Lattice for a Higgs Factory

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Synchrotron Radiation

Synchrotron radiation

$$U_0 = \frac{4\pi}{3} \frac{r_e mc^2}{\rho} \gamma^4$$

Beam power given by RF

$$P_b = U_0 I / e$$

Limits the total beam current to 26 mA

For our example, E_0=125 GeV, ρ=12.2 km, U_0=1.923 GeV

I=26 mA, limited by P_b=50 MW in the design.

Luminosity

Bunch luminosity

$$L_b = f_{rev} \frac{N_b^2}{4\pi\sigma_x\sigma_y} R_h$$

where R_h is a geometrical reduction from the hourglass effect and is written as

$$R_h = \sqrt{\frac{2}{\pi}} a e^{a^2} K_0(a^2), a = \frac{\beta_y^*}{\sqrt{2}\sigma_z}$$

Total luminosity:

$$L = n_b L_b$$

Beam-Beam Limit

Given the beam-beam parameter

$$\xi_y = \frac{r_e N_b \beta_y^*}{2\pi\gamma\sigma_y(\sigma_x + \sigma_y)}$$

The luminosity can be re-written as

$$L = \frac{cI\gamma\xi_y}{2r_e^2 I_A \beta_y^*} R_h$$

where I_A=17045 A. Smaller β_y* is absolutely necessary. For example, in the design we have I=26 mA, E_0=125 GeV, ξ_y=0.1, R_h=0.824, β_y*=2 mm, gives 2.90x10^{34} $cm^{-2}s^{-1}$ in luminosity. So what is the beam-beam limit for Higgs factory?

Scaling of Luminosity

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If there is a beam-beam limit as suggested by the simulation and beam power is also limited, the luminosity can be re-written as

$$L = \frac{3c}{8\pi r_e^3}\frac{\xi_y \rho}{\gamma^3 \beta_y^*}\frac{P_b}{P_A}R_h$$

where $P_A = mc^2 I_A/e$ = 8.7 GW. This scaling was given by B. Richter, Nucl. Instr. Meth. 136 (1976) 47-60.

Beamstrahlung Effects

Beam lifetime due to large single photon emission (for 30 minutes, V.I. Telnov, 2012)

$$\frac{N_b}{\sigma_x \sigma_z} < 0.1\eta \frac{\alpha}{3\gamma r_e^2}$$

Large RF-buckets and large momentum aperture η, α is the fine-structure constant.

Large σ_z and σ_x. Favors longer and larger horizontal beam size.

Limits bunch population N_b

Are there any reasonable solutions?

Analysis of Design Constraints

To achieve the beam-beam parameter and assuming $\beta_y=\kappa_\beta\beta_x$ and $\varepsilon_y=\kappa_\varepsilon\varepsilon_x$ we have

$$\frac{N_b}{\varepsilon_x}=\frac{2\pi\gamma\xi_y}{r_e}\sqrt{\frac{\kappa_e}{\kappa_\beta}}$$

To have adequate beam lifetime (due to beamstrahlung)

$$\frac{N_b}{\sqrt{\varepsilon_x}}<0.1\eta\frac{\alpha\sigma_z}{3\gamma r_e^2}\sqrt{\frac{\beta_y^*}{\kappa_\beta}}$$

Clearly, smaller coupling κ_e is better and larger momentum acceptance η is better but they have their own limits.

Low Emittance

Given a momentum acceptance η, we solve

$$\varepsilon_x < \left(\frac{0.1\eta\alpha\sigma_z}{6\pi\gamma^2\xi_y r_e}\right)^2 \frac{\beta_y^*}{\kappa_e} = 5.5nm$$

Note that it does not depends on $\kappa_{\beta.}$ We can use κ_β to adjust the bunch population N_b or the number of bunches n_b. Clearly, there are many possible solutions.

But is there any self-consistent one? η=3%, κ_e=0.1%, σ_z=2.41 mm

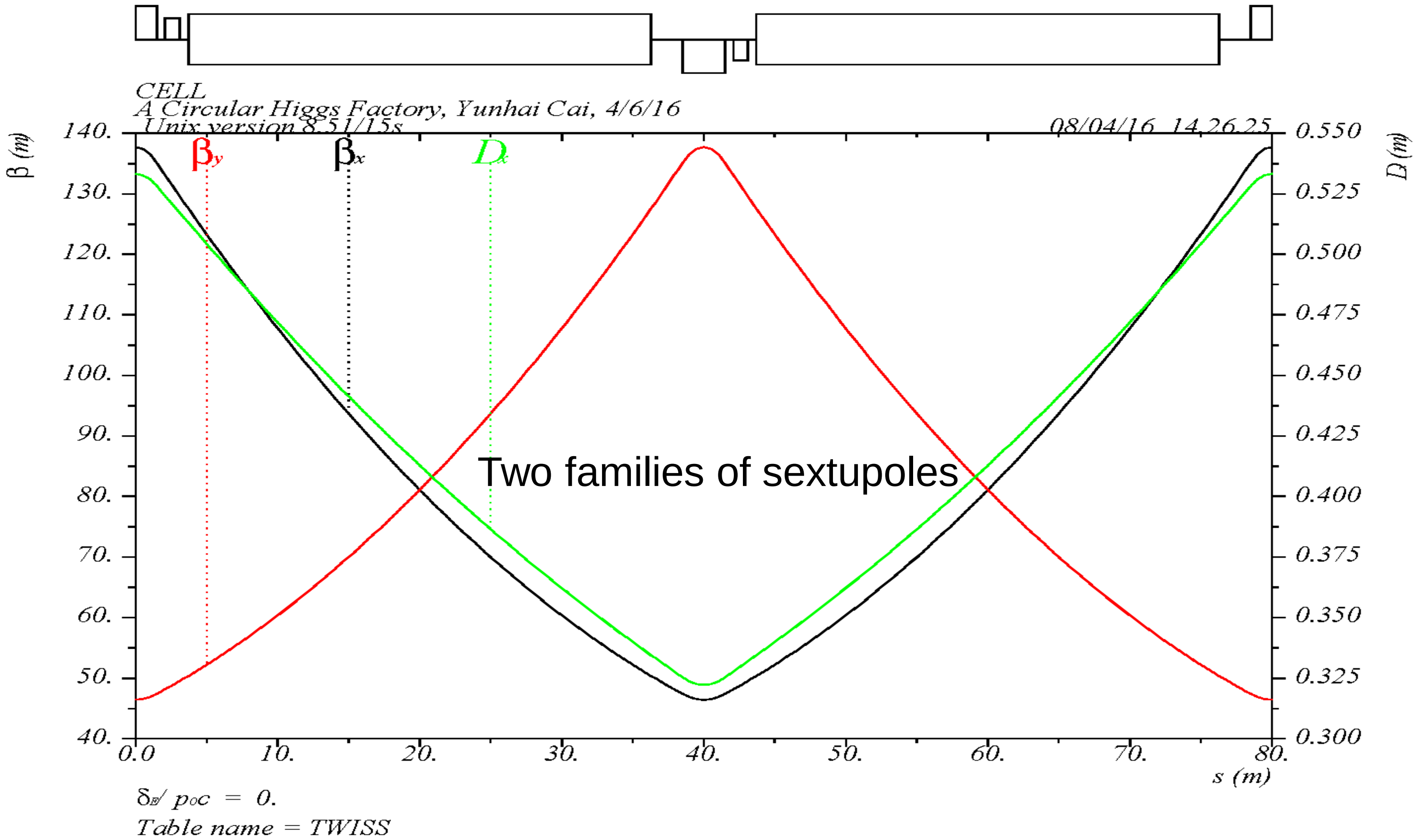
The requirement of accommodating beamstrahlung is translated to design a low emttance lattice.

