Longitudinal polarization at TLEP-Z?

Main interest: measure EW couplings at the Z peak most of which provide measurements of \( \sin^2\theta_{\text{lept}}^W = e^2/g^2 (m_z) \)
-- not to be confused with -- \( \sin^2\theta_W = 1- m_w^2/m_z^2 \)

Useful references from the past:
«polarization at LEP» CERN Yellow Report 88-02
Precision Electroweak Measurements on the Z Resonance
GigaZ @ ILC by K. Moenig
Measuring $\sin^2 \theta_W^{\text{eff}} (m_Z)$

\[ \sin^2 \theta_W^{\text{eff}} \equiv \frac{1}{4} (1 - g_V/g_A) \]

\[ g_V = g_L + g_R \]

\[ g_A = g_L - g_R \]

<table>
<thead>
<tr>
<th>$A_{tb}^{0,l}$</th>
<th>0.23099 ± 0.00053</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{tb}(P_T)$</td>
<td>0.23159 ± 0.00041</td>
</tr>
<tr>
<td>$A_{tb}(\text{SLD})$</td>
<td>0.23098 ± 0.00026</td>
</tr>
<tr>
<td>$A_{tb}^{0,b}$</td>
<td>0.23221 ± 0.00029</td>
</tr>
<tr>
<td>$A_{tb}^{0,c}$</td>
<td>0.23220 ± 0.00081</td>
</tr>
<tr>
<td>$Q_{tb}^{\text{had}}$</td>
<td>0.2324 ± 0.0012</td>
</tr>
</tbody>
</table>

Average $0.23153 \pm 0.00016$

$\chi^2/\text{d.o.f.} = 11.8/5$

\[ m_{\text{H}} \quad 10^2 - 10^3 \quad \text{GeV} \]

$\Delta \alpha_{\text{had}}^{(5)} = 0.02781 \pm 0.00036$

$\text{m}_t = 178.0 \pm 4.3 \text{ GeV}$
relations to the well measured

\[ G_F \; m_Z \; \alpha_{\text{QED}} \]

at first order:

\[ \Delta \rho = \frac{\alpha}{\pi} \left( \frac{m_{\text{top}}}{m_Z} \right)^2 \]

- \[ \frac{\alpha}{4\pi} \log \left( \frac{m_h}{m_Z} \right)^2 \]

\[ \varepsilon_3 = \cos^2\theta_w \; \frac{\alpha}{9\pi} \log \left( \frac{m_h}{m_Z} \right)^2 \]

\[ \delta_{\text{vb}} = 20/13 \; \frac{\alpha}{\pi} \left( \frac{m_{\text{top}}}{m_Z} \right)^2 \]

complete formulae at 2d order including strong corrections are available in fitting codes

e.g. ZFITTER, GFITTER
$A_{fb}^{0,l}$: $0.23099 \pm 0.00053$

$A_{j}(P_{t})$: $0.23159 \pm 0.00041$

$A_{j}(SLD)$: $0.23098 \pm 0.00026$

$A_{fb}^{0,b}$: $0.23221 \pm 0.00029$

$A_{fb}^{0,c}$: $0.23220 \pm 0.00081$

$Q_{fb}^{had}$: $0.2324 \pm 0.0012$

Average: $0.23153 \pm 0.00016$

$\chi^2/d.o.f.: 11.8/5$

$\Delta \alpha^{(S)}_{had} = 0.02758 \pm 0.00035$

$m_{t} = 178.0 \pm 4.3$ GeV
Extracting physics from $\sin^2\theta_{\text{lept}}^W$

1. Direct comparison with $m_Z$

$$\sin^2\theta_{\text{lept}}^W \cos^2\theta_{\text{lept}}^W = \frac{\pi \alpha (M_Z^2)}{\sqrt{2} \ G \ F \ m_Z^2} \cdot \frac{1}{1 + \Delta \rho} \cdot \frac{1}{1 - \frac{E_3}{\Delta \theta_{\text{lept}}^W}}$$

Uncertainties in $m_{\text{top}}$, $\Delta \alpha(m_Z)$, $m_H$, etc....

$\Delta \sin^2\theta_{\text{lept}}^W \sim \Delta \alpha(m_Z) / 3 \sim 10^{-5}$ if we can reduce $\Delta \alpha(m_Z)$ (see P. Janot)

2. Comparison with $m_w/m_Z$

Compare above formula with similar one:

$$\sin^2\theta_w \cos^2\theta_w = \frac{\pi \alpha (M_Z^2)}{\sqrt{2} \ G \ F \ m_Z^2} \cdot \frac{1}{1 - \left( -\frac{\cos^2\theta_w}{\sin^2\theta_w} \Delta \rho + 2 \frac{\sin^2\theta_w}{\sin^2\theta_w} \varepsilon_3 + \frac{\varepsilon_1^2 - \varepsilon_2^2}{\varepsilon_3} \varepsilon_2 \right)}$$

Where it can be seen that $\Delta \alpha(m_Z)$ cancels in the relation.

