



# Tolerance studies and Dispersion Free Steering for FCC-ee

Sandra Aumon, CERN  
On behalf of the FCC-ee Lattice Design team

Acknowledgements to  
Andreas Doblhammer, Bernhard Holzer,  
Bastian Haerer, Katsunobu Oide,  
Jorg Wenninger.



# Outline

- FCC-ee beam parameters – Saw-tooth effect
- Emittance tuning for electron machines
- Lattice errors and challenges for the tolerance study
- Status Review Meeting & Recommendations
- BPMs errors
- Dispersion Free Steering for FCC-ee
- Summary

## This study relies on

- FCC-ee Lattice design option: **Katsunobu Oide**
- Lattice Design & Chromaticity correction W-function matching in the Arcs: **Bastian Haerer**
- IR design and local chromaticity correction at the IP: **Anton Bogomyagkov**
- Tapering : **Andreas Doblhammer (Poster)**

	<b>Z</b>	<b>W</b>	<b>H</b>	<b>tt</b>
Beam energy [GeV]	45.5	80	120	175
Beam current [mA]	1450	152	30	6.6
Bunches / beam	91500	5260	780	81
Bunch population [ $10^{11}$ ]	0.33	0.6	0.8	1.7
Transverse emittance $\epsilon$				
- Horizontal [nm]	0.09	0.26	0.61	1.3
- Vertical [nm]	0.001	0.001	0.0012	0.0025
Momentum comp. [ $10^{-5}$ ]	0.7	0.7	0.7	0.7
Betatron function at IP $\beta^*$				
- Horizontal [mm]	1000	1000	1000	1000
- Vertical [mm]	2	2	2	2
Energy loss / turn [GeV]	0.03	0.33	1.67	7.55
Total RF voltage [GV]	0.2	0.8	3	10

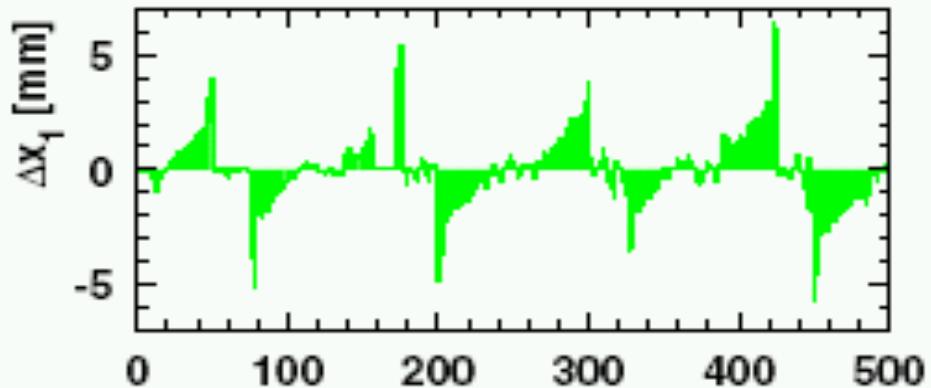
- Coupling ratio  
 $V.\text{emit}/H.\text{emit} \sim 2/1000$   
and  $1/1000$



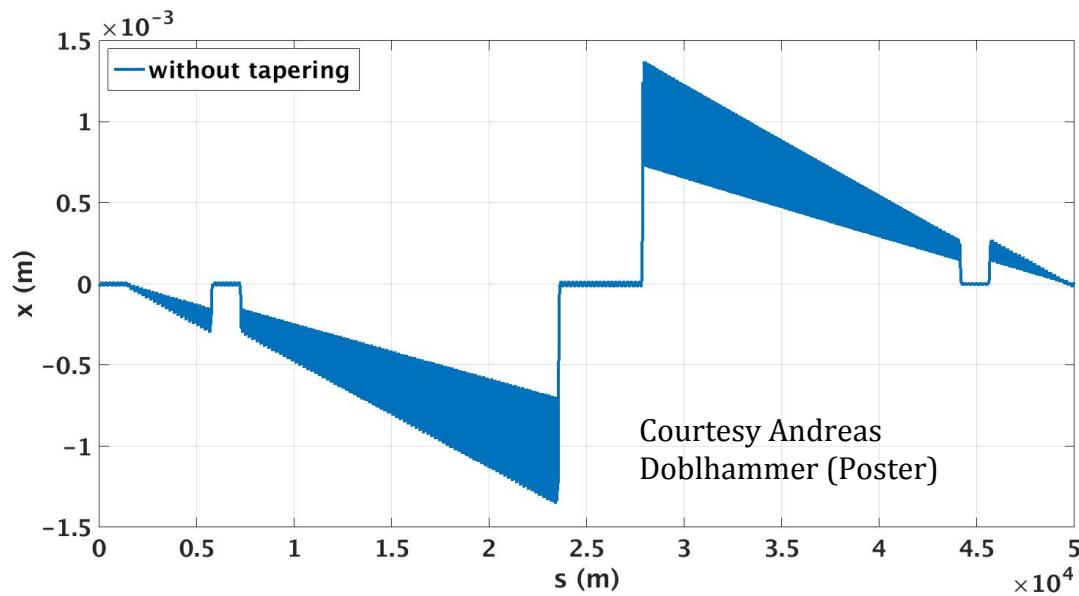
Light source  
emittance type

$$P = \frac{cC_\gamma \beta^3 E^4}{2\pi R \rho} \longrightarrow P_{175}/P_{45} \approx 200$$

# Saw-tooth Effect



- LEP time
- Saw-tooth =+/-5mm



- 175 GeV, 2 RF sections
- Large chromaticity
- $Q'x = -573.6506369$
- $Q'y = -852.4978106$
- Sextupoles 100 times stronger
- Tapering needed to keep the beam as centered as possible
- More details Andreas's poster

# Emittance tuning for electron machine

- Horizontal emittance

$$\epsilon_x = \frac{C_g}{J_x} \gamma^2 \theta^3 F \quad F_{FODO} = \frac{1}{2\sin\psi} \frac{5+3\cos\psi}{1-\cos\psi} \frac{L}{l_B}$$

L: cell length  
l<sub>B</sub>: dipole length  
 $\psi$ : phase advance/cell

- Vertical emittance

$$\epsilon_y = \left( \frac{dp}{p} \right)^2 (\gamma D^2 + 2\alpha D D' + \beta D'^2)$$

Alignment errors, rolls angle and coupling spoil the vertical emittance and compromise the coupling of 1/1000 (2/1000)

-> Coupling and Dy should be under control.

# Lattice errors and challenges for the tolerance study

- **Quadrupoles off-set:** dipolar kick \*

$$B_x = k(y + \Delta y) = ky + \underline{k\Delta y}$$

Constant term

Vertical dipole -> vertical dispersion

- **Sextupoles off-set \***

$$\begin{aligned} B_x &= kx(y + \Delta y) = kxy + \underline{kx\Delta y} \\ B_y &= k(x^2 - (y + \Delta y)^2) = k(x^2 - y^2) - \underline{2ky\Delta y} - \underline{(\Delta y^2)} \end{aligned}$$

Skew quad (coupling) + vertical dipole

- **Quadrupole roll:** skew quad, coupling of the planes
- **Dipole roll:** vertical dipole