Normally, the emittance scales as γ^2. This relation requires a scaling of γ^{-4}, indicating a difficulty to design a machine with much higher energy than 125 GeV.

Design Parameters

	LEP2	CHF
Beam Energy [GeV]	104.5	125
Circumference [km]	26.70	105.29
Beam current [mA]	4	26
Number of bunches	4	736
Bunch population [10^{10}]	57.5	7.8
Horizontal emittance [nm]	48	4.5
Vertical emittance [pm]	250	4.5
Momentum compaction factor	18.5×10^{-5}	2.5×10^{-5}
β_x^* [mm]	1500	200
β_y^* [mm]	50	2
Hourglass factor	0.98	0.82
SR power [MW]	11	50
Bunch length [mm]	16.1	2.41
Beam-beam parameter	0.07	0.10
Luminosity [10^{34} cm^{-2}s^{-1}]	**0.0125**	**2.90**

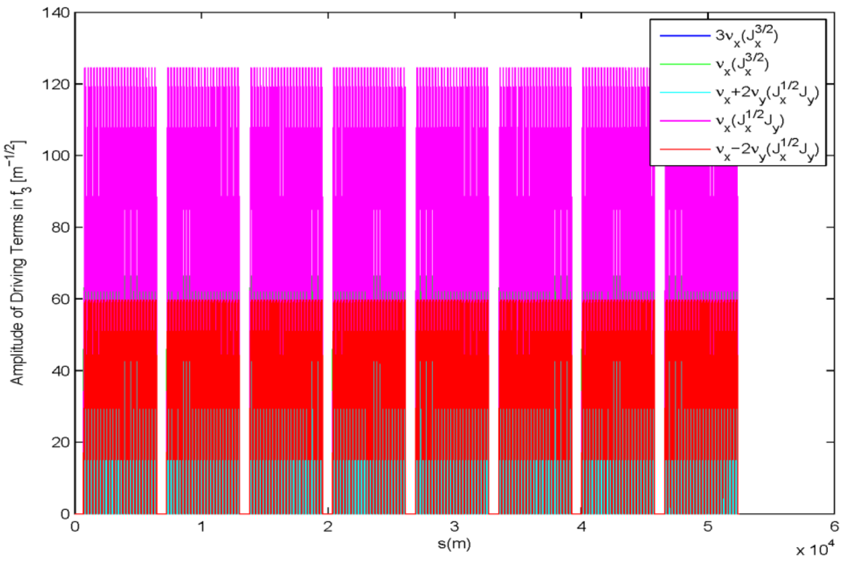
$60^0/60^0$ Arc Cell



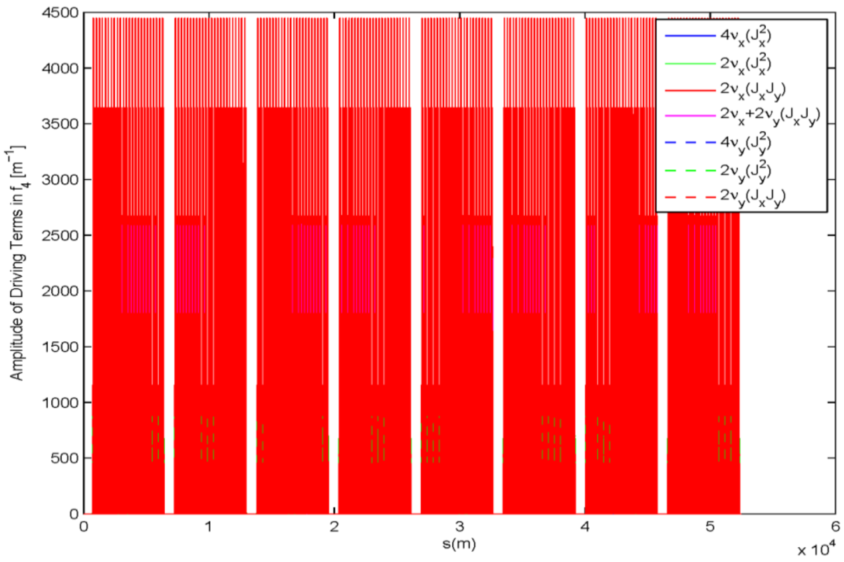
6 cells make an achromat (unit transformation).

Quasi (4^{th} Order) Achromat

3^{rd} order driving terms



4^{th} order driving terms



The cancellation occurs in every 6 cells in arcs.
Only non-vanishing resonance is $2\nu_x$-$2\nu_y$.

Yunhai Cai, "Single-particle dynamics in electron storage rings with extremely low emittance," Nucl. Instr. Meth., A645:168–174, 2011.

Arc Design

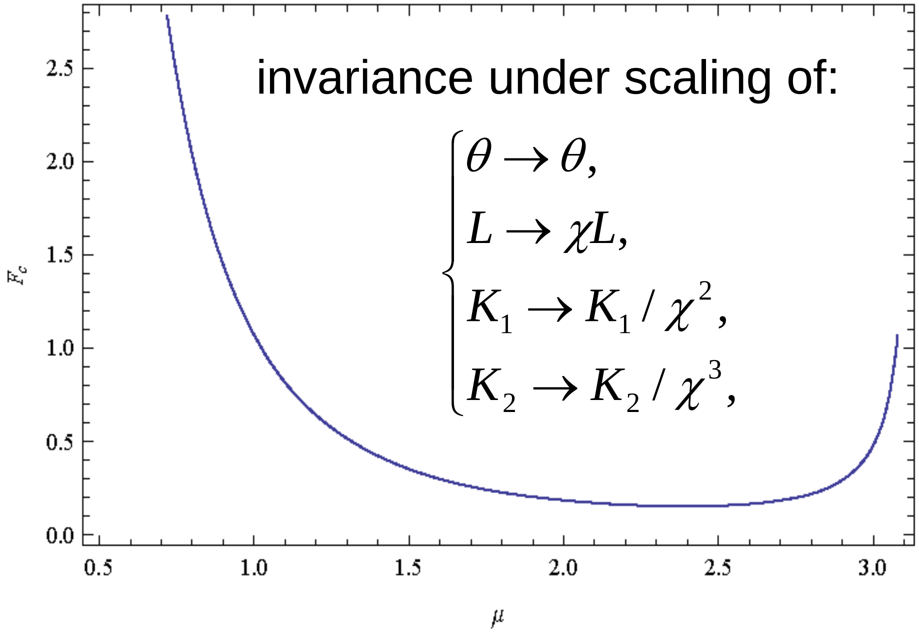
Emittance:

$$\varepsilon_x = F_c \frac{C_q \gamma^2}{J_x} \theta^3$$

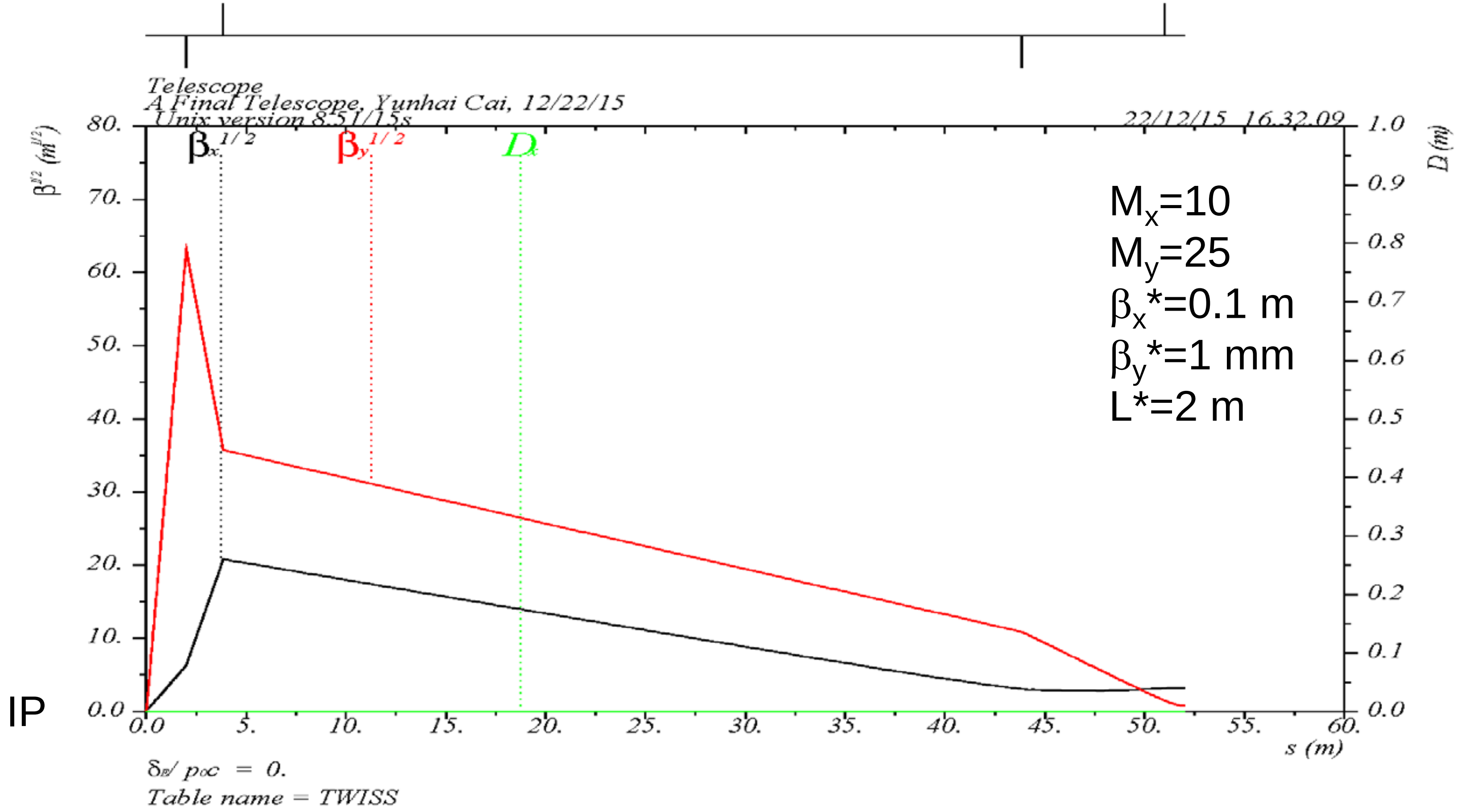
where θ is the bending angle in a cell and the form factor:

$$F_c = \frac{1 - \frac{3}{4}\sin^2\frac{\mu}{2} + \frac{1}{60}\sin^4\frac{\mu}{2}}{4\sin^2\frac{\mu}{2}\sin\mu}$$

Here μ is the phase advance per a cell.



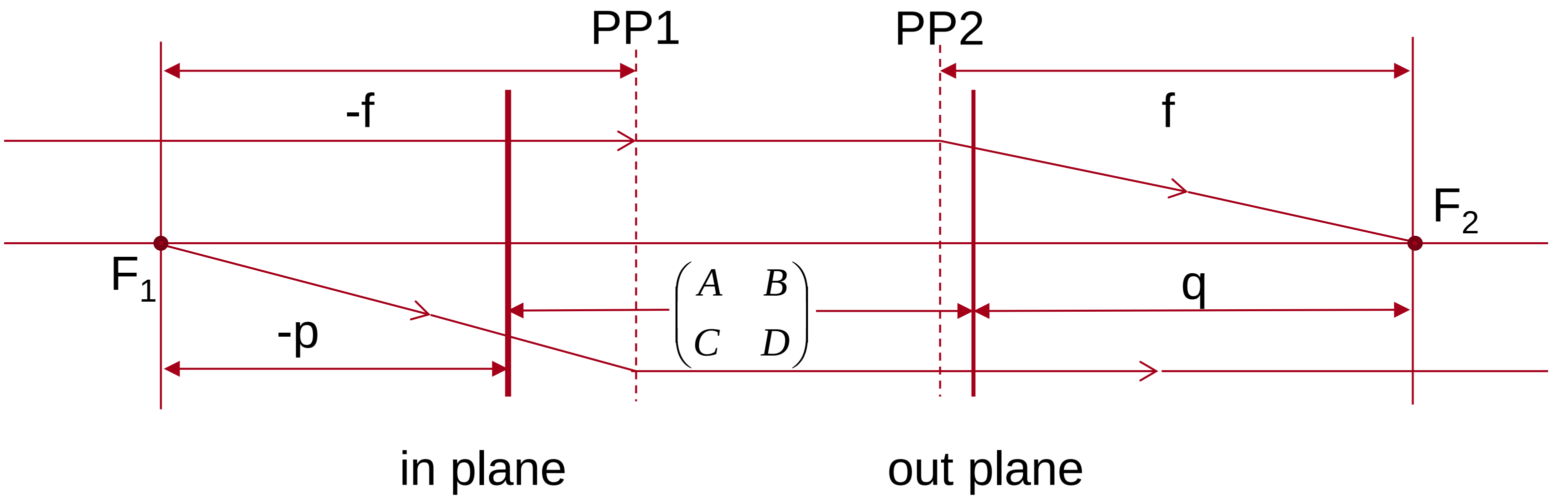
Final Telescope in Particle Accelerators



