FCC Week 2016
11 Apr. 2016
Rome
K. Oide (KEK)

Design constraints & assumptions

- C = 100 km, fits to the FCC-hh tunnel as much as possible.
- 2 IPs / ring.
- 30 mrad crossing angle at the IP with crab waist.
- Common lattice for all energies.
- $\varepsilon_x \leq 1.3$ nm @ 175 GeV.
- $\pm 2\%$ momentum acceptance at 175 GeV.
- Vertical emittance less than 1 pm at 175 GeV.
- $\beta_{x,y}^* = (1 \text{ m}, 2 \text{ mm})$ at 175 GeV, $(0.5 \text{ m}, 1 \text{ mm})$ at 45.6 GeV.
- Suppress the synchrotron radiation to the IP below 100 keV, up to 500 m upstream (as suggested by H. Burkhardt).
- “tapering” to cure the sawtooth at high energy.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circumference [km]</td>
<td>99983.76</td>
</tr>
<tr>
<td>Number of IPs / ring</td>
<td>2</td>
</tr>
<tr>
<td>Crossing angle at IP [mrad]</td>
<td>30</td>
</tr>
<tr>
<td>Solenoid with compensation at IP</td>
<td>±2 T × 1 m</td>
</tr>
<tr>
<td>$l^*$ [m] (asymmetric version)</td>
<td>2.2 / 2.9</td>
</tr>
<tr>
<td>Critical energy of photons to IP</td>
<td>&lt; 100 keV @ 175 GeV, up to 510 m upstream</td>
</tr>
<tr>
<td>IR Optics</td>
<td>asymmetric</td>
</tr>
<tr>
<td>Local chromaticity correction</td>
<td>Y</td>
</tr>
<tr>
<td>Crab sexts</td>
<td>integrated with LCCS</td>
</tr>
<tr>
<td>Arc cell</td>
<td>FODO, 90°/90°</td>
</tr>
<tr>
<td>Arc sextuple families</td>
<td>292 (paired)</td>
</tr>
<tr>
<td>mom. comp. $[10^{-5}]$</td>
<td>0.70</td>
</tr>
<tr>
<td>Tunes (x/y)</td>
<td>387.08 / 387.14</td>
</tr>
<tr>
<td><strong>Ebeam [GeV]</strong></td>
<td>45.6 / 175</td>
</tr>
<tr>
<td>SR energy loss per turn [GeV]</td>
<td>0.0346 / 7.47</td>
</tr>
<tr>
<td>Current / beam [mA]</td>
<td>1450 / 6.6</td>
</tr>
<tr>
<td>$P_{SR,\text{tot}}$ [MW]</td>
<td>100.3 / 98.6</td>
</tr>
<tr>
<td>$\varepsilon_x$ [nm]</td>
<td>0.86 / 1.26</td>
</tr>
<tr>
<td>$\beta_x^*$ [m]</td>
<td>0.5 (1) / 1 (0.5)</td>
</tr>
<tr>
<td>$\beta_y^*$ [mm]</td>
<td>1 (2) / 2 (1)</td>
</tr>
<tr>
<td>RF frequency [MHz]</td>
<td>400</td>
</tr>
<tr>
<td>$\sigma_{6,SR}$ [%]</td>
<td>0.038 / 0.141</td>
</tr>
<tr>
<td>$\sigma_{Z,SR}$ [mm]</td>
<td>2.8 @ $V_c = 78$ MV / 2.4 @ $V_c = 9.04$ GV</td>
</tr>
<tr>
<td>Synchrotron tune</td>
<td>-0.0158 @ $V_c = 78$ MV / -0.0657 @ $V_c = 9.04$ GV</td>
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</table>
The separation of 3(4) rings is about 12 m: wide tunnel and two tunnels are necessary around the IR, for ±1.2 km. A more compact layout/optics around the IP is also possible (A. Bogomyagkov).

Beams must cross over through the common RF (@ tt) to enter the IP from inside. Only a half of each ring is filled with bunches.
Above are the optics for $tt$, $\beta^{*}_{x/y} = 1 \text{ m} / 2 \text{ mm}$.

