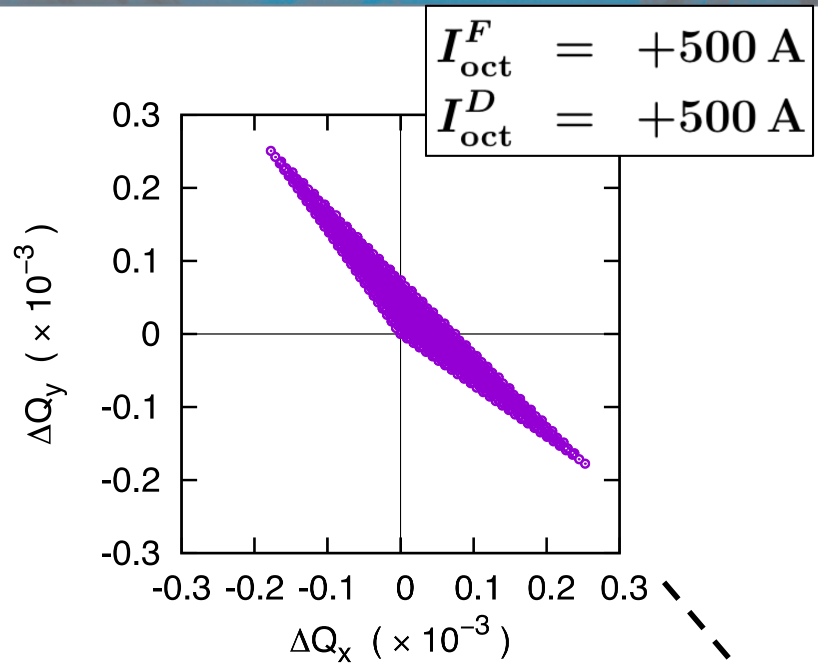
Tune spread provides Landau damping

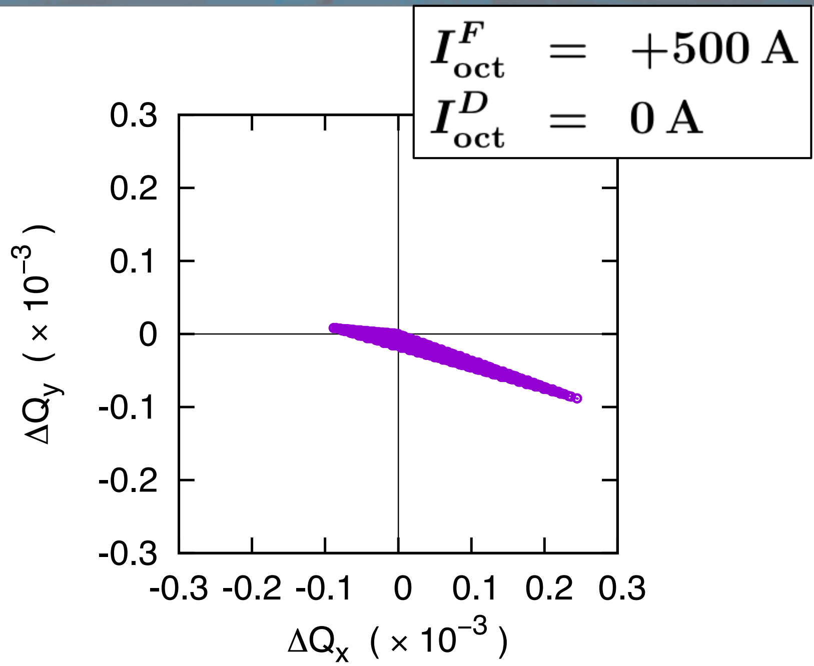
Tune Footprint and Stability Diagram

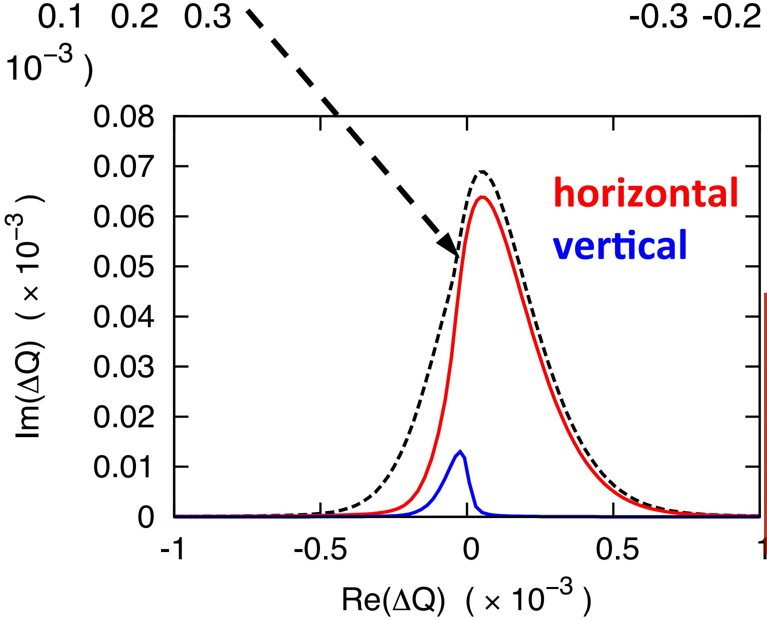


