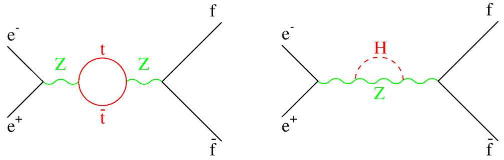
Direct measurement of $\alpha_{QED}(m_Z^2)$ @ FCC-ee

- **Outline**
  - Why measure $\alpha_{QED}(m_Z^2)$?
    - Physics behind precision
    - Impact of $\alpha_{QED}(m_Z^2)$ precision
  - Sensitivity of of $e^+e^- \rightarrow \mu^+\mu^-$ to $\alpha_{QED}(m_Z^2)$
    - Cross section
    - Forward-backward asymmetry
    - Statistical uncertainty and sensitivity
  - Determination of $\alpha_{QED}(m_Z^2)$
  - Systematic uncertainties
    - A few highlights and follow-ups
  - Conclusions

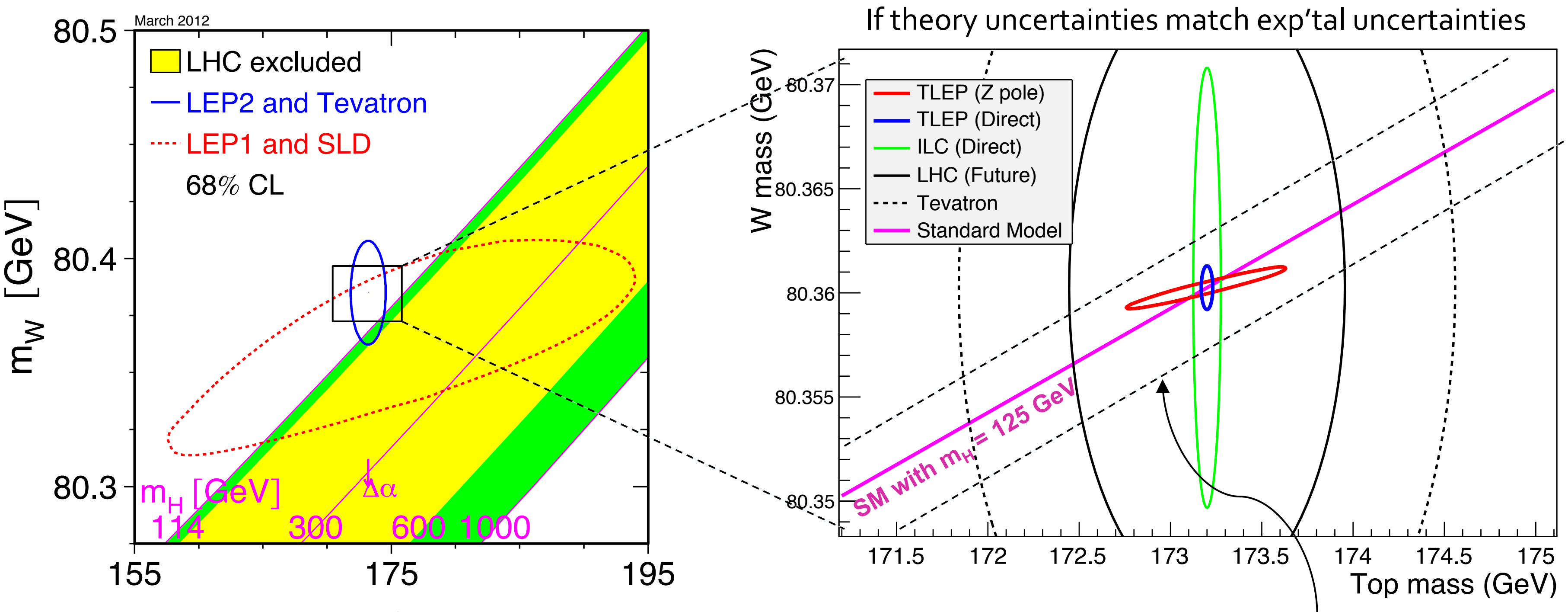
See arXiv:1512:05544
(Published as JHEP 2016(2) 1-22)

Physics behind precision

- EWPO measurements allow a prediction of m_{top}, m_{W}, m_H, $\sin^2\theta_W$ in the SM
  - Compare with FCC-ee direct measurements
    - The standard model has nowhere to go



If theory uncertainties match exp'tal uncertainties

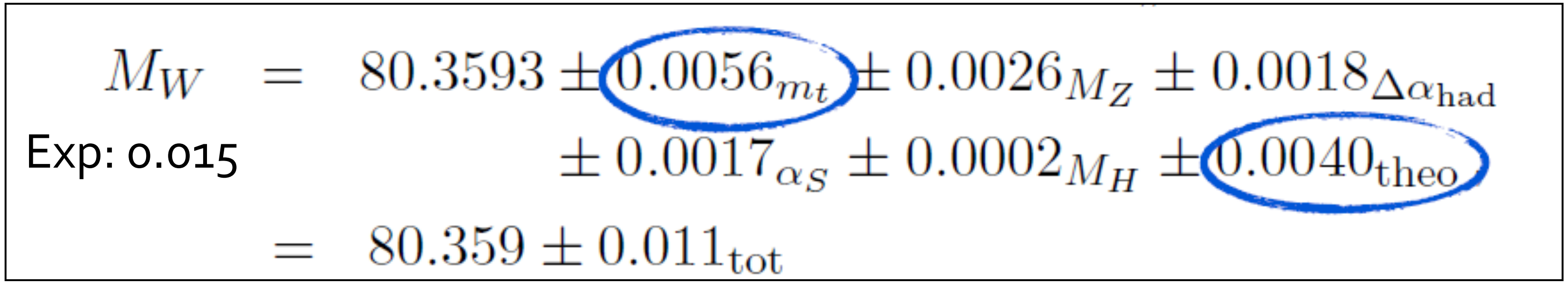


Without m_Z@FCC-ee, the SM line would have a 2.2 MeV width

- Sensitivity to new physics ?

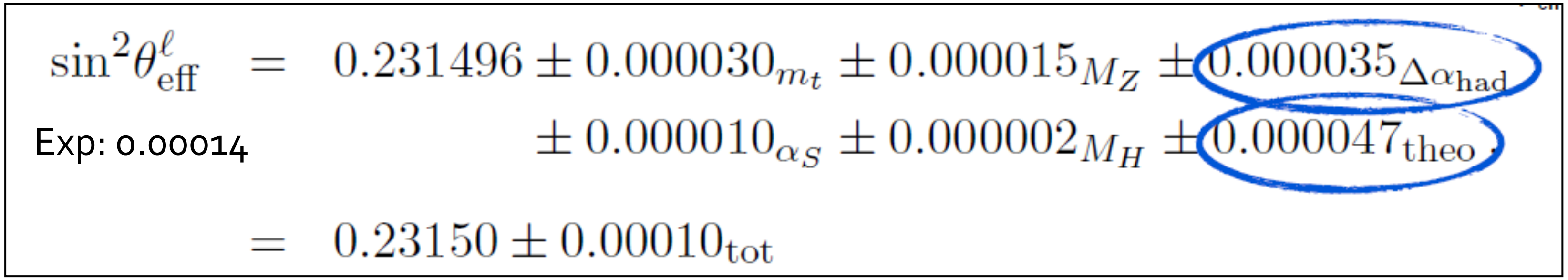
Impact of $\alpha_{QED}(m_Z^2)$ precision

- **Uncertainty on these predictions are of two origins**
  - **Parametric**
  - **Higher orders**
- **For m_W and $\sin^2\theta_W$ today (see Sven Heinemeyer's talk on Tuesday)**



$$\begin{aligned} M_W &= 80.3593 \pm 0.0056_{m_t} \pm 0.0026_{M_Z} \pm 0.0018_{\Delta\alpha_{\text{had}}} \\ &\quad \pm 0.0017_{\alpha_S} \pm 0.0002_{M_H} \pm 0.0040_{\text{theo}} \\ &= 80.359 \pm 0.011_{\text{tot}} \end{aligned}$$