Source emittance  
growth

\* SY Lee "Accelerator Physics", Javier Barcelona Presentation

# Lattice errors and challenges for the tolerance study

## Why is it critical for FCC-ee?

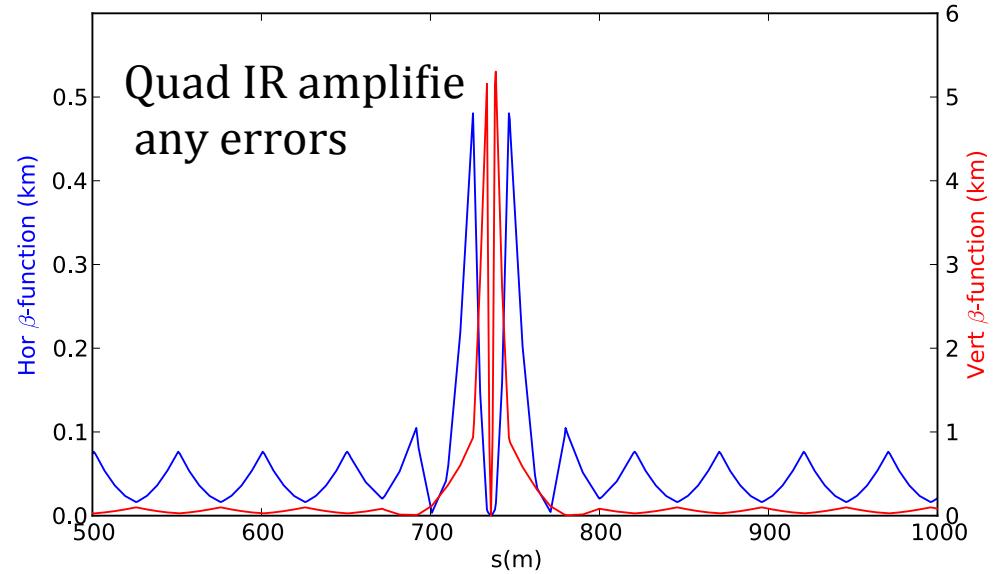
Closed Orbit Distortion (COD)  
due to a kick (or misaligned quadrupole)

$$y_{\text{co}}(s) = \theta_0 G(s, s_0)$$

$$G_0 = \frac{\sqrt{\beta_0 \beta(s)}}{2 \sin(\pi \mu)} \cos(\pi \mu - |\psi(s_0) - \psi(s)|)$$

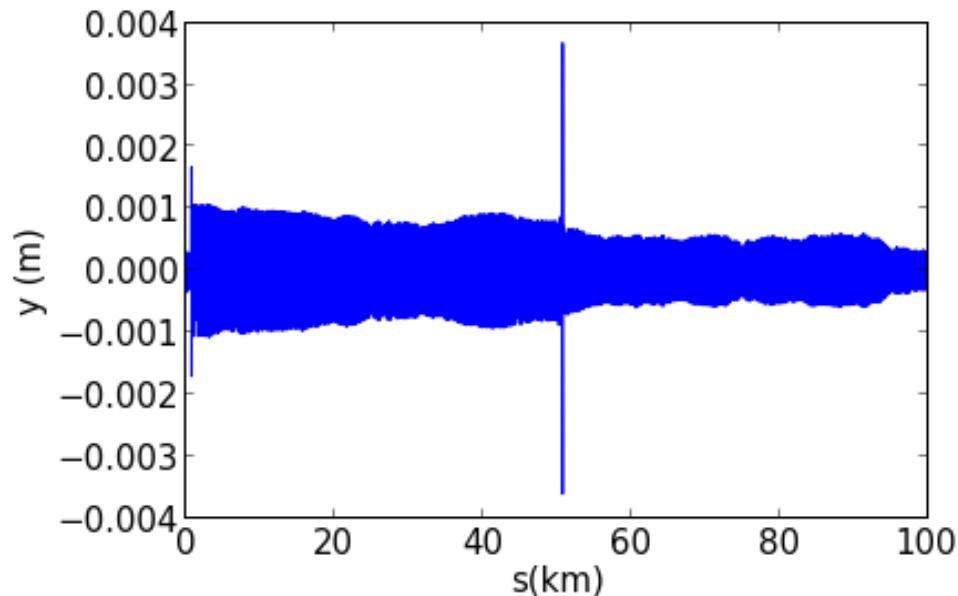
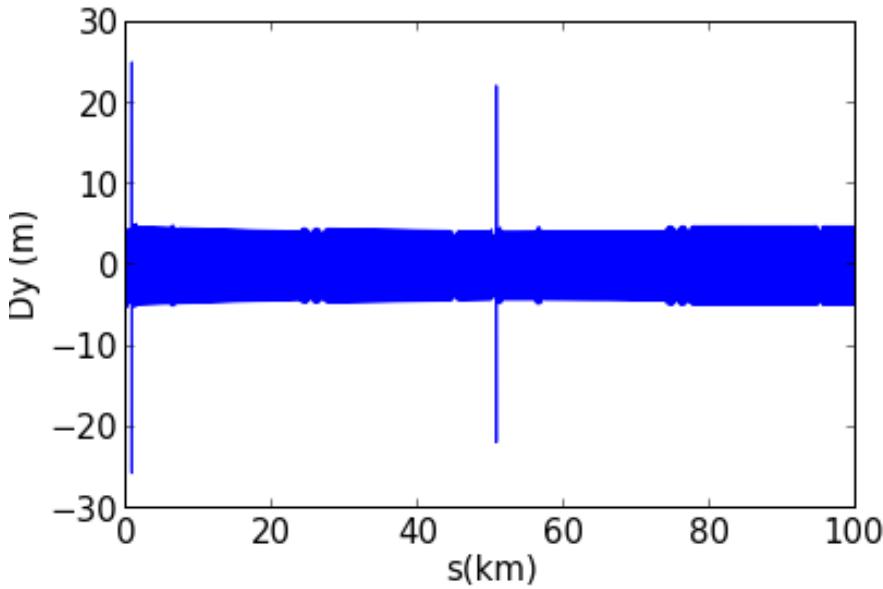
Dispersion response  
kick (or misaligned quadrupole)

$$\Delta D_i^l = K_i L_i \Delta u_i \frac{\sqrt{\beta_l \beta_i}}{\sin(\pi Q)} \cos(|\mu_l - \mu_i| - \pi Q) \theta_j$$



# Lattice errors and challenges for the tolerance study

- 2 microm RMS vertically misaligned quadrupoles, no sextupole
- 175GeV



To keep the vertical emittance within the requirement , **Vert. Dispersion  $\sim$ 1mm**

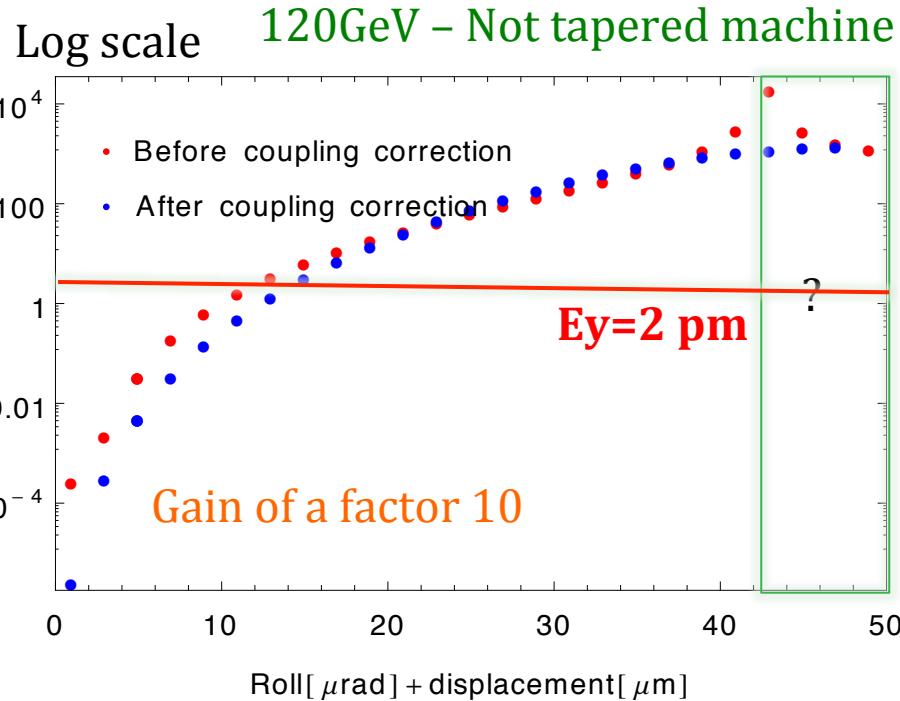
## Beam Physics Challenges

- FCC-ee is a **collider with beam parameters of a Light Source** (flat beam, extreme low vertical emittance)-> **Hor~nm, Vert. pm!!**
- **How to conserve 2pm Vert. Emittance with spurious dispersion**
- How to keep coupling ratio  $E_y/E_x$  of 1/1000 or 2/1000 with strong sextupole field ( $x100$  in  $k_2$  larger than LEP)?
- How to keep the vertical dispersion **to 1mm RMS**

