Longitudinal polarization at TLEP-Z?

Main interest: measure EW couplings at the Z peak most of which provide measurements of **$\sin^2\theta^{\ell ept}_{W}$** $= e^2/g^2\ (m_z)$

-- not to be confused with -- $\sin^2\theta_W = 1 - m_w^2/m_z^2$

Useful references from the past:

«polarization at LEP» CERN Yellow Report 88-02

Precision Electroweak Measurements on the Z Resonance

Phys.Rept.427:257-454,2006

http://arxiv.org/abs/hep-ex/0509008v3

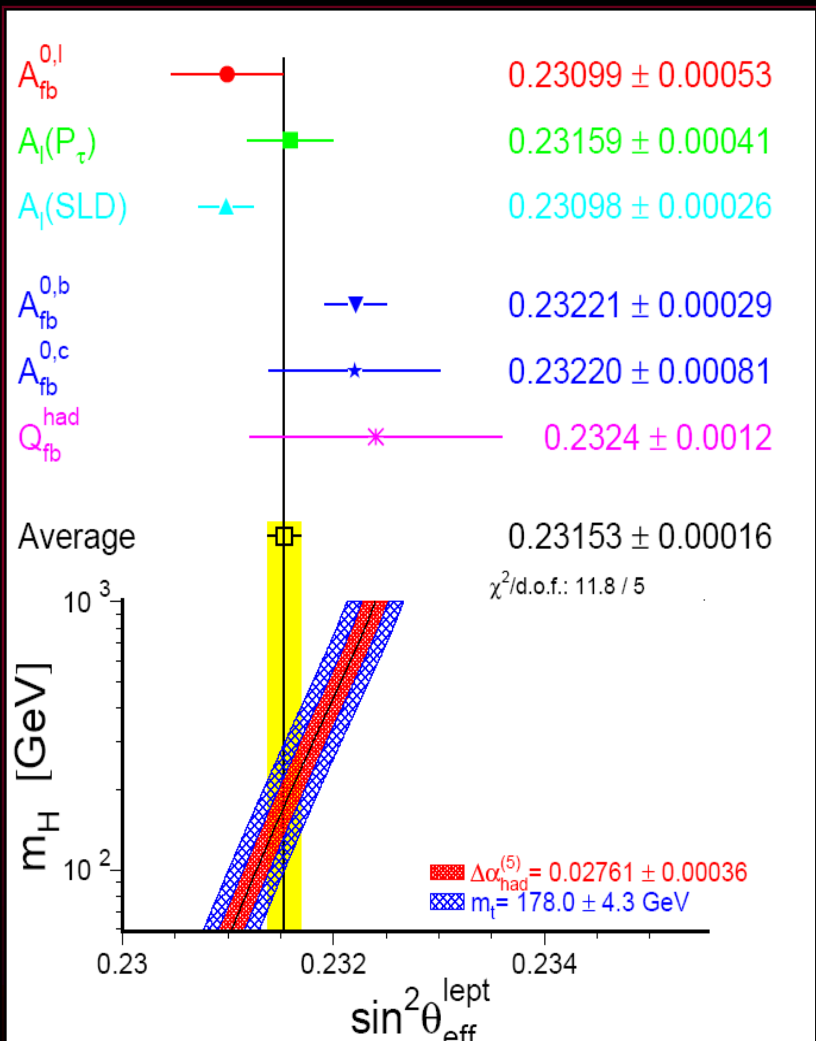
GigaZ @ ILC by K. Moenig

Measuring $\sin^2\theta_W^{eff}$ (m_Z)

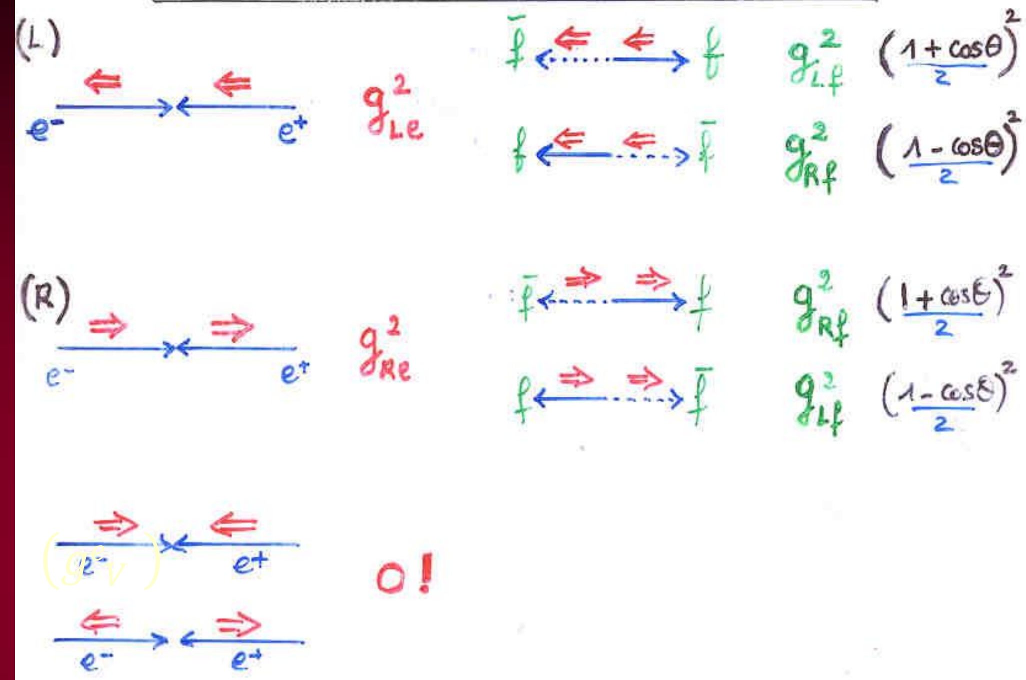
$$\sin^2\theta_W^{eff} \equiv \frac{1}{4}(1 - g_V/g_A)$$