The limiting error is the error on $m_w$.

For $\Delta m_w = 0.5 \text{ MeV}$ this corresponds to $\Delta \sin^2\theta_{\text{lept}}^W = 10^{-5}$
Assume for now ONE experiment at ECM=91.2

Luminosity «baseline» with beta*=1mm : 2.1 \(10^{36}\)/cm\(^2\)/s = 2 pb\(^{-1}\)/s,
Sigma\_had = 31 \(10^{-33}\)cm\(^2\) \(\rightarrow\) 6.5 \(10^{11}\) qq events/10\(^7\) year/exp.

Consider 3 years of 10\(^7\) s
2 \(10^{12}\) Z\(\rightarrow\) qq events (typical exp at LEP was 4.10\(^6\))
4 \(10^{11}\) Z\(\rightarrow\) bb
10\(^{11}\) Z\(\rightarrow\) \(\mu\mu\), \(\tau\tau\) each
Will consider today the contribution of the Center-of-mass energy systematic errors

Today: step I, compare
ILC measurement of $A_{LR}$ with $10^9 Z$ and $P_{e^-}=80\%, P_{e^+}=30\%$

FCC-ee measurement of $A_{FB}^{\mu\mu}$ and $A_{FB}^{pol}(\tau)$ with $2.10^{12} Z$
Both measure the weak mixing angle as **defined** by the relation

\[ A_\ell = \frac{\left( g_e^L \right)^2 - \left( g_e^R \right)^2}{\left( g_e^L \right)^2 + \left( g_e^R \right)^2} \]

with \( g_e^L = \frac{1}{2} \sin^2 \theta_{\text{lept}}^W \) and \( g_e^R = -\sin^2 \theta_{\text{lept}}^W \)

\[ A_{\text{LR}} = A_e \]

\[ A_{\text{FB}}^{\mu\mu} = \frac{3}{4} A_e A_\mu = \frac{3}{4} A_\ell^2 \]

---

**A\text{FB}^{\mu\mu}** is measured using muon pairs (5% of visible Z decays) and unpolarized beams

**A\text{LR}** is measured using all statistics of visible Z decays with beams of alternating longitudinal polarization

both with very small experimental systematics

---

**parametric sensitivity**

\[ \frac{dA_{\text{FB}}^{\mu\mu}}{d\sin^2 \theta_{\text{lept}}^W} = 1.73 \quad \text{vs} \quad \frac{dA_{\text{LR}}}{d\sin^2 \theta_{\text{lept}}^W} = 7.9 \]

---

**sensitivity to center-of-mass energy** (w.r.t. \( m_Z \)) is larger for **A\text{FB}^{\mu\mu}**

\[ \frac{\partial A_{\text{FB}}^{\mu\mu}}{\partial \sqrt{s}} = 0.09/\text{GeV} \quad \text{vs} \quad \frac{\partial A_{\text{LR}}}{\partial \sqrt{s}} = 0.019/\text{GeV} \]

“an 80 MeV uncertainty in Ecm corresponds to a 1% error on A_{\text{LR}}” (relative error)

But of course **A\text{FB}^{\mu\mu}** benefits from much larger statistics and Ecm precision of circular collider
Most sensitive observable is $A_{LR}$, so only this is discussed

$$A_{LR} = \frac{1}{\mathcal{P}} \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = A_e = \frac{2v_e a_e}{\nu_e^2 + a_e^2}$$

$v_e/a_e = 1 - 4 \sin^2 \theta_{eff}^l$

independent of the final state

Statistical error with $10^9$ Zs: $\Delta A_{LR} = 4 \cdot 10^{-5}$

(for $\mathcal{P}_{e^-} = 80\%$, $\mathcal{P}_{e^+} = 0$)

Crucial ingredient: polarisation measurement

Error from polarisation: $\Delta A_{LR}/A_{LR} = \Delta \mathcal{P}/\mathcal{P}$

- only electron polarisation with $\Delta \mathcal{P}/\mathcal{P} = 0.5\% \Rightarrow \Delta A_{LR} = 8 \cdot 10^{-4}$
  (Still factor three to SLD, but few million Zs are sufficient)
Measurement of $A_{LR}$