Opposite polarity

Tune Footprint and Stability Diagram





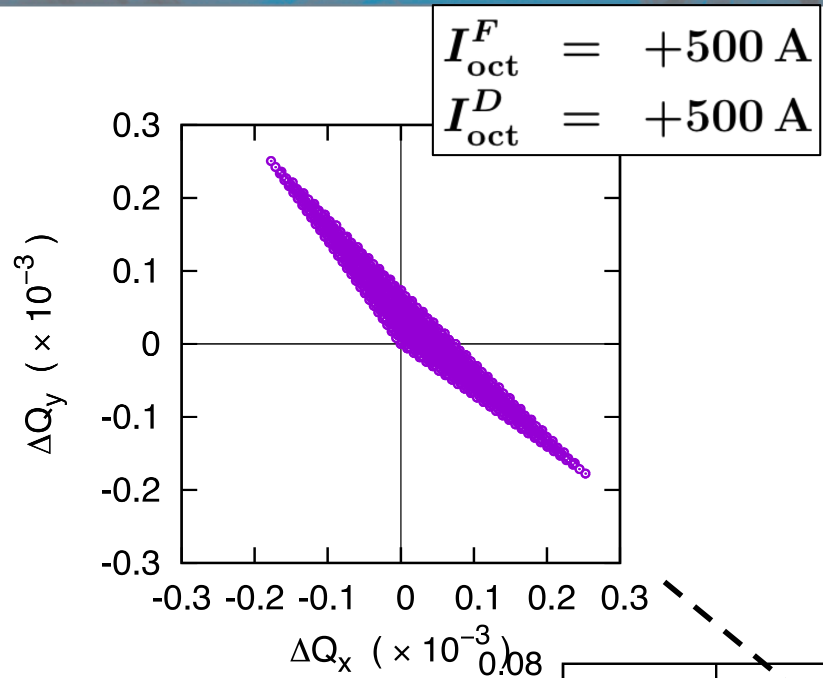


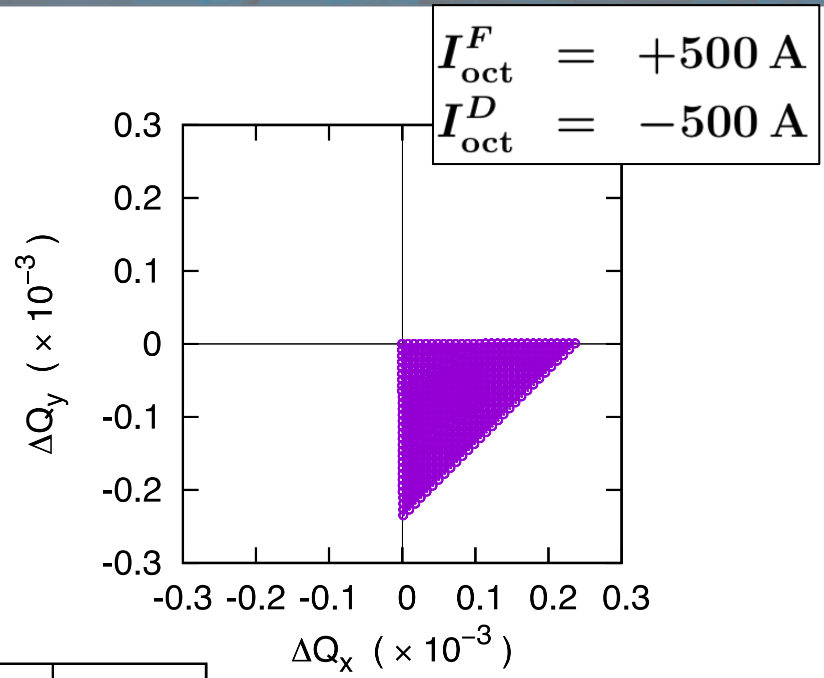
Flexibily with two knobs:

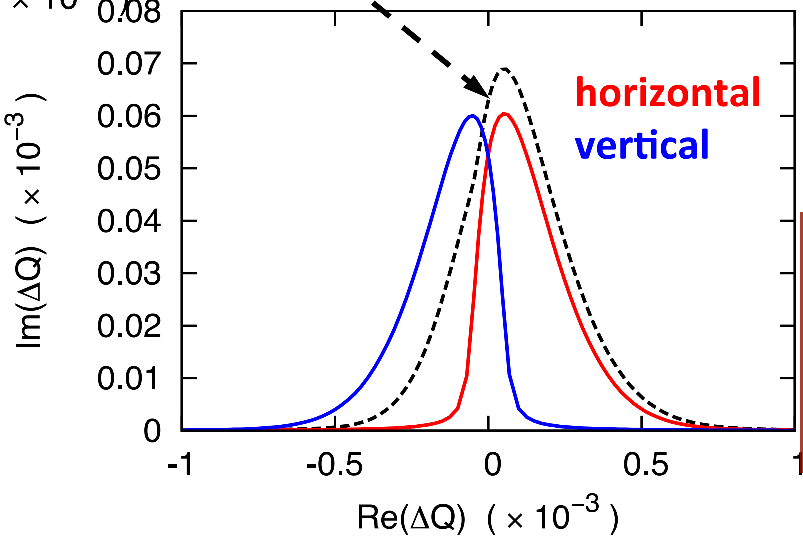
X ⟷ Y

\+ ⟷ −

Tune Footprint and Stability Diagram







Flexibily with two knobs:

X ⟺ Y

\+ ⟺ –

FCC Landau Octupole Scheme

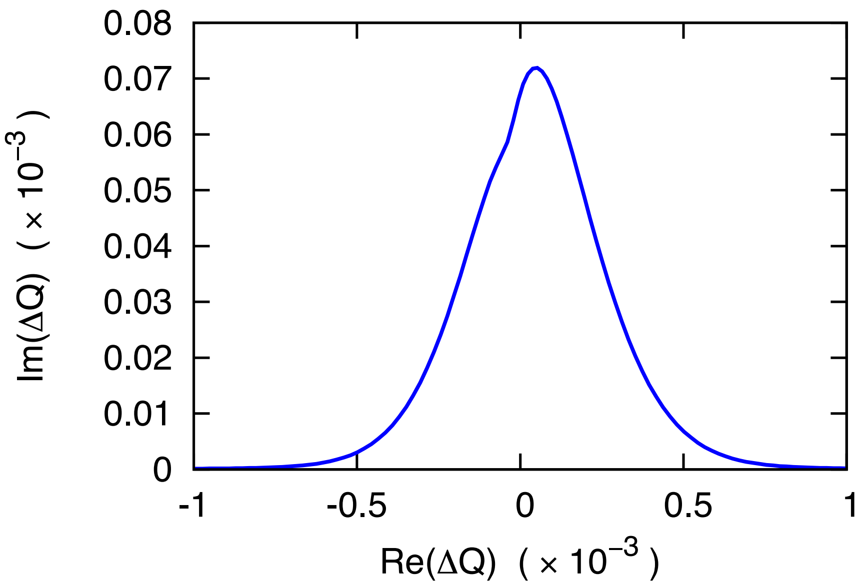
The estimation above for FCC−ΔQ was the Ring Size Scaling from LHC.

