

# $\alpha_s$ : status and FCC-ee prospects

**FCC week 2016**

Rome, 14<sup>th</sup> April 2016

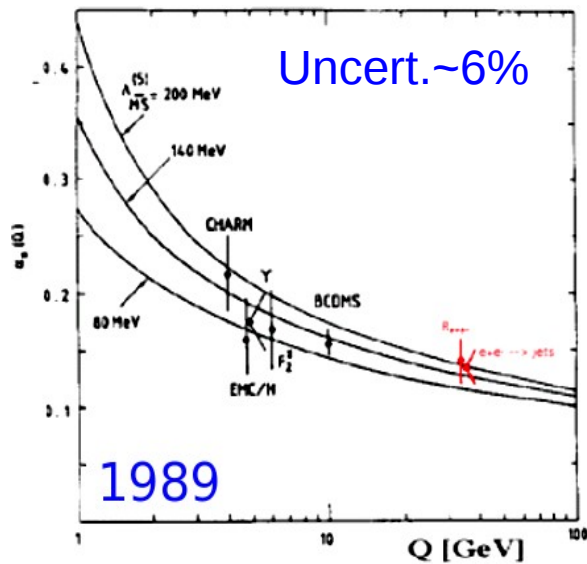
**David d'Enterria<sup>(\*)</sup>**

**CERN**

(\*) D. d'Enterria, M. Srebre, " $\alpha_s$ ,  $V_{cs}$ , and CKM unitarity test from hadronic W boson decays at NNLO", [arXiv:1603.06501](https://arxiv.org/abs/1603.06501)  
D. d'Enterria, P.Z. Skands (eds.), Proceeds. "High-precision  $\alpha_s$  from LHC to FCC-ee", CERN Oct. 2015; [arXiv:1512.05194](https://arxiv.org/abs/1512.05194)

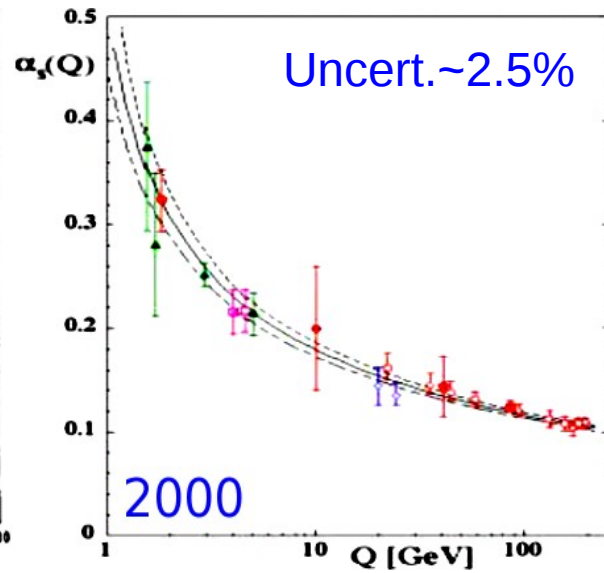
# QCD coupling $\alpha_s$

- Determines **strength of the strong interaction** between quarks & gluons
- **Single free parameter in QCD** (in the  $m_q \rightarrow 0$  limit)
- Determined at a ref. scale ( $Q=m_Z$ ), decreases as  $\alpha_s \sim \ln(Q^2/\Lambda^2)^{-1}$ ,  $\Lambda \sim 0.2$  GeV



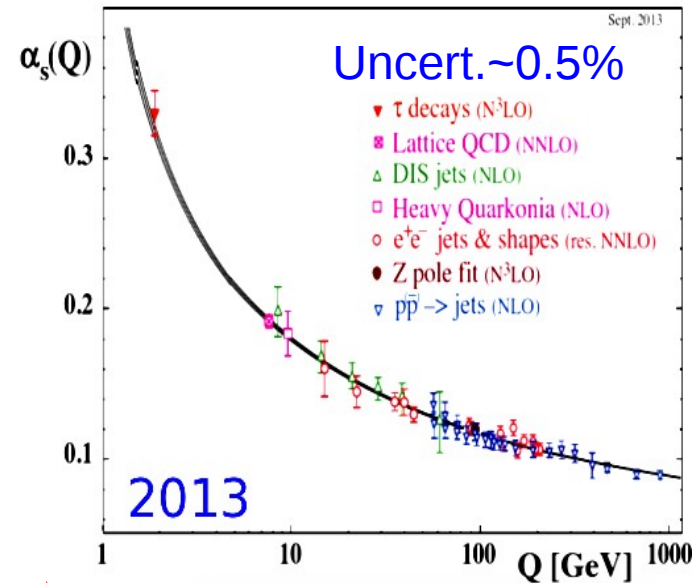
$$\alpha_s(M_Z) = 0.110^{+0.006}_{-0.008} \text{ (NLO)}$$

G. Altarelli, Ann. Rev. Nucl. Part. Sci. 39, 1989



$$\alpha_s(M_Z) = 0.1184 \pm 0.0031 \text{ (NNLO)}$$

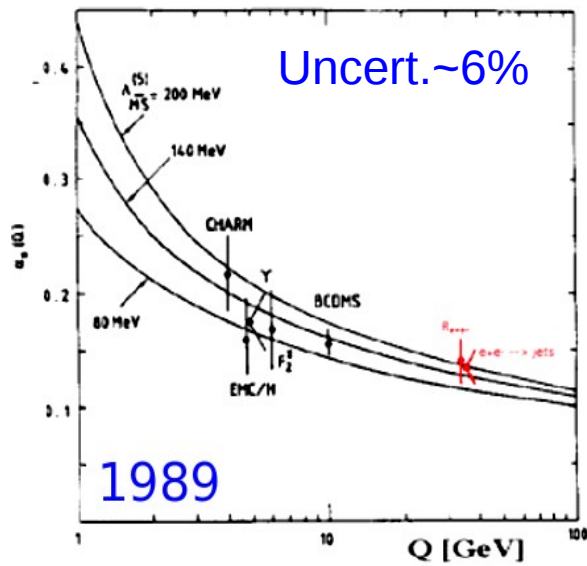
S. B. , J. Phys. G 26, 2000



$$\alpha_s(M_Z) = 0.1185 \pm 0.0006 \text{ (NNLO)}$$

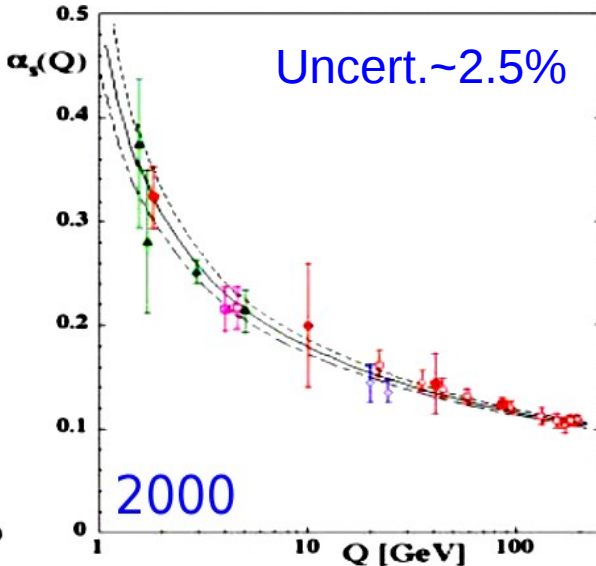
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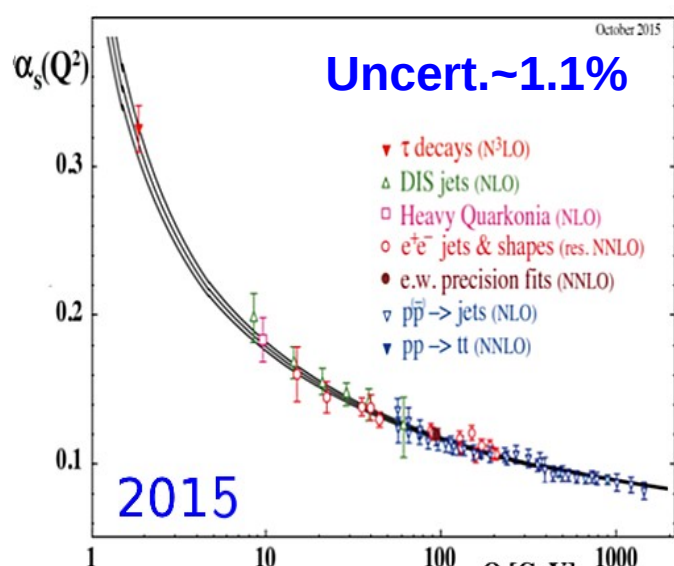
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S. B., J. Phys. G 26, 2000



$$\alpha_s(M_Z) = 0.1181 \pm 0.0013 \text{ (NNLO)}$$

➔ **Least precisely known** of all interaction **couplings** !

$$\delta\alpha \sim 10^{-10} \ll \delta G_F \ll 10^{-7} \ll \delta G \sim 10^{-5} \ll \delta\alpha_s \sim 10^{-3}$$

# Importance of the QCD coupling $\alpha_s$

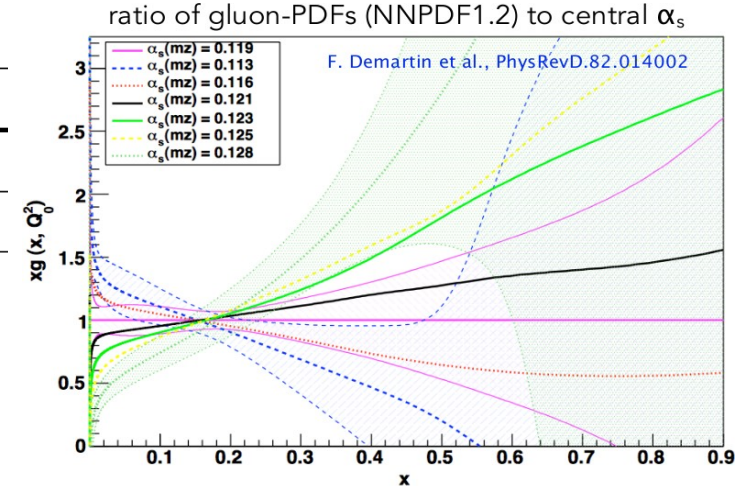
➔ Impacts all LHC x-sections & QCD decays, chiefly for Higgs:

Uncertainties (update of [LHC HXSWG 2013] for  $\sqrt{s} = 14$  TeV)