$$g_V = g_L + g_R$$

$$g_A = g_L - g_R$$



Helicity effects in $e^+e^- \to f\bar{f}$



Pol'd BEAM $\Rightarrow$

$$A_{LR} = \frac{\sigma_L^{tot} - \sigma_R^{tot}}{\sigma_L^{tot} + \sigma_R^{tot}} = \frac{g_{Le}^2 - g_{Re}^2}{g_{Le}^2 + g_{Re}^2} \equiv \mathcal{A}_e = \frac{2\, g_{Ve}\, g_{Ae}}{g_{Ve}^2 + g_{Ae}^2}$$

$$A_{FB}^{Pol\,f} = \frac{\sigma_L^{Ff} - \sigma_L^{Bf} - (\sigma_R^{Ff} - \sigma_R^{Bf})}{\sigma_L^{Ff} + \sigma_L^{Bf} + \sigma_R^{Ff} + \sigma_R^{Bf}} = \frac{3}{4}\mathcal{A}_f$$

no Pol available:

$$A_{FB} = \frac{\sigma_U^{Ff} - \sigma_U^{Bf}}{\sigma_U^{Ff} + \sigma_U^{Bf}} = \frac{3}{4}\mathcal{A}_e \mathcal{A}_f$$

Pol analyses τ:

$$\langle P_f \rangle = \frac{\sigma_U^R - \sigma_U^L}{\sigma_U^R + \sigma_U^L} = -\mathcal{A}_f$$

$$A_{FB}^{Pol} = \frac{\sigma_U^{RF} - \sigma_U^{LF} - (\sigma_U^{RB} - \sigma_U^{LB})}{\sigma_U^{RF} + \sigma_U^{LF} + \sigma_U^{RB} + \sigma_U^{LB}} = -\frac{3}{4}\mathcal{A}_e$$

EWRCs

relations to the well measured

$G_F \; m_Z \; \alpha_{QED}$

at first order:

$\Delta\rho$: $\equiv \varepsilon_1$

$$\Gamma_\ell = (1+\Delta\rho)\,\frac{G_F\, M_Z^3}{24\pi\sqrt{2}}\left(1+\left(\frac{g_{V\ell}}{g_{A\ell}}\right)^2\right)\left(1+\frac{3}{4}\frac{\alpha}{\pi}\right)$$

$$\Delta\rho = \alpha/\pi\,(m_{top}/m_Z)^2 - \alpha/4\pi\,\log(m_h/m_Z)^2$$

ε_3

$$\sin^2\theta_W^{eff}\cos^2\theta_W^{eff} = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}\,G_F\,m_Z^2}\,\frac{1}{1+\Delta\rho}\,\frac{1}{1-\frac{\varepsilon_3}{\cos^2\theta_W}}$$

$$\varepsilon_3 = \cos^2\theta_W\,\alpha/9\pi\,\log(m_h/m_Z)^2$$

δ_{vb}

$$\Gamma_b = (1+\delta_{vb})\,\Gamma_d\,(1-\text{mass corrections}\;\alpha\, m_b^2/M_Z^2)$$

$$\delta_{vb} = 20/13\,\alpha/\pi\,(m_{top}/m_Z)^2$$

ε_2

$$M_W^2 = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}\,G_F\sin^2\theta_W^{eff.}}\cdot\frac{1}{(1-\varepsilon_3+\varepsilon_2)}$$

$\sin^2\theta_W^{eff}$ is defined from

$$\sin^2\theta_W^{eff} = \frac{1}{4}\left(1-\frac{g_{V\ell}}{g_{A\ell}}\right) = \sin^2\theta_W^{eff}{}_{lept}$$

obtained from asymmetries at the Z.