Exp: 0.015



$$\begin{aligned} \sin^2\theta^\ell_{\text{eff}} &= 0.231496 \pm 0.000030_{m_t} \pm 0.000015_{M_Z} \pm 0.000035_{\Delta\alpha_{\text{had}}} \\ &\quad \pm 0.000010_{\alpha_S} \pm 0.000002_{M_H} \pm 0.000047_{\text{theo}} \\ &= 0.23150 \pm 0.00010_{\text{tot}} \end{aligned}$$

Exp: 0.00014

Impact of $\alpha_{QED}(m_Z^2)$ precision

FCC-ee

- Uncertainty on these predictions are of two origins
  - Parametric
  - Higher orders (QCD, EW, mixed)
- Reduced uncertainties at the FCC-ee

$$
\begin{aligned}
M_W &= 80.3593 \pm 0.0001_{m_t} \pm 0.0001_{M_Z} \pm 0.0018_{\Delta\alpha_{\rm had}} \\
&\quad \pm 0.0002_{\alpha_S} \pm 0.0000_{M_H} \pm 0.0040_{\rm theo} \\
&= 80.359 \pm 0.005_{\rm tot}
\end{aligned}
$$

Exp: 0.0005

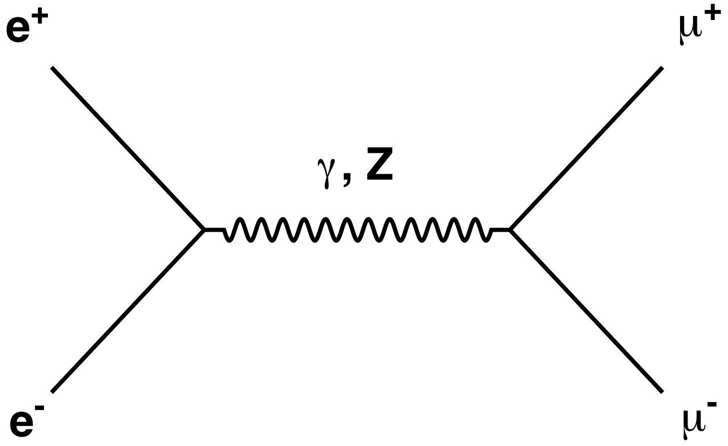
$$
\begin{aligned}
\sin^2\theta^{\ell}_{\rm eff} &= 0.231496 \pm 0.000001_{m_t} \pm 0.000001_{M_Z} \pm 0.000035_{\Delta\alpha_{\rm had}} \\
&\quad \pm 0.000001_{\alpha_S} \pm 0.000000_{M_H} \pm 0.000047_{\rm theo} \\
&= 0.23150 \pm 0.00006_{\rm tot}
\end{aligned}
$$

Exp: 0.000006

- Must reduce current exp'tal uncertainty on $\alpha_{QED}(m_Z^2)$ by a factor ~4-5
  - New generation of theoretical calculations is necessary to gain a factor 10 in precision

Sensitivity of $e^+e^- \rightarrow \mu^+\mu^-$ to $\alpha_{QED}(m_Z^2)$

- The $e^+e^- \rightarrow \mu^+\mu^-$ cross section



$$\mathcal{G} = \frac{c_\gamma^2}{s},$$
$$\mathcal{Z} = \frac{c_Z^2(v^2+a^2)^2 \times s}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2},$$
$$\mathcal{I} = \frac{2c_\gamma c_Z v^2 \times (s-m_Z^2)}{(s-m_Z^2)^2 + m_Z^2\Gamma_Z^2},$$

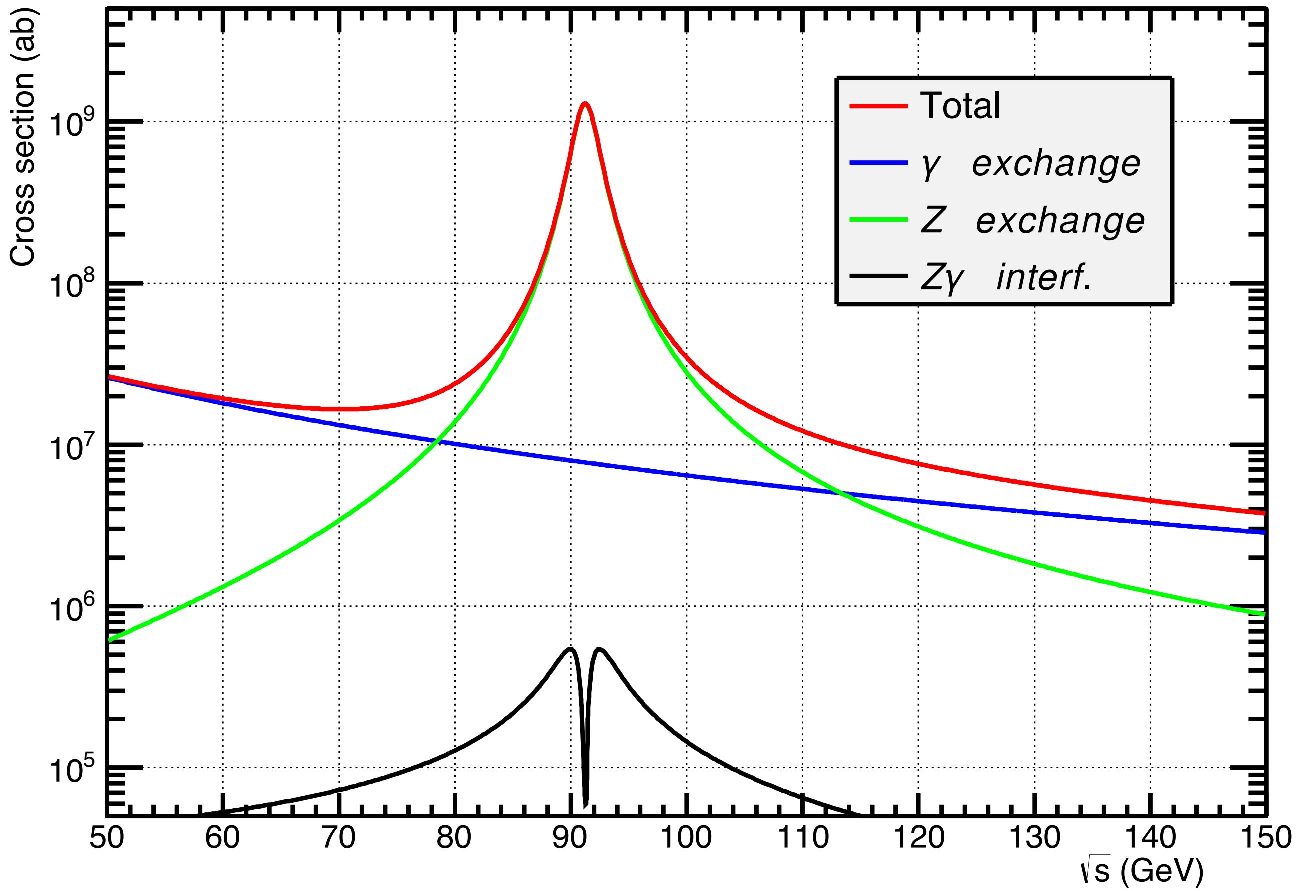
$$c_\gamma = \sqrt{\frac{4\pi}{3}\alpha_{\rm QED}(s)}, \quad c_Z = \sqrt{\frac{4\pi}{3}\frac{m_Z^2}{2\pi}\frac{G_F}{\sqrt{2}}}, \quad a = -\frac{1}{2}, \quad v = a \times (1 - 4\sin^2\theta_W),$$