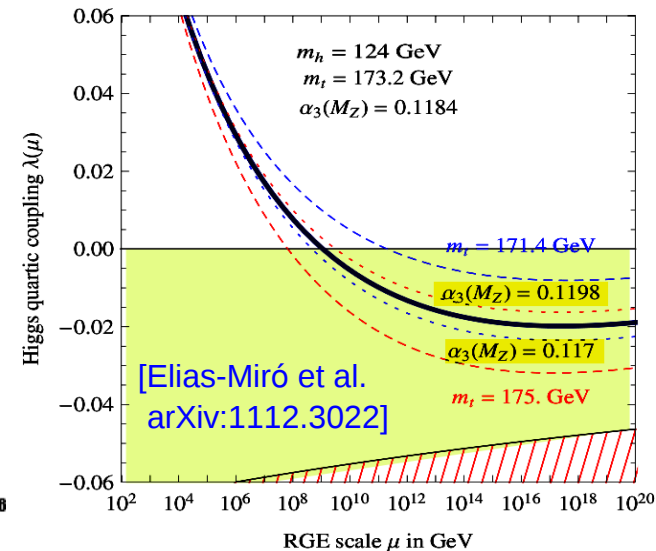
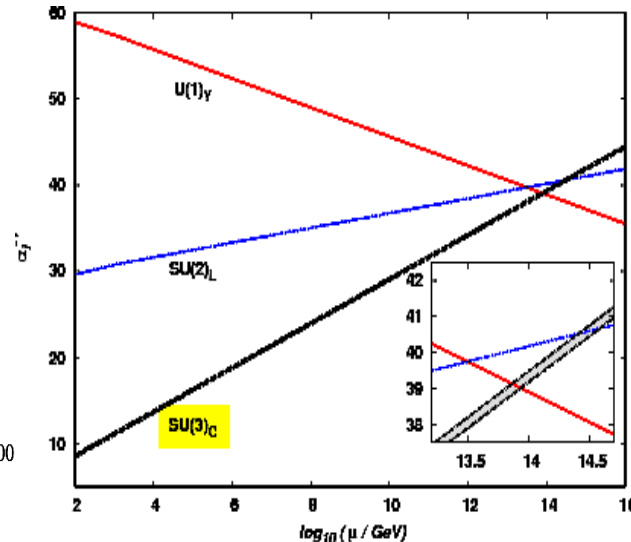
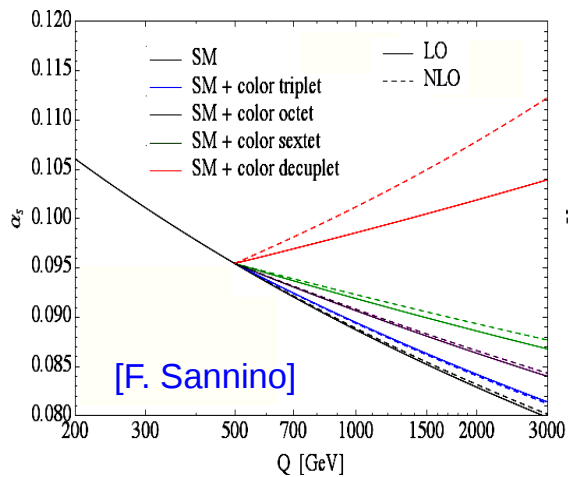
Process	$\sigma$ (pb)	$\delta\alpha_s(\%)$	PDF + $\alpha_s(\%)$	Scale(%)
ggH	49.87	$\pm 3.7$	-6.2 +7.4	-2.61 + 0.32
ttH	0.611	$\pm 3.0$	$\pm 8.9$	-9.3 + 5.9

Channel	$M_H$ [GeV]	$\delta\alpha_s(\%)$	$\Delta m_b$	$\Delta m_c$
H $\rightarrow$ c $\bar{c}$	126	$\pm 7.1$	$\pm 0.1\%$	$\pm 2.3\%$
H $\rightarrow$ gg	126	$\pm 4.1$	$\pm 0.1\%$	$\pm 0\%$

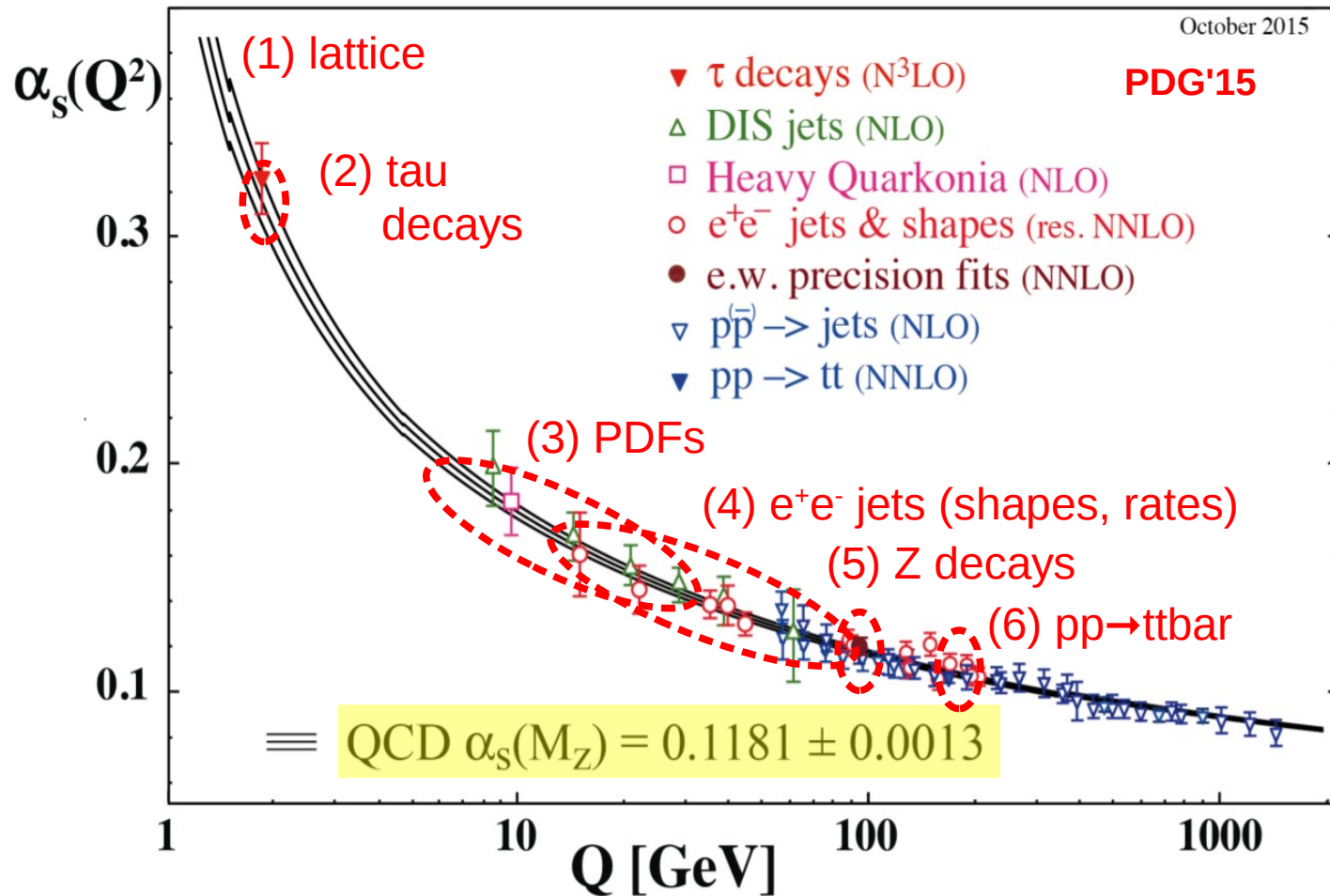


➔ Impacts new physics: new colored sectors, GUT, EW vacuum stability



# Status of $\alpha_s$ determination (PDG 2015)

- Determined by comparing 6 experimental observables to pQCD NNLO, N<sup>3</sup>LO predictions, and performing a global average of their propagated values at the Z pole scale:



# (1) $\alpha_s$ from lattice QCD

- Comparison of short-distance quantities (Wilson loops,  $Q\bar{Q}$  potential, vacuum polariz.,...) computed at NNLO in pQCD, to lattice QCD “data”:

$$K^{\text{NP}} = K^{\text{PT}} = \sum_{i=0}^n c_i \alpha_s^i$$

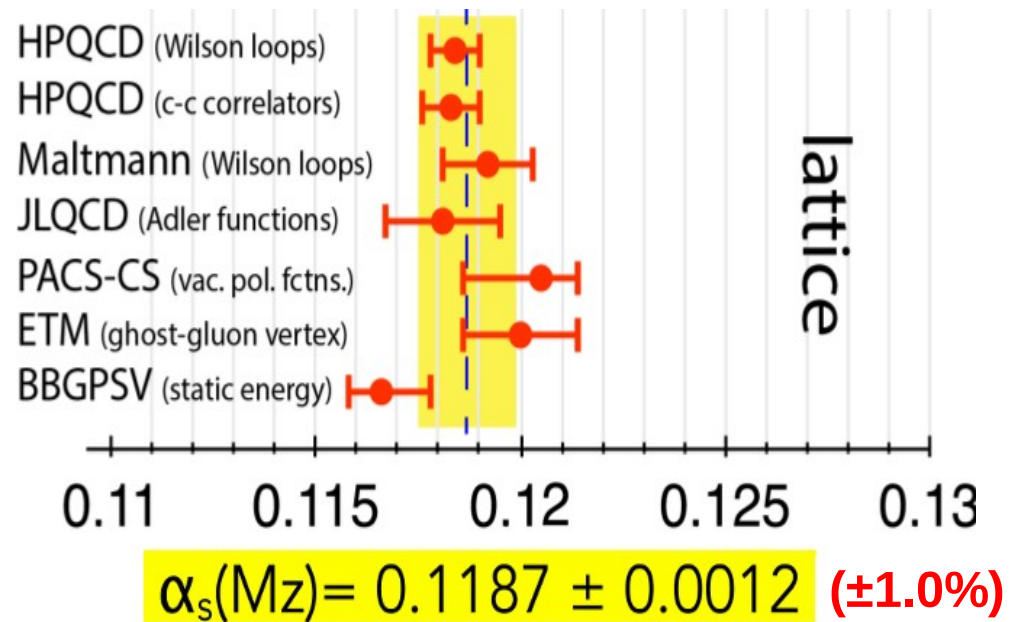
[FLAG Collab. <http://itpwiki.unibe.ch/flag>]

- Currently, it's extraction with **smallest uncertainties:  $\pm 1\%$**  (lattice spacing & statistics).

Extracted value depends on observables:

Uncertainty **increased**:

**2013 ( $\pm 0.4\%$ )  $\rightarrow$  2015 ( $\pm 1.0\%$ )**



- Future prospects:

- **Uncertainty in  $\alpha_s$  could be halved** with (much) better numerical data.
- Reaching  **$\pm 0.1\%$  requires 4<sup>th</sup>-loop** perturbation theory (~10 years?)



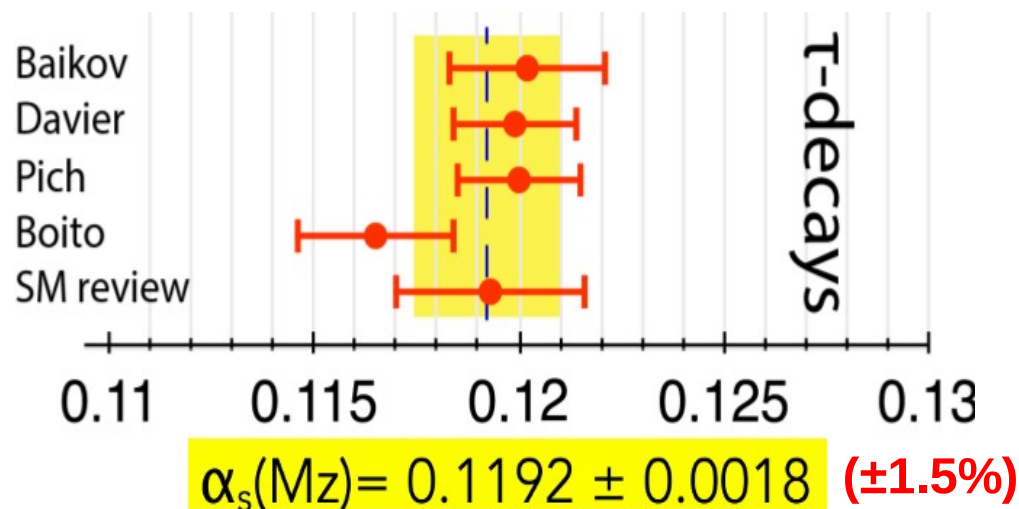
# (2) $\alpha_s$ from hadronic $\tau$ -lepton decays

➔ Computed at **N<sup>3</sup>LO**:  $R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} = S_{\text{EW}} N_C (1 + \sum_{n=1}^4 c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_{\text{np}})$

➔ Experimentally:  $R_{\tau, \text{exp}} = 3.4697 \pm 0.0080$  ( $\pm 0.23\%$ )

➔ Various pQCD approaches (**FOPT vs CIPT**) & treatment of non-pQCD contributions, yield different results.