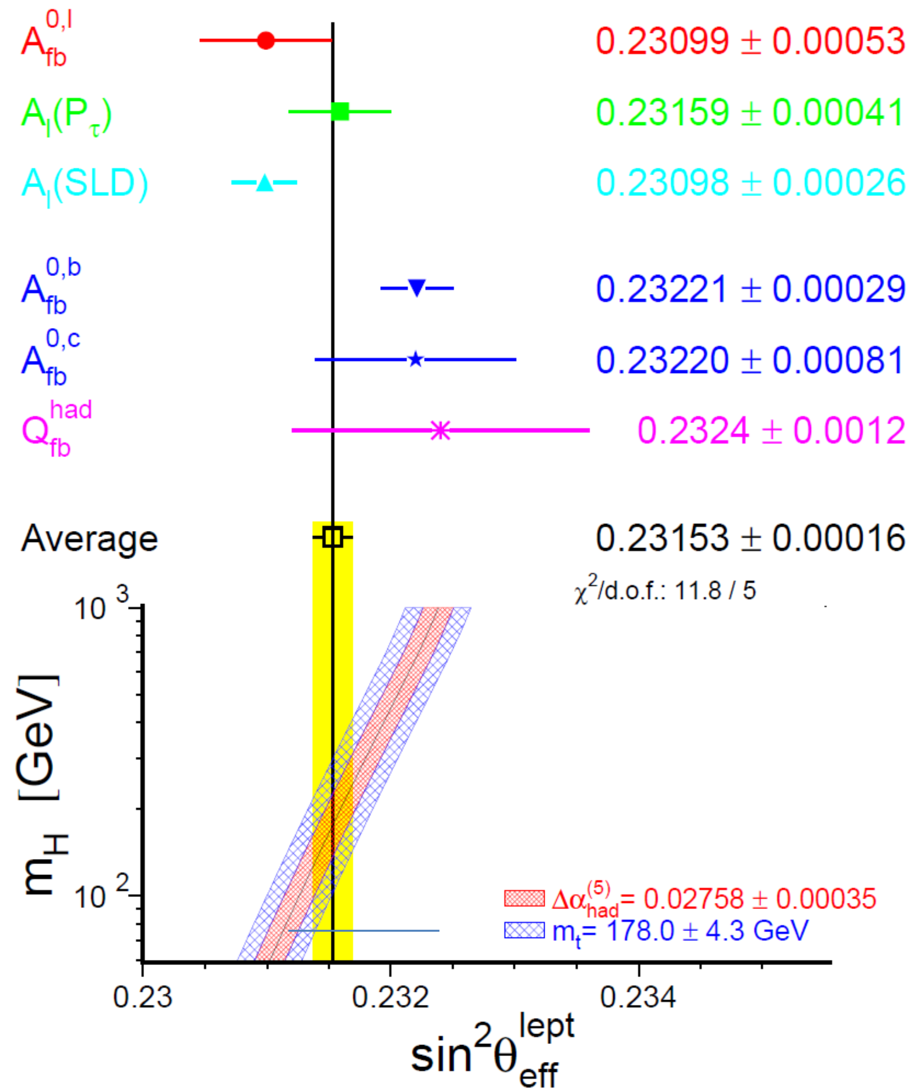
also Δr

$$m_W^2 = \frac{\pi\alpha}{\sqrt{2}\,G_F}\cdot\frac{1}{\left(1-\frac{M_W^2}{M_Z^2}\right)}\,\frac{1}{(1-\Delta r)}$$

$$\Delta r = \Delta\alpha - \frac{\cos^2\theta_W}{\sin^2\theta_W}\Delta\rho + 2\,\frac{\cos^2\theta_W}{\sin^2\theta_W}\,\varepsilon_3 + \frac{c^2-s^2}{s^2}\,\varepsilon_2$$

complete formulae at 2d order including strong corrections are available in fitting codes

e.g. ZFITTER , GFITTER



$A_{fb}^{0,l}$ 0.23099 ± 0.00053
$A_l(P_\tau)$ 0.23159 ± 0.00041
A_l(SLD) 0.23098 ± 0.00026
$A_{fb}^{0,b}$ 0.23221 ± 0.00029
$A_{fb}^{0,c}$ 0.23220 ± 0.00081
Q_{fb}^{had} 0.2324 ± 0.0012
Average 0.23153 ± 0.00016
χ^2/d.o.f.: 11.8 / 5
10^3
10^2
m_H [GeV]
$\Delta\alpha_{had}^{(5)}$ = 0.02758 ± 0.00035
m_t = 178.0 ± 4.3 GeV
0.23
0.232
0.234
$\sin^2\theta_{eff}^{lept}$

Extracting physics from $\sin^2\theta^{lept}_W$

1. Direct comparison with m_Z

$$\sin^2\theta_W^{eff}\cos^2\theta_W^{eff} = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}\,G_F\,m_Z^2}\,\frac{1}{1+\Delta\rho}\,\frac{1}{1-\frac{\varepsilon_3}{\cos^2\theta_W}}$$

Uncertainties in m_{top}, $\Delta\alpha(m_z)$, m_H, etc….

$\Delta\sin^2\theta^{lept}_W$ ~ $\Delta\alpha(m_z)/3$ = 10^{-5} if we can reduce $\Delta\alpha(m_z)$ (see P. Janot)

2. Comparison with m_W/m_Z

Compare above formula with similar one:

$$\sin^2\theta_W\cos^2\theta_W = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}\,G_F\,m_Z^2}\cdot\frac{1}{1-\left(-\frac{\cos^2\theta_W}{\sin^2\theta_W}\Delta\rho + 2\frac{\cos^2\theta_W}{\sin^2\theta_W}\varepsilon_3 + \frac{c^2-s^2}{s^2}\varepsilon_2\right)}$$

Where it can be seen that **$\Delta\alpha(m_z)$** cancels in the relation.

The limiting error is the error on m_W.

For **Δm_W= 0.5 MeV this corresponds to $\Delta\sin^2\theta^{lept}_W = 10^{-5}$**

Assume for now ONE experiment at ECM=91.2

Luminosity «baseline» with beta*=1mm : 2.1 $10^{36}/cm^2/s$ = 2 pb^{-1}/s,
Sigma_had = 31 $10^{-33} cm^2$ ➔ 6.5 10^{11} qq events/10^7 year/exp.