- Electron bunches: 1 $\leftrightarrow$ 2 $\equiv$ 3 $\equiv$ 4
- Positron bunches: 1 $\equiv$ 2 $\Rightarrow$ 3 $\equiv$ 4
- Cross sections: $\sigma_1$, $\sigma_2$, $\sigma_3$, $\sigma_4$
- Event numbers: $N_1$, $N_2$, $N_3$, $N_4$

\[
\begin{align*}
\sigma_1 &= \sigma_u (1 - P^- c \Lambda_{LR}) \\
\sigma_2 &= \sigma_u (1 + P^+ c \Lambda_{LR}) \\
\sigma_3 &= \sigma_u \\
\sigma_4 &= \sigma_u [1 - P^+ c P^- c + (P^+ c - P^- c) \Lambda_{LR}]
\end{align*}
\]

Verifies polarimeter with experimentally measured cross-section ratios

Statistics:

- $\Delta A_{LR} = 0.0025$ with about $10^6 Z^0$ events,
- $\Delta A_{LR} = 0.000015$ with $10^{11} Z$ and 40% polarization in collisions.

$\Delta \sin^2 \theta_{w^{\text{eff}}}^{\text{(stat)}} = O(2 \cdot 10^{-6})$
• with positron polarisation $\mathcal{P}_{\text{eff}} = \frac{\mathcal{P}_{e^+} + \mathcal{P}_{e^-}}{1 + \mathcal{P}_{e^+} + \mathcal{P}_{e^-}}$

$\Rightarrow$ gain a factor four for $\mathcal{P}_{e^-}/\mathcal{P}_{e^+} = 80\%/60\%$ due to error propagation (even when error is $100\%$ correlated between the polarimeters the gain is a factor three)

• even better with Blondel scheme:

$$\sigma = \sigma_u [1 - \mathcal{P}_{e^+}\mathcal{P}_{e^-} + A_{LR}(\mathcal{P}_{e^+} - \mathcal{P}_{e^-})]$$

$$A_{LR} = \sqrt{\frac{(\sigma_{++} + \sigma_{--} - \sigma_{+-} - \sigma_{-+})(-\sigma_{++} + \sigma_{--} - \sigma_{+-} + \sigma_{-+})}{(\sigma_{++} + \sigma_{--} + \sigma_{+-} + \sigma_{-+})(-\sigma_{++} + \sigma_{--} + \sigma_{+-} - \sigma_{-+})}}$$

can measure $A_{LR}$ independent from polarimeters with very small loss in precision and only $10\%$ of the luminosity on the small cross sections

Conclude that $\Delta \sin^2 \theta_{\text{lept}}^W \sim 10^{-5}$
Will consider two sources of errors

-- statistics
-- uncertainty on center-of-mass energy (relative to the Z mass)

main inputs taken from
arXiv:hep-ex/0509008v3 precision measurements on the Z resonance

there are other uncertainties but they are very small for $A_{FB}$
This is a lower limit estimate for $A_{LR}$; the systematics related to knowledge of
the beam polarization (80% for e-, 30% for e+) should also be taken into account
<table>
<thead>
<tr>
<th></th>
<th>$A_{FB}^{\mu\mu}$ @ FCC-ee</th>
<th>$A_{LR}$ @ ILC</th>
</tr>
</thead>
<tbody>
<tr>
<td>visible Z decays</td>
<td>$10^{12}$</td>
<td>$10^{9}$</td>
</tr>
<tr>
<td>muon pairs</td>
<td>$10^{11}$</td>
<td>90%</td>
</tr>
<tr>
<td>$\Delta A_{FB}^{\mu\mu}$ (stat)</td>
<td>$3 \times 10^{-6}$</td>
<td>$4.2 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\Delta E_{cm}$ (MeV)</td>
<td>0.1</td>
<td>2.2</td>
</tr>
<tr>
<td>$\Delta A_{FB}^{\mu\mu} (E_{CM})$</td>
<td>$9.2 \times 10^{-6}$</td>
<td>$4.1 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\Delta A_{FB}^{\mu\mu}$</td>
<td>$1.0 \times 10^{-5}$</td>
<td>$5.9 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\Delta \sin^{2}\theta_{W}$</td>
<td>$5.9 \times 10^{-6}$</td>
<td>$7.5 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

$\Delta \sin^{2}\theta_{W}$ from $A_{FB}^{\mu\mu}$

- LEP: $2.1 \times 10^{-7}$
- SLC: $5.1 \times 10^{-5}$
- W.A.: $1.6 \times 10^{-4}$

$\Delta \alpha = 0.00035$ (NLO)
The forward backward tau polarization asymmetry is very clean. Dependence on $E_{CM}$ same as $A_{LR}$ negl.

ALEPH data 160 pb$^{-1}$ (80 s @ FCC-ee !)