## Algorithm & Methodology Challenges

- **Getting powerful algorithms of lattice correction** (orbit-dispersion matrix response, and coupling correction) inspired from light source experience (LOCO) and LHC or other colliders
- Computation time  $\sim L^4$
- Apply adapt them to 100km collider with a emittance ratio of 1 or 2/1000
- **MADX itself is not sufficient and limited**

# Rolls & Displacements errors on quadrupoles

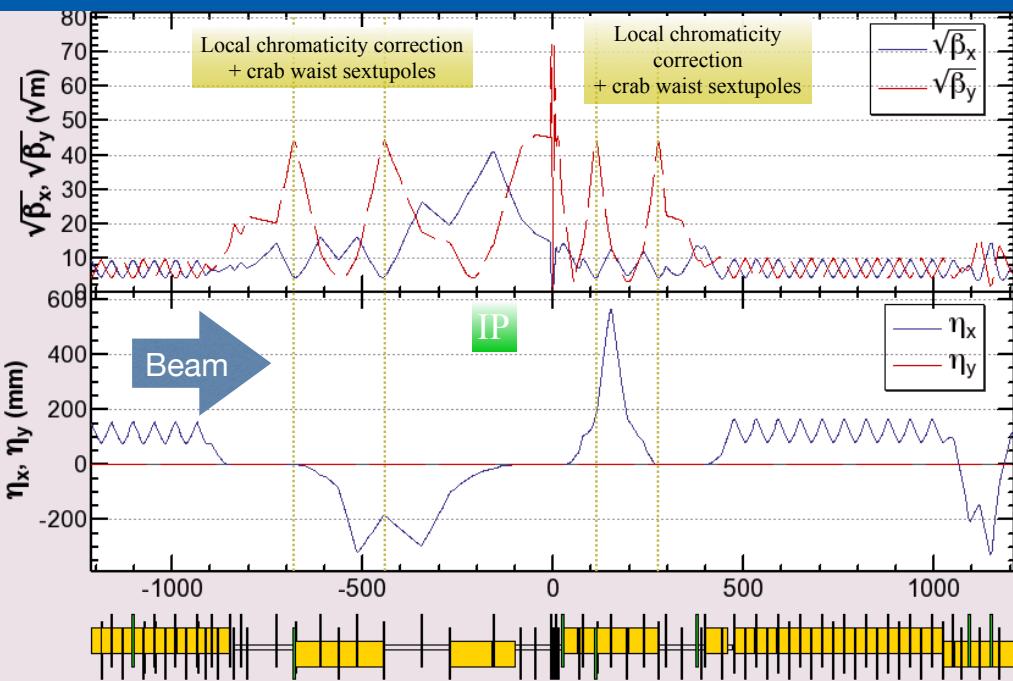


- 12-folds lattice
- Coupling correction from the non-diagonal element+vert. dispersion
- Vertical dispersion RMS  $\sim 50\text{mm}$   
( $1/2\text{m}$  at the IP)

## Recommendations by the review committee:

- 1 - More iterations should be apply for coupling
- 2 - BPMs errors should be taken into account
- 3 - For LEP operation experience: max effort to keep the vertical dispersion as low as possible

# Racetrack Layout



**B. Haerer lattice option (See his talk)**

2 Ips

$$\beta^*_{x/y} = 1 \text{ m} / 2 \text{ mm.}$$

No local chromaticity correction

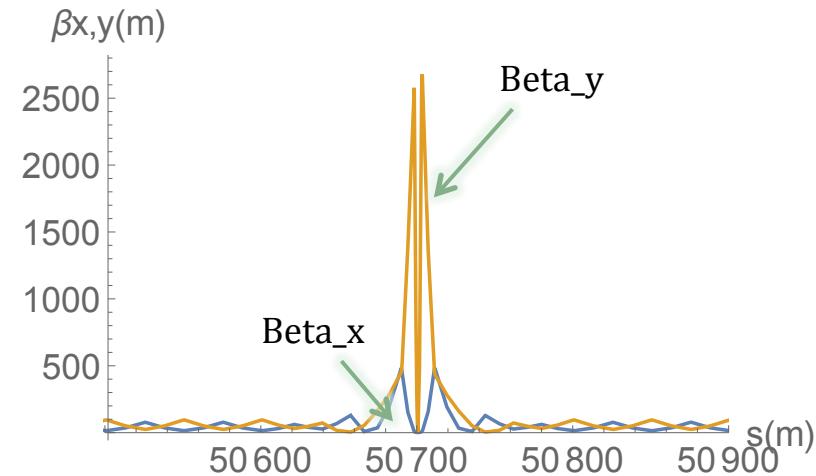
Will be combined with A. Bogomyagkov's IR  
in the future

Racetrack follows official design

**K. Oide lattice option (K.Oide 's talk)**

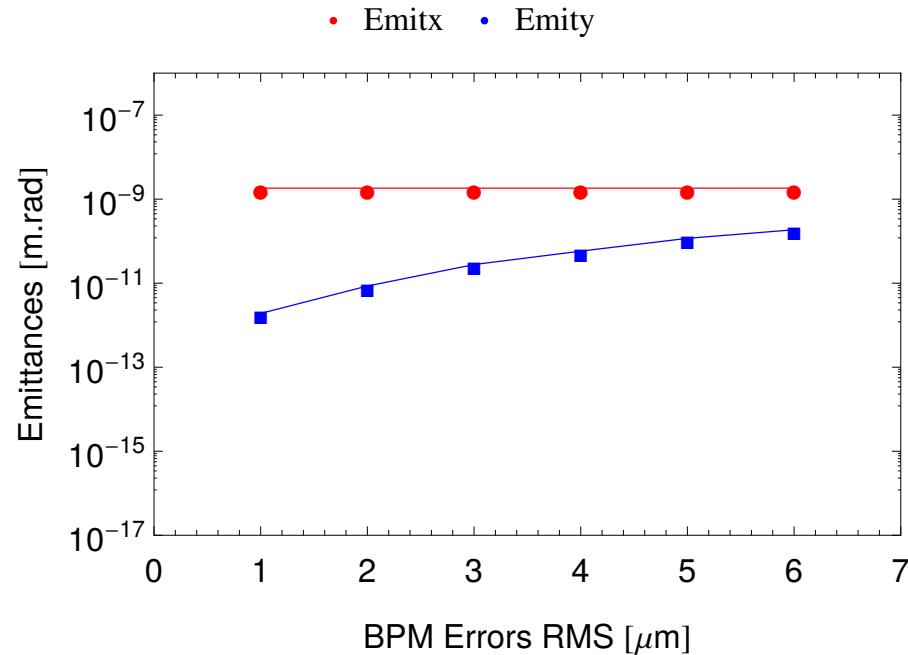
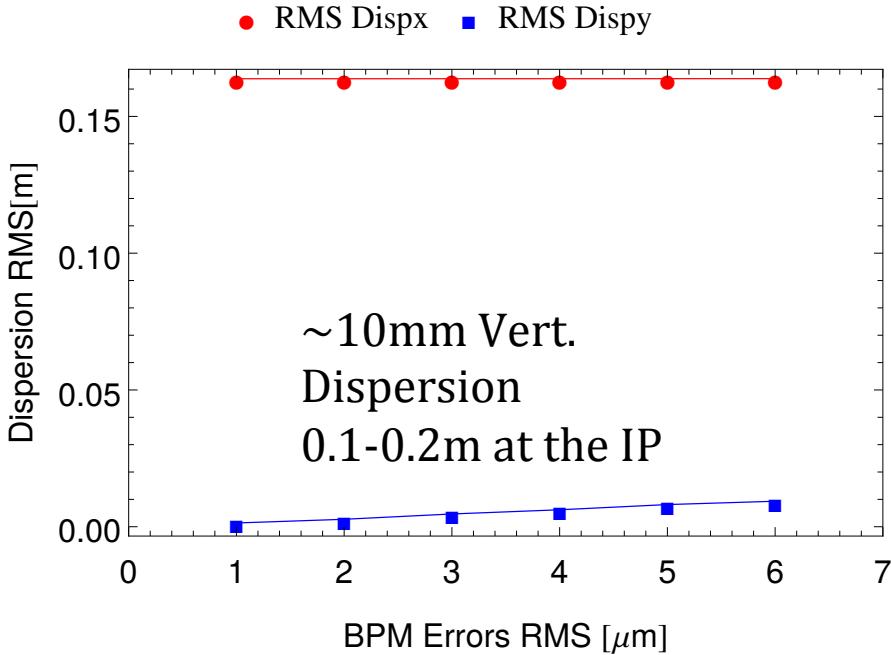
2 IPs

