Arc Optics, global Q’ correction and emittance variation

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Arc Lattice

Courtesy Michael Hauschild

FCC-ee Week 2016
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Arc lattice and global Q’ correction and emittance variation
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FCC-ee Layout

- 100 km circumference
- 2 IPs / Experiments
- 6 Long Straight Sections
- 2 Extended Straight Sections
Interaction Region

- 2 IPs
  - $\beta^* = 1\text{m}/2\text{mm}$
  - $L^* = 2\text{ m}$
- Optimisation of the arc lattice:
  - no local CCS in the interaction regions
Arc FODO cell

Completely symmetric!
Layout already considers space for absorbers, flanges etc.!

\[ \begin{align*}
N_{\text{dipoles}} &= 6152 \text{ (192 half bend)} \\
(\rho &\approx 9.79 \text{ km}, \theta = 1.02 \text{ mrad}, B = 60 \text{ mT})
\end{align*} \]

Phase advance: \( \Psi = 90^\circ/60^\circ \)
Optical functions

Arc FODO cell

Mini straight section
Horizontal emittance

- Lattices with modified cell length were studied to obtain required horizontal emittances

\[
\epsilon = \left( \frac{\delta p}{p} \right)^2 \left( \gamma D^2 + 2\alpha D D' + \beta D'^2 \right)
\]

\[
\hat{D} = \frac{L_{cell}^2}{\rho} \cdot \left( 1 + \frac{1}{2} \sin \left( \frac{\psi_{cell}}{2} \right) \right) \div \sin^2 \left( \frac{\psi_{cell}}{2} \right)
\]

<table>
<thead>
<tr>
<th>Beam energy</th>
<th>Cell length L</th>
<th>Emittance (\epsilon_x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>175 GeV</td>
<td>50 m</td>
<td>1.00 nm rad</td>
</tr>
<tr>
<td>120 GeV</td>
<td>50 m</td>
<td>0.47 nm rad</td>
</tr>
<tr>
<td>80 GeV</td>
<td>100 m</td>
<td>1.74 nm rad</td>
</tr>
<tr>
<td>45.5 GeV</td>
<td>300 m</td>
<td>14.15 nm rad</td>
</tr>
</tbody>
</table>

More information:
B. Haerer, FCC-ee: Lattice optimization and emittance tuning, FCC-Week 2015, Washington, D.C.
Global Q’' correction

Sextupole magnet for LEP

Hall full of LEP magnets waiting to be installed in November 1987
Chromaticity

- Change of the tune with energy deviation

- Textbook: \( \Delta Q = \xi \cdot \Delta p / p \)

- In our case not precise enough: \( \delta = \Delta p / p \)

\[
Q(\delta) = Q_0 + \frac{\partial Q}{\partial \delta} \delta + \frac{1}{2} \frac{\partial^2 Q}{\partial \delta^2} \delta^2 + \frac{1}{6} \frac{\partial^3 Q}{\partial \delta^3} \delta^3 + \ldots
\]
Natural chromaticity compared

|        | \(\beta^*_y = 1\) mm | \(\beta^*_y = 2\) mm | \(\Delta Q (\delta p = 0.1\%)
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>(Q_x)</td>
<td>503.08</td>
<td>505.08</td>
<td></td>
</tr>
<tr>
<td>(Q_x')</td>
<td>-584.26</td>
<td>-587.67</td>
<td>-1.18</td>
</tr>
<tr>
<td>(Q_x'')</td>
<td>-3818.40</td>
<td>-3847.84</td>
<td>-0.01</td>
</tr>
<tr>
<td>(Q_x^{(3)})</td>
<td>(-1.43 \times 10^8)</td>
<td>(-1.52 \times 10^8)</td>
<td>-0.20</td>
</tr>
<tr>
<td>(Q_x^{(4)})</td>
<td>(1.45 \times 10^{13})</td>
<td>(-1.41 \times 10^{13})</td>
<td>-9.40</td>
</tr>
<tr>
<td>(Q_y)</td>
<td>335.14</td>
<td>337.14</td>
<td></td>
</tr>
<tr>
<td>(Q_y')</td>
<td>-2059.23</td>
<td>-860.42</td>
<td>-1.72</td>
</tr>
<tr>
<td>(Q_y'')</td>
<td>(-4.18 \times 10^6)</td>
<td>(-1.04 \times 10^6)</td>
<td>-2.09</td>
</tr>
<tr>
<td>(Q_y^{(3)})</td>
<td>(-1.19 \times 10^{11})</td>
<td>(-0.21 \times 10^{11})</td>
<td>-27.75</td>
</tr>
<tr>
<td>(Q_y^{(4)})</td>
<td>(-4.53 \times 10^{15})</td>
<td>(-0.53 \times 10^{15})</td>
<td>-351.67</td>
</tr>
</tbody>
</table>
Montague functions