Uncertainty slightly increased:  
2013 ( $\pm 1.3\%$ )  $\rightarrow$  2015 ( $\pm 1.5\%$ )



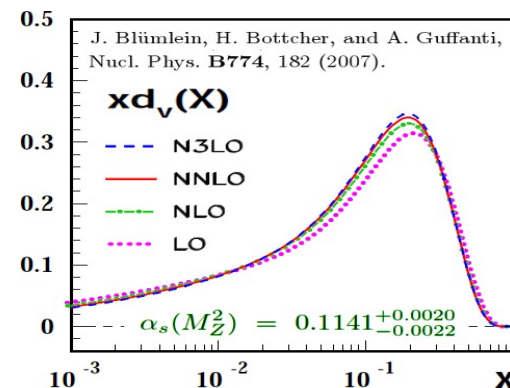
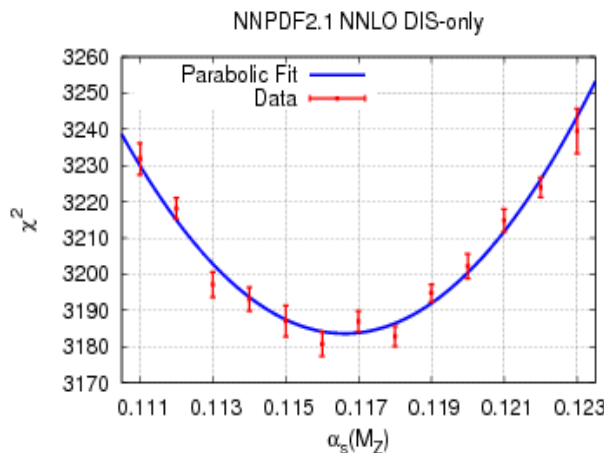
➔ Future prospects:

- Better understanding of **FOPT vs. CIPT differences**.
- **Better spectral functions** needed (high stats & precision): B-factories (BELLE-2), future (ILC, **FCC-ee**).

# (3) $\alpha_s$ from proton structure functions

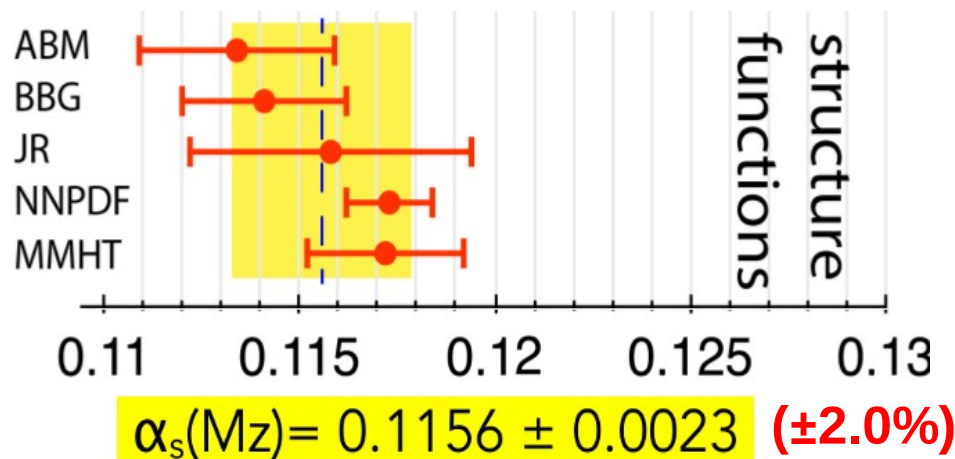
- Computed at **N<sup>2,3</sup>LO**:  $F_2(x, Q^2) = x \sum_{n=0}^{\infty} \frac{\alpha_s^n(\mu_R^2)}{(2\pi)^n} \sum_{i=q,g} \int_x^1 \frac{dz}{z} C_{2,i}^{(n)}(z, Q^2, \mu_R^2, \mu_F^2) f_{i/p}\left(\frac{x}{z}, \mu_F^2\right) + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$
- Experimentally: Multiple  $F_2(x, Q^2)$ ,  $F_2^c(x, Q^2)$ ,  $F_L(x, Q^2)$ , PDFs(x, Q<sup>2</sup>)

- Different **approaches**:  
 Non-singlet fits,  
 singlet+non-singlet fits,  
 global fits of PDFs, ...



Uncertainty slightly increased:  
 2013 ( $\pm 1.7\%$ )  $\rightarrow$  2015 ( $\pm 2.0\%$ )

- Future prospects:
  - Full-NNLO PDF fits (including tbar, jets,...)



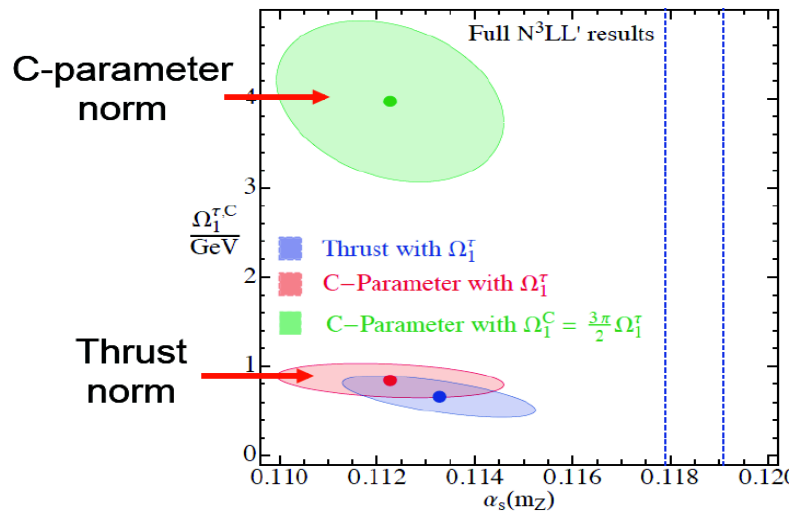


# (4) $\alpha_s$ from $e^+e^-$ jets (event shapes & rates)

➤ Computed at  $N^{2,3}LO+N^{(2)}LL$  accuracy.

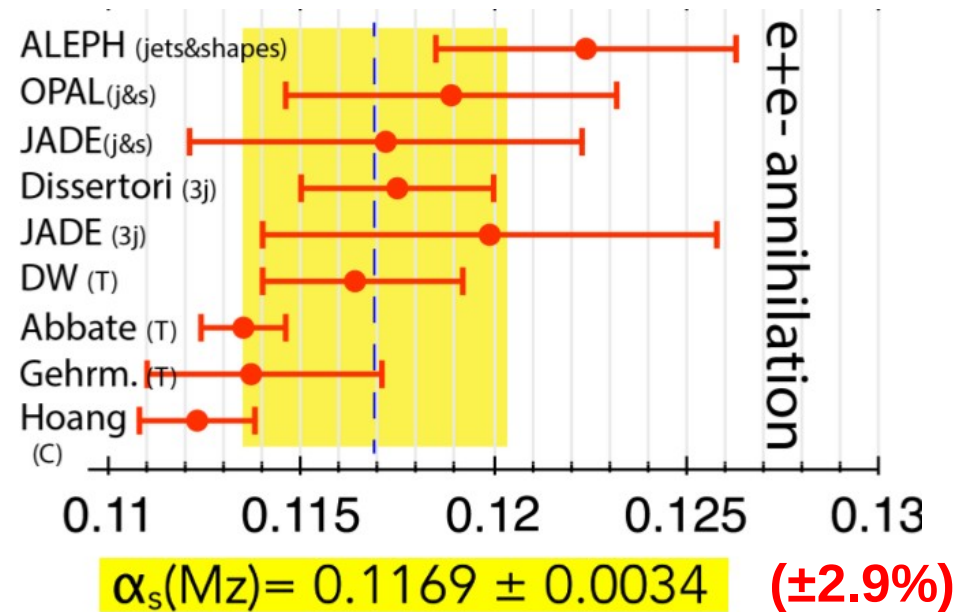
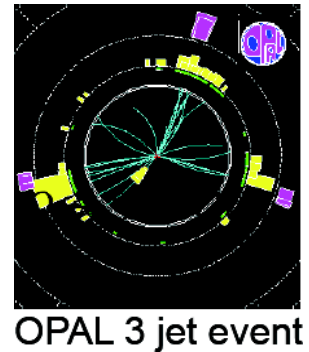
➤ Experimentally (LEP):  
Thrust, C-parameter, jet shapes  
3-jet x-sections

➤ Results sensitive to non-pQCD  
(hadronization) accounted for  
via MCs or analytically:



$$\tau = 1 - \max_{\hat{n}} \frac{\sum |\vec{p}_i \cdot \hat{n}|}{\sum |\vec{p}_i|}$$