- 2 IP/ring.
- The optics for straight sections except for the IR are tentative, customizable for infection/extraction/collimation, etc.
• The optics in the interaction region are asymmetric.
• The synchrotron radiation from the upstream dipoles are suppressed below 100 keV up to 450 m from the IP.
• The crab sextuples are integrated in the local chromaticity correction in the vertical plane.
Synchrotron radiation toward the IP @ 175 GeV

$\nu_c (\text{keV})$  $1062$ $930$  $204$ $449$ $292$ $9.1$  $100$ $100$  $691$ $1472$ $296$ $533$  
$P_{SR} (\text{kW})$  $16.6$ $15.3$  $1.7$ $5.4$ $2.3$ $0.003$  $0.67$ $0.34$  $11.2$ $38.7$ $1.9$ $6.9$  

$\nu_c < 100 \text{ keV}$ up to 510 m from the IP.
A more compact layout / optics (AB Lattice) has been developed by A. Bogomyagkov.

- The deviation from FCC-hh is reduced to 5 m (9.5 m), the maximum excursion 7.8 m (11.9 m), the wide tunnel region ±730 m (1,200 m).

- Local chromaticity correction for both X and Y can be installed.

- A stronger dipoles are necessary for upstream of the IP (100 keV up to ~200 m, 200 keV up to ~300 m).
Favoured design at the moment. (it is not clear that a luminometer can fit inside the compensating solenoid)

- The effect of the solenoids are locally compensated within ±2 m around the IP.
- The final quads are shielded.
Solenoid compensation / shielding at the IR

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M. Koratzinos
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SC final focus quadrupole at BINP

Main contributors are Ivan Okunev and Pavel Vobly

Two versions of the FF twin-aperture iron yoke quad prototype with 2 cm aperture and 100 T/m gradient are in production.

Saddle-shaped coils, complicated in production, the first coil failed. New winding device is in development.

Straight coil, successfully wound and tested (650 A instead of the nominal 400 A)

E. Levitchev
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Also prototyping of CCT quadrupoles has started at CERN (M. Koratzinos, G. Kirby).

Straight coil, successfully wound and tested (650 A instead of the nominal 400 A).

E. Levitchev
Another QD0 prototype

A new version of QD0 was developed at BINP recently and a single-aperture prototype was manufactured.

Main parameters:
Max. gradient 100 T/m
Max. current 1100 A
Length 40 cm
Aperture 2 cm
NbTi 1.8 x 1.4 mm²
Saddle-type coils

During the first cryo-test (01.02.16) the current of 1060 A was achieved after 3 quenches.

A. Bogomyagkov, E. Levechev
HOM trapping by the cavity structure at IP

- HOM is trapped in the IP beam pipe, if all beam pipes are narrower than the IP, which needs to be larger that 40 mm (M. Sullivan).
- Heating, esp. at Z.
- Leak of HOM to the detector, through the thin Be beam pipe at the IP.
Asymmetric L*: larger outgoing beam pipe & thinner final quads

- The HOM can escape to the outside through the outgoing beam pipe, which has a diameter not smaller than IP.
- The outgoing final quad becomes thinner and stronger (E. Levichev, S. Sinyatkin).
Even with the asymmetric L*, the optics, so as the chromaticity, look similar.

The solenoid compensation is unchanged: locally compensated up to 2.2 m from the IP.

Longer L* downstream may give a space for a luminometer.
基本上一个90/90度的FODO细胞。

- 四极磁场QF/QD分别为3.5 m/1.8 m长，以减少同步辐射。它们还取决于四极磁场和束管的设计（A. Milanese, F. Zimmermann）。
- 六级磁场对-I变换。
- 292个六极磁场对每半个环。

- 基本上一个90/90度的FODO细胞。
- 四极磁场QF/QD分别为3.5 m/1.8 m长，以减少同步辐射。它们还取决于四极磁场和束管的设计（A. Milanese, F. Zimmermann）。
- 六极磁场对-I变换。
- 292个六极磁场对每半个环。
The RF section (175 GeV)

Beams cross over through the RF section.