- Chromatic variables

\[ B = \frac{1}{\beta} \frac{\partial \beta}{\partial \delta} \quad A = \frac{\partial \alpha}{\partial \delta} - \frac{\alpha}{\beta} \frac{\partial \beta}{\partial \delta} \]

- W-vector

\[ \vec{W} = \frac{1}{2} (B + iA) \]

\[ = \frac{1}{2} \sqrt{A^2 + B^2} e^{i2\psi} \]

Final Focus Quad: \( \Delta A \approx -\beta_0 k_0 l_q \)

Sextupole: \( \Delta A \approx -\beta_0 k'_0 D_x l_s \)

Rotates with twice the phase advance!
FCC-ee sextupole scheme

Horizontal plane: 2 families
Vertical plane: 3 families

\[ \mu_x = 90^\circ = \frac{\pi}{2} \]
\[ \mu_y = 60^\circ = \frac{\pi}{3} \]
\[ \mu_x = 180^\circ = \pi \ (\rightarrow -I \ transformation) \]
W functions in the half-ring
W functions: 0-10 km

ARC1 (SARC)  SS2 (LSS)  ARC2 (LARC)

Ss1 (LSS)
Beta functions

- Relative energy deviation $\delta p/p = \pm 0.001$

After chromaticity correction!
Momentum acceptance

- At 175 GeV beam energy 2 % energy acceptance is required due to energy loss by beamstrahlung at the IP

  → Matching of the W functions is not sufficient

- The tune is a function of the energy deviation

  \[ Q(\delta) = Q_0 + \frac{\partial Q}{\partial \delta} \delta + \frac{1}{2} \frac{\partial^2 Q}{\partial \delta^2} \delta^2 + \frac{1}{6} \frac{\partial^3 Q}{\partial \delta^3} \delta^3 + ... \]

Modify tune function to increase momentum acceptance by

- Choice of \( \beta^*_y \)
- Modifying the linear chromaticity
- Different sextupole schemes
- Different optimisation methods
Choice of $\beta^*_y$

- $\beta^*_y = 1$ mm: $[-0.21\%, 0.07\%]$  
- $\beta^*_y = 2$ mm: $[-0.29\%, 0.11\%]$  

$+43\%$
Optimising the momentum acceptance

$Q'_y = 0 \ [-0.15\%, \ 0.07\%]$  
$Q'_y = 15 \ [-0.21\%, \ 0.07\%]$
Different sextupole schemes

6 sextupole families per arc per plane

6 sextupole families + 12 free sextupole pairs per arc per plane

Vary individual sextupole pairs to flatten $Q(\Delta p/p)$

TODO: Dynamic aperture studies!!!
Downhill-Simplex-Optimisation

Algorithm implemented in Fortran calls MAD-X and optimises strengths of 6/6, 12/12 and 54/54 families

Largest momentum acceptance: 6/6 families per plane

[[-0.38%, 0.54%]
Summary of the last year

• Sextupole schemes for global Q’ correction have been studied systematically

• Methods to increase momentum acceptance have been developed with considerable success!

• Sophisticated sextupole scheme for the arcs is worth to be studied
  → Momentum acceptance has been increased by >5 times
  → Relaxes requirements for the local CCS
Outlook

Interaction region designed by A. Bogomyagkov:

- Combine optimised arcs with local chromaticity correction scheme (CCS)
- Maybe remove local CCS of horizontal plane
- Dynamic aperture studies!
Thank you for your attention!

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Phase advance FD – 1st Sext.

\[ \mu_x = m^*\pi \]

\[ \mu_y = n^*\pi \]

(m, n integer)