(~ -0.037)

- Photon exchange (G) proportional to $\alpha^2(s)$
- Z exchange (Z) proportional to G_F^2
- Interference term proportional to $\alpha(s)$ G_F
  - Need to choose √s judiciously to maximize sensitivity to $\alpha(s)$
  - If √s is close to m_Z, the √s → m_Z extrapolation uncertainty is negligible

Sensitivity of $e^+e^- \rightarrow \mu^+\mu^-$ to $\alpha_{QED}(m_Z^2)$

- **The $e^+e^- \rightarrow \mu^+\mu^-$ cross section**



Sensitivity of $e^+e^- \rightarrow \mu^+\mu^-$ to $\alpha_{QED}(m_Z^2)$

- **The $e^+e^- \rightarrow \mu^+\mu^-$ angular distribution**
  - Absolute cross section measurement might be challenging to the required precision
    - Uncertainty of the integrated luminosity determination
  - Rely of a self-normalizing quantity, the forward-backward asymmetry $A_{\rm FB}^{\mu\mu} = \frac{\sigma^{\rm F}_{\mu\mu} - \sigma^{\rm B}_{\mu\mu}}{\sigma^{\rm F}_{\mu\mu} + \sigma^{\rm B}_{\mu\mu}}$,

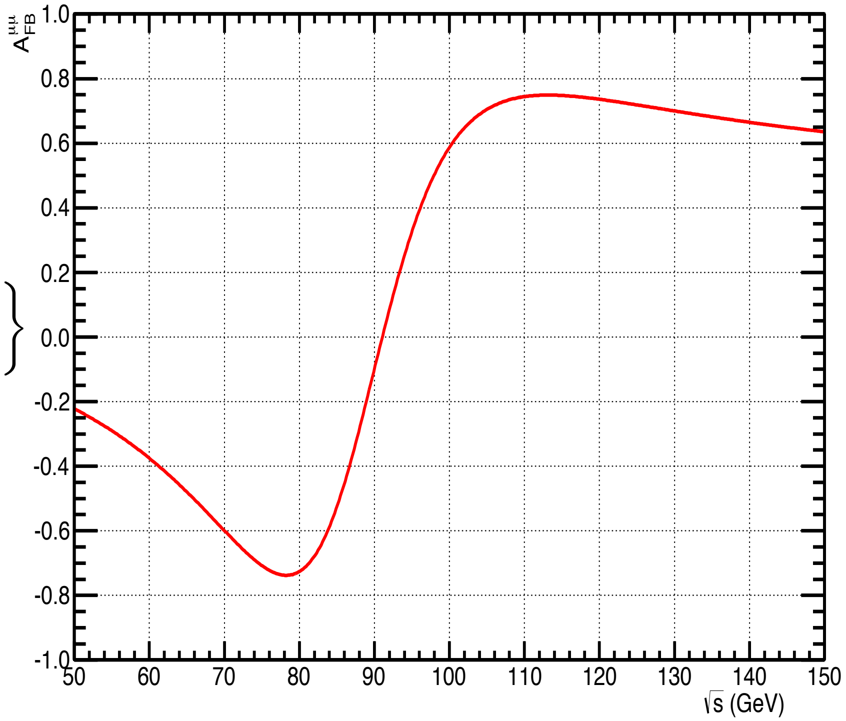
$$\frac{d\sigma_{\mu\mu}}{d\cos\theta}(s) \propto G_1(s) \times (1+\cos^2\theta) + G_3(s) \times 2\cos\theta,$$

$$A_{\rm FB}^{\mu\mu}(s) = \frac{3}{4}\frac{G_3(s)}{G_1(s)}.$$

$$G_1(s) = \mathcal{G} + \mathcal{I} + \mathcal{Z} \quad \text{and} \quad G_3(s) = \frac{a^2}{v^2}\left\{\mathcal{I} + \frac{4v^4/a^4}{(1+v^2/a^2)^2}\mathcal{Z}\right\}$$

$$\boxed{A_{\rm FB}^{\mu\mu} = A_{\rm FB,0}^{\mu\mu} + \frac{3}{4}\frac{a^2}{v^2}\frac{\mathcal{I}}{\mathcal{G}+\mathcal{Z}}.}$$

$$A_{\rm FB,0}^{\mu\mu} = (3/4) \times 4v^2a^2/(a^2+v^2)^2 \simeq 0.016.$$



Sensitivity of $e^+e^- \longrightarrow \mu^+\mu^-$ to $\alpha_{QED}(m_Z^2)$

- **Variation of A_{FB} with α_{QED}**

$$\boxed{A_{\rm FB}^{\mu\mu} = A_{\rm FB,0}^{\mu\mu} + \frac{3}{4}\frac{a^2}{v^2}\frac{\mathcal{I}}{\mathcal{G}+\mathcal{Z}}.}$$

$dZ/d\alpha = 0$ **$dI/d\alpha = I/\alpha$** **$dG/d\alpha = 2G/\alpha$**

- For a small variation $\Delta\alpha$

$$\boxed{\Delta A_{\rm FB}^{\mu\mu} = \frac{\Delta\alpha}{\alpha} \times \frac{3}{4}\frac{a^2}{v^2}\frac{\mathcal{I}(\mathcal{Z}-\mathcal{G})}{(\mathcal{G}+\mathcal{Z})^2} = \left(A_{\rm FB}^{\mu\mu} - A_{\rm FB,0}^{\mu\mu}\right) \times \frac{\mathcal{Z}-\mathcal{G}}{\mathcal{Z}+\mathcal{G}} \times \frac{\Delta\alpha}{\alpha}.}$$

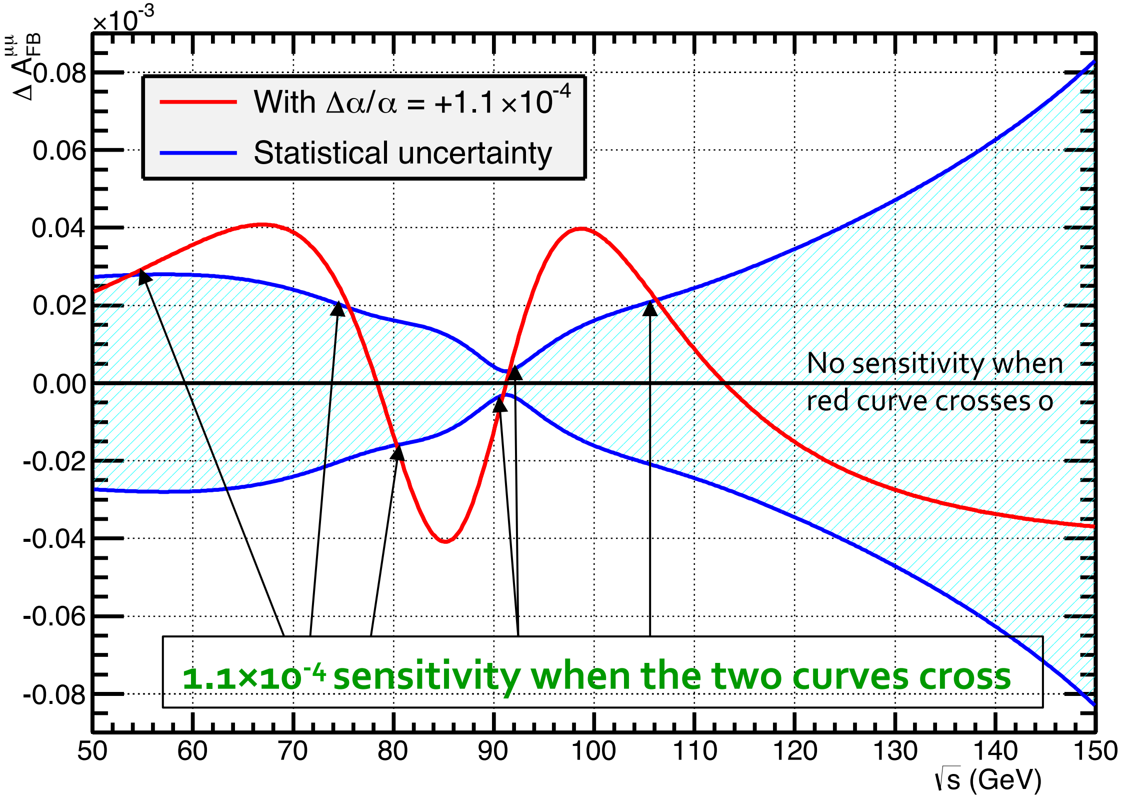
$$\frac{\Delta\alpha}{\alpha} = \frac{\Delta A_{\rm FB}^{\mu\mu}}{A_{\rm FB}^{\mu\mu} - A_{\rm FB,0}^{\mu\mu}} \times \frac{\mathcal{Z}+\mathcal{G}}{\mathcal{Z}-\mathcal{G}} \simeq \frac{\Delta A_{\rm FB}^{\mu\mu}}{A_{\rm FB}^{\mu\mu}} \times \frac{\mathcal{Z}+\mathcal{G}}{\mathcal{Z}-\mathcal{G}},$$