- The usage of the straights on the both sides of the RF is to be determined.
- If the nominal strengths of quads are symmetrical in the common section, it matches to the optics of both beam.
- This section is compatible with the RF staging scenario. For lower energy, the common RF and cross over will not be necessary.

An electrostatic separator, combined with a dipole magnet.
The change of the orbit due to energy loss along the arc causes serious deformation on the optics, causing the loss of the dynamic aperture.

Everything can be cured almost completely by “tapering”, i.e. scaling the strengths of all magnets along the local energy of the beam: this is one of the best merits of a double-ring collider (F. Zimmermann).
Dynamic Aperture satisfies the requirements.

175 GeV, $\beta^*_{x,y} = (1 \text{ m}, 2 \text{ mm})$

45.6 GeV, $\beta^*_{x,y} = (0.5 \text{ m}, 1 \text{ mm})$

Requirements assuming the same horizontal emittance as the collider and 1% coupling from the booster:

$\Delta p/p > \pm 2\%$, $\Delta x > 15\sigma_x$, $\Delta y > 15\sigma_y$ @ 175 GeV, $\Delta p/p > \pm 2\%$, $\Delta x > 15\sigma_x$, $\Delta y > 18\sigma_y$ @ 45.6 GeV (See M. Aiba’s talk).
<table>
<thead>
<tr>
<th>Effects</th>
<th>included?</th>
<th>significance for DA in FCC-ee @ 175 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>synchrotron motion</td>
<td>yes</td>
<td>essential</td>
</tr>
<tr>
<td>radiation damping (turn by turn)</td>
<td>yes</td>
<td>essential, aperture↑</td>
</tr>
<tr>
<td>radiation damping (each element, esp. quads)</td>
<td>yes (no fluctuation yet)</td>
<td>essential, aperture↓</td>
</tr>
<tr>
<td>“tapering”</td>
<td>yes</td>
<td>essential</td>
</tr>
<tr>
<td>crab waist</td>
<td>yes</td>
<td>yes, aperture↓</td>
</tr>
<tr>
<td>solenoids</td>
<td>yes</td>
<td>minimal, if locally compensated</td>
</tr>
<tr>
<td>Maxwellian fringe field</td>
<td>yes</td>
<td>small</td>
</tr>
<tr>
<td>kinematical terms</td>
<td>yes</td>
<td>small</td>
</tr>
<tr>
<td>beam-beam</td>
<td>yes (weak-strong)</td>
<td>yes, esp. on lifetime (D. Zhou)</td>
</tr>
<tr>
<td>errors/misalignments</td>
<td>not yet</td>
<td>essential, correction schemes must be developed</td>
</tr>
</tbody>
</table>
The dynamic aperture for the AB lattice is under optimization, and looks promising so far.

P. Piminov, A. Bogomyagkov
A negative field gradient in the main dipole of the unit cell provides:

- longer cell length for a given emittance / better packing factor
- larger momentum compaction (longer bunch length for a same RF voltage)
- larger energy spread
- larger dispersion
- weaker sextupoles

Suggested by E. Levechev
An example of combined function: $J_z = 0.6 \, @ \, 175 \, \text{GeV}$

<table>
<thead>
<tr>
<th>$J_z$</th>
<th>0.6</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td># of FODO cells</td>
<td>1062</td>
<td>1442</td>
</tr>
<tr>
<td>Length of dipole (m)</td>
<td>33.9</td>
<td>23.1</td>
</tr>
<tr>
<td>H dispersion at SF (cm)</td>
<td>29.6</td>
<td>16.3</td>
</tr>
<tr>
<td>1 turn energy loss (GV)</td>
<td>7.09</td>
<td>7.74</td>
</tr>
<tr>
<td>momentum spread (%)</td>
<td>0.24</td>
<td>0.14</td>
</tr>
<tr>
<td>momentum compaction ($10^{-6}$)</td>
<td>12.8</td>
<td>7.2</td>
</tr>
<tr>
<td>bunch length (mm)</td>
<td>5.0</td>
<td>2.4</td>
</tr>
<tr>
<td>RF voltage (GV)</td>
<td>9.6</td>
<td>9.4</td>
</tr>
<tr>
<td>synchrotron tune</td>
<td>-0.10</td>
<td>-0.068</td>
</tr>
</tbody>
</table>

![Graphs showing $J_z = 0.6$ and $J_z = 2$](FCCee_t_72_2.sad, FCCee_t_74_11.sad)
Dynamic aperture of combined function lattice.