M_x=10
M_y=25
β_x*=0.1 m
β_y*=1 mm
L*=2 m

Parameters: L_1=2 m, d_1=1.85541 m, L=40 m, d_2=7.14276 m, L_2=1 m, f_1=1.36174 m, f_2=2.51748 m, f_3=15.11842 m, f_4= 17.01195 m

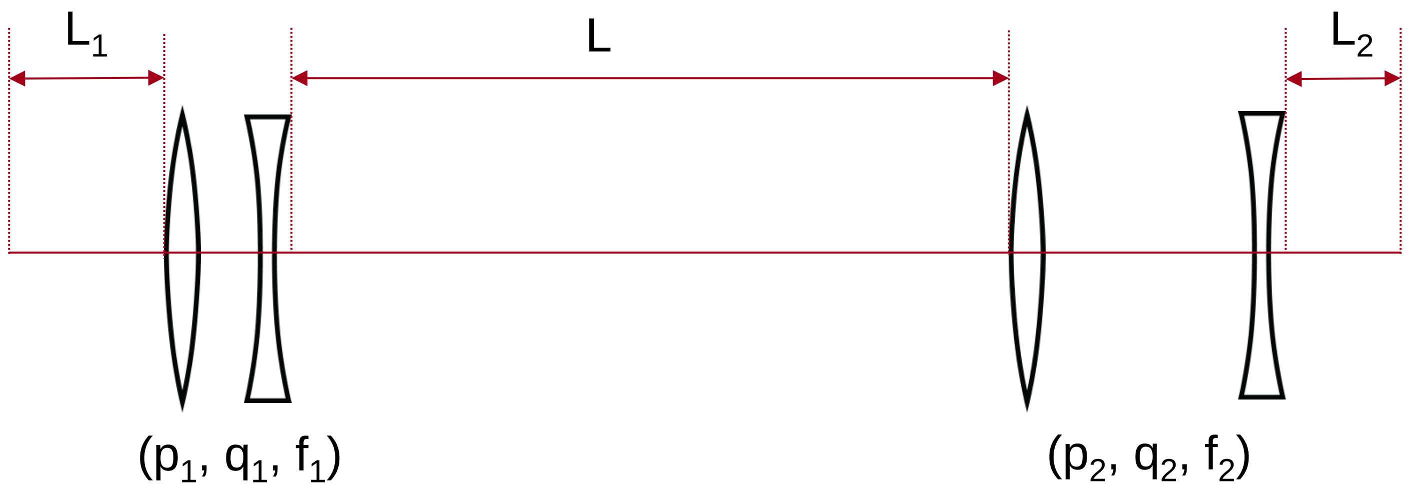
Matrix Method in Paraxial Optics



It can be easily shown, using the paraxial approximation and symplectic condition of the R-matrix,

$$\begin{cases} p = \dfrac{D}{C} \\ q = -\dfrac{A}{C} \\ f = -\dfrac{1}{C} \end{cases} \quad \text{and} \quad R = \begin{pmatrix} \dfrac{q}{f} & \dfrac{pq + f^2}{f} \\ -\dfrac{1}{f} & -\dfrac{p}{f} \end{pmatrix}$$

Necessary Conditions for Final Telescope



$$\begin{cases} p_2 = q_1 - L \\ q_2 = L_2 + M^2(L_1 + p_1) \\ f_2 = M f_1 \end{cases}$$

p is the distance to the first focal point
q is for the second focal point
f is the effective focusing length
sign convention: >0 for right and <0 left

M is the magnification factor

The subsystems do not have to be doublets or thin lens. Moreover, these conditions are valid both in the horizontal and vertical planes.

Approximated Transfer Maps

A map up to the second order in the vertical plane:

$$\begin{cases} M_3 = -My + \dfrac{ML(L_1+p)}{f^2} y\delta + \dfrac{f^2(L_2+M^2L_1)+M^2L(L_1+p)^2}{Mf^2} p_y\delta \\ M_4 = -\dfrac{1}{M} p_y - \dfrac{L}{Mf^2} y\delta - \dfrac{L(L_1+p)}{Mf^2} p_y\delta \end{cases}$$

An approximated map:

$$\begin{cases} M_3 = -My + ML_1 p_y\delta \\ M_4 = -\dfrac{1}{M} p_y \end{cases}$$

$L_1+p=0$, implies that the first focus point of the final doublet is the interaction point. That is a very good approximation. Also assumed M>>1

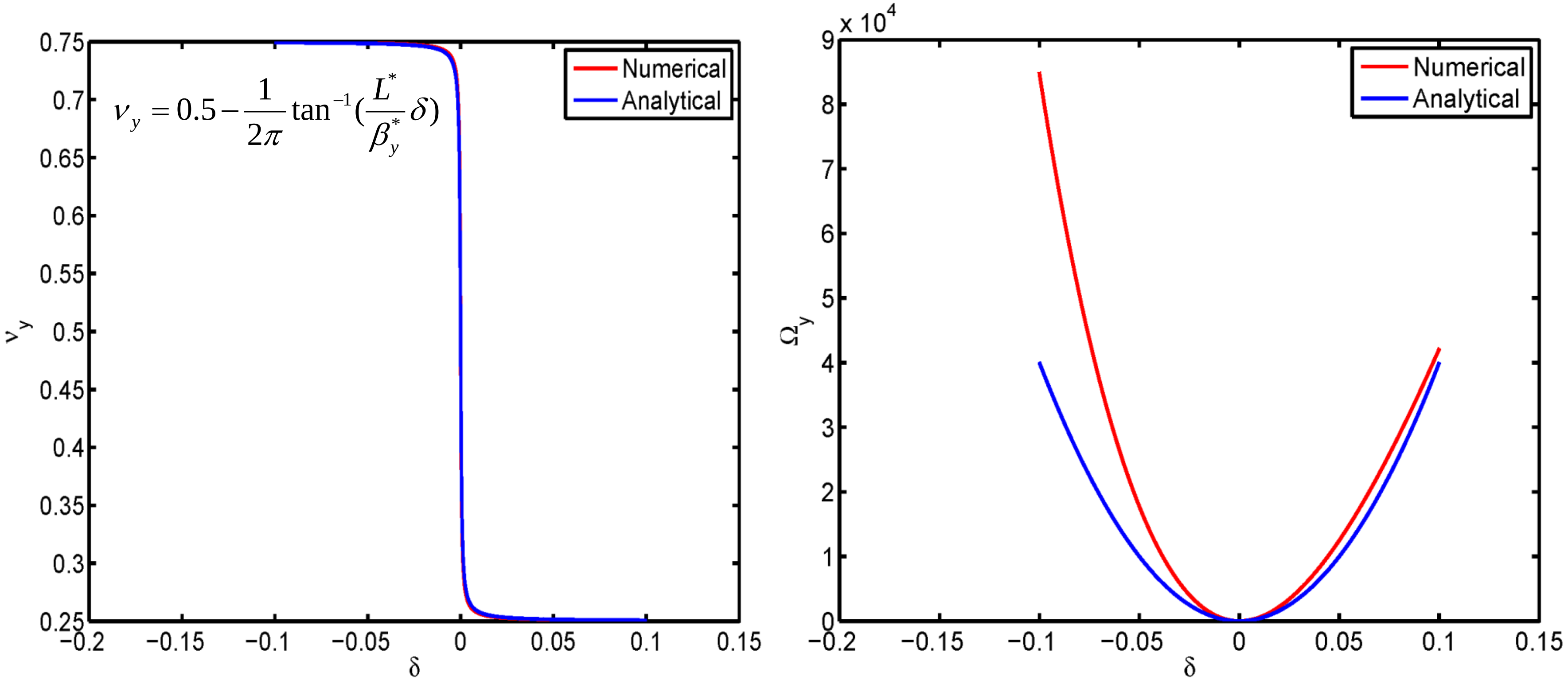
Chromatic Aberrations in Final Telescope