Consider 3 years of 10^7 s
2 10^{12} Z→ $\bar{q}q$ events (typical exp at LEP was 4.10^6)
4 10^{11} Z→ $\bar{b}b$
10^{11} Z→ $\mu\mu$, $\tau\tau$ each

Will consider today the contribution of the Center-of-mass energy systematic errors

Today: step I, compare

ILC measurement of A_{LR} with 10^9 Z and P_{e-} =80%, P_{e+} =30%

FCC-ee measurement of $A_{FB}^{\mu\mu}$ and $A_{FB}^{Pol}(\tau)$ with 2.10^{12} Z

A_{LR} (P) and A_{FB} ($\mu\mu$)

Both measure the weak mixing angle as **defined** by the relation $A_\ell = \frac{(g^e_L)^2-(g^e_R)^2}{(g^e_L)^2+(g^e_R)^2}$

with $(g^e_L) = ½ - \sin^2\theta^{lept}_W$ and $(g^e_R) = -\sin^2\theta^{lept}_W$ $A_\ell \approx 8(1/4 - \sin^2\theta^{lept}_W)$

$A_{LR} = A_e$

$A_{FB}^{\mu\mu} = ¾ A_e A_\mu = ¾ A_\ell^2$

-- $A_{FB}^{\mu\mu}$ is measured using muon pairs (5% of visible Z decays) and unpolarized beams

-- A_{LR} is measured using all statistics of visible Z decays with beams of alternating longitudinal polarization

both with very small experimental systematics

-- **parametric sensitivity** $\frac{dA_{FB}^{\mu\mu}}{d\sin^2\theta^{lept}_W} = 1.73$ **vs** $\frac{dA_{LR}}{d\sin^2\theta^{lept}_W} = 7.9$

-- **sensitivity to center-of-mass energy** (w.r.t. m_Z) is larger for $A_{FB}^{\mu\mu}$

$$\frac{\partial A_{FB}^{\mu\mu}}{\partial\sqrt{s}} = 0.09/\text{GeV} \quad \textbf{vs} \quad \frac{\partial A_{LR}}{\partial\sqrt{s}} = 0.019/\text{GeV}$$

"an 80 MeV uncertainty in Ecm corresponds to a 1% error on A_{LR}" (relative error)

But of course $A_{FB}^{\mu\mu}$ benefits from much larger statistics and Ecm precision of circular collider

K. Moening:

$\sin^2\theta^l_{eff}$

Most sensitive observable is A_{LR}, so only this is discussed

$$A_{LR} = \frac{1}{\mathcal{P}}\frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \mathcal{A}_e = \frac{2v_e a_e}{v_e^2 + a_e^2}$$

$$v_e/a_e = 1 - 4\sin^2\theta^l_{eff}$$

independent of the final state

Statistical error with 10^9 Zs: $\Delta A_{LR} = 4 \cdot 10^{-5}$

(for $\mathcal{P}_{e^-} = 80\%$, $\mathcal{P}_{e^+} = 0$)

Crucial ingredient: polarisation measurement

Error from polarisation: $\Delta A_{LR}/A_{LR} = \Delta\mathcal{P}/\mathcal{P}$

- only electron polarisation with $\Delta\mathcal{P}/\mathcal{P} = 0.5\% \Rightarrow \Delta A_{LR} = 8 \cdot 10^{-4}$
  (Still factor three to SLD, but few million Zs are sufficient)

Measurement of A_{LR}

electron bunches	$1\Leftarrow$	2	3	$4\Leftarrow$
positron bunches	1	$2\Rightarrow$	3	$4\Rightarrow$
cross sections	σ_1	σ_2	σ_3	σ_4
event numbers	N_1	N_2	N_3	N_4

$$\sigma_1 = \sigma_u (1 - P^-_e A_{LR})$$

$$\sigma_2 = \sigma_u (1 + P^+_e A_{LR})$$

$$\sigma_3 = \sigma_u$$

$$\sigma_4 = \sigma_u [1 - P^+_e P^-_e + (P^+_e - P^-_e) A_{LR}]$$

Verifies polarimeter with experimentally measured cross-section ratios

statistics

ΔA_{LR} = 0.0025 with about 10^6 Z^0 events,

ΔA_{LR} = 0.000015 with 10^{11} Z and 40% polarization in collisions.

$\Delta \sin^2\theta_W^{eff}$ (stat) = O(2.10^{-6})

- with positron polarisation $\mathcal{P}_{\text{eff}} = \frac{\mathcal{P}_{e^+}+\mathcal{P}_{e^-}}{1+\mathcal{P}_{e^+}\mathcal{P}_{e^-}}$

  $\Rightarrow$ gain a factor four for $\mathcal{P}_{e^-}/\mathcal{P}_{e^+} = 80\%/60\%$ due to error propagation (even when error is 100% correlated between the polarimeters the gain is a factor three)

- even better with Blondel scheme:

$$\sigma = \sigma_u \left[1 - \mathcal{P}_{e^+}\mathcal{P}_{e^-} + A_{\text{LR}}(\mathcal{P}_{e^+} - \mathcal{P}_{e^-})\right]$$

$$A_{\text{LR}} = \sqrt{\frac{(\sigma_{++} + \sigma_{-+} - \sigma_{+-} - \sigma_{--})(-\sigma_{++} + \sigma_{-+} - \sigma_{+-} + \sigma_{--})}{(\sigma_{++} + \sigma_{-+} + \sigma_{+-} + \sigma_{--})(-\sigma_{++} + \sigma_{-+} + \sigma_{+-} - \sigma_{--})}}$$

can measure A_{LR} independent from polarimeters with very small loss in precision and only 10% of the luminosity on the small cross sections