175 GeV, $\beta^*_{x,y} = (0.5 \text{ m}, 1 \text{ mm})$

- The dynamic aperture is comparable to the flat-dipole lattice.
- Looking for beam-beam simulation and hardware solution of the dipole.
Some related talks during this workshop

<table>
<thead>
<tr>
<th>Topic</th>
<th>Speaker</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF system parameters for Z, W, H and tt</td>
<td>O. Brunner</td>
<td>Tue. 8:30</td>
</tr>
<tr>
<td>Beam dynamics: RF requirements for the FCC-hh and FCC-ee options</td>
<td>E. Shaposhnikova</td>
<td>Tue. 8:50</td>
</tr>
<tr>
<td>Beam-beam simulations of FCC-ee with Lifetrac</td>
<td>D. Shatilov</td>
<td>Wed. 8:30</td>
</tr>
<tr>
<td>Beam-beam simulations and ecloud in e+ ring</td>
<td>K. Ohmi</td>
<td>Wed. 8:50</td>
</tr>
<tr>
<td>Interplay of the beam-beam effect and the lattice nonlinearity</td>
<td>D. Zhou</td>
<td>Wed. 9:10</td>
</tr>
<tr>
<td>A new beam-beam effect in collisions with crossing angle</td>
<td>V. Telnov</td>
<td>Wed. 9:30</td>
</tr>
<tr>
<td>Challenges for FCC-ee MDI</td>
<td>M. Boscolo</td>
<td>Wed. 8:30</td>
</tr>
<tr>
<td>Synchrotron radiation background</td>
<td>H. Burkhardt</td>
<td>Wed. 8:50</td>
</tr>
<tr>
<td>FCC-ee Interaction Region Layout</td>
<td>M. Sullivan</td>
<td>Wed. 9:10</td>
</tr>
<tr>
<td>Backgrounds in the FCC-ee detector and consequences for the trigger and DAQ</td>
<td>E. Perez</td>
<td>Wed. 9:30</td>
</tr>
<tr>
<td>FCC-ee warm magnets design</td>
<td>A. Milanese</td>
<td>Wed. 14:10</td>
</tr>
<tr>
<td>FCC-ee IR optics solutions</td>
<td>A. Bogomyagkov</td>
<td>Wed. 15:30</td>
</tr>
<tr>
<td>Arc optics, global Q’ correction and emittance variation</td>
<td>B. Harer</td>
<td>Wed. 15:50</td>
</tr>
<tr>
<td>Lattice for a Higgs factory</td>
<td>Y. Cai</td>
<td>Wed. 16:40</td>
</tr>
<tr>
<td>Tolerance studies and coupling correction for FCC-ee</td>
<td>S. Aumon</td>
<td>Thu. 10:30</td>
</tr>
<tr>
<td>Dynamic aperture</td>
<td>L. Medrano</td>
<td>Thu. 10:50</td>
</tr>
<tr>
<td>FCC-ee injector complex incl. Booster</td>
<td>Y. Papaphilippou</td>
<td>Thu. 11:10</td>
</tr>
<tr>
<td>Preliminary inector linac design</td>
<td>S. Polozov</td>
<td>Thu. 13:50</td>
</tr>
<tr>
<td>Design of the booster ring optics</td>
<td>O. Etisken</td>
<td>Thu. 14:05</td>
</tr>
<tr>
<td>Top-up injection schemes</td>
<td>M. Aiba</td>
<td>Thu. 14:35</td>
</tr>
</tbody>
</table>

