A new beam-beam effect in collisions with crossing angle

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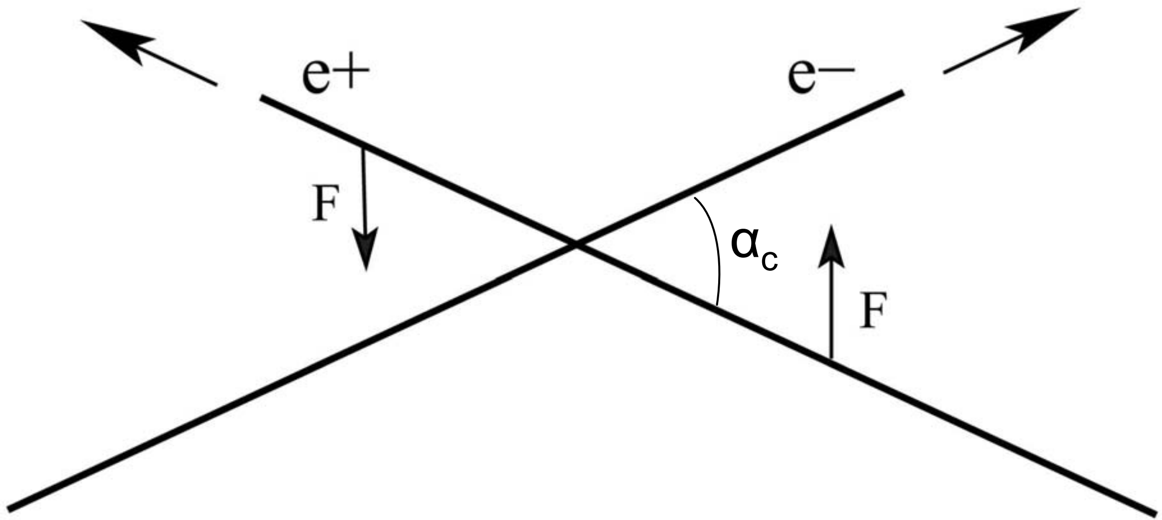
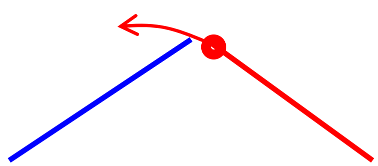
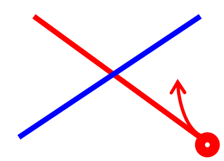
The effect: rotation of the beams

During the collision beams attract each other. The total deflection of the particle at the beam center (z=0) is zero, but the head and the tail of the bunch are deflected horizontally in opposite directions (like in a crab cavity).