The simpler map leads to:

$$\begin{cases} \beta_y(\delta) = M^2 \beta_y^* [1 + (\frac{L^*}{\beta_y^*}\delta)^2] \\ \alpha_y(\delta) = \frac{L^*}{\beta_y^*}\delta \\ \psi_y(\delta) = \pi - \tan^{-1}(\frac{L^*}{\beta_y^*}\delta) \end{cases}$$

Back to the common notation, we use L_1=L*. We have captured the most important chromatic aberrations in the final telescope. The first-order terms agree with the well-known results derived using the perturbation method in a periodical system. The large second-order term in the beta function was discussed extensively by Karl Brown. Here we show that the phase advance has to be corrected up to third order for a very small vertical beta function at the IP.

Chromatic Bandwidth of Telescope

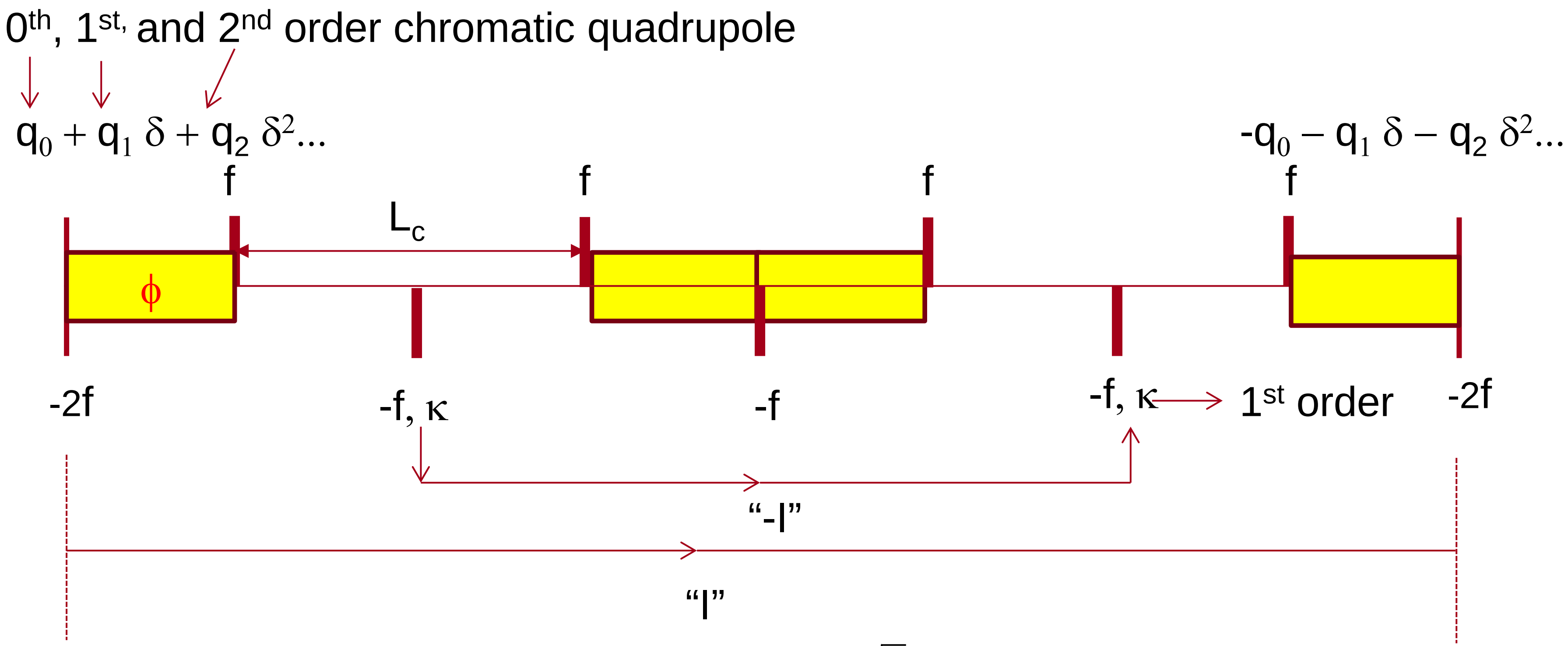


Mismatch Parameters:

$$\xi(\delta) = \frac{1}{2}[\gamma_0\beta(\delta) - 2\alpha_0\alpha(\delta) + \beta_0\gamma(\delta)]$$