- **Statistical uncertainty on A_{FB}**

$$\boxed{\sigma\left(A_{\rm FB}^{\mu\mu}\right) = \sqrt{\frac{1-A_{\rm FB}^{\mu\mu\,2}}{\mathcal{L}\sigma_{\mu\mu}}}.}$$

Sensitivity of $e^+e^- \longrightarrow \mu^+\mu^-$ to $\alpha_{QED}(m_Z{}^2)$

- **Comparison of the sensitivity and the statistical uncertainty**
  - Statistical uncertainty: one year with target luminosities (86 ab^{-1} at the Z pole)
    - Compared with ΔA_{FB} for current α_{QED} uncertainty



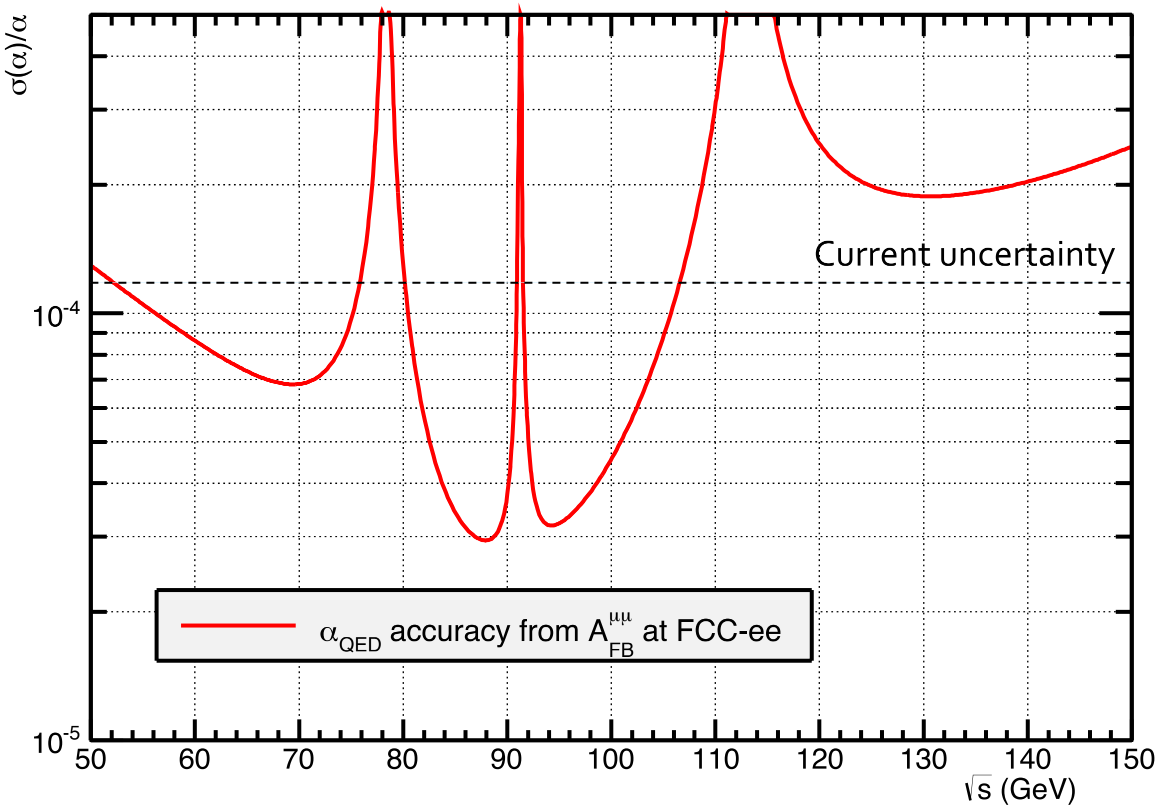
No sensitivity when red curve crosses 0

1.1×10^{-4} sensitivity when the two curves cross

- ΔA_{FB} changes sign for $\sqrt{s} = m_Z$ and when Z = G (78 and 112 GeV): no sensitivity to α_{QED}

Sensitivity of $e^+e^- \rightarrow \mu^+\mu^-$ to $\alpha_{QED}(m_Z^2)$

- Turning the previous plot in a $\sigma(\alpha)/\alpha$ plot, for a year of running at any $\sqrt{s}$



- Optimal centre-of-mass energies for a 3×10^{-5} uncertainty on α_{QED}
  - One year at $\sqrt{s_-}$ = 87.9 GeV or one year at $\sqrt{s_+}$ = 94.3 GeV
  - Even better: six months at $\sqrt{s_-}$ and six months at $\sqrt{s_+}$

Determination of $\alpha_{\rm QED}(m_Z^2)$

- **Two measurements**
  - **Two asymmetries at two $\sqrt{s}$: $A_{FB}(s_-)$ and $A_{FB}(s_+)$**

$$\boxed{\alpha_- \equiv \alpha_{\rm QED}(s_-) \text{ and } \alpha_+ \equiv \alpha_{\rm QED}(s_+),}$$

- **Running from $\sqrt{s}_{-\pm}$ to m_Z gives two determinations of α_0**

$$\boxed{\frac{1}{\alpha_0} = \frac{1}{\alpha_\pm} + \beta \log \frac{s_\pm}{m_Z^2},}$$

- **Solve for $\alpha_0 = \alpha_{\rm QED}(m_Z^2)$**

$$\boxed{\frac{1}{\alpha_0} = \frac{1}{2}\left(\frac{1-\xi}{\alpha_-} + \frac{1+\xi}{\alpha_+}\right), \quad \text{where} \quad \xi = \frac{\log s_- s_+ / m_Z^4}{\log s_- / s_+} \simeq 0.045,}$$

- **With potential of almost exact cancellations for correlated effects at $s_\pm$**

$$\frac{\Delta\alpha_0}{\alpha_0} \simeq \quad 0.528 \frac{\Delta A_{\rm FB}}{A_{\rm FB}}(s_-) \oplus 0.563 \frac{\Delta A_{\rm FB}}{A_{\rm FB}}(s_+)$$