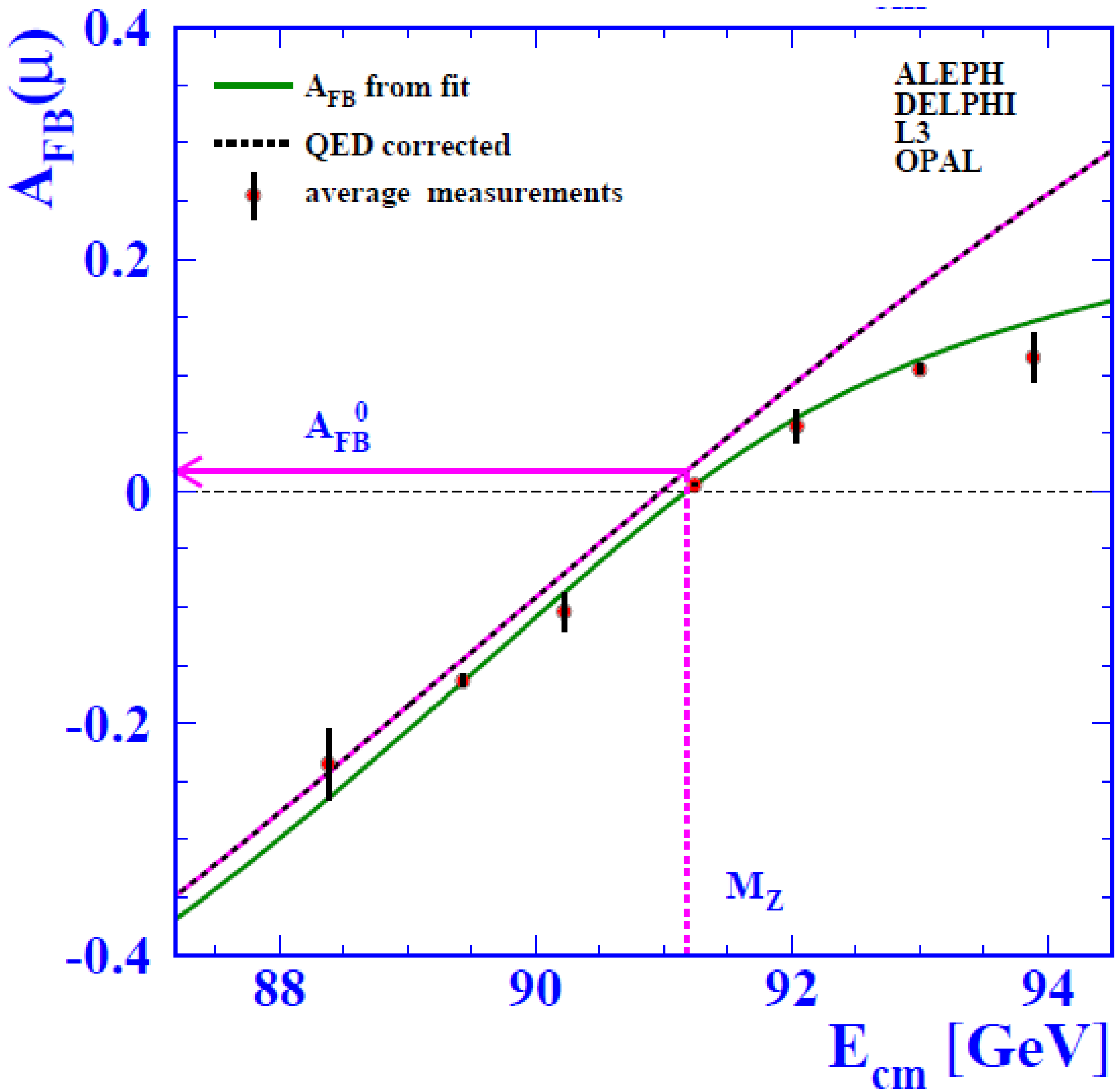
Conclude that $\Delta\sin^2\theta^{\text{lept}}_W \sim 10^{-5}$

Will consider two sources of errors

-- statistics
-- uncertainty on center-of-mass energy (relative to the Z mass)

main inputs taken from
arXiv:hep-ex/0509008v3 precision measurements on the Z resonance
Phys. Rep. 427:257-454,2006

there are other uncertainties but they are very small for A_{FB}
This is a lower limit estimate for A_{LR} ; the systematics related to knowledge of the beam polarization (80% for e-, 30% for e+) should also be taken into account



$A_{FB}(\mu)$
0.4
0.2
0
-0.2
-0.4
A_{FB} from fit
QED corrected
average measurements
ALEPH
DELPHI
L3
OPAL
A_{FB}^0
M_Z
88
90
92
94
E_{cm} [GeV]