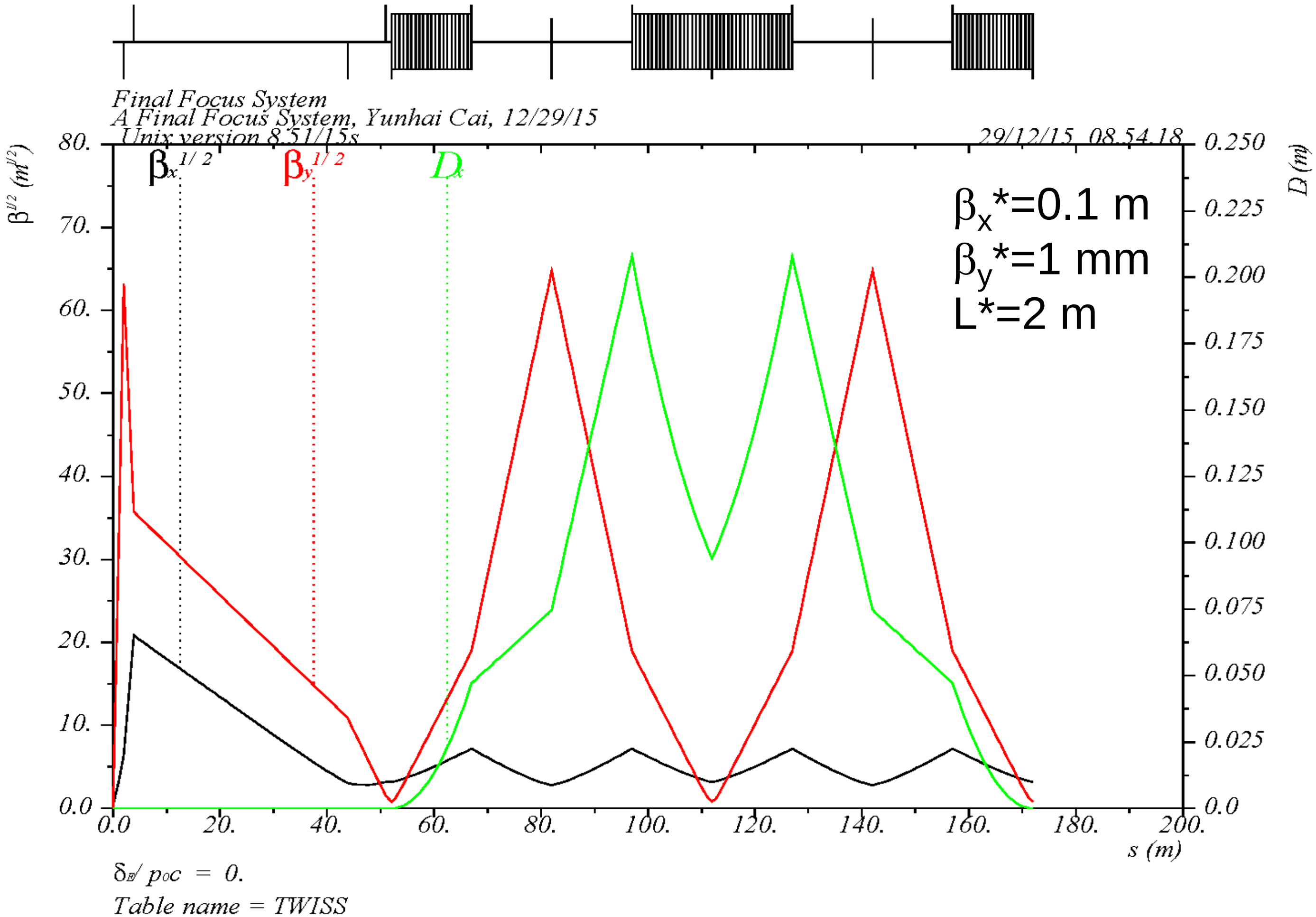
$$\Omega(\delta) = \xi(\delta) + \sqrt{\xi(\delta)^2 - 1}, W = \Omega'$$

Chromatic Correction System in Vertical Plane



- Consists of four $90^0/90^0$ cells, $f = L_c / 2\sqrt{2}$
- Dipoles for dispersion at sextupole and paired by 180^0 to cancel their dispersion outside
- Multipole for nonlinear chromaticity correction
- Multipole paired asymmetrically to cancel their geometric aberration

Optics of Final Focus System in Vertical Plane



L_c=30 m and $\phi=\pi/500$

1st Chromatic Correction

The 1st order chromaticity is given by,

$$\nu_y' = -\frac{L_c[16(2+\sqrt{2})+(5+3\sqrt{2})L_c^2\kappa\phi]+4M^2L^*}{8\pi M^2\beta_y^*}$$

It is zeroed out by a pair of sextupoles with its integrated and dimensionless strength,

$$\kappa L_c^2 = -\frac{(32+16\sqrt{2})L_c + 4M^2L^*}{(5+3\sqrt{2})L_c\phi}$$

We have used the simplified map.

Nonlinear Chromatic Correction

The nonlinear chromaticity has a simple pattern,

$$\nu_y'' = A_0 q_0 + B_0,$$

$$\nu_y''' = A_0 q_1 + B_1,$$

$$\ldots$$

$$\nu_y^{(n)} = A_0 q_{n-2} + B_{n-2},$$

$$\ldots$$

where

$$A_0 = \frac{L_c^2[512(3+\sqrt{2}) + 32(16+11\sqrt{2})L_c^2\kappa\phi + (43+30\sqrt{2})L_c^4\kappa^2\phi^2]}{32\pi M^2\beta_y^*}$$

Most importantly, B_i does not have dependence on q_j for j<=i. So the equation can be solved sequentially.

Nonlinear Chromatic Correction

Then the 2nd order chromaticity can be set to zero by a pair of quadrupoles with its integrated strength,

$$q_0 = \frac{(1460+1030\sqrt{2})L_c - (17+13\sqrt{2})M^2 L^*}{(43+30\sqrt{2})(M^2 L^*)^2}$$

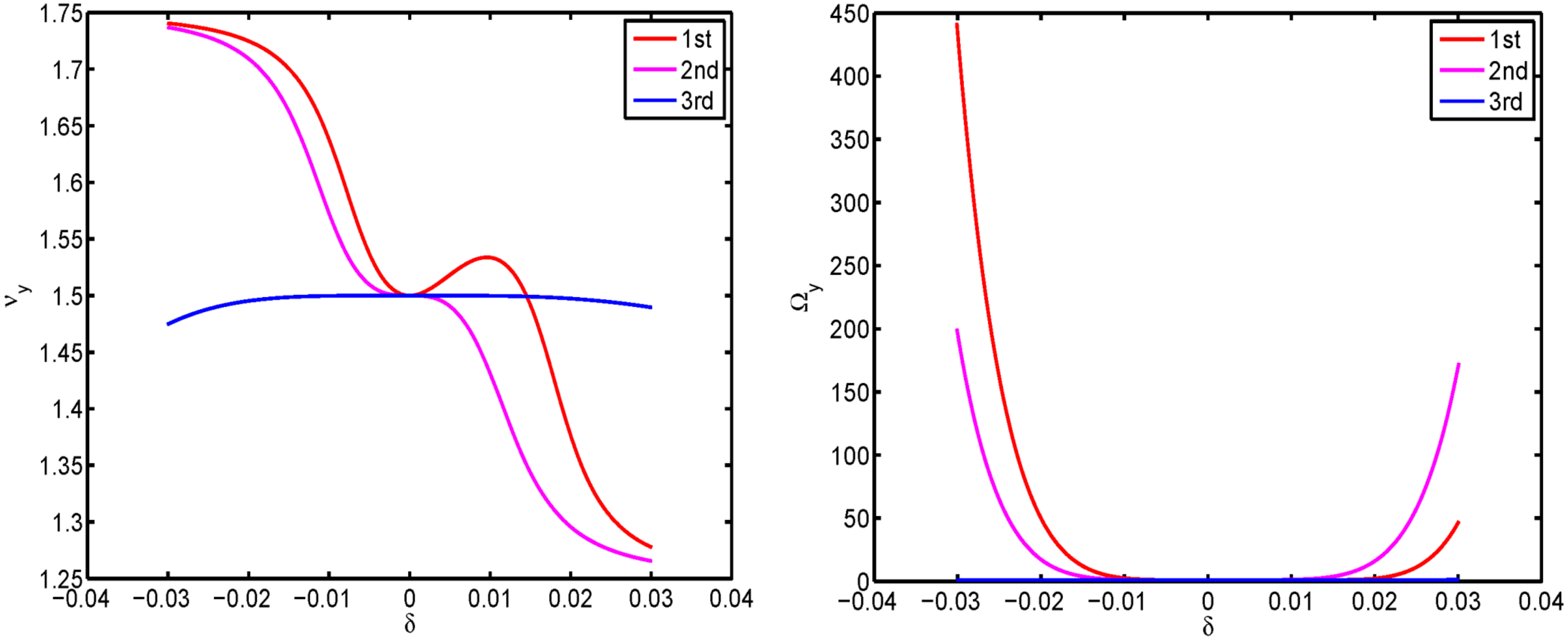
The quadrupole strength is very small if M >> 1.