$$\beta^*_{x/y} = 1 \text{ m} / 2 \text{ mm.}$$



# BPM Error Tolerance

- Racetrack type lattice , **175GeV**, 2RF, fully tapered
- Very low tolerance, emittance growth driven by vertical dispersion.
- Preferable to correct the vert Disp. via a **DISPERSION FREE STEERING (DFS)** rather than by a orbit correction



# Dispersion Free Steering: Principle

- Build a matrix for vertical orbit ( $u$ ) & dispersion ( $D_u$ ) response under a corrector kick ( $\alpha_l$ )

$$\begin{pmatrix} (1 - \alpha) \vec{u} \\ \alpha \vec{D}_u \end{pmatrix} + \begin{pmatrix} (1 - \alpha) \mathbf{A} \\ \alpha \mathbf{B} \end{pmatrix} \vec{\theta} = 0$$

Response build with  
1micro.rad

- Orbit response  $A_{i,j} = \frac{\sqrt{\beta_i \beta_j}}{2 \sin(\pi Q_y)} \cos(|\mu_i - \mu_j| - \pi Q_y)$

- Dispersion response  $B_{ij} = \left\{ \sum_l^{quad} \frac{K_l L_l \beta_l}{4 \sin(\pi Q)^2} \cos(|\mu_i - \mu_l| - \pi Q) \cos(|\mu_l - \mu_j| - \pi Q) \right. \right.$   

$$- \sum_m^{sext} \frac{K_{2,m} D_{x,m} L_m \beta_m}{4 \sin(\pi Q)^2} \cos(|\mu_i - \mu_m| - \pi Q) \cos(|\mu_m - \mu_j| - \pi Q)$$
  

$$\left. \left. - \frac{\cos(|\mu_i - \mu_j| - \pi Q)}{\sin(\pi Q)} \right\} \sqrt{\beta_l \beta_j} \right.$$

“Dispersion Free Steering for YASP and dispersion for TI8”,  
J. Wenninger, LHC-Performance-Note-005.

# DFS optimization with singular value decomposition

- Without SVD, the correction does not work – mandatory step.
- SVD principle

$$\mathbf{T} = \mathbf{UWV}^t = \mathbf{U} \begin{pmatrix} w_1 & 0 & \cdots & 0 \\ 0 & w_2 & & \\ \cdots & & \cdots & 0 \\ 0 & \cdots & 0 & w_M \end{pmatrix} \mathbf{V}^t,$$

- W is a diagonal matrix with  $w_i$  are the singular values.
- Establish a cutoff to eliminate the singular value to optimize the correction –  
**Compromise between noise in the correction and efficiency**  
**More singular value -> more local correction, more noise**  
**Less singular value-> global correction, IP**

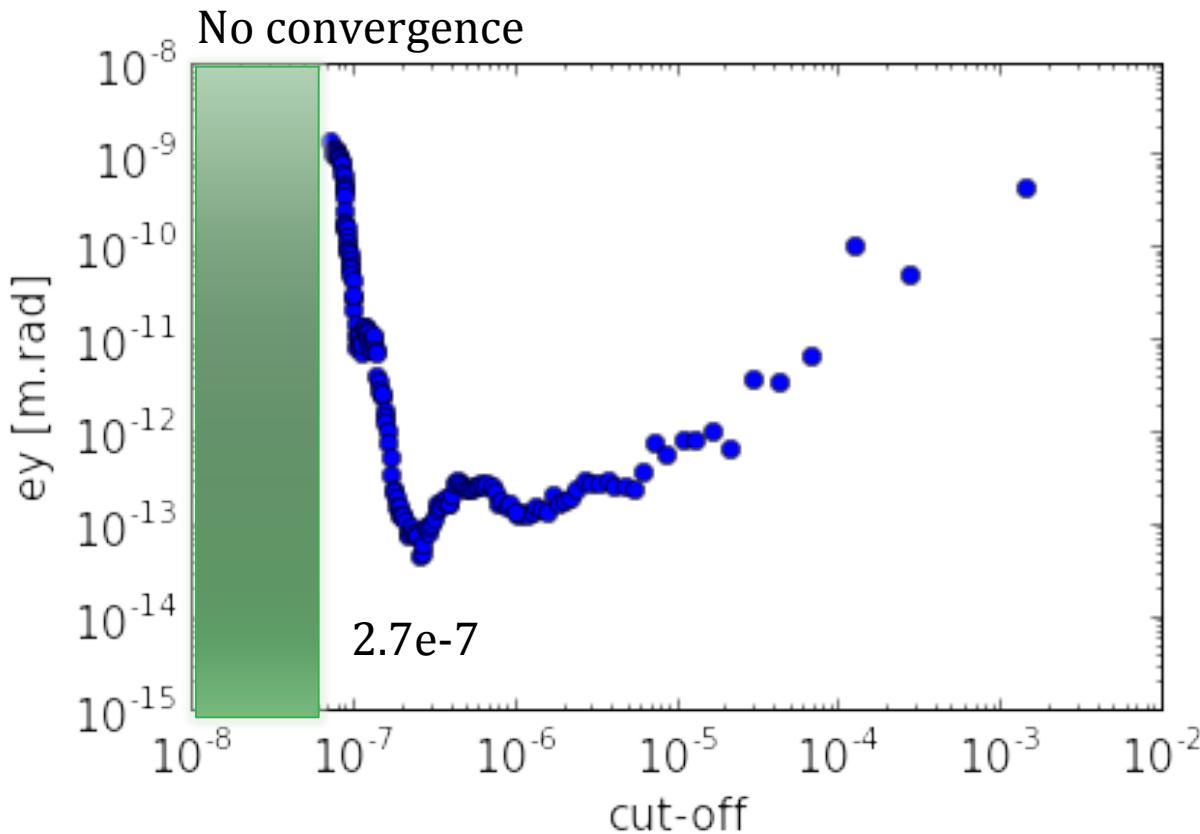
## First approach with small misalignments

Matrix Size  
(2006,2006)

- Scan in number of singular values considered
- **2 microm** vertically misaligned quads
- Pure vertical dispersion correction (0 weight on the orbit)
- 20 iterations of DFS
- Chromaticity correction is applied (Bastian's scheme)
- Tapering, which includes a re-matching optics
- Computation of the emittance.