$$C = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_i |\vec{p}_i|)^2}$$

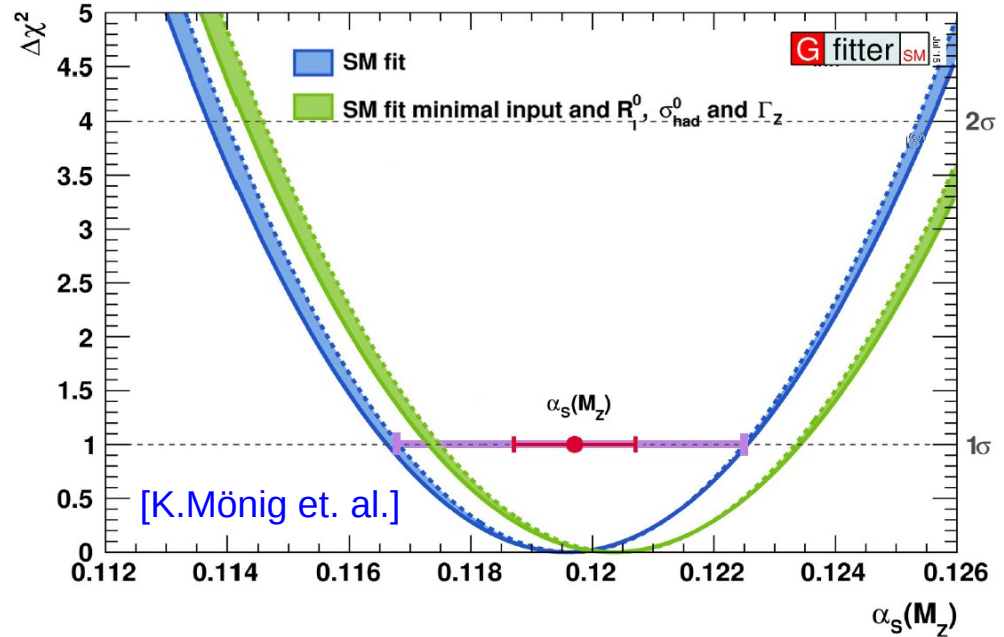
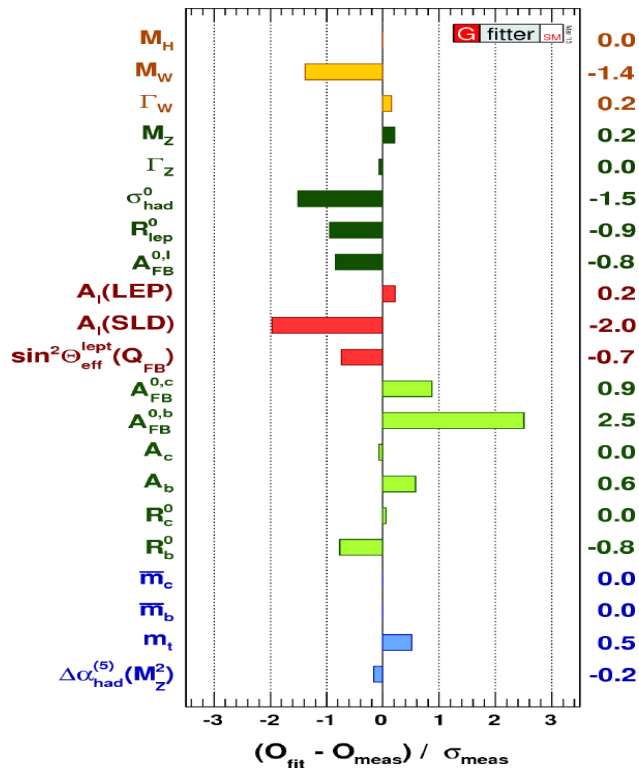


➤ Prospects:

- New  $e^+e^-$  data: low- $\sqrt{s}$  (Belle-II) for shapes, high- $\sqrt{s}$  (FCC-ee) for rates
- Jet rates with improved resummation: NNLL or  $N^3LL$

# (5) $\alpha_s$ from hadronic Z decays

- Computed at **N<sup>3</sup>LO**:  $R_Z \equiv \frac{\Gamma(Z \rightarrow h)}{\Gamma(Z \rightarrow l)} = R_Z^{\text{EW}} N_C (1 + \sum_{n=1}^4 c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5) + \delta_m + \delta_{\text{np}})$
- Experim.:  $\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV } (\pm 0.1\%)$ ,  $R_\ell^0 = \frac{\Gamma_{\text{had}}}{\Gamma_\ell}$ ,  $\sigma_{\text{had}}^0 = \frac{12\pi}{m_Z} \frac{\Gamma_e \Gamma_{\text{had}}}{\Gamma_Z^2}$ ,  $\sigma_\ell^0 = \frac{12\pi}{m_Z} \frac{\Gamma_\ell^2}{\Gamma_Z^2}$
- After Higgs discovery,  $\alpha_s$  can be directly determined from **full fit of SM**:

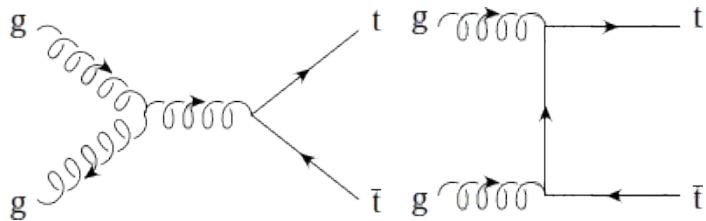


$$\alpha_s(M_Z) = 0.1196 \pm 0.0030 \quad (\pm 2.5\%)$$

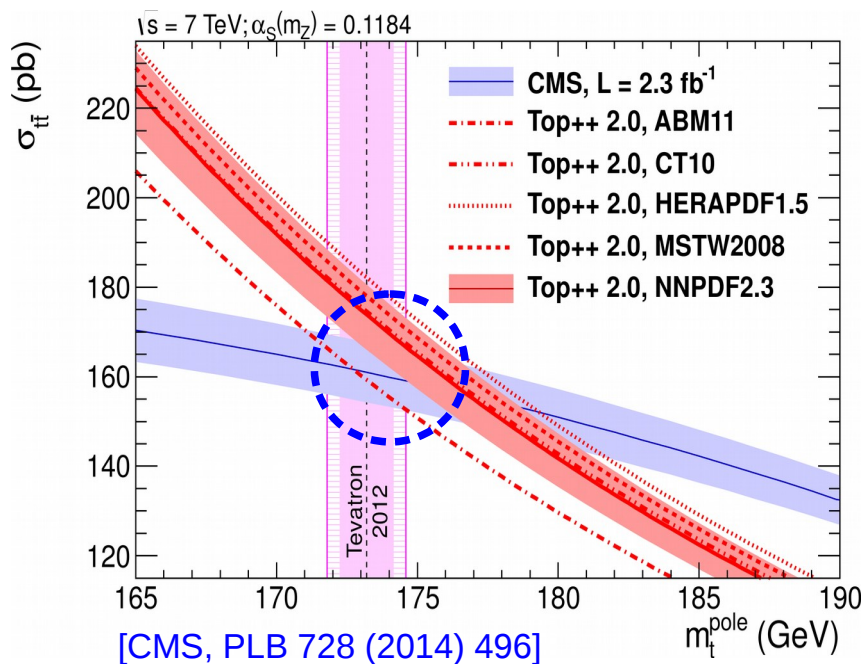
- Prospects:
  - Huge Z stats at **FCC-ee** will lead to:  $\delta \alpha_s < 0.3\%$
  - Improved parametric ( $\sin^2 \theta_{\text{eff}}^{\text{lept}}, m_W, m_t$ ) and TH (**N<sup>4</sup>LO, EW, mixed**) uncert.

# (6) $\alpha_s$ from top-pair p-p cross sections

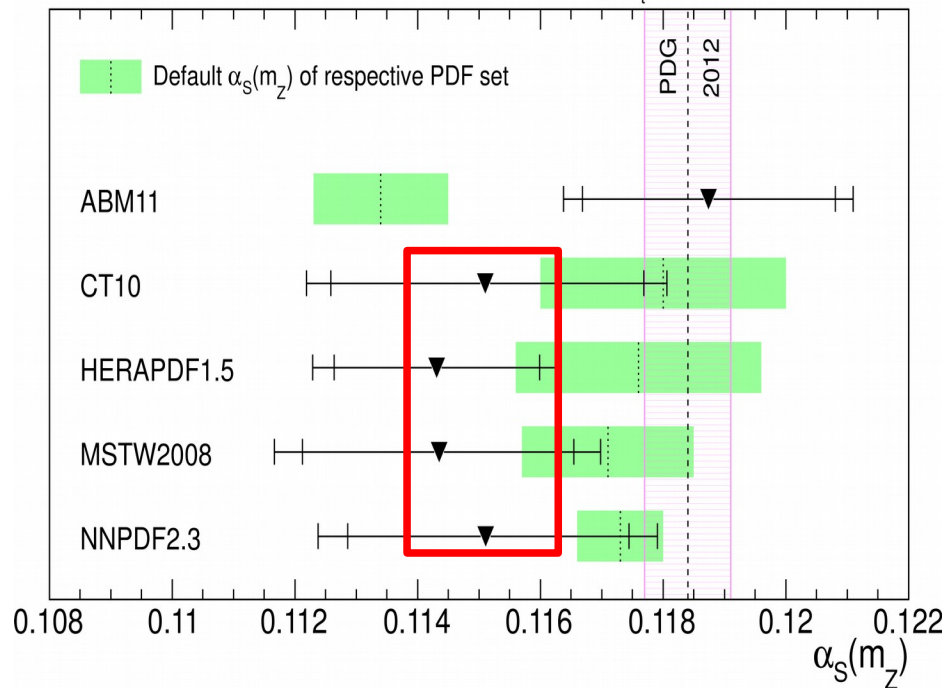
- Total top-antitop cross section (known at NNLO+NNLL) is the 1<sup>st</sup> p-p collider observable to constrain  $\alpha_s$  at NNLO accuracy:



Data-theory x-section comparison for varying PDF+ $\alpha_s$  as a function of  $m_{top}$ :



CMS,  $\sqrt{s} = 7 \text{ TeV}, L = 2.3 \text{ fb}^{-1}$ ; NNLO+NNLL for  $\sigma_{tt}; m_t^{\text{pole}} = 173.2 \pm 1.4 \text{ GeV}$

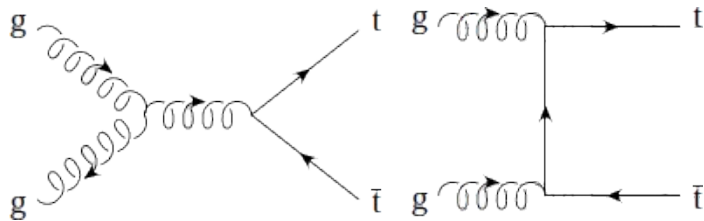


Precise measurement dominated by associated PDF uncertainty ( $\pm 2.5\%$ )