The 3rd order can be set to zero by a pair of chromatic quadrupoles with its integrated strength,

$$q_1 \approx -\frac{12458+8809\sqrt{2}}{(7298+5160\sqrt{2})L_c}$$

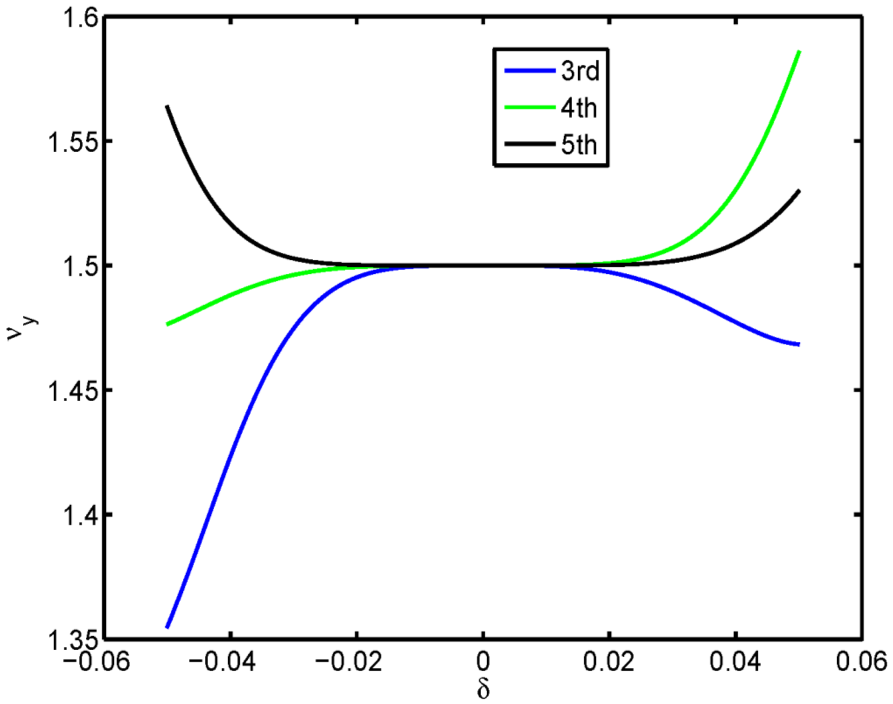
In practice, it can be generated by a sextupole and a dipole in the final telescope. This sextupole at the imaging point is commonly called the Brinkmann sextupole.

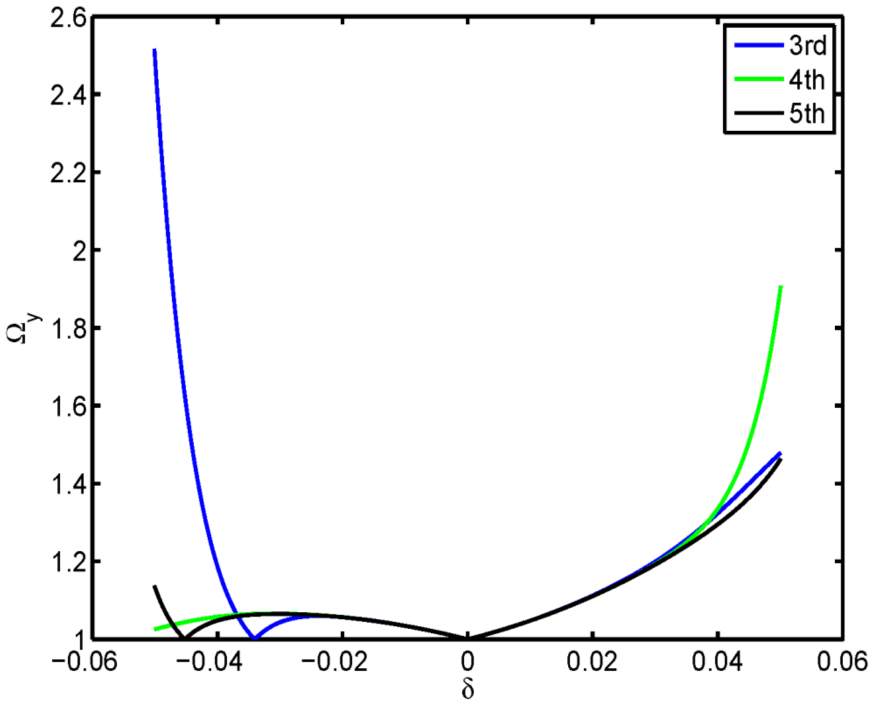
Chromatic Bandwidths and Corrections



For ±2% of the bandwidth, it is sufficient to correct up to the third order of momentum deviation δ. It shows that the third-order compensation is critical for a large bandwidth.

Further Improve Bandwidths





To double the bandwidth to ±4%, we have to correct up to the fifth order of momentum deviation δ. In practice, it implies that we need to use octupoles and decapoles.

Fourth-Order Geometric-Chromatic Aberrations

Although the sextupoles are paired in “-I” to cancel their geometric aberration, they generate the geometric-chromatic aberrations. At the fourth order, we find a Lie operator,