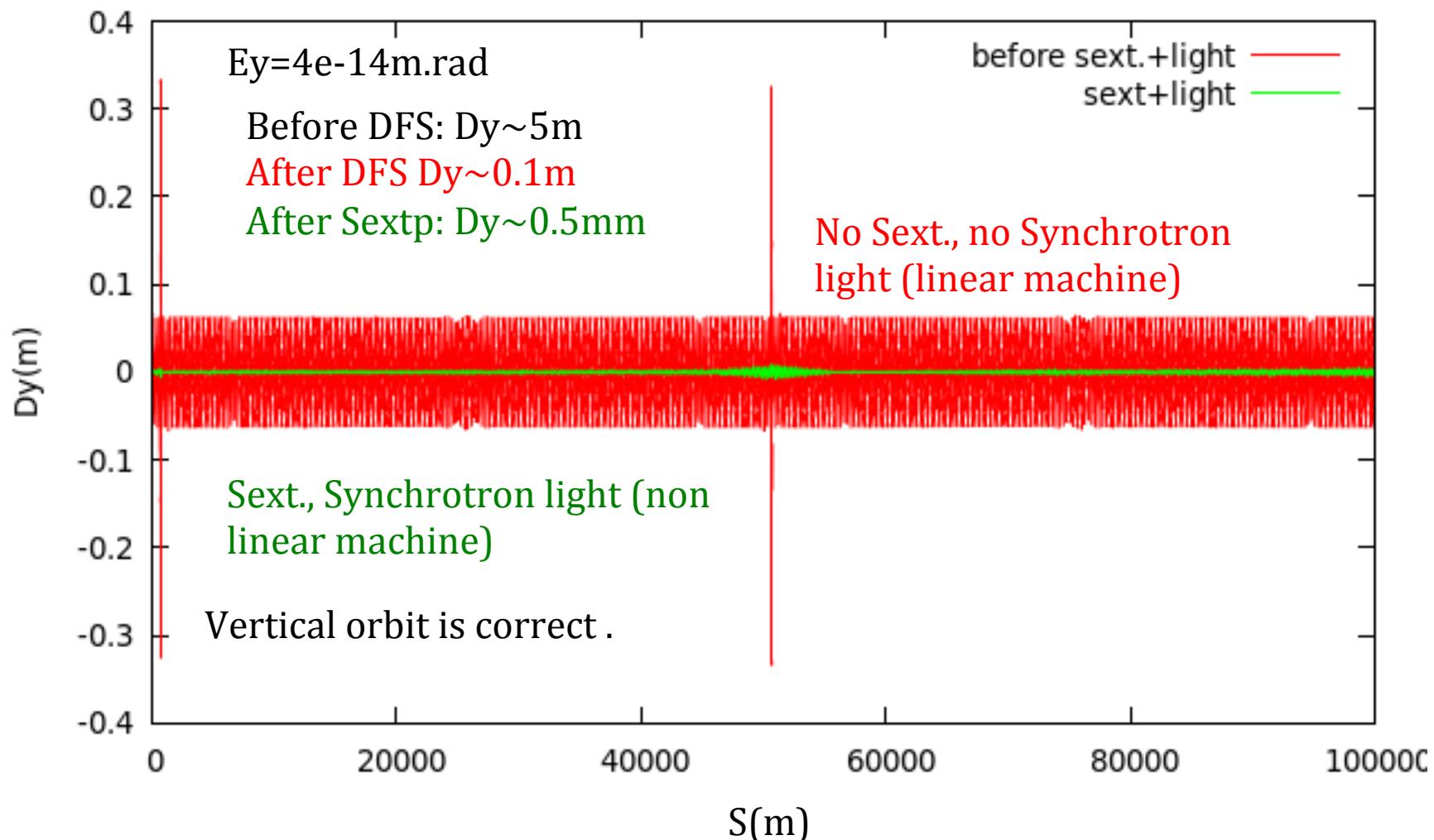
# DFS optimization with singular value decomposition

DFS, chromaticity correction+W function optimization  
(See Bastian Haerer's talk)



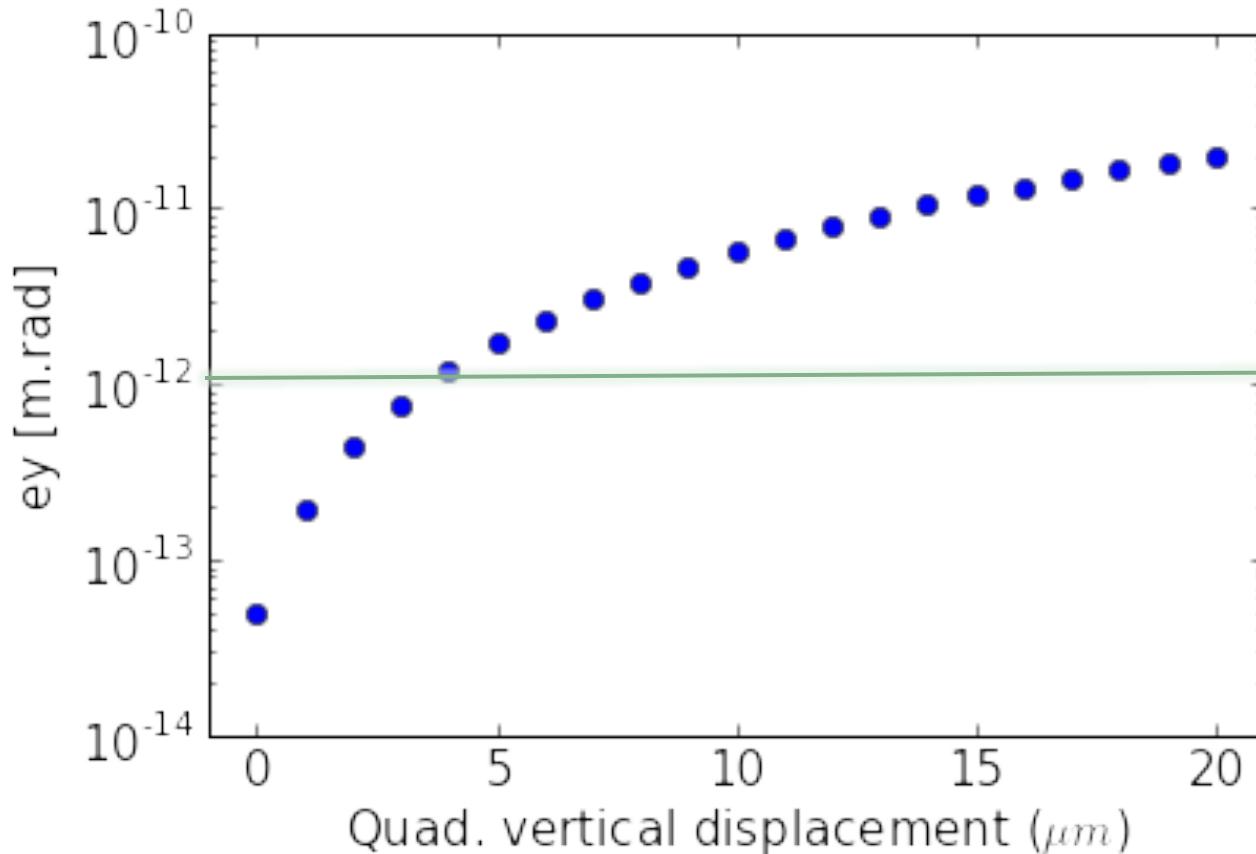
Cut-off:  
smallest considered  
singular value/largest  
singular value