	$A_{FB}^{\mu\mu}$ @ FCC-ee		A_{LR} @ ILC
visible Z decays	10^{12}	visible Z decays	10^{9}
muon pairs	10^{11}	beam polarization	90%
$\Delta A_{FB}^{\mu\mu}$ (stat)	$3\ 10^{-6}$	ΔA_{LR} (stat)	$4.2\ 10^{-5}$
$\Delta\ E_{cm}$ (MeV)	0.1		2.2
$\Delta A_{FB}^{\mu\mu}$ (E_{CM})	$9.2\ 10^{-6}$	ΔA_{LR} (E_{CM})	$4.1\ 10^{-5}$
$\Delta A_{FB}^{\mu\mu}$	$1.0\ 10^{-5}$	ΔA_{LR}	$5.9\ 10^{-5}$
$\Delta \sin^2\theta^{lept}_W$	$5.9\ 10^{-6}$		$7.5\ 10^{-6}$

from $A_{FB}^{\mu\mu}$ LEP 2.10^7Z SLC, 5.10^5 Z $\Delta\alpha$= 0.00035 **$\Delta\alpha$= 0.00003**

$\Delta \sin^2\theta^{lept}_W$ $5.3\ 10^{-4}$ $2.6\ 10^{-4}$ $1.2\ 10^{-4}$ **$1.\ 10^{-5}$**

W.A. $1.6\ 10^{-4}$

The forward backward tau polarization asymmetry is very clean.
Dependence on E_{CM} same as A_{LR} negl.

ALEPH data 160 pb^{-1} (80 s @ FCC-ee !)

Already at systematic level of 5 10^{-4}
6 10^{-5} on
much improvement possible
by using dedicated selection
e.g. tau→ π ν to avoid had. model

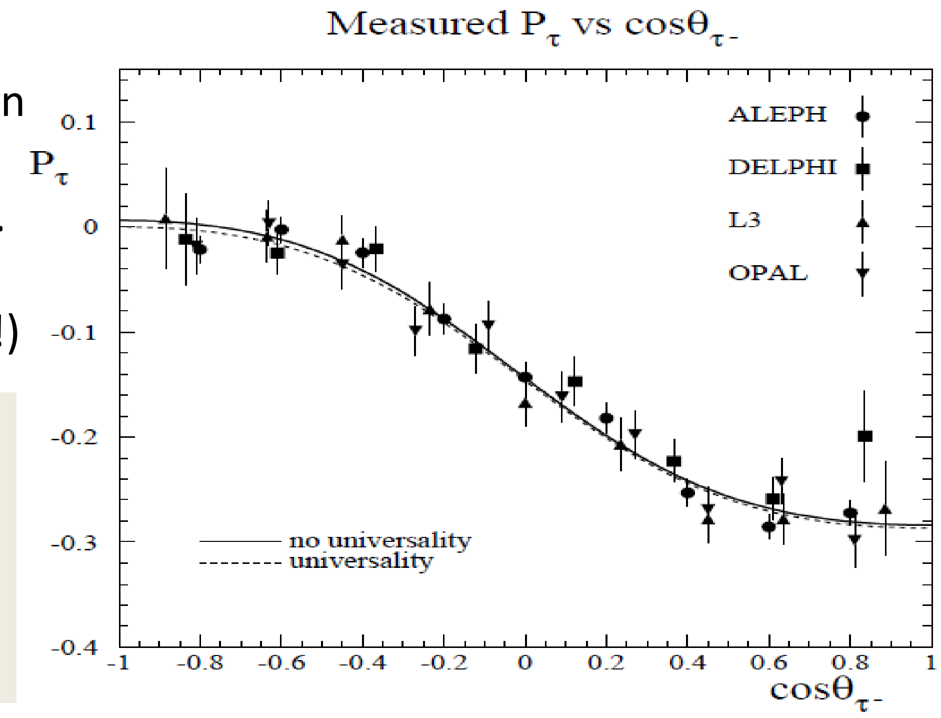


Figure 4.7: The values of $\mathcal{P}_\tau$ as a function of $\cos\theta_{\tau^-}$ as measured by each of the LEP experiments. Only the statistical errors are shown. The values are not corrected for radiation, interference or pure photon exchange. The solid curve overlays Equation 4.2 for the LEP values of $\mathcal{A}_\tau$ and $\mathcal{A}_e$. The dashed curve overlays Equation 4.2 under the assumption of lepton universality for the LEP value of $\mathcal{A}_\ell$.

	ALEPH		DELPHI		L3		OPAL	
	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$	$\delta\mathcal{A}_\tau$	$\delta\mathcal{A}_e$
ZFITTER	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
τ branching fractions	0.0003	0.0000	0.0016	0.0000	0.0007	0.0012	0.0011	0.0003
two-photon bg	0.0000	0.0000	0.0005	0.0000	0.0007	0.0000	0.0000	0.0000
had. decay model	0.0012	0.0008	0.0010	0.0000	0.0010	0.0001	0.0025	0.0005

Table 4.2: The magnitude of the major common systematic errors on $\mathcal{A}_\tau$ and $\mathcal{A}_e$ by category for each of the LEP experiments.

