A new beam-beam effect in collisions with crossing angle

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The effect: rotation of the beams

During the collision beams attract each other. The total deflection of the particle at the beam center \((z=0)\) is zero, but the head and the tail of the bunch are deflected horizontally in opposite directions (like in a crab cavity).

\[
\alpha_c
\]

\[\begin{align*}
\text{the head is attracted only after beam crossing} \\
\text{the tail is attracted only before the collision}
\end{align*}\]
The deflection angle $\theta_x$

\[ P_\perp = \int_{-\infty}^{\infty} \frac{4e^2 N}{\sqrt{2\pi} \sigma_z c \alpha_c(z + z_0)} e^{-\frac{(2z + z_0)^2}{2\sigma_z^2}} dz \]

\[ \theta_x = \frac{P_\perp}{\gamma mc} = \frac{4r_e N}{\gamma \sigma_z \alpha_c} f\left(\frac{z_0}{\sigma_z}\right) \]

\[ f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{-u^2/2}}{x + u} du \]

\[ f(x) = x \text{ at } x \ll 1 \]

\[ x = \frac{z_0}{\sigma} \]

beam shape
During a collision the particles get transverse kicks dependent on the longitudinal position like in a crab cavity (but more nonlinear on z). The head and the tail are deflected in opposite directions, the maximum deflection angle

$$\theta_x = \frac{4r e N}{\gamma \sigma_z \alpha_c} \cdot 0.7$$

The opposite bunch acts as a perturbation (a small permanent magnet, which strength depends on the longitudinal position).

If a short piece of the bunch gets at the IP the deflection angle $\theta_x$, then its displacement at some point in the ring $\Delta x_k = \sqrt{\beta_i \beta_k} \theta_x \sin(\psi_k - \psi_i)$

and after $k$ turns its position at the IP, (in the case of 2 IP) will be

$$\Delta x = \beta_x^* \theta_x \left[ \sum_k \sin 2\pi k \nu \right]$$

(1)

the displacement will undergo beating (depends on $k_{\text{max}}$).

Alternative formula for the displacement of equilibrium orbit (possibly it is not applicable because kick’s amplitude varies with synchrotron oscillation period, which is smaller than the damping time

$$\Delta x = \frac{\beta_x^* \theta_x}{2 \sin \pi \nu} \left( \cos(\pi \nu) + \cos(\pi \nu / 2) \right)$$

(2)
D. Shatilov suggests \( n + 0.55 \) (close to \( \frac{1}{2} \) integer) oscillations between two IP, in this case one gets

\[
\Delta x = \beta_x^* \theta_x * 0.6 \quad \text{beating, formula (1)}
\]
\[
\Delta x = \beta_x^* \theta_x * 0.08 \quad \text{displacement, formula (2)}
\]

Which one is more correct? Possibly one has to calculate in some way the displacement of equilibrium orbit taking into account variation of kick’s amplitude due to synchrotron oscillations.

**Meaning of \( \Delta x \).** The deflection angle \( \theta_x \) depends on the longitudinal position. The IPs for head-head and the tail-tail are shifted on \( 2 \Delta x \). The required \( \beta_y \approx 2 \Delta x_{\text{max}} \). The max. luminosity in the crab-waist scheme \( L \sim 1/\beta_y \). Namely this is the reason why \( \Delta x \) is important!

One can decrease \( \Delta x \) in some way by acceptable level by selecting phases between 2 IP (that is bad, of course – IPs are dependent), but in any case the head and the tail of the beam oscillate in the ring with the amplitude exceeding its horizontal beam size \( (\Delta x/\sigma_x \sim \theta_x/\theta_x^*) \) which will lead to the dilution of the horizontal emittance due to wakes (see the next slide). The beam in the ring has the tilt (angle) \( \Delta x/\sigma_z \sim (\theta_x/\theta_x^*)(\sigma_x/\sigma_z) \).
### Comparison of $\Delta x_1$ with a natural $\sigma_x$

<table>
<thead>
<tr>
<th></th>
<th>FCC-ee</th>
<th>c–$\tau$</th>
<th>Sup-KEKB</th>
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<tbody>
<tr>
<td>$E_0$, GeV</td>
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<td>$N$, $10^{11}$</td>
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<td>$\sigma_z$, cm</td>
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<td>$\alpha_c$, rad</td>
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<td>0.06</td>
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<td>$\beta_x$, cm</td>
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<tr>
<td>$\sigma_x$, $\mu$m</td>
<td>8</td>
<td>18</td>
<td>10</td>
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<tr>
<td>$\theta_x$, $10^{-4}$</td>
<td>0.46</td>
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<td>$\beta_x \theta_x$, $10^{-3}$</td>
<td>2.3</td>
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<td>0.8</td>
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<tr>
<td>$\beta_x \theta_x / \sigma_x$</td>
<td>2.9</td>
<td>0.54</td>
<td>0.5</td>
</tr>
</tbody>
</table>

One can see that $\Delta x \approx \beta_x \theta_x$ is comparable with $\sigma_x$ and for FCC-ee it is even larger by a factor of 3!
Consequences, conclusion

1. The luminosity in the scheme with the crab-waist scheme does not depend on \( \sigma_x \) directly, but \( L \sim 1/\beta_y \), where the minimum value of \( \beta_y = \sigma_x/\alpha_c \), therefore \( L \sim 1/\sigma_x \).

2. In the crab-waist scheme it is important that the waist position coincides with the beam axis, but due to the beam deflection the collision points for the head and the tail of the bunch are shifted (on \( x \)) which can lead to appearance of some resonances.

3. The head and the tail of the bunch bunch in the ring undergo horizontal betatron oscillations with the amplitude comparable with the natural \( \sigma_x \) and beams are tilted. This can cause transverse wake fields with undesirable consequences (further increase of the horizontal emittance).

4. In the above consideration we did not consider synchrotron oscillations, it seems they do not change the picture. Their stochastic effects seems small.
Consequences, conclusions (cont.)

5. Installation of compensating crab-cavities after the IP can reduce (partially) the deflection effect.

6. The effect is small for \[
\frac{3r_e N}{\gamma \sigma_z \alpha_c} \frac{\sigma_x}{\beta_x} < 1
\] - it is an additional constraint on beam parameters in the crab-waist scheme

7. This effect needs further analyses by experts.

8. This effect determines the sign of the crossing angle at the IP2, in one case the second IP compensate the kick in the IP1, in the other - kicks add up (D. Shatilov)