the head is attracted
only after beam crossing



the tail is attracted
only before the collision

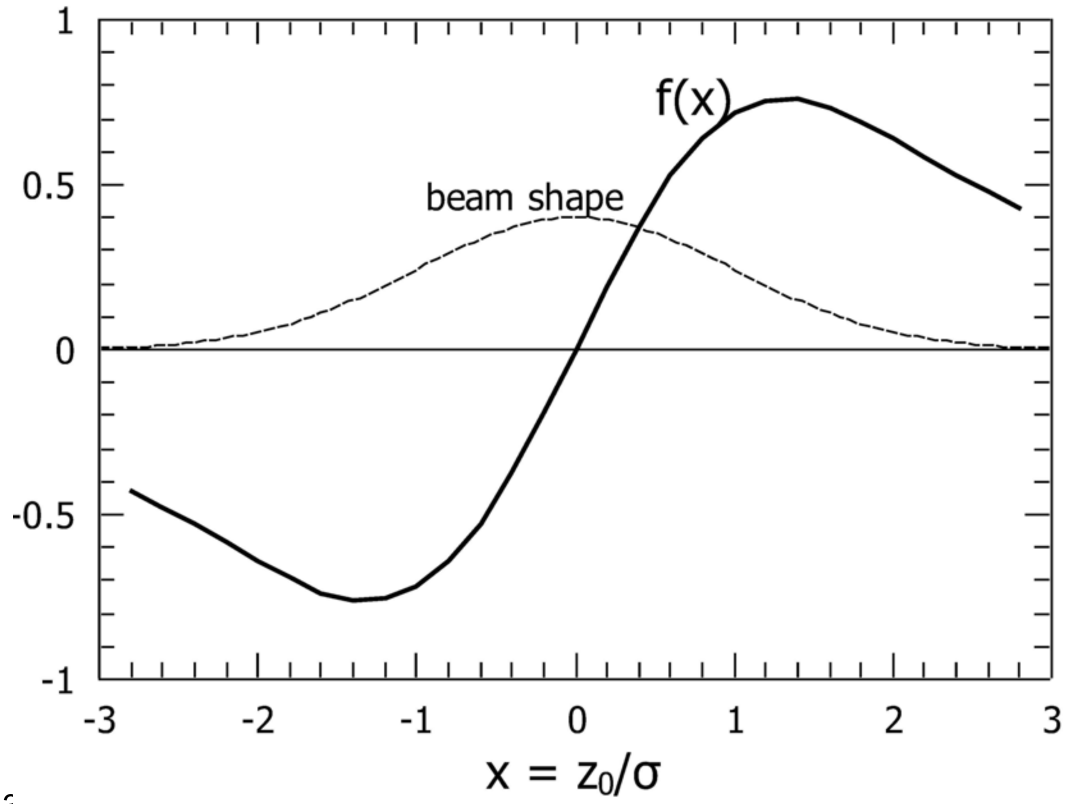
The deflection angle θ_x

$$P_\perp = \int_{-\infty}^{\infty} \frac{4e^2 N}{\sqrt{2\pi}\sigma_z c\alpha_c (z+z_0)} e^{-\frac{(2z+z_0)^2}{2\sigma_z^2}} dz$$

$$\underline{\theta_x = \frac{P_\perp}{\gamma mc} = \frac{4r_e N}{\gamma \sigma_z \alpha_c} f\left(\frac{z_0}{\sigma_z}\right)}$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{-u^2/2}}{x+u} du$$

$$f(x) = x \text{ at } x \ll 1$$



During a collision the particles get transverse kicks dependent on the longitudinal position like in a crab cavity (but more nonlinear on z). The head and the tail are deflected in opposite directions, the maximum deflection angle

$$\theta_x = \frac{4 r_e N}{\gamma \sigma_z \alpha_c} \cdot 0.7$$

The opposite bunch acts as a perturbation (a small permanent magnet, which strength depends on the longitudinal position).

If a short piece of the bunch gets at the IP the deflection angle θ_x, then its displacement at some point in the ring $\Delta x_k = \sqrt{\beta_i \beta_k}\theta_x \sin(\psi_k - \psi_i)$

and after *k* turns its position at the IP, (in the case of 2 IP) will be

$$\Delta x = \beta_x^* \theta_x \left[\sum_k \sin 2\pi k \nu_x + \sum_k \sin(2\pi k \nu_x - \pi \nu_x) \right] \qquad (1)$$

the displacement will undergo beating (depends on k_{max}).

Alternative formula for the displacement of equilibrium orbit (possibly it is not applicable because kick's amplitude varies with synchrotron oscillation period, which is smaller than the damping time

$$\Delta x = \frac{\beta_x^* \theta_x}{2 \sin \pi \nu_x} (\cos(\pi \nu_x) + \cos(\pi \nu_x / 2)) \qquad (2)$$

D. Shatilov suggests n+0.55 (close to ½ integer) oscillations between two IP, in this case one gets

$$\Delta x = \beta_x^* \theta_x * 0.6$$ beating, formula (1)

$$\Delta x = \beta_x^* \theta_x * 0.08$$ displacement, formula (2)

Which one is more correct? Possibly one has to calculate in some way the displacement of equilibrium orbit taking into account variation of kick's amplitude due to synchrotron oscillations.