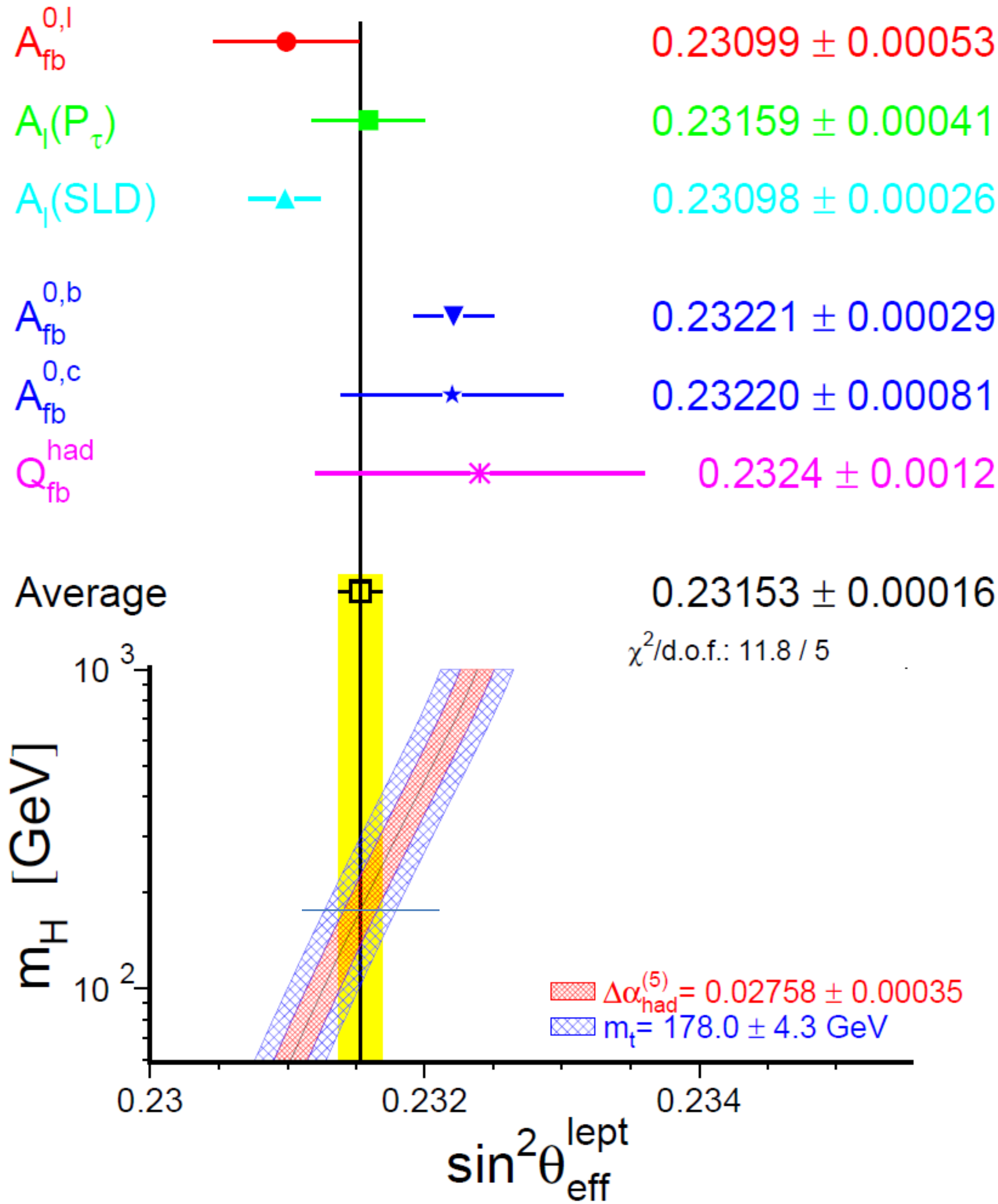
Concluding remarks for today:

1. The Energy uncertainty on the muon Forward Backward uncertainty
in FCC-ee is < that encurred in LC for A_LR measurement

2. At FCC-ee the Forward backward asymmetry for muons and the tau polarization FB asymmetry should give a result at least as good as that given by ALR at ILC with GIGAZ

3. All exceeds the theoretical precision from $\Delta\alpha(m_Z)$ or the comparison with m_W
But this precision on $\Delta \sin^2\theta^{lept}_W$ can only be exploited at FCC-ee!

3. from A_{FB}^b we should extract the b-quark couplings, not the lepton coupling
IF there is a case for longitudinal polarization it should come from this.