## DFS optimization with singular value decomposition

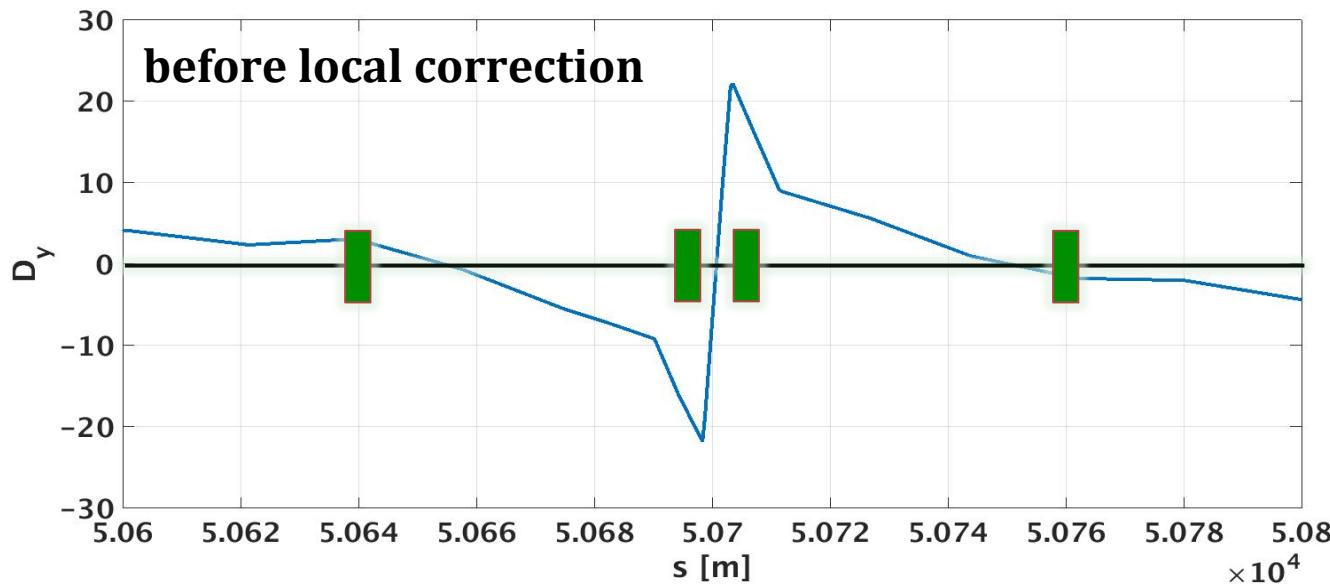


# DFS optimization with singular value decomposition

According to the SEED, 4-6microm



# Local Dispersion Correction at the IP



- Clear indications show that the IRs has to be treated separately from the arcs.
- Vert. Dispersion excursion is driving E<sub>y</sub> to high value.
- Using a local bump at the IP, correct locally the Vert. Dispersion  
**From 2% emittance ratio (E<sub>y</sub>/E<sub>x</sub>) -> 0.5% emittance ratio**
- **Next step: should be fully integrated to the correction algorithm**

- **Lattice errors huge impact on Dy & Ey, due to the beta-function from the IRs, very sensitive**
- **Developing lattice correction methods, adapting them to 100km machine is still on going**
- To overcome BPM errors, a Dispersion Free Steering has been implemented.
- .... still needs quite some optimizations (DFS directly with sextupoles)
- Next step: what do we gain by treating IP & Arcs ?
- **Coupling correction is another very challenging next step.**
- Alignment techniques from Light sources should be considered.



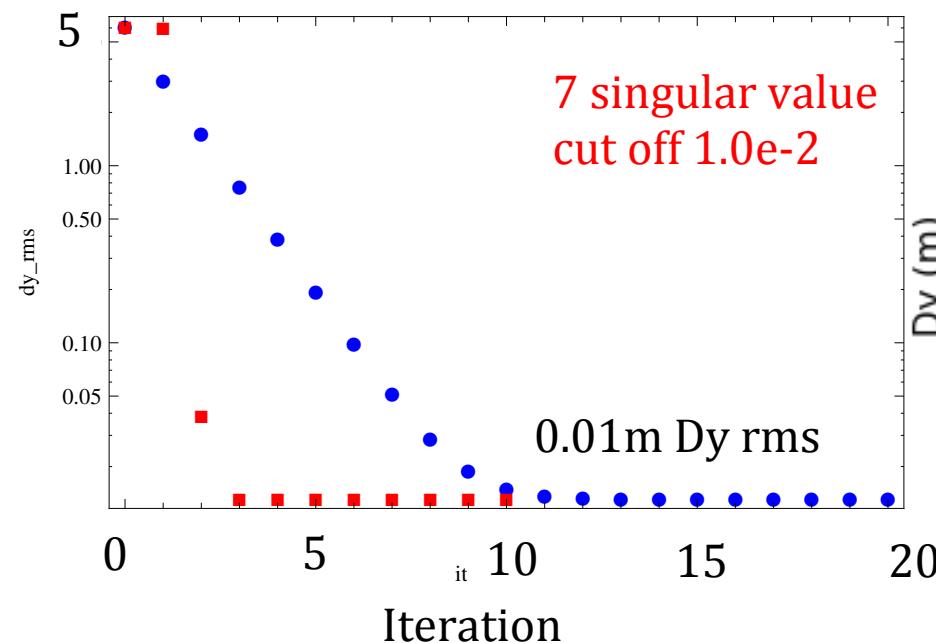
# Thank you for your attention!



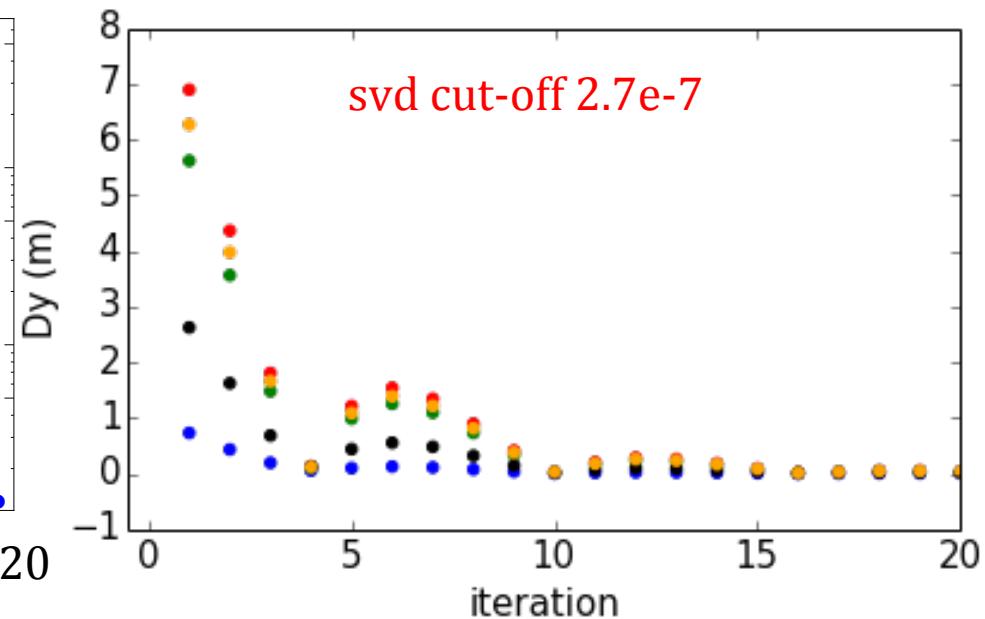
# DFS optimization with singular value decomposition

- Vertical Dispersion Convergence (No sextupole, no SR)