$$\alpha_s(M_Z^2) = 0.1151^{+0.0028}_{-0.0027}$$

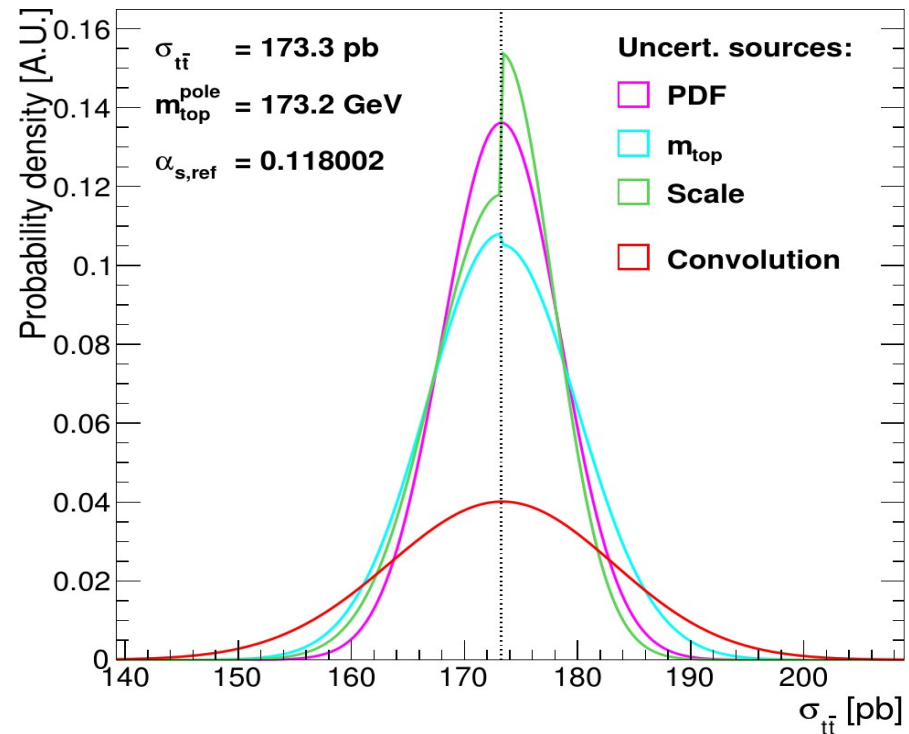
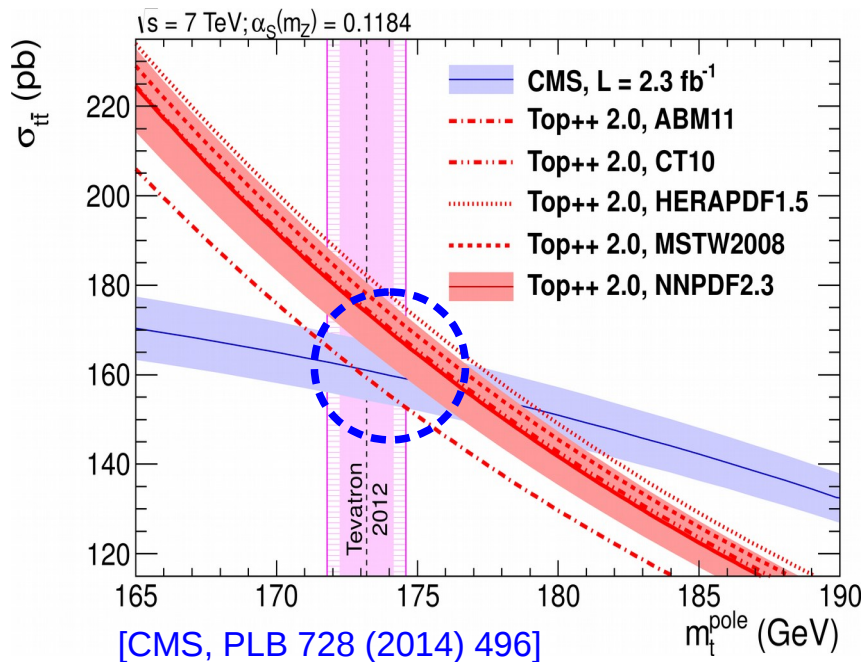
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[G.Salam et al. arXiv:1512.05194]

Data-theory x-section comparison for varying PDF+ $\alpha_s$  as a function of  $m_{top}$ :



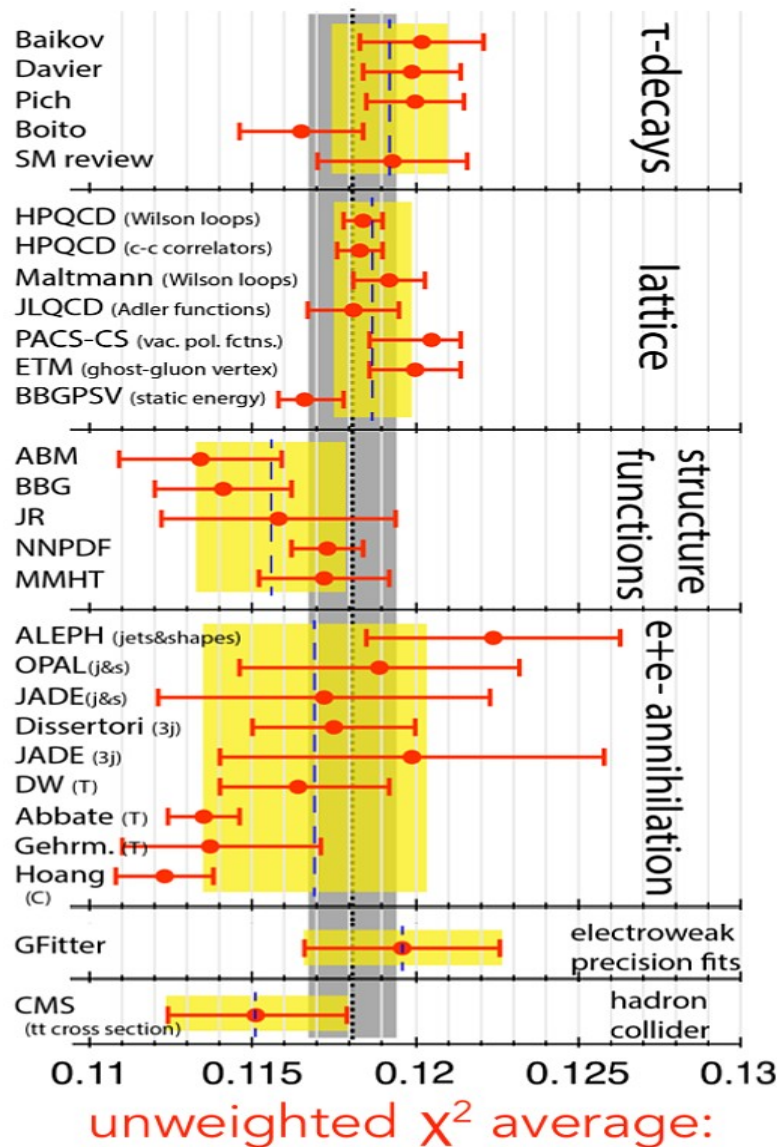
Inclusion of full set of t-tbar data increases  $\alpha_s(m_Z)$  & uncertainty:

$$\alpha_s(M_Z) = 0.1186 \pm 0.0033 (\pm 2.9\%)$$



# PDG 2015 $\alpha_s$ world average (NNLO)

[S.Bethke, Feb'16]



class averages:

$$\alpha_s(M_Z) = 0.1192 \pm 0.0018 \quad (\pm 1.5\%)$$

$$\alpha_s(M_Z) = 0.1187 \pm 0.0012 \quad (\pm 1.0\%)$$

$$\alpha_s(M_Z) = 0.1156 \pm 0.0023 \quad (\pm 2.0\%)$$

$$\alpha_s(M_Z) = 0.1169 \pm 0.0034 \quad (\pm 2.9\%)$$

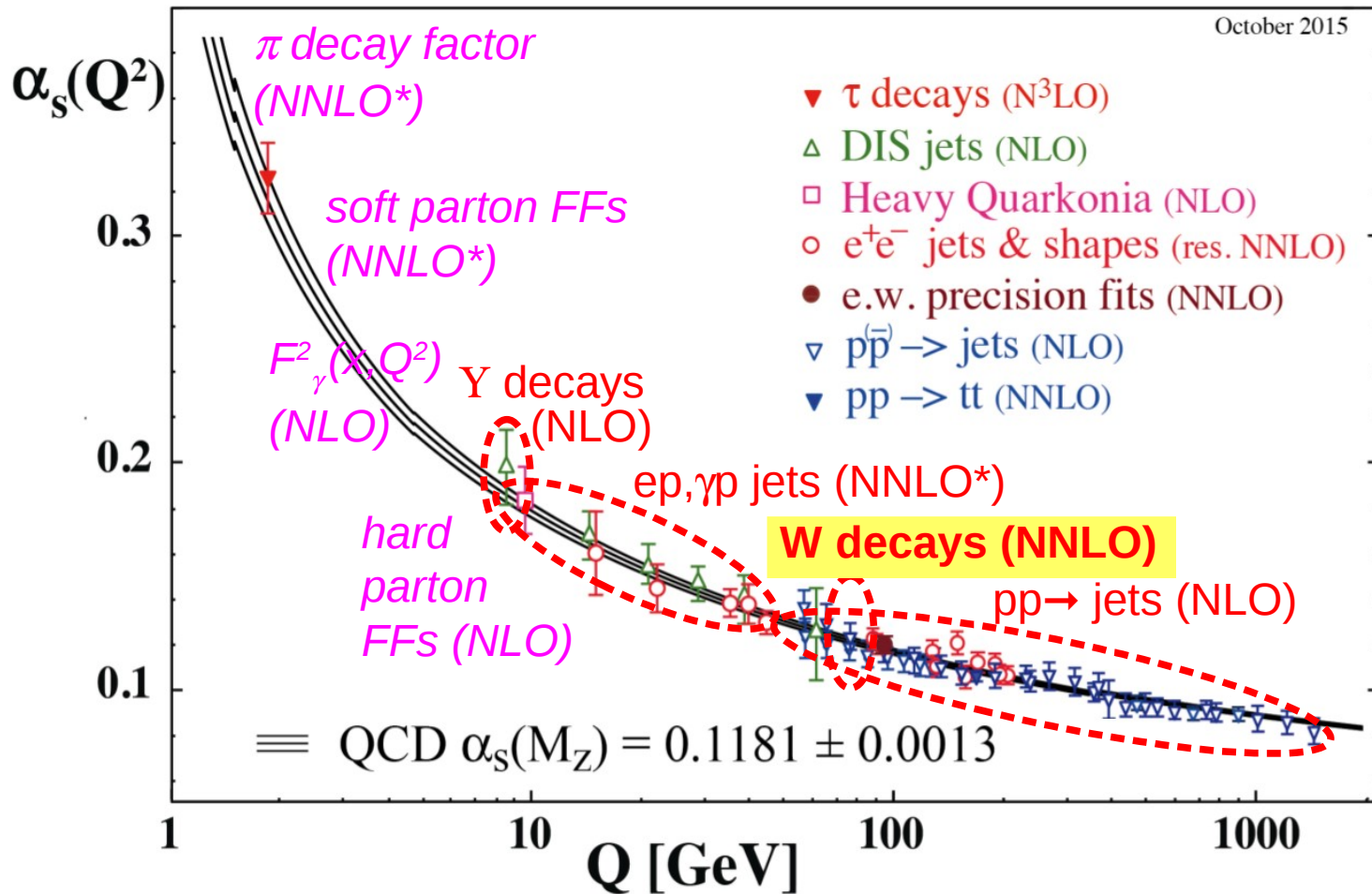
$$\alpha_s(M_Z) = 0.1196 \pm 0.0030 \quad (\pm 2.5\%)$$

$$\alpha_s(M_Z) = 0.1151 \pm 0.0033 \quad (\pm 2.9\%)$$

$$\alpha_s(M_Z) = 0.1181 \pm 0.0013 \quad (\pm 1.1\%)$$

# Other $\alpha_s$ extractions (not yet in world average)

- There exist at least 8 other classes of  $\alpha_s$  observables (mostly of lower NLO, NNLO\* accuracy) used to extract the QCD coupling:





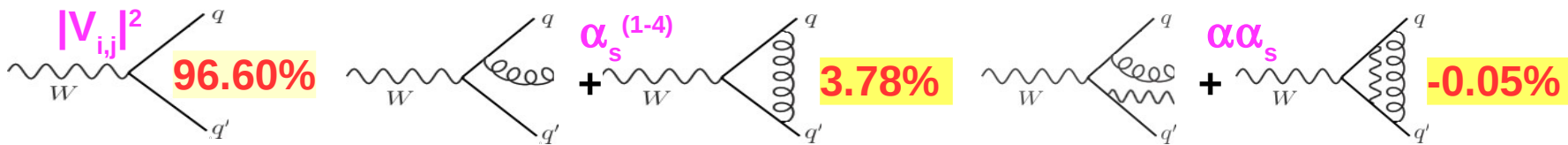
# Hadronic W width & branching ratio

[D.d'E, M.Srebre, arXiv:1603.06501]

- Width (BR) known at N<sup>3</sup>LO (NNLO). Small sensitivity to  $\alpha_s$  (beyond Born)

$$\Gamma_{W,\text{had}} = \frac{\sqrt{2}}{4\pi} G_F m_W^3 \sum_{\text{quarks } i,j} |V_{i,j}|^2 \left[ 1 + \sum_{k=1}^4 \left( \frac{\alpha_s}{\pi} \right)^k + \delta_{\text{electroweak}}(\alpha) + \delta_{\text{mixed}}(\alpha\alpha_s) \right]$$

[EWK: -0.35%]



- TH improvements: finite quark-mass effects included (LO), updated PDG values, careful evaluation of parametric ( $V_{i,j}$ ,  $m_W$ ) & theoretical uncert.