Already at systematic level of $5 \times 10^{-4}$ 6 $10^{-5}$ on much improvement possible by using dedicated selection e.g. tau$\rightarrow$ $\pi$ $\nu$ to avoid had. model

---

![Graph of Measured $P_\tau$ vs $\cos\theta_\tau$]

Figure 4.7: The values of $P_\tau$ as a function of $\cos\theta_\tau$ as measured by each of the LEP experiments. Only the statistical errors are shown. The values are not corrected for radiation, interference or pure photon exchange. The solid curve overlays Equation 4.2 for the LEP values of $A_\tau$ and $A_e$. The dashed curve overlays Equation 4.2 under the assumption of lepton universality for the LEP value of $A_e$.

<table>
<thead>
<tr>
<th>Source</th>
<th>$\delta A_\tau$</th>
<th>$\delta A_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH</td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
<tr>
<td>DELPHI</td>
<td>0.0003</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>0.0016</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>0.0007</td>
<td>0.0012</td>
</tr>
<tr>
<td></td>
<td>0.0011</td>
<td>0.0003</td>
</tr>
<tr>
<td>L3</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>0.0005</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>0.0007</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>OPAL</td>
<td>0.0012</td>
<td>0.0008</td>
</tr>
<tr>
<td></td>
<td>0.0010</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>0.0012</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>0.0025</td>
<td>0.0005</td>
</tr>
</tbody>
</table>

Table 4.2: The magnitude of the major common systematic errors on $A_\tau$ and $A_e$ by category for each of the LEP experiments.
Concluding remarks for today:

1. The Energy uncertainty on the muon Forward Backward uncertainty in FCC-ee is < that encurred in LC for A_LR measurement

2. At FCC-ee the Forward backward asymmetry for muons and the tau polarization FB asymmetry should give a result at least as good as that given by ALR at ILC with GIGAZ

3. All exceeds the theoretical precision from $\Delta\alpha(m_Z)$ or the comparison with $m_W$
   But this precision on $\Delta\sin^2\theta^\text{lep}_W$ can only be exploited at FCC-ee!

3. from $A_{FB}^b$ we should extract the b-quark couplings, not the lepton coupling
   IF there is a case for longitudinal polarization it should come from this.
\[ A_{\text{fb}}^{0,l} = 0.23099 \pm 0.00053 \]
\[ A_{\text{fb}}^{0,b} = 0.23221 \pm 0.00029 \]
\[ A_{\text{fb}}^{0,c} = 0.23220 \pm 0.00081 \]
\[ Q_{\text{had}} = 0.2324 \pm 0.0012 \]

Average:
\[ \sin^2 \theta_{\text{eff}} = 0.23153 \pm 0.00016 \]
\[ \chi^2 / \text{d.o.f.} = 11.8 / 5 \]

\[ m_h = 178.0 \pm 4.3 \text{ GeV} \]
Going through the observables

the weak mixing angle as defined by the relation

\[ A_\ell = \frac{2 g_V^e g_A^e}{(g_V^e)^2 + (g_A^e)^2} = \frac{(g_L^e)^2 - (g_R^e)^2}{(g_L^e)^2 + (g_R^e)^2} \]

with \((g_L^e) = \frac{1}{2} - \sin^2 \theta_{\text{lept}}^W\) and \((g_R^e) = -\sin^2 \theta_{\text{lept}}^W\)

\[ A_\ell \approx 8(1/4 \cdot -\sin^2 \theta_{\text{lept}}^W) \text{ very sensitive to } \sin^2 \theta_{\text{lept}}^W ! \]

\[ A_{LR} = A_e \text{ measured from } (\sigma_{\text{vis},L} - \sigma_{\text{vis},R}) / (\sigma_{\text{vis},L} + \sigma_{\text{vis},R}) \]

(\text{Total visible cross-section had } + \mu \mu + \tau \tau \text{ (35 nb) for 100% Left Polarization})

\[ A_{FB}^{\mu \mu} = \frac{3}{4} A_e A_\mu = \frac{3}{4} A_\ell^2 \]

\[ G_{Vf} = \sqrt{R_f} \left( T_3^f - 2 Q_f K_f \sin^2 \theta_W \right) \]

\[ G_{Af} = \sqrt{R_f} T_3^f \]

\[ A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \]

\[ A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \frac{1}{\langle |P_e| \rangle} \]

\[ A_{LRFB} = \frac{(\sigma_F - \sigma_B)_L - (\sigma_F - \sigma_B)_R}{(\sigma_F + \sigma_B)_L + (\sigma_F + \sigma_B)_R} \frac{1}{\langle |P_e| \rangle} \]

\[ A_{FB}^{0 \mu} = \frac{3}{4} A_e A_f \]

\[ A_{LR}^{0} = A_e \]

\[ A_{LRFB}^{0} = \frac{3}{4} A_f \]

\[ \langle P_{\tau}^{0} \rangle = -A_{\tau} \]

\[ A_{FB}^{\text{pol},0} = -\frac{3}{4} A_e. \]