RMS Dy (m) • 1/2kick • full kick



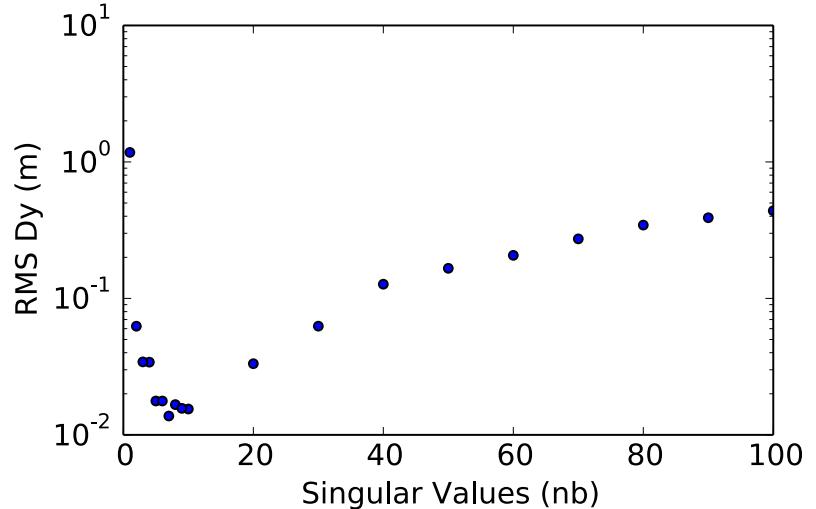
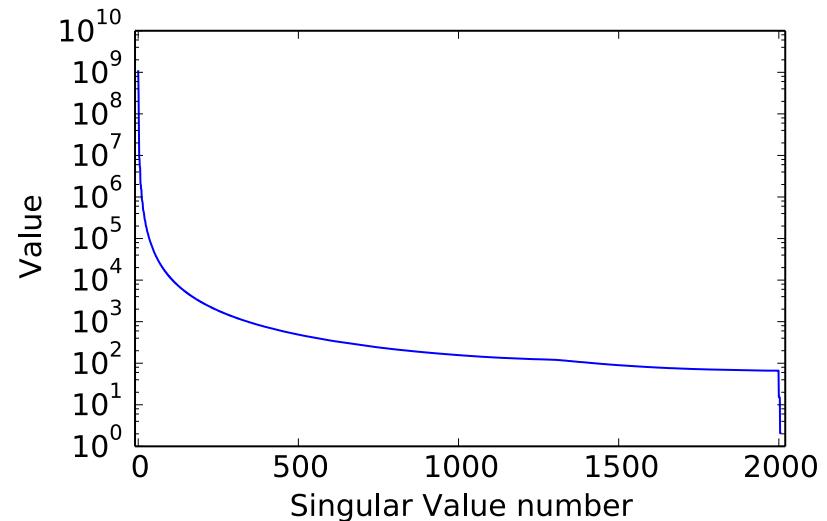
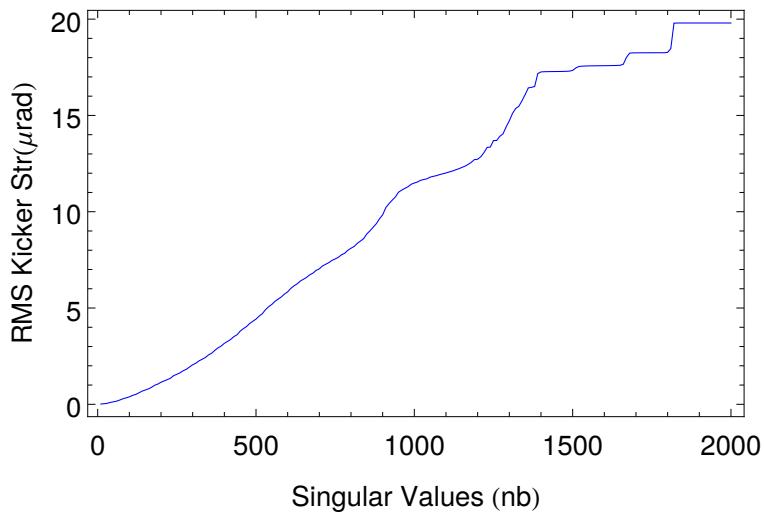
Repeatable on 50 seeds..



After sextupoles:  
 $Ey \sim 1e-10 \text{ m.rad}$

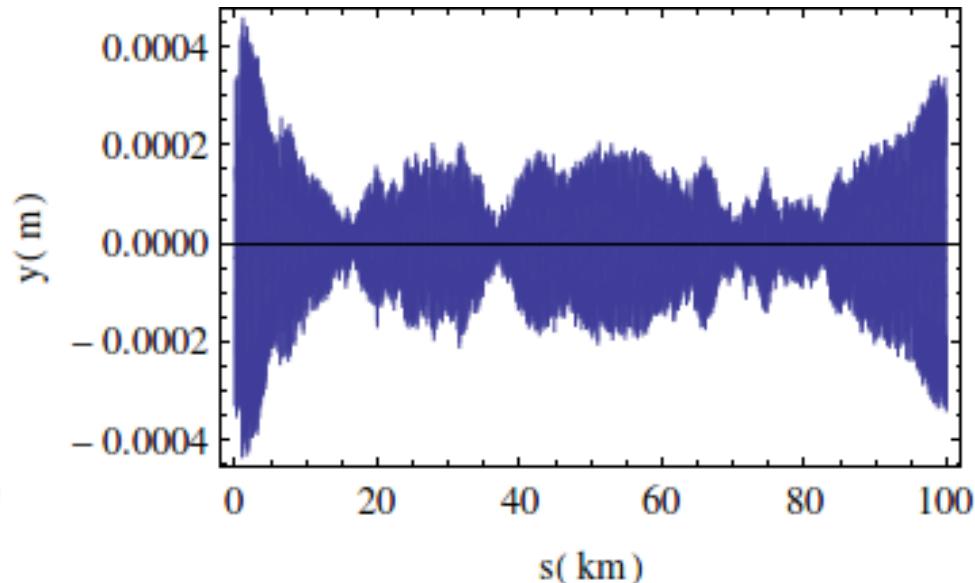
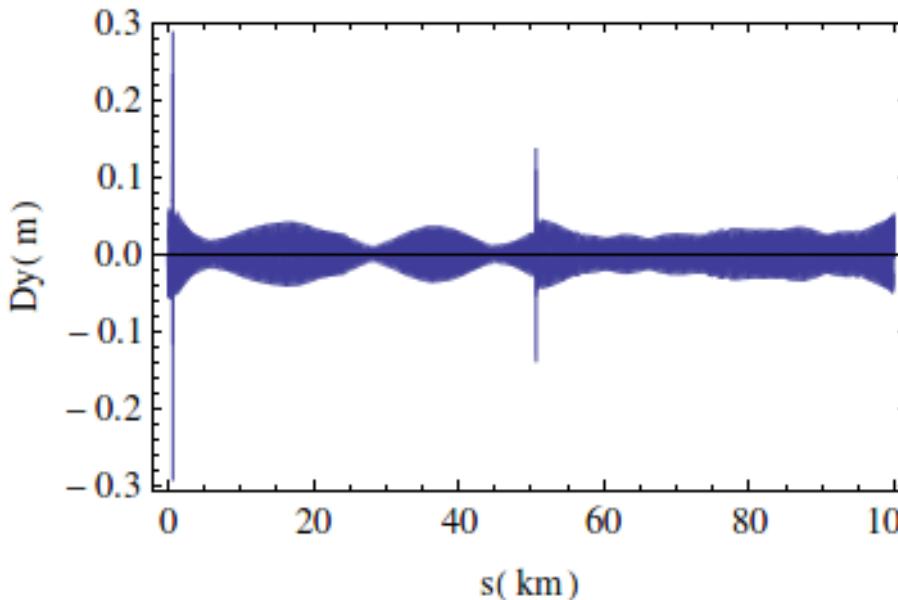
After sextupoles:  
 $Ey \sim 2e-13 \text{ m.rad}$