- Calculation dominated by  $\pm 1.5\%$  parametric (mostly  $V_{cs}$ ) uncertainty:

$$\Gamma_W (\text{MeV}) = 1428.67 \pm 22.40_{(\text{par})} \pm 0.04_{(\text{th})} \quad (\text{exp. CKM})$$

$$1411.40 \pm 0.96_{(\text{par})} \pm 0.04_{(\text{th})} \quad (\text{CKM}=1)$$

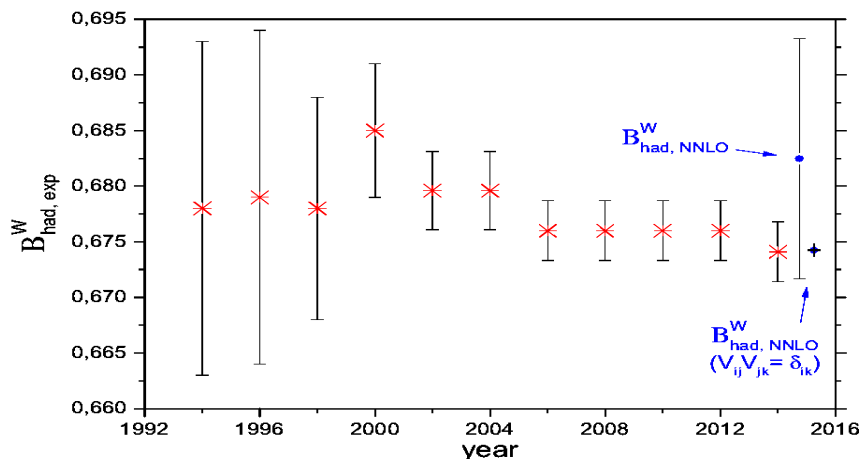
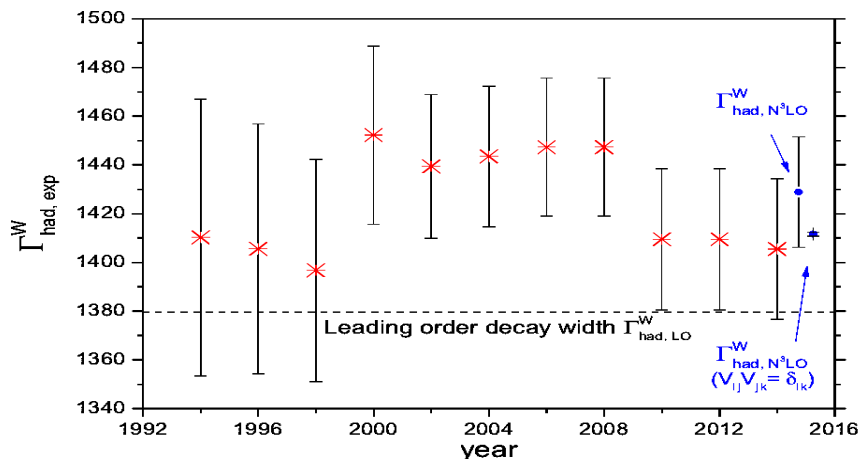
$$\text{BR}_W = \Gamma_W / \Gamma_{\text{tot}} = 0.6820 \pm 0.0110_{(\text{par})} \pm 0.0002_{(\text{th})} \quad (\text{exp. CKM})$$

$$0.6742 \pm 0.0001_{(\text{par})} \pm 0.0002_{(\text{th})} \quad (\text{CKM}=1)$$

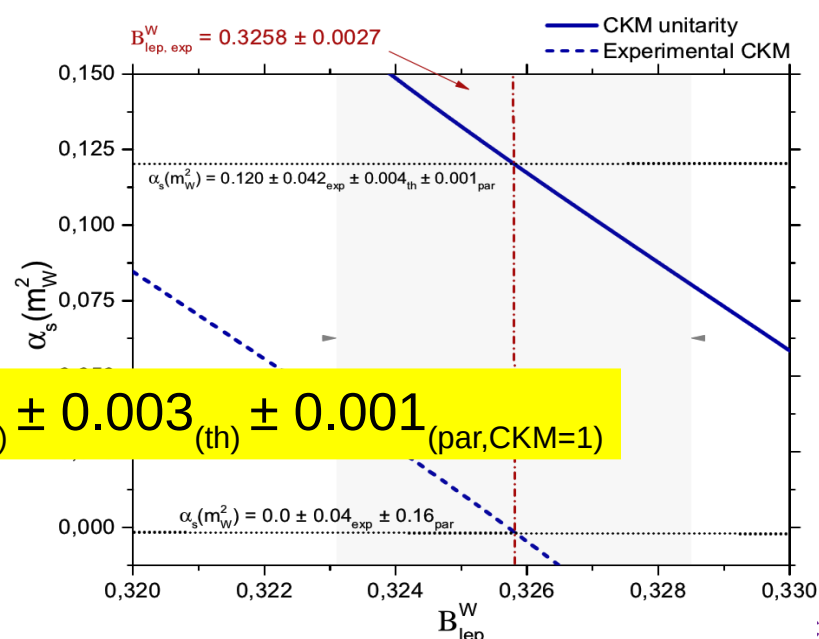
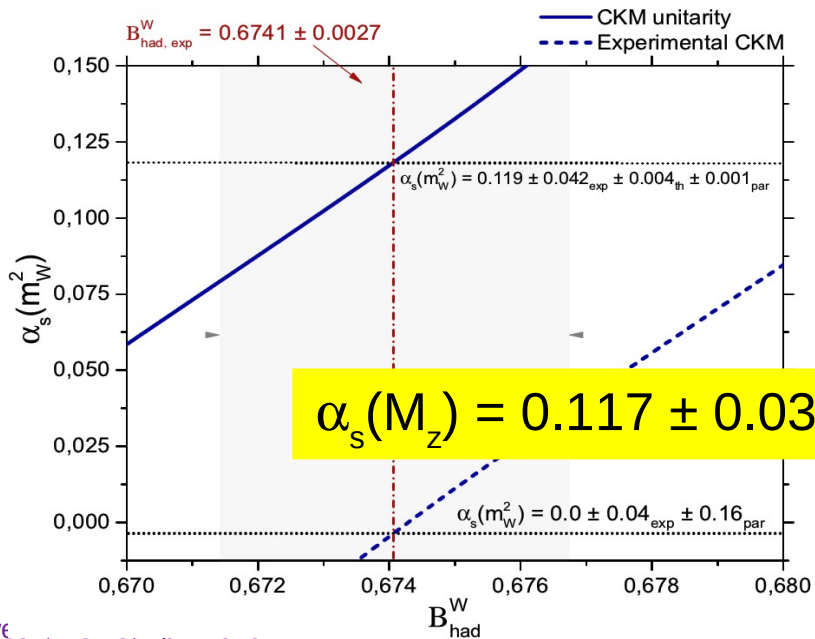
- TH uncertainty (missing  $\alpha_s^5$  terms, non-pQCD  $(\Lambda_{\text{QCD}}/m_W)^4$  power corr., finite quark masses beyond LO, CKM matrix renorm. scheme):  $\pm 0.03\%$

# $\alpha_s$ from hadronic W decays

▶ TH vs. EXP:  $\Gamma_W = 1405 \pm 29$  MeV ( $\pm 2\%$ ),  $BR_W = 0.6741 \pm 0.0027$  ( $\pm 0.4\%$ )



▶ Extraction with large exp. & parametric uncertainties ( $\pm 25\%$ ) today:



# $\alpha_s$ from hadronic W decays

[D.d'E, M.Srebre, arXiv:1603.06501]

► LHC prospects:

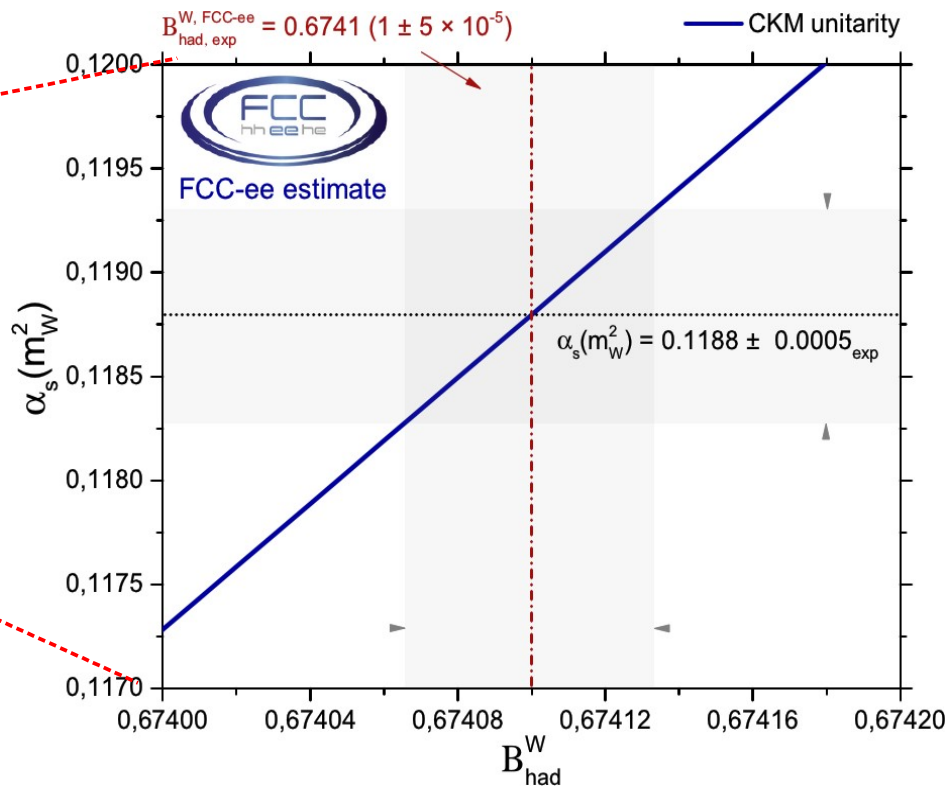
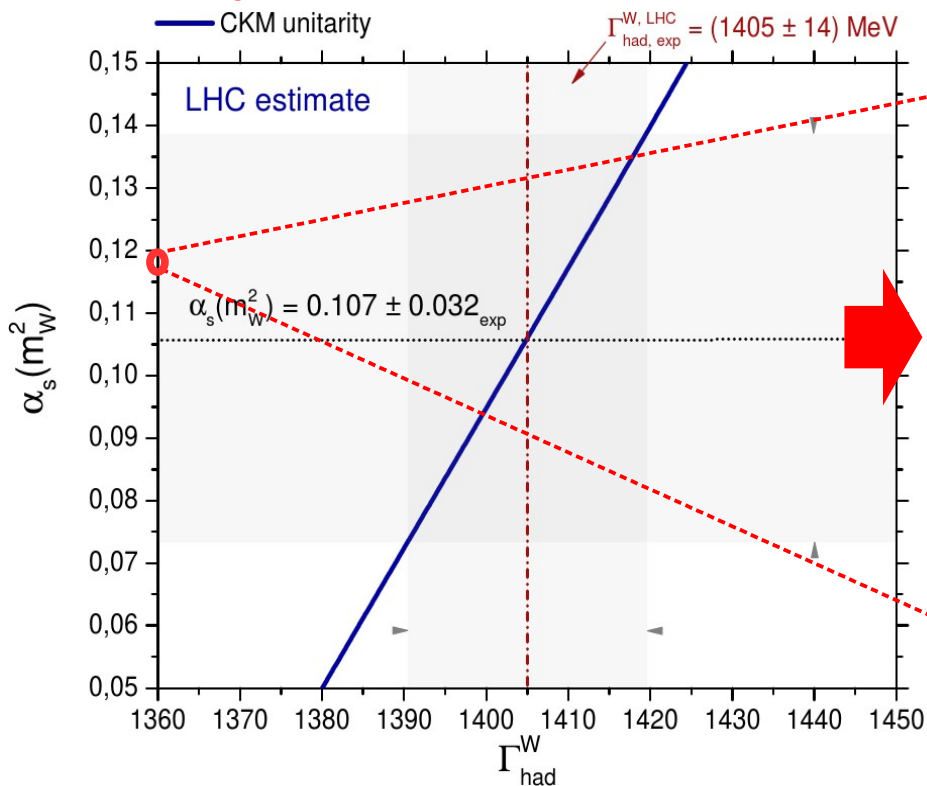
Improved measurement of  $\Gamma_W$ :