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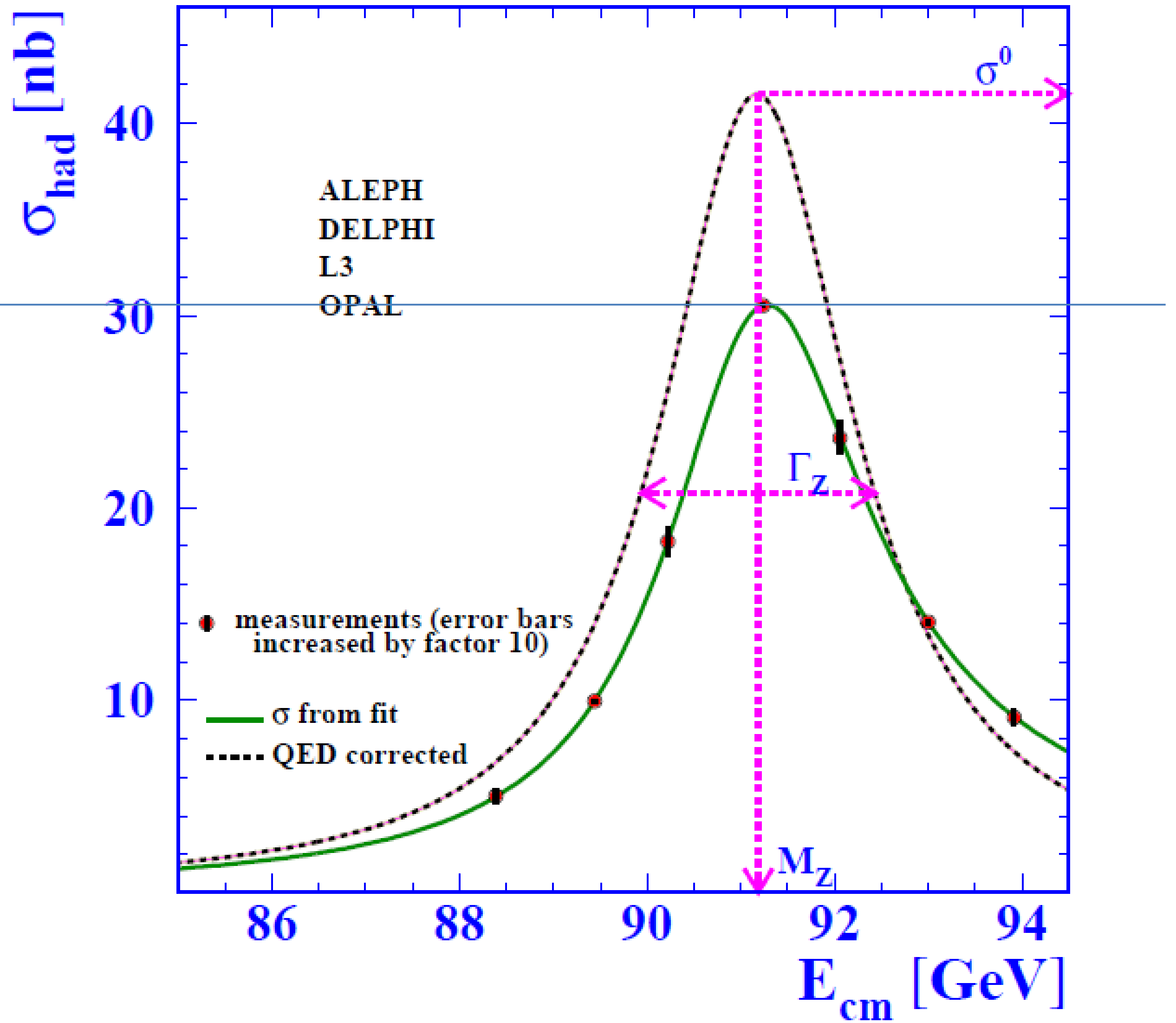
10^2
m_H [GeV]
$\Delta\alpha_{had}^{(5)}$ = 0.02758 ± 0.00035
m_t = 178.0 ± 4.3 GeV
0.23
0.232
0.234
$\sin^2\theta_{eff}^{lept}$



σ_had [nb]
40
30
20
10
σ^0
ALEPH
DELPHI
L3
OPAL
Γ_Z
M_Z
measurements (error bars
increased by factor 10)
σ from fit
QED corrected
86
88
90
92
94
E_cm [GeV]

Going through the observables

the weak mixing angle as **defined** by the relation

$$A_\ell = \frac{2 g^e_V\, g^e_A}{(g^e_V)^2 + (g^e_A)^2} = \frac{(g^e_L)^2 - (g^e_R)^2}{(g^e_L)^2 + (g^e_R)^2}$$

with $(g^e_L) = ½ - \sin^2\theta^{\ell ept}_W$ and $(g^e_R) = -\sin^2\theta^{\ell ept}_W$

$A_\ell \approx 8(1/4 - \sin^2\theta^{lept}_W)$ very sensitive to $\sin^2\theta^{lept}_W$!

$A_{LR} = A_e$ measured from $(\sigma_{vis,L} - \sigma_{vis,R}) / (\sigma_{vis,L} - \sigma_{vis,R})$

(total visible cross-section had + $\mu\mu$ + $\tau\tau$ (35 nb) for 100% Left Polarization

$A_{FB}^{\mu\mu} = ¾\, A_e A_\mu = ¾\, A_\ell^2$

$$\mathcal{G}_{Vf} = \sqrt{\mathcal{R}_f}\,(T_3^f - 2 Q_f \mathcal{K}_f \sin^2\theta_W)$$

$$\mathcal{G}_{Af} = \sqrt{\mathcal{R}_f}\, T_3^f\,.$$

$$A_{FB}^{0,f} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f$$

$$A_{LR}^0 = \mathcal{A}_e$$

$$A_{LRFB}^0 = \frac{3}{4}\mathcal{A}_f$$

$$\langle \mathcal{P}_\tau^0 \rangle = -\mathcal{A}_\tau$$

$$A_{FB}^{pol,0} = -\frac{3}{4}\mathcal{A}_e\,.$$

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}\frac{1}{\langle|\mathcal{P}_e|\rangle}$$

$$A_{LRFB} = \frac{(\sigma_F - \sigma_B)_L - (\sigma_F - \sigma_B)_R}{(\sigma_F + \sigma_B)_L + (\sigma_F + \sigma_B)_R}\frac{1}{\langle|\mathcal{P}_e|\rangle}\,.$$