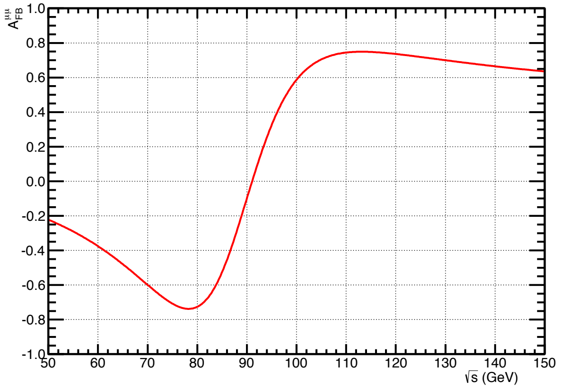
$$\boxed{\begin{array}{l} \Delta A_{FB}/A_{FB}(s_-) > 0 \\ \Delta A_{FB}/A_{FB}(s_+) < 0 \end{array}}$$

Systematic uncertainties

- **Summary of the study**

Type	Source	Uncertainty
Experimental	E_{beam} calibration	1×10^{-5}
	E_{beam} spread	$< 10^{-7}$
	Acceptance and efficiency	negl.
	Charge inversion	negl.
	Backgrounds	negl.
Parametric	m_{Z} and Γ_{Z}	1×10^{-6}
	$\sin^2 \theta_{\mathrm{W}}$	5×10^{-6}
	G_{F}	5×10^{-7}
Theoretical	QED (ISR, FSR, IFI)	$< 10^{-6}$
	Missing EW higher orders	few 10^{-4}
	New physics in the running	0.0
Total	Systematics	1.2×10^{-5}
(except missing EW higher orders)	Statistics	3×10^{-5}

Knowledge of the beam energy is crucial



- **Strong dependence of A_{FB} with √s (and m_Z)**

$$\boxed{A_{\rm FB}^{\mu\mu}(s, m_{\rm Z}) \propto (s - m_{\rm Z}^2)/(s m_{\rm Z}^2).}$$

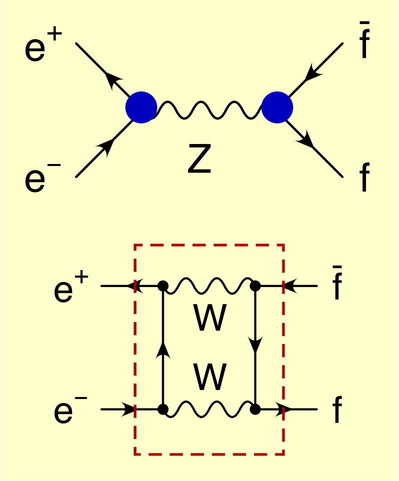
- Uncertainty of the beam energy measurements
  - Total ~ 50 keV
    - of which 45 keV correlated and 23 keV uncorrelated between energy points
  - Uncorrelated variables
    - $D = \sqrt{s} - m_{\rm Z}$ and $\Sigma = (\sqrt{s} + m_{\rm Z})/2$
    - $\sigma_D = 46\,{\rm keV}$ and $\sigma_\Sigma = 94\,{\rm keV}$
- Propagation to A_{FB}

$$\frac{\sigma(A_{\rm FB}^{\mu\mu})}{A_{\rm FB}^{\mu\mu}} \simeq \frac{1}{\sqrt{s}m_{\rm Z}} \sqrt{\left(s + m_{\rm Z}^2 - \sqrt{s}m_{\rm Z}\right)^2 \frac{\sigma_D^2}{D^2} \cancel{+ \left(s + m_{\rm Z}^2 + \sqrt{s}m_{\rm Z}\right)^2 \frac{\sigma_\Sigma^2}{\Sigma^2}}},$$

- Propagation to α_{QED} : $\frac{\sigma(\alpha_\pm)}{\alpha_\pm} \simeq \frac{\sigma_{D_\pm}}{D_\pm}$, with $D_\pm = \sqrt{s_\pm} - m_{\rm Z}$, (uncorrelated)

$$\frac{\sigma(\alpha_0)}{\alpha_0} \simeq \frac{1}{2}\sqrt{(1-\xi)^2 \frac{\sigma^2(\alpha_-)}{\alpha_-^2} + (1+\xi)^2 \frac{\sigma^2(\alpha_+)}{\alpha_+^2}} \simeq 1 \times 10^{-5}.$$

Missing EW higher orders: few 10^{-4}



- ☐ See presentation on Tuesday (Sven Heinemeyer)
  - Current status: full one-loop calculation available
    - Box EW correction : 8-9 × 10^{-4}
    - Vertex EW correction : ~10^{-3}
    - Uncertainty : few 10^{-4}
  - Higher-order corrections computable with current techniques
    - At the level of a few 10^{-4}
    - Uncertainty : few 10^{-5} - This is what we need.
  - New techniques might be needed for 3-loop corrections
    - At the level of a few 10^{-5}
    - Uncertainty : < 10^{-5}

- ☐ Significant precision improvement needed for all EWPO predictions
  - Need to set up a consistent international effort now
    - To benefit from the experience of our experts
    - To train a new generation of theorists

Efficiency and acceptance: negligible

- At lowest order in QED (no ISR/FSR/IFI)
  - Any cosθ-dependent efficiency and acceptance ε(cosθ) can be unfolded as follows

$$\frac{\mathrm{d}N^{\pm}}{\mathrm{d}\cos\theta} \propto \left\{1+\cos^2\theta \mp \frac{8}{3}A_{\mathrm{FB}}^{\mu\mu}\cos\theta\right\} \times \varepsilon(\cos\theta), \Rightarrow A_{\pm}(\cos\theta) \equiv \frac{N_-(\cos\theta)-N_+(\cos\theta)}{N_-(\cos\theta)+N_+(\cos\theta)} = \frac{4}{3}\frac{2\cos\theta}{1+\cos^2\theta}A_{\mathrm{FB}}^{\mu\mu}$$

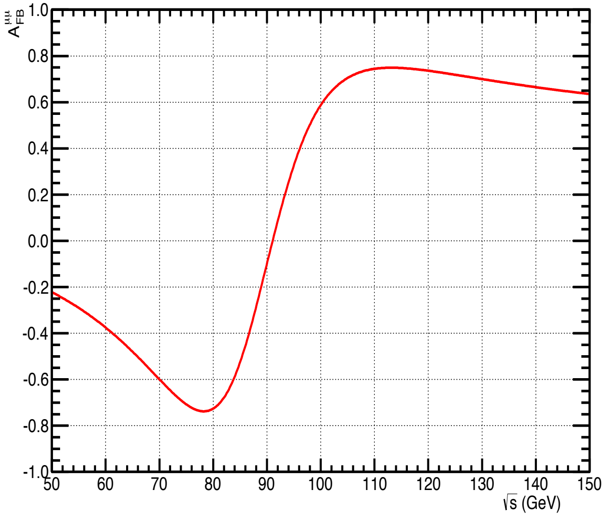
  - A_{FB} is thus obtained from μ^+/μ^- asymmetry in each cosθ bin

$$\boxed{A_{\mathrm{FB}}^{\mu\mu} = \frac{3}{4}\frac{1+\cos^2\theta}{2\cos\theta} \times A_{\pm}(\cos\theta)}$$

- Effect of QED initial-state radiation (no FSR/IFI)
  - ISR changes the muon angular distribution
    - Because of the longitudinal boost
    - Because of the reduction of √s ($\Delta A_{FB} < 0$)
  - Both effect can be dealt with in case of one ISR photon