... and more ...
Some related posters during this workshop

<table>
<thead>
<tr>
<th>Title</th>
<th>Speaker</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam dump for the FCC-ee collider</td>
<td>A. Apyan</td>
<td>Wed. 17:30</td>
</tr>
<tr>
<td>Superconducting Cavity Design for FCC-ee</td>
<td>S. Zadeh</td>
<td>Wed. 17:30</td>
</tr>
<tr>
<td>Superconducting sputtered Nb3Sn films for SRF applications</td>
<td>K. Ilyina</td>
<td>Wed. 17:30</td>
</tr>
<tr>
<td>Tapering options for the FCC-ee collider</td>
<td>A. Doblhammer</td>
<td>Wed. 17:30</td>
</tr>
</tbody>
</table>

... and more ...
Summary

- **Optics for FCC-ee are presented, considering:**
  - 2 IPs/ring, with 30 mrad crossing angle.
  - Local chromaticity correction with crab waist.
  - Suppression of synchrotron radiation in the IR below 100 keV up to 510 m from the IP.
  - Solenoid at IP & its compensation.
  - Possible asymmetric L* for wider outgoing beam pipe at the IP.
  - Element-by-element synchrotron radiation.
  - Tapering of all magnets according to the local beam energy to suppress sawtooth.
  - Common RF sections with cross-over of two beams (at least at tt).
  - Optimization of dynamic aperture with hundreds of sextuple families.
  - Geometrical fitting to the FCC-hh tunnel.
  - Combined function dipole in the arc will bring a number of merits, if realized.

- **Resulting dynamic aperture almost satisfies the requirements.**

- Things need further investigation:
  - Field quality, more realistic profile of magnetic field.
  - Tolerances / tuning scheme for machine errors, misalignments.
  - and more...
Backups
The crab waist scheme shifts the vertical waist of a beam by

\[ \Delta s = -\frac{x^*}{2\theta_x}. \]  

Thus the associated transformation is

\[ y^* \rightarrow y^* - p^*_y \Delta s = y^* + \frac{p^*_y x^*}{2\theta_x}, \]  

which is performed by a Hamiltonian at the IP:

\[ H^* = \frac{x^* p^*_y^2}{4\theta_x}. \]  

If there are the phase relations between the IP and the sextupoles:

\[ \Delta \psi_x = 2\pi \quad \text{and} \quad \Delta \psi_y = 2.5\pi, \]  

then the variables at the IP \((x^*, p^*_y)\) are expressed in those at the sext \((x, y)\):

\[ x^* = \sqrt{\frac{\beta^*_x}{\beta_x}} x, \quad p^*_y = \frac{y}{\sqrt{\beta^*_y \beta_y}}. \]  

Thus the Hamiltonian at the IP is equivalent to a Hamiltonian at the sext:

\[ H = \frac{xy^2}{4\theta_x \beta^*_y \beta_y} \sqrt{\frac{\beta^*_x}{\beta_x}}, \]  

which can be approximated by a Hamiltonian of a sextupole:

\[ H_s = \frac{k_2}{6} (x^3 - 3xy^2), \]  

with

\[ k_2 = -\frac{1}{2\theta_x \beta^*_y \beta_y} \sqrt{\frac{\beta^*_x}{\beta_x}}. \]
Chromogeometric aberration of CCS

Consider the transfer matrix between sexts of YCCS with momentum dependence:

\[
m_y = \left( \frac{-1 + 8\delta^2}{\delta / \ell (-4 - 4\sqrt{2} + (14 + 9\sqrt{2})\delta)} \right) -2\delta\ell(-4 - 4\sqrt{2} + (6 + 5\sqrt{2})\delta) + O(\delta^3),
\]

and similar for \( m_x \). Then the emittance increment due to the sextupole kick is calculated as:

\[
R\delta^2 \equiv \frac{\Delta\varepsilon_y}{\varepsilon_y} = \frac{256\delta^2}{\beta_x\beta_y^2\eta_x^2\ell^2} (2\sqrt{2}\beta_y + \ell\xi_y)^2 \times \left((6 - 4\sqrt{2})\beta_y^2\varepsilon_x\ell^2 + ((9 - 6\sqrt{2})\beta_y^2 - 2\ell^2)\beta_x\beta_y\varepsilon_y + ((2\sqrt{2} - 1)\beta_y^2 + (14 + 8\sqrt{2}\ell^2)\beta_x^2\varepsilon_x\right),
\]

where \( \beta_x, \beta_y, \) and \( \eta_x \) are the values at the sextupole, \( \ell \) the separation of quads, and we have assumed

\[
-k_2\beta_y\eta_x = \xi_y + 2\sqrt{2}\beta_y/\ell.
\]

If we plug in the numbers:

\[
\beta_x = 15 \text{ m}, \ \varepsilon_x = 1.2 \text{ nm}, \ \varepsilon_y = 2.4 \text{ pm}, \ \eta_x = 0.16 \text{ m}, \ \ell = 58 \text{ m}, \ \xi_y = 3,500,
\]

we get the following graph.