$\delta\alpha_s \sim 10\%$

► FCC-ee prospects:

Huge  $ee \rightarrow WW$  stats ( $10^8$ ,  $\times 10^3$  LEP):

$\delta\alpha_s < 0.2\%$



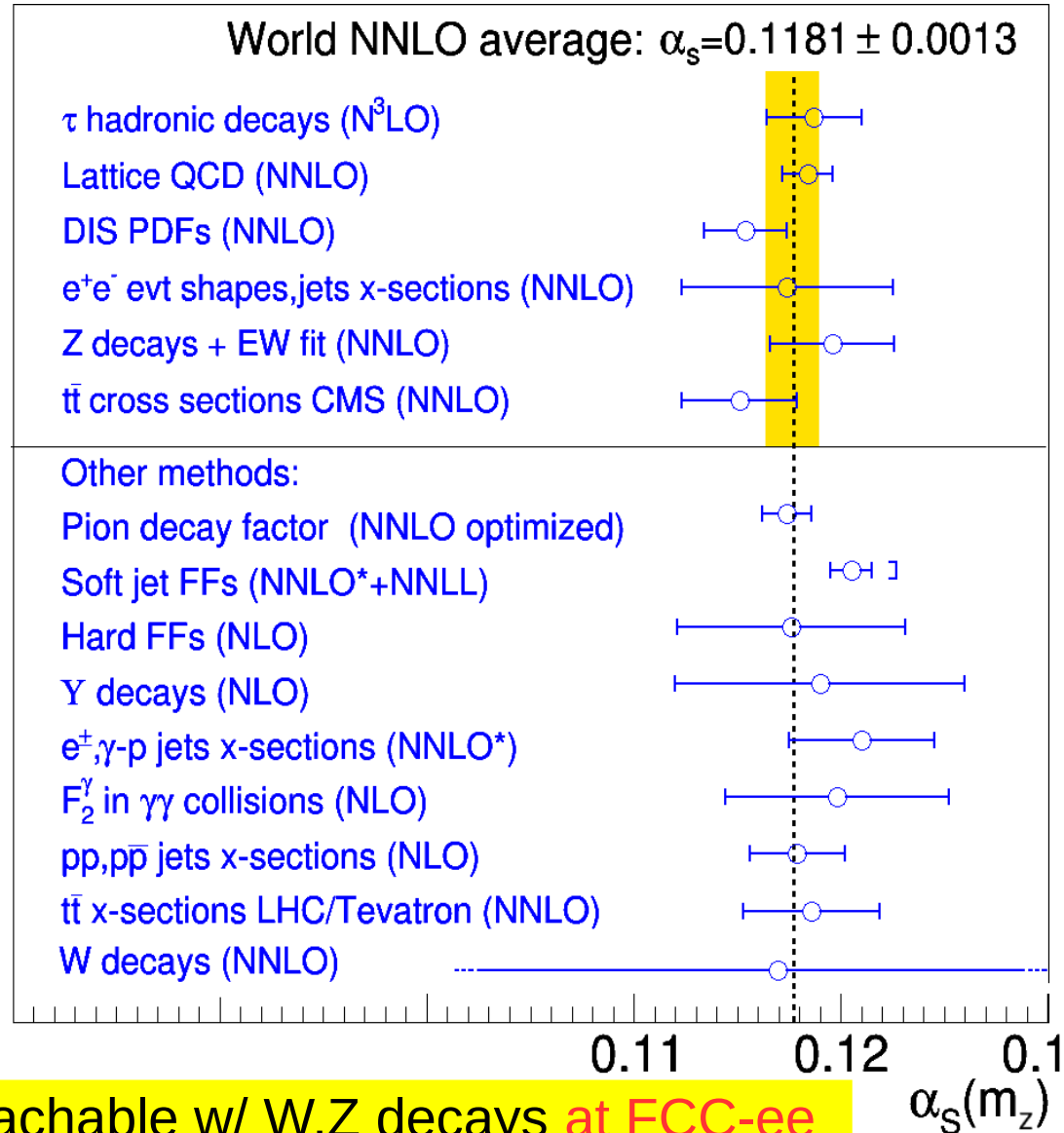
► Combined FCC-ee from W & Z decays:

$$\alpha_s(M_Z) = 0.1190 \pm 0.0002$$

$\delta\alpha_s(\text{exp}) < 0.15\%$ . Parametric ( $V_{ij}$ ,  $m_W$ ) & TH uncert. to be improved too.

# Summary: $\alpha_s$ status & FCC-ee prospects

- World-average QCD coupling at NNLO:
  - Determined from **6 observables** with **1.1% uncertainty** (least well-known coupling).
  - Impacts all **LHC x-sections & QCD decays**.
  - Role **beyond SM**:  
Couplings unification  
EWK vacuum stability  
New? colored sectors
- Detailed study of NNLO extraction via W decays:  $\pm 25\%$  exp. uncertainty today



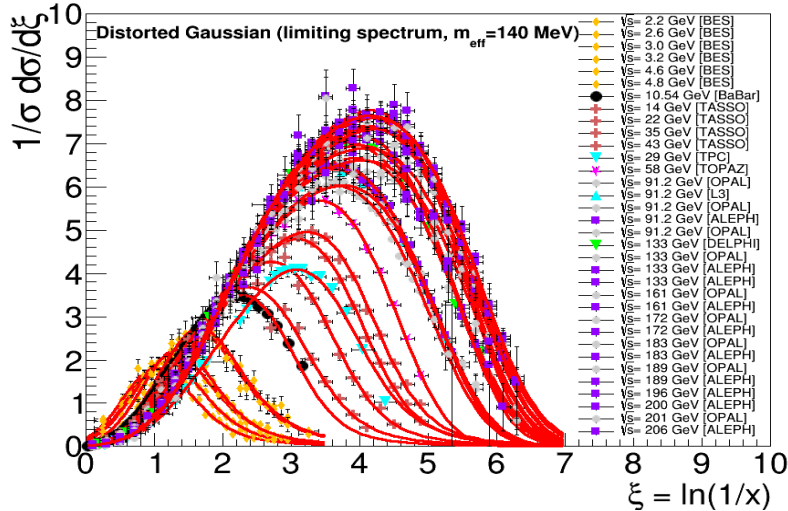
■ **O(0.1%) uncertainty** only reachable w/ W,Z decays at FCC-ee

# Backup slides

# Other $\alpha_s$ extractions (NLO, NNLO\*)

➔ Soft parton-to-hadron FFs (NNLO\*+NNLL):

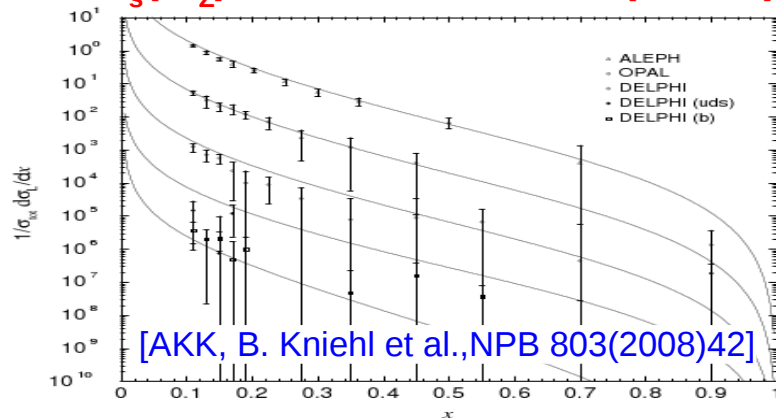
$$\alpha_s(m_z) = 0.1205 \pm 0.0022 (\pm 2\%)$$



[D.d'E., R.Perez-Ramos, arXiv:0505.02624]

➔ Hard parton-to-hadron FFs (NLO):

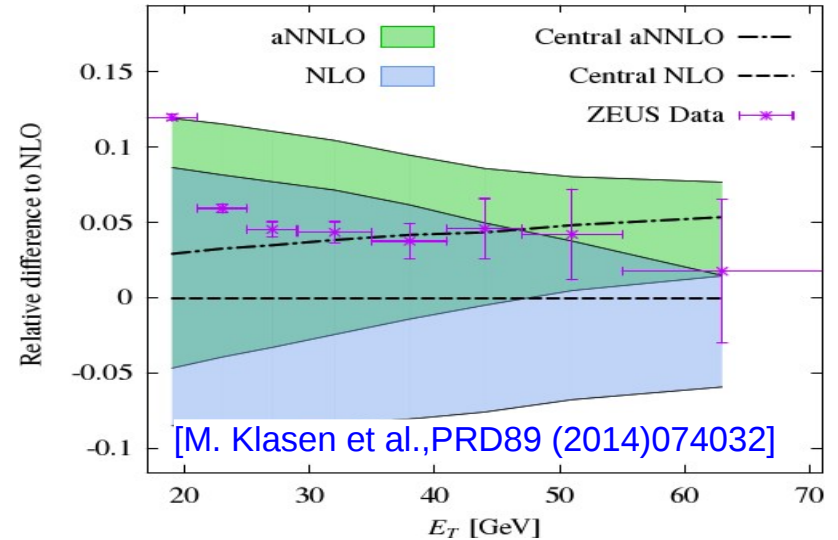
$$\alpha_s(m_z) = 0.1176 \pm 0.0055 (\pm 4.7\%)$$



[AKK, B. Kniehl et al., NPB 803(2008)42]

➔ Jet x-sections in  $e, \gamma$ -p (NNLO\*):

$$\alpha_s(m_z) = 0.112 \pm 0.002 \pm 0.003 (\pm 4\%)$$



[M. Klasen et al., PRD89 (2014)074032]