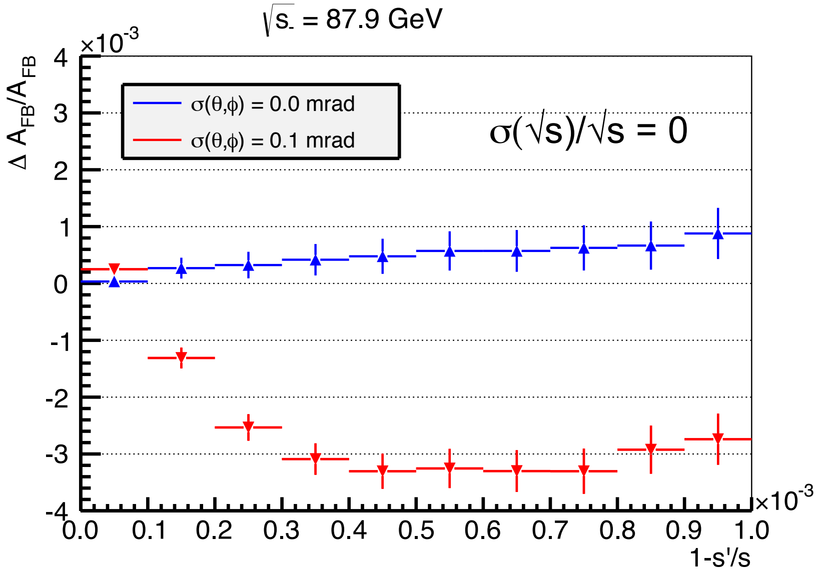
$$\cos\theta^* = \frac{\sin(\theta^+-\theta^-)}{\sin\theta^+ + \sin\theta^-}. \qquad \text{and} \qquad \frac{s'}{s} = \frac{\sin\theta^+ + \sin\theta^- - |\sin(\theta^+ + \theta^-)|}{\sin\theta^+ + \sin\theta^- + |\sin(\theta^+ + \theta^-)|}$$

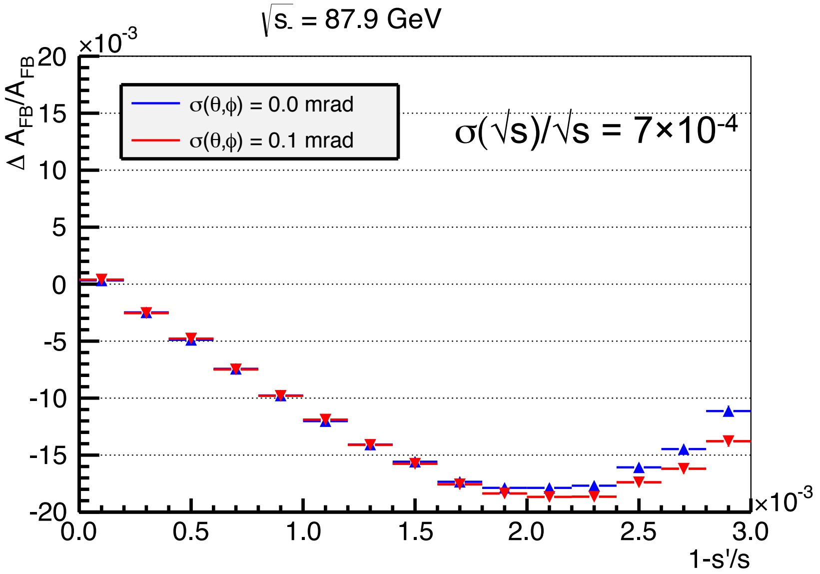
  - Measure $A_{\pm}$(s'/s, cosθ*) in each cosθ* bin
    - And fit for $A_{FB}^{\mu\mu}$(s'/s)



Initial-state radiation (no FSR/IFI)

- The s'/s and cosθ* determination is no longer exact when
  - Two ISR photons are radiated
  - The muon angular resolution of the detector is not perfect
  - The beam energy spread is not negligible (comment from Mogens Dam)
    - All generate important biasses on the measurement of $A_{FB}(s'/s)$





- Changes the asymmetry by up to 2% at √s = 87.9 GeV !

Initial-state radiation (no FSR/IFI)

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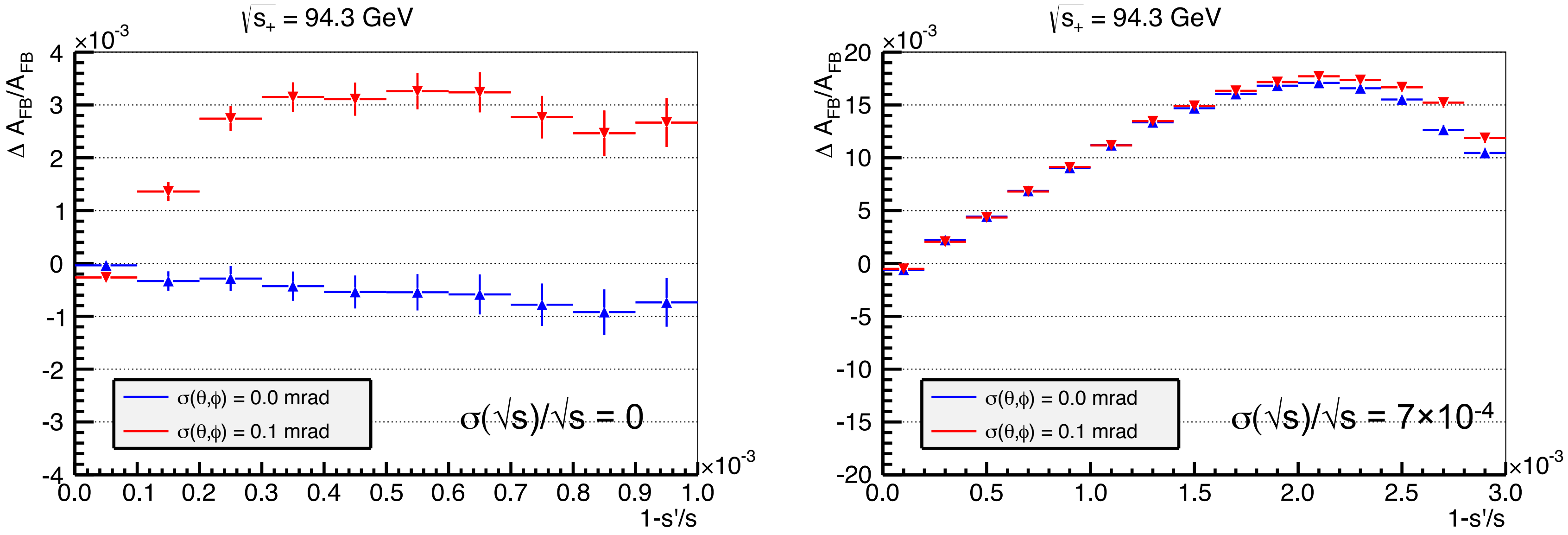