# SVD no sextupoles



Keep the 7<sup>th</sup> largest singular values  
Sv\_max/sv~1.0e-3

# DFS applied to a misaligned lattice

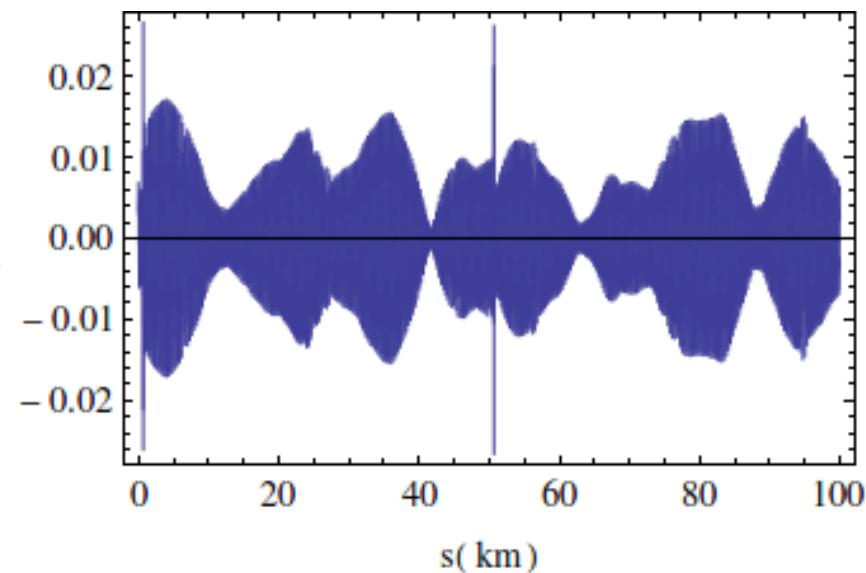
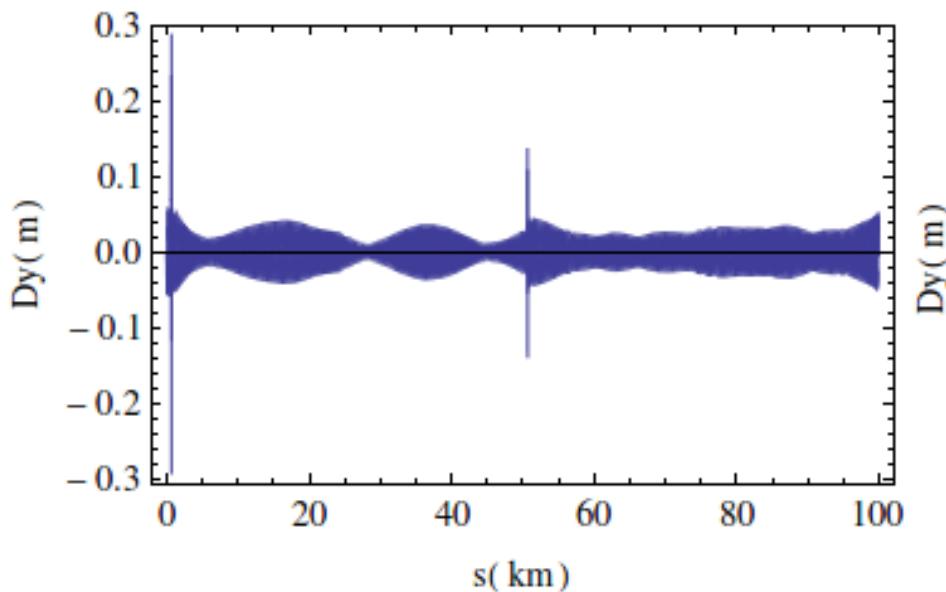


Emittance ratio  $\sim 2\%$

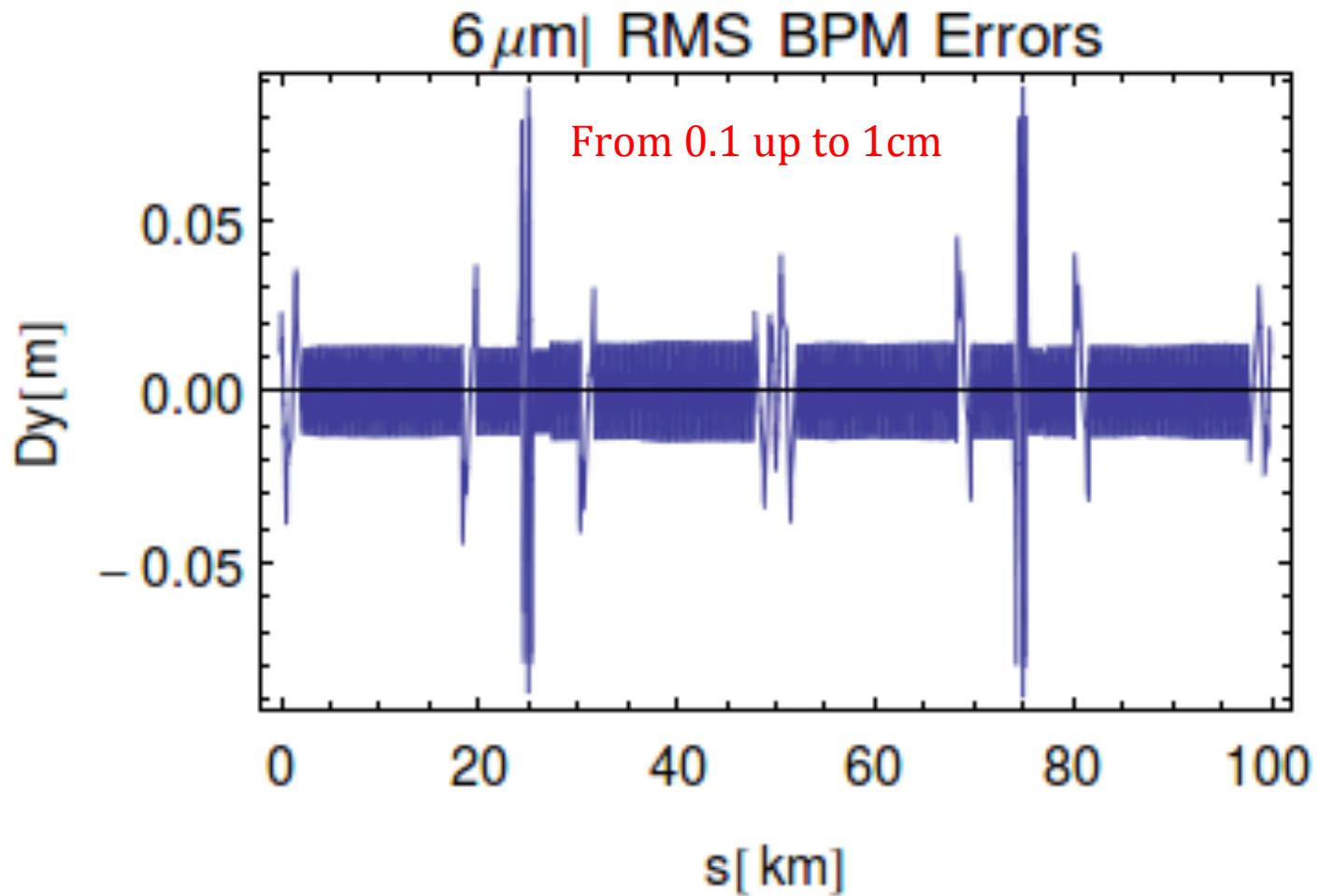
Dispersion at the IP drive the emittance to large value

# Local Dispersion Correction

- ... extremely efficient:  
it brings the vertical dispersion at the same order of magnitude as the arcs.
- **2%-> 0.5%** in emittance ratio.
- DFS extra iterations do not change the kicker strengths in the IRs



# Motivations



# Dispersion Free Steering: Motivations

- **Dispersion Free Steering** (used in LEP): simultaneous correction of the dispersion and of the orbit with orbit correctors.
- Method to minimize the vertical emittance at LEP
- For Vert. Disper, BPM errors do not matter.
- No DFS in MADX (only MAD8)  
YASP (J. Wenninger)

$$\varepsilon_y \simeq \varepsilon_{y0} + \kappa \varepsilon_x + rE^2(D_y^{\text{rms}})^2,$$

# Status Review Meeting & Recommendations Oct 2015

- 120 GeV
- 1m/2mm beta\*, 2 Ips
- RF spread around the ring
- $Q_x = 419.08 / Q_y = 333.14$
- $Q'_x = -573.6506369$
- **$Q'_y = -852.4978106$**