➔  $\pi$  decay factor (N<sup>3</sup>LO, RGOPT):

$$\alpha_s(m_z) = 0.1174 \pm 0.0017 (\pm 1.5\%)$$

[Kneur&Neveu, PRD81(2010)125012]

➔  $Y$  decay (NLO): [Mambrilla et al. PRD75(07)074014]

$$\alpha_s(m_z) = 0.1190 \pm 0.007 (\pm 6\%)$$

➔ Photon structure function  $F_\gamma^2$  (NLO):

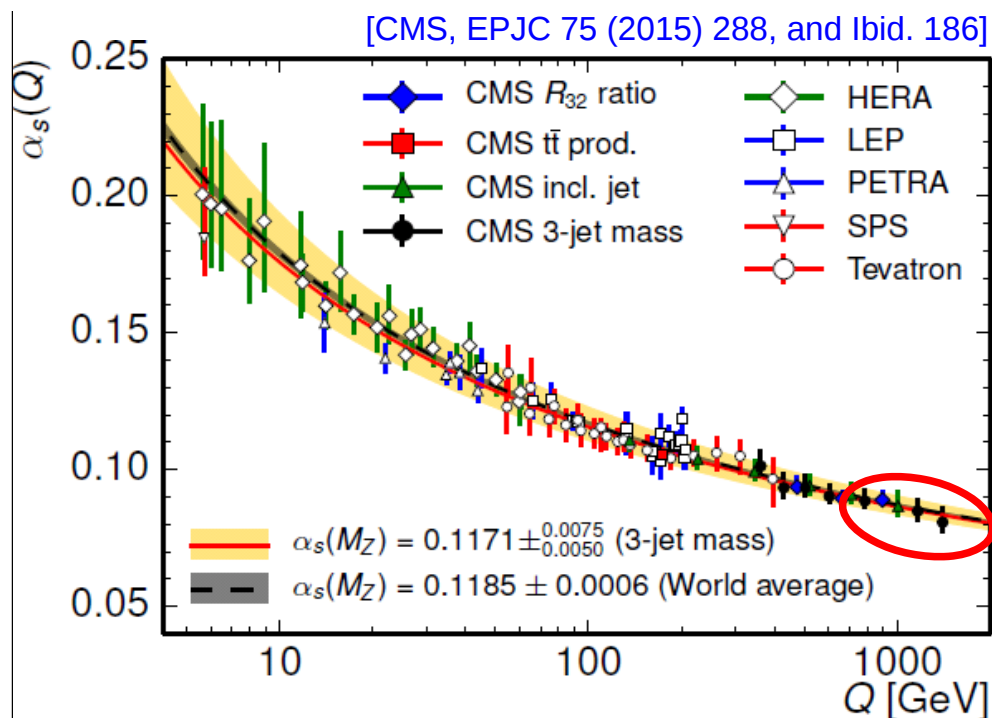
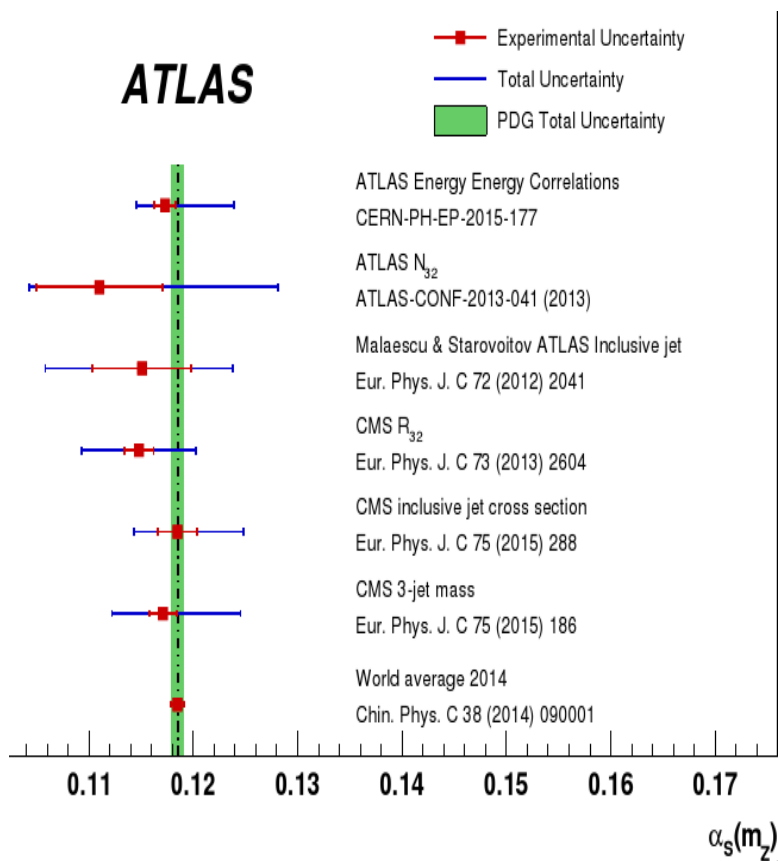
$$\alpha_s(m_z) = 0.1198 \pm 0.0054 (\pm 4.5\%)$$

[M.Klasen et al. PRL89 (2002)122004]



# $\alpha_s$ coupling from LHC pp $\rightarrow$ jets (NLO)

- Ratio of 3-jets of 2-jets, 3-jet mass & inclusive jets x-sections constrain  $\alpha_s$  (NLO only) up to so-far unprobed scales  $Q \sim 1.4$  TeV:

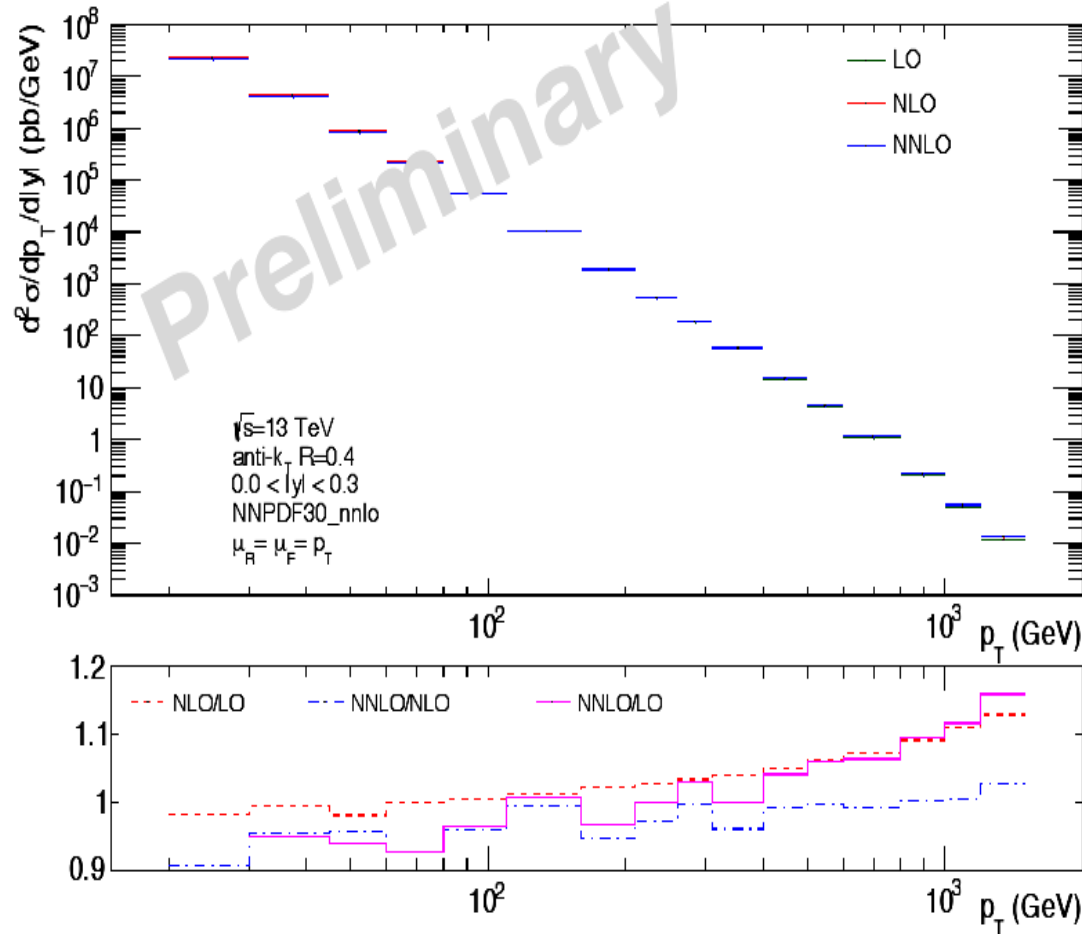


➔ 1<sup>st</sup> time asymptotic freedom is probed at the TeV scale!

➔ Upcoming NNLO jet cross sections (changes x-section by -10%, +1%. Reduced scale uncertainty): Important pQCD breakthrough!

# $\alpha_s$ from p-p jets cross sections (NNLO)

[J.Pires]



moderate corrections of -10% that rise to 1% at high- $p_T$  with respect to NLO

- ▶ NNLO jet cross sections will have a **large impact in many  $\alpha_s$  extractions:**  
**PDFs, FFs, jets in DIS/photoprod.,....**

# $\alpha_s$ from pion and $\Upsilon$ decays

[J.L.Kneur]

$$F_\pi^2(\text{pert})_{\overline{\text{MS}}} = N_c \frac{m^2}{2\pi^2} \left[ -L + \frac{\alpha_s}{4\pi} (8L^2 + \frac{4}{3}L + \frac{1}{6}) \right. \\ \left. + (\frac{\alpha_s}{4\pi})^2 [f_{30}(n_f)L^3 + f_{31}(n_f)L + f_{32}(n_f)L + f_{33}(n_f)] + \mathcal{O}(\alpha_s^3) \right]$$

$$L \equiv \ln \frac{m}{\mu}, n_f = 2(3)$$

$$\bar{\alpha}_s(m_Z) = 0.1174_{-0.0005}^{+0.0010}(\text{rgopt th}) \pm .0010|_{(F_\pi/F_0)} \pm .0005_{\text{evol}}$$

Issues:

- ▶ Too low scale for pQCD?
- Optimization approach,...
- ▶ Intriguing agreement with world average.

$$\alpha_s = 0.1174 \pm 0.0017 \quad (\pm 1.5\%)$$

$$R_\gamma \equiv \frac{\Gamma(\Upsilon(1S) \rightarrow \gamma X)}{\Gamma(\Upsilon(1S) \rightarrow X)} = \frac{36 e_b^2 \alpha N}{5 \alpha_s D},$$

[J. Soto]

$$N, D = 1 + \mathcal{O}(\alpha_s) + \mathcal{O}(v^2) + \mathcal{O}\left(\frac{v^4}{\alpha_s}\right)$$

$$\alpha_s(\text{NLO}) = 0.1190 \pm 0.007 \quad (\pm 6\%)$$

$$+ \mathcal{O}(\alpha_s^2) + \mathcal{O}(\alpha_s v^2) + \mathcal{O}\left(\alpha_s \frac{v^4}{\alpha_s}\right) + \mathcal{O}(v^4) + \mathcal{O}\left(\frac{v^6}{\alpha_s}\right)$$

- A NNLO extraction of  $\alpha_s$  appears feasible in the coming years, the key ingredients being:
  - More precise data for the  $\Upsilon(1S)$  photon spectrum (and total hadronic width)
  - Non-trivial higher order perturbative calculations