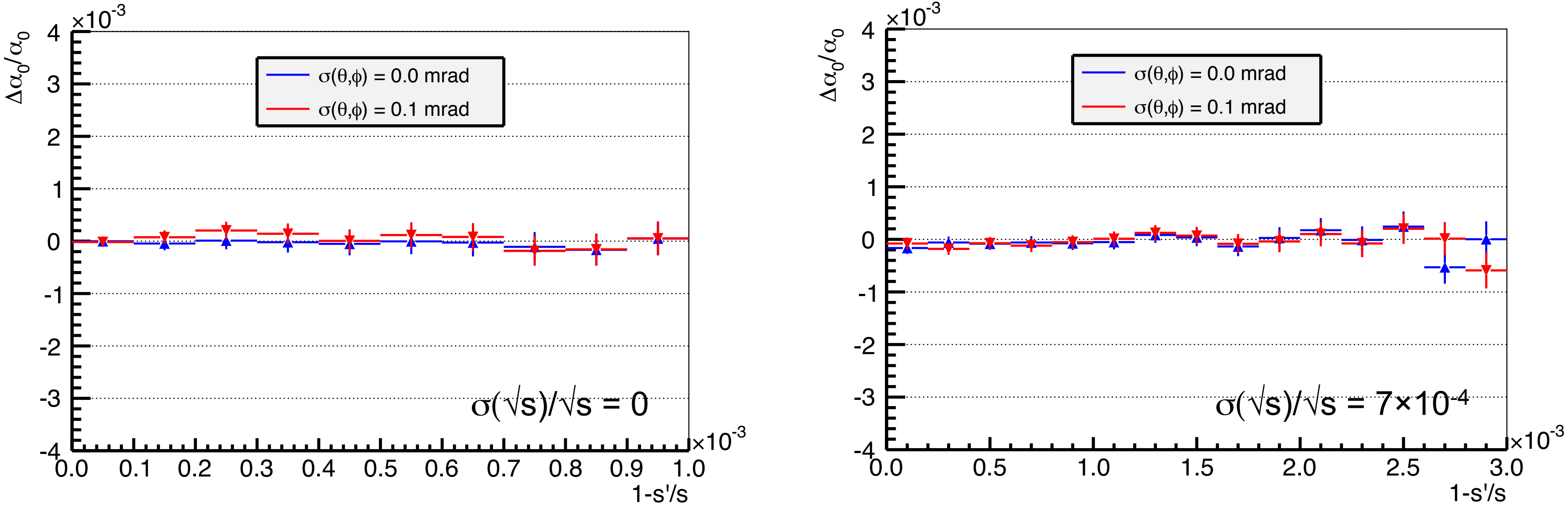
- Changes the asymmetry by up to 2% at √s = 87.9 GeV !
  - But the absolute effect on A_{FB} is universal : it is ~identical at √s = 94.3 GeV

$\Delta A_{FB}/A_{FB}(s_-) > 0$ and $\Delta A_{FB}/A_{FB}(s_+) < 0$

Initial-state radiation (no FSR/IFI)

- Cancellation of the biasses when combining the two measurements

$$\frac{\Delta\alpha_0}{\alpha_0} \simeq \quad 0.528\frac{\Delta A_{\mathrm{FB}}}{A_{\mathrm{FB}}}(s_-) \quad + \quad 0.563\frac{\Delta A_{\mathrm{FB}}}{A_{\mathrm{FB}}}(s_+)$$

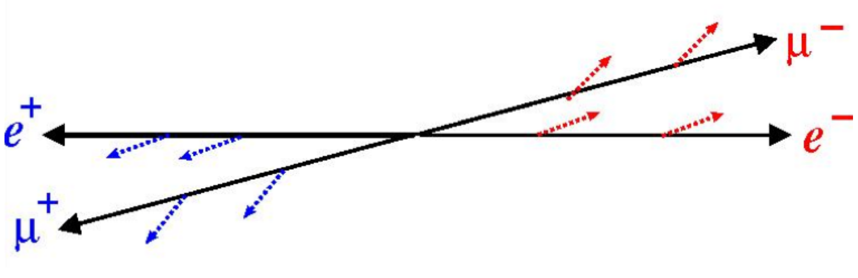


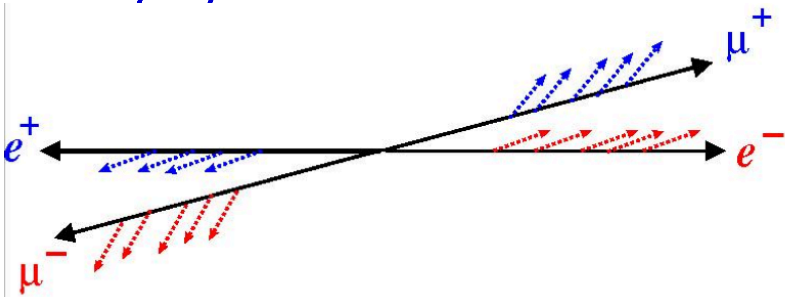
- Total bias on $\alpha_{QED}(m_Z^2)$ of the order of 8×10^{-6}
  - And is well predictable by QED soft photon prediction: negligible uncertainty

- Note: √s-spread also changes the average √s due to non flat cross section
  - Additional bias of 8×10^{-5} on $\alpha_{QED}(m_Z^2)$: must know √s-spread to better than 10%

Initial-Final-state Interference (IFI)

- **See also Staszek Jadach's talk on Tuesday**
  - Angular distribution modified with another totally asymmetric function





IFI ≈ $Q_{\mu-} \times Q_{e-}$ or $Q_{\mu+} \times Q_{e+} > 0$

IFI ≈ $Q_{\mu+} \times Q_{e-}$ or $Q_{\mu-} \times Q_{e+} < 0$

$$\frac{d\sigma_{\mu\mu}}{d\cos\theta^*}(s') \propto \left\{1+\cos^2\theta^* + \frac{8}{3}A_{\rm FB}(s')\cos\theta^*\right\} \times \underbrace{\left\{1+f\left(\frac{s'}{s}\right)\log\frac{1+\cos\theta^*}{1-\cos\theta^*}\right\}}_{F(s'/s,\ \cos\theta^*)=F} ; \text{(+FSR)}$$

Creates yet another asymmetry

- Effect is large (unlike what I inferred in arXiv:1512:05544)
  - It is larger for tighter cuts on s'/s

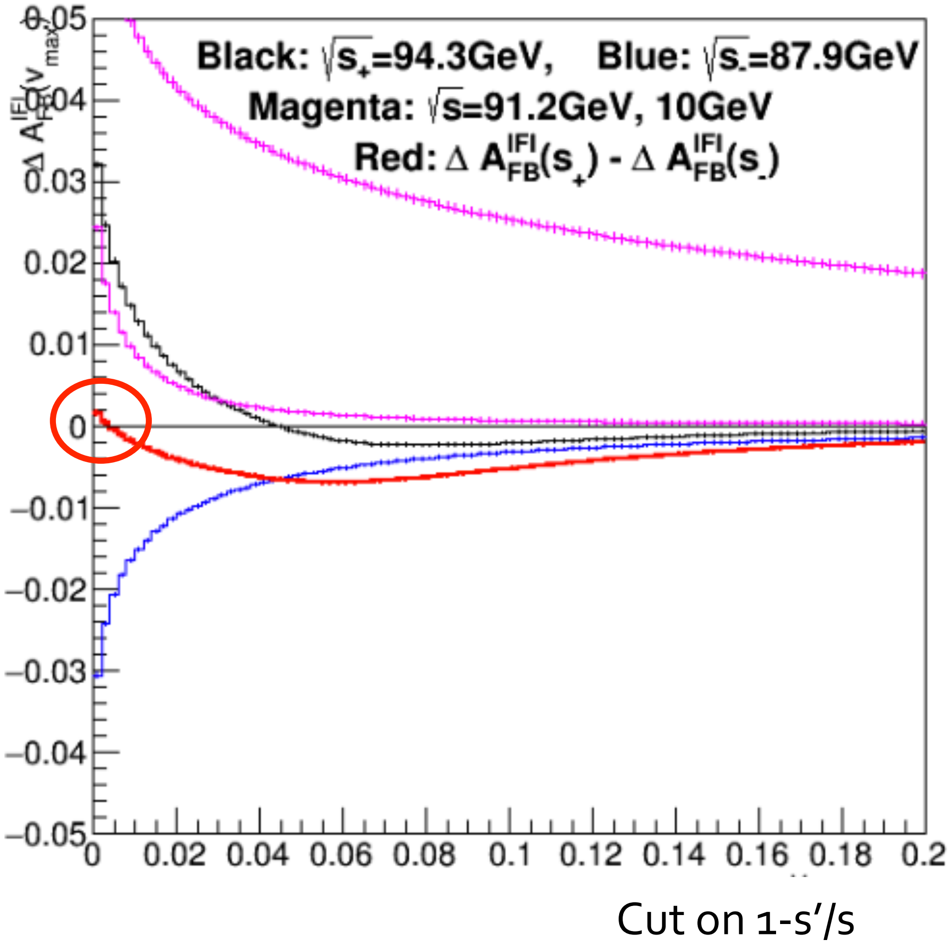
- Cancellation still occurs between s_- and s_+
  - But the simple trick with $A_\pm(\cos\theta)$ needs to be refined