Meaning of Δx. The deflection angle θ_x depends on the longitudinal position. The IPs for head-head and the tail-tail are shifted on $2\Delta x$. The required $\beta_y \approx 2\Delta x_{max.}$ The max. luminosity in the crab-waist scheme $L \sim 1/\beta_y$. Namely this is the reason why Δx is important!

One can decrease Δx in some way by acceptable level by selecting phases between 2 IP (that is bad, of course – IPs are dependent), but in any case the head and the tail of the beam oscillate in the ring with the amplitude exceeding its horizontal beam size ($\Delta x/\sigma_x \sim \theta_x/\theta_x^*$) which will lead to the dilution of the horizontal emittance due to wakes (see the next slide). The beam in the ring has the tilt (angle) $\Delta x/\sigma_z \sim (\theta x/\theta x^*)(\sigma_x/\sigma_z)$.

Comparison of Δx_1 with a natural σ_x

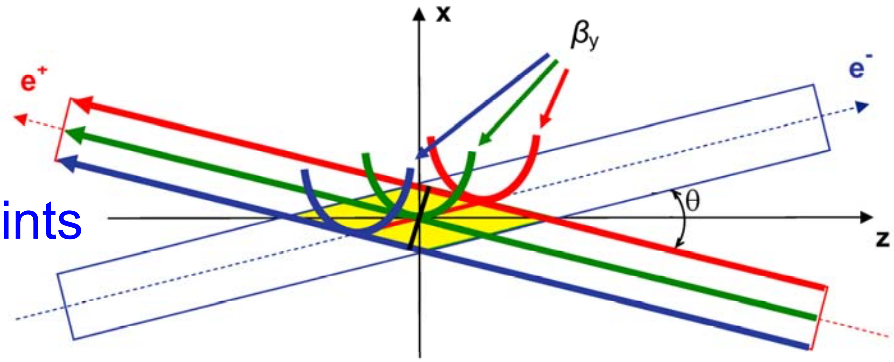
	FCC-ee	c–τ	Sup-KEKB
E_0, GeV	45	2	4
N, 10^{11}	1.	0.72	0.65
σ_z, cm	0.64	1	0.5
α_c, rad	0.03	0.06	0.083
β_x, cm	50	4	3.2
σ_x μm	8	18	10
θ_x, 10^{-4}	0.46	2.4	1.55
$\beta_x\theta_x$, 10^{-3}	2.3	0.95	0.8
$\beta_x\theta_x/\sigma_x$	2.9	0.54	0.5

One can see that $\Delta x \approx \beta_x^* \theta_x$ is comparable with σ_x and for FCC-ee it is even larger by a factor of 3!

Consequences, conclusion

1. The luminosity in the scheme with the crab-waist scheme does not depend on σ_x directly, but $L \sim 1/\beta_y$, where the minimum value of $\beta_y = \sigma_x/\alpha_c$, therefore $L \sim 1/\sigma_x$.

2. In the crab-waist scheme it is important that the waist position coincides with the beam axis, but due to the beam deflection the collision points for the head and the tail of the bunch are shifted (on x) which can lead to appearance of some resonances.



3. The head and the tail of the bunch bunch in the ring undergo horizontal betatron oscillations with the amplitude comparable with the natural σ_x and beams are tilted. This can cause transverse wake fields with undesirable consequences (further increase of the horizontal emittance).

4. In the above consideration we did not consider synchrotron oscillations, it seems they do not change the picture. Their stochastic effects seems small.

Consequences, conclusions (cont.)

5. Installation of compensating crab-cavities after the IP can reduce (partially) the deflection effect.

6. The effect is small for $$\frac{3r_e N}{\gamma \sigma_z \alpha_c} < \frac{\sigma_x}{\beta_x}$$ - it is an additional constraint on beam parameters in the crab-waist scheme

7. This effects needs further analyses by experts.

8. This effect determines the sign of the crossing angle at the IP2, in one case the second IP compensate the kick in the IP1, in the other - kicks add up (D. Shatilov)