Although the optimum is at around \( \beta_y = 600 \text{ m} \), the increment is small up to \( \beta_y \lesssim 8,000 \text{ m} \).

- Above are just tentative optics.
- Usage of these sections is to be determined.
A rough estimation of radiation by arc quads

- The radiation power:
  \[ P \propto \gamma^2 B^2 \ell \]

- Ratio of powers by dipoles and quadrupoles per unit cell:
  - dipole:
    \[ P_d \propto \gamma^2 \left( \frac{B \ell_{\text{cell}}}{B \rho} \right)^2 \left( \frac{B \rho}{\ell_{\text{cell}}} \right)^2 \ell_{\text{cell}} \propto \gamma^4 \frac{\theta^2}{\ell_{\text{cell}}} \]
  - quadrupole:
    \[ P_q \propto \frac{\gamma^2}{2} \left( \frac{B' \Delta x \ell_q}{B \rho} \right)^2 \left( \frac{B \rho}{\ell_q} \right)^2 \ell_q \propto \gamma^4 \frac{k_1^2 \Delta x^2}{2} \frac{\ell}{\ell_q} \]
  - ratio:
    \[ \frac{P_q}{P_d} = \frac{(k_1 \ell_{\text{cell}})^2 \beta_{xq} n^2 \varepsilon_x}{2 \ell_{\text{cell}} \theta^2 \ell_q}, \quad \Delta x^2 = n^2 \beta_{xq} \varepsilon_x \]

- In the case of a 90° cell, \( k_1 \ell_{\text{cell}} = 2\sqrt{2}, \beta_{xq}/\ell_{\text{cell}} = 1 + \frac{1}{\sqrt{2}} \), then:
  \[ \frac{P_q}{P_d} = (4 + 2\sqrt{2}) \frac{n^2 \varepsilon_x}{\theta^2 \ell_q} \]

- or a particle with an amplitude of \( n \sigma_x \) will receive an energy loss per every turn:
  \[ \frac{\Delta p_1}{p_0} = P_q \times \frac{U_0}{E} = (4 + 2\sqrt{2}) \frac{n^2 \varepsilon_x}{\theta^2 \ell_q} \alpha_{\varepsilon} \quad (\alpha_{\varepsilon}: \text{long. damping per turn}) \]

- which causes a synchrotron motion with a momentum amplitude ±\( \Delta p/p_0 \):
  \[ \frac{\Delta p}{p_0} = \frac{1}{2\pi \nu_s} \frac{\Delta p_1}{p_0} = \left(2 + \sqrt{2}\right) \frac{n^2 \varepsilon_x}{\pi \theta^2 \ell_q \nu_s} \alpha_{\varepsilon} \]
A rough estimation of radiation by arc quads (cont’d)

\[ \varepsilon_x = 2 \text{ nm}, \theta = \frac{2\pi}{1240}, \alpha_\varepsilon/\nu_s = 0.41 \text{ gives} \]

\[ \frac{\Delta p}{p_0} = 0.58\% \left( \frac{n}{10} \right)^2 \left( \frac{0.6 \text{ m}}{\ell_q} \right) \]

If we plug-in the number for FCC-ee-tt:

* only damping, no fluctuation, is taken into account in simulations in these slides.

The effect on the dynamic aperture

\[ \frac{\Delta x}{\sigma_x} \]

\[ \frac{\Delta p_x}{\sigma_{p_x}} \]

\[ \Delta \varepsilon / \sigma_\varepsilon \]

\[ \ell_q = 0.6 \text{ m} \]

\[ \ell_q = 3 \text{ m} \]