Initial-Final-state Interference (IFI)

- **Plot from Staszek's talk**
  - In this plot, A_{FB} is determined as

$$A_{\rm FB}^{\mu\mu} = \frac{\sigma^{\rm F}_{\mu\mu} - \sigma^{\rm B}_{\mu\mu}}{\sigma^{\rm F}_{\mu\mu} + \sigma^{\rm B}_{\mu\mu}},$$

  - Relative effect on $A_{FB}(s_\pm)$ of the order of 10% for a tight cut on s'/s
  - Relative effect on the difference of the order of 0.2% [~used to determine $\alpha_{QED}(m_Z^2)$]
  - Requires IFI to be predicted to a precision better than 1% to reach the required precision on $\alpha_{QED}(m_Z^2)$

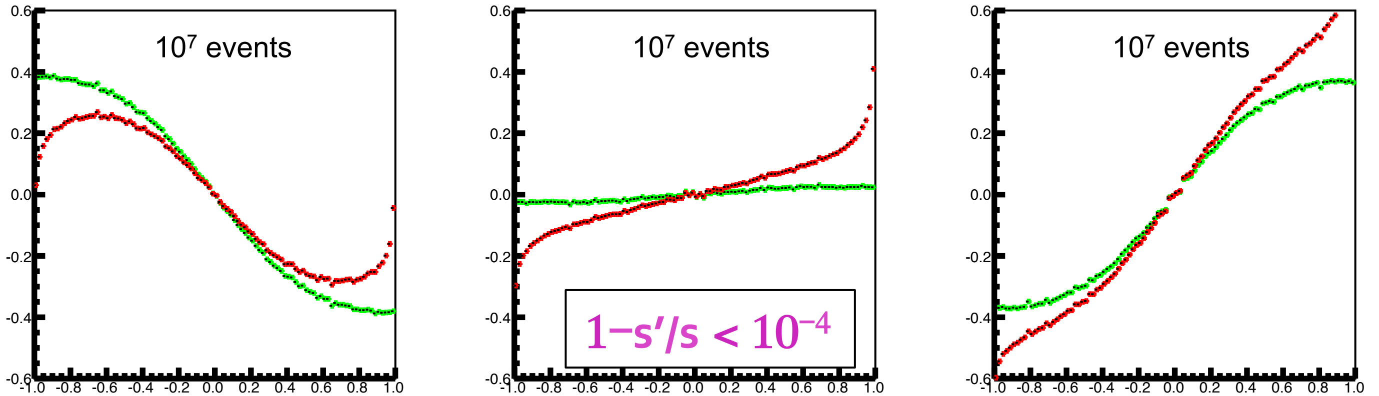


Initial-Final-state Interference (IFI)

- **Best would be to fit IFI effect away from the data – at least partially**

√s = 87.9 GeV   √s = 91.2 GeV   √s = 94.3 GeV

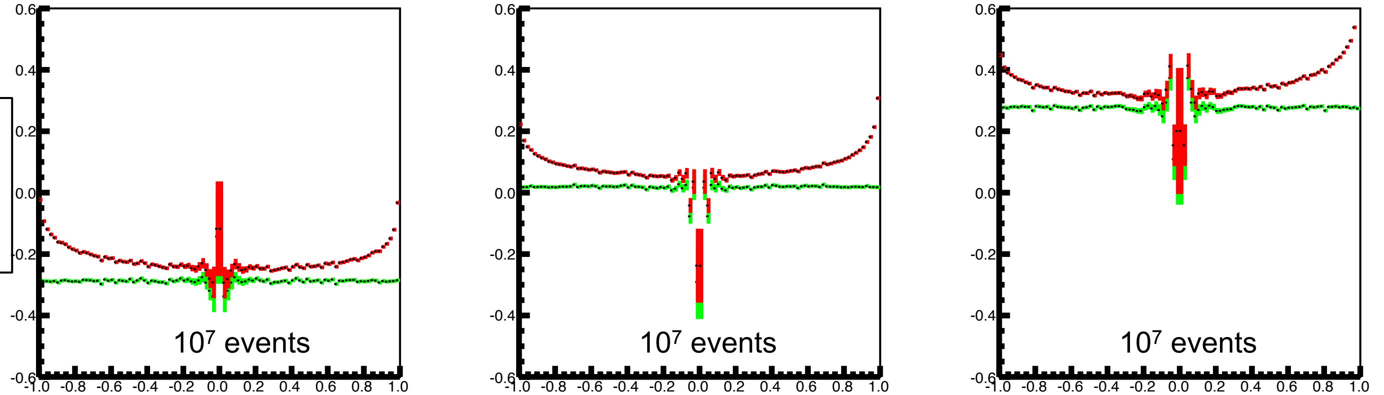
A±(cosθ)



IFI Off
IFI On

10^7 events produced with KKMC by Scott Yost and Staszek Jadach

$A_\pm(\cos\theta)$
$\times\, ¾\,(1+\cos^2\theta)$
$/\; 2\cos\theta$



IFI Off
IFI On

- **Characteristic effect similar at all √s : simultaneous fit may bring an elegant solution**

Initial-Final-state Interference (IFI)

- Once IFI is fit from the data (to be done)

$$N_\pm(\cos\theta) \approx N_o \times \varepsilon(\cos\theta) \times \left\{1+\cos^2\theta^* \pm \frac{8}{3}A_{\rm FB}(s')\cos\theta^*\right\} \times \left\{1 \pm \underbrace{f\left(\frac{s'}{s}\right)\log\frac{1+\cos\theta^*}{1-\cos\theta^*}\Bigg\}}_{\text{F(s'/s, } \cos\theta*\text{)=F}} ;$$

$$\boxed{A_{FB} = \frac{3}{4}\,\frac{1+\cos^2\vartheta}{2\cos\vartheta}\,\frac{A_\pm - F}{1-A_\pm F} \qquad \text{with} \quad A_\pm = \frac{N_-(\cos\vartheta)-N_+(\cos\vartheta)}{N_-(\cos\vartheta)+N_+(\cos\vartheta)}}$$

- I still need to find my way (analytically or not) towards a satisfactory fit
  - And benefit from the full cancellation of the IFI bias between s_+ and s_-

- The IFI functional form can also be taken from Monte Carlo
  - In this case, it has to be predicted with a precision of a few 10^{-3}
    - ➨ Towards a precision on $\alpha_{QED}(m_Z^2)$ of few 10^{-5}

Conclusions

- The FCC-ee can deliver a direct measurement of $\alpha_{QED}(m_Z^2)$ to 3×10^{-5}
  - With an accuracy 4-5 times smaller than current measurements of $\Delta\alpha_{had}$
    - Needed to exploit EWPO measurements at ANY e^+e^- collider

- The FCC-ee is the only e^+e^- collider able to do so (because it is circular)
  - First key breakthrough: The large integrated luminosities
    - Target luminosities with 4 IP's allow this measurement to be done in a year
  - Second key breakthrough: The ultra-precise measurement of the beam energy
    - Two orders of magnitude better than at linear colliders
      - ➨ Still the dominant experimental systematic uncertainty at the FCC-ee
      - ➨ Note: The beam energy spread must also be measured to ~10% or better

- The only obstacle today (beyond building the collider) is THEORY
  - Pure QED corrections (in particular IFI) can probably be fit from the data
    - Or will need to be predicted with a precision of a few 10^{-3}
  - Electroweak corrections to A_{FB} need to be computed to higher orders
    - Today, full one-loop calculation available, uncertainty ~ few 10^{-4}
  - A consistent international effort must be setup for all EWPO predictions.