The FCC studies of X.Buffat, O.Boine-Frankenheim et.al.:

The instability rise time ≈ 100 turns at 3.3 TeV.
This means Im(ΔQ) ≈ 0.1×10^{-3} at 50 TeV
(no collimators here, thus incomplete)

The TMCI Re(ΔQ) ≈ 10^{-3} at 3.3 TeV.
This means Re(ΔQ) ≈ 0.06×10^{-3} at 50 TeV.

<u>Need to be further specified</u>



Overview FCC Landau Octupoles

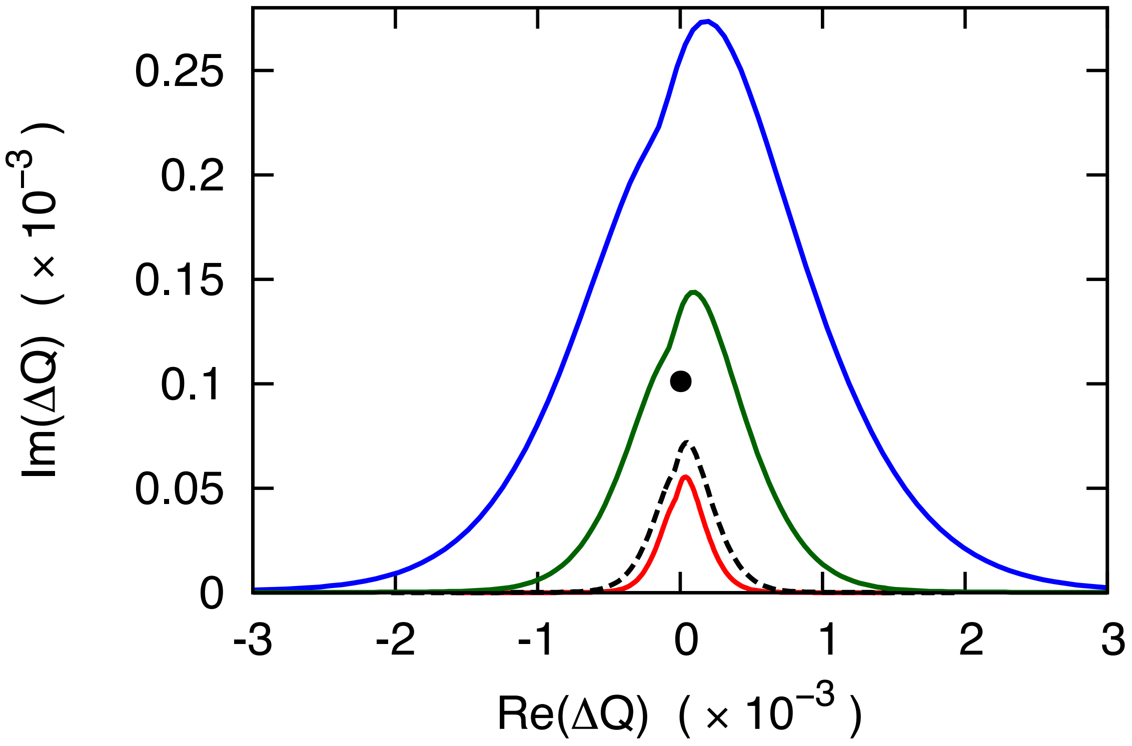
Blue: ΔQ_{coh}–Damping as in LHC.
3646 Octupoles.

Green: enough damping for the (•) studied impedances (no collimators). **1828** octupoles.

Black Dashed: $N_{MO} = N_{MQ} =$ **814** (figures above)

Red: N_{MO} per length as in LHC.
627 octupoles.

LHC: 168 octupoles.
LHC octupole magnets are assumed here.



Stability Diagram:
stable below the line,
unstable above the line.

Conclusions

- For the sufficient stability in FCC, much more octupoles than the LHC N_{MO}/length are needed.
- Different scenario for the combination of octupoles with the feedback systems are possible.
- The **two-knobs (I_{oct}^F, I_{oct}^D)** octupole scheme provides a good flexibility for x− and y− tune shifts and damping.
- Further detailed studies/simulations with impedances −octupoles−feedback are needed.