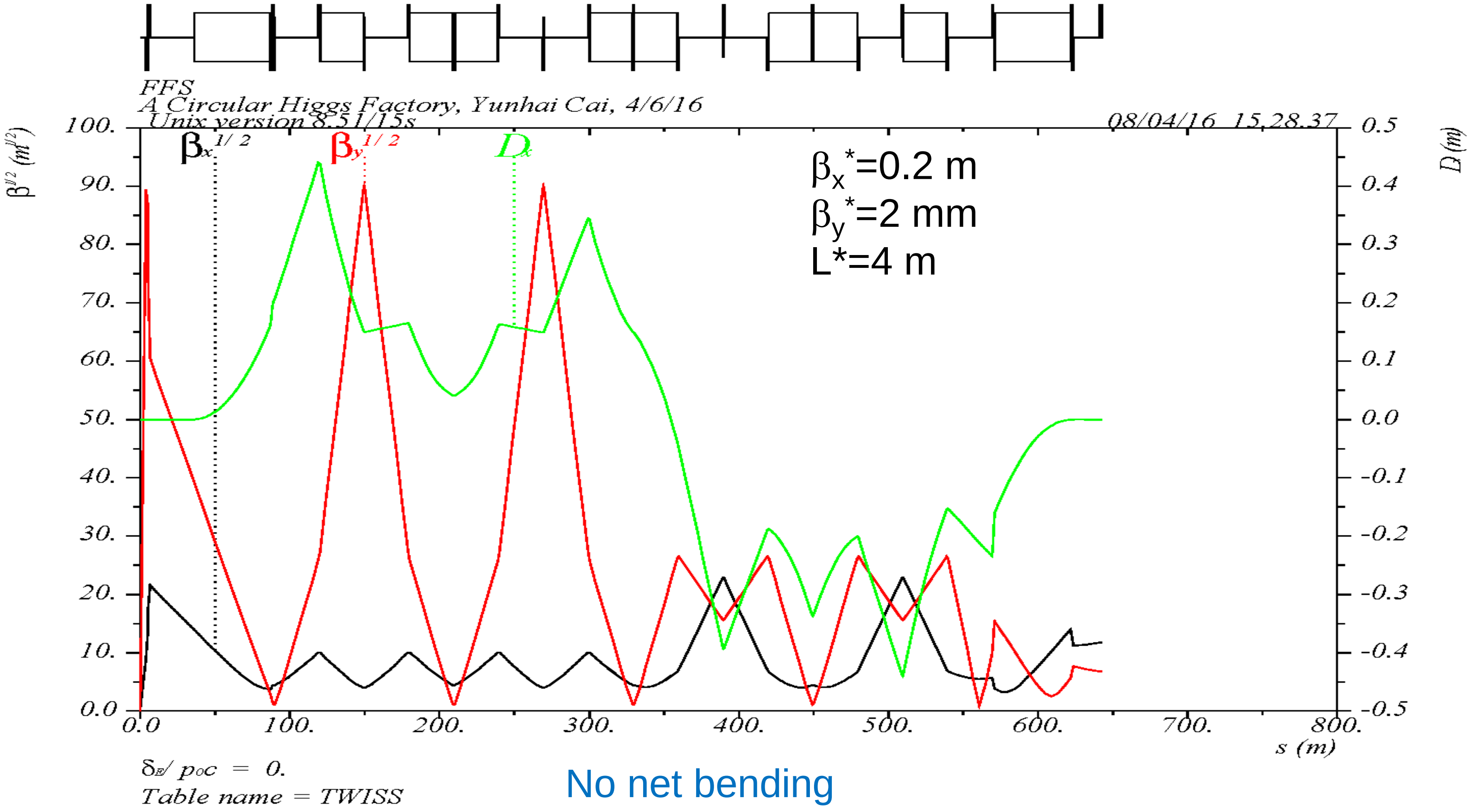
$$f_4 = -\kappa L_c^2[(2\sqrt{2}-3)xp_x^2 + 2yp_x p_y + (2\sqrt{2}+3)xp_y^2]\delta$$

These terms excite the third-order resonances for the off-momentum particles. Fortunately, their dynamical effects can be cancelled by an asymmetric arrangement of the dipole and sextupole magnets across the interaction region. This asymmetry also cancel the residue slope of the second-order dispersion,

$$\eta_{p_x}^{'} = -4(2+\sqrt{2})\phi$$

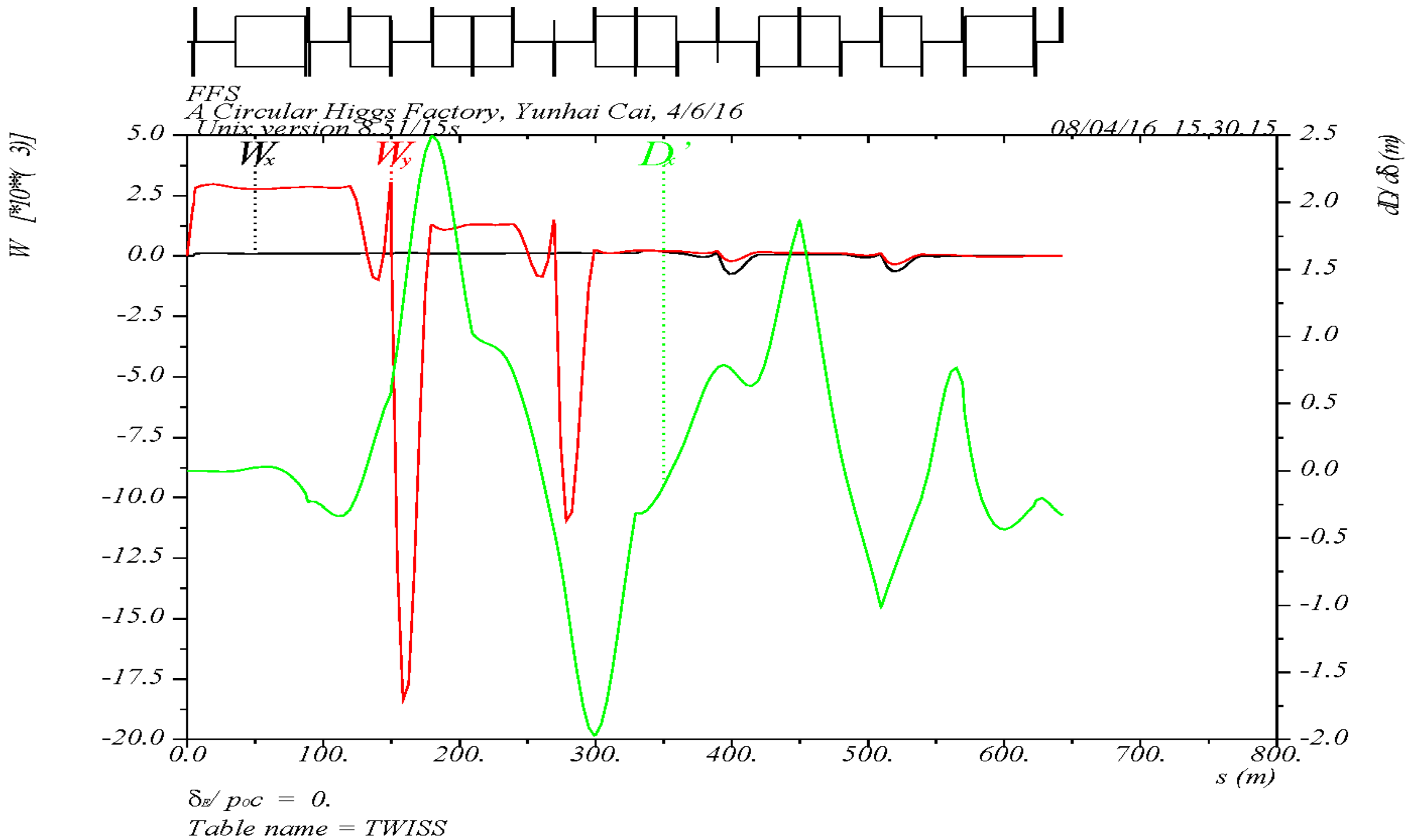
found in the chromatic correction module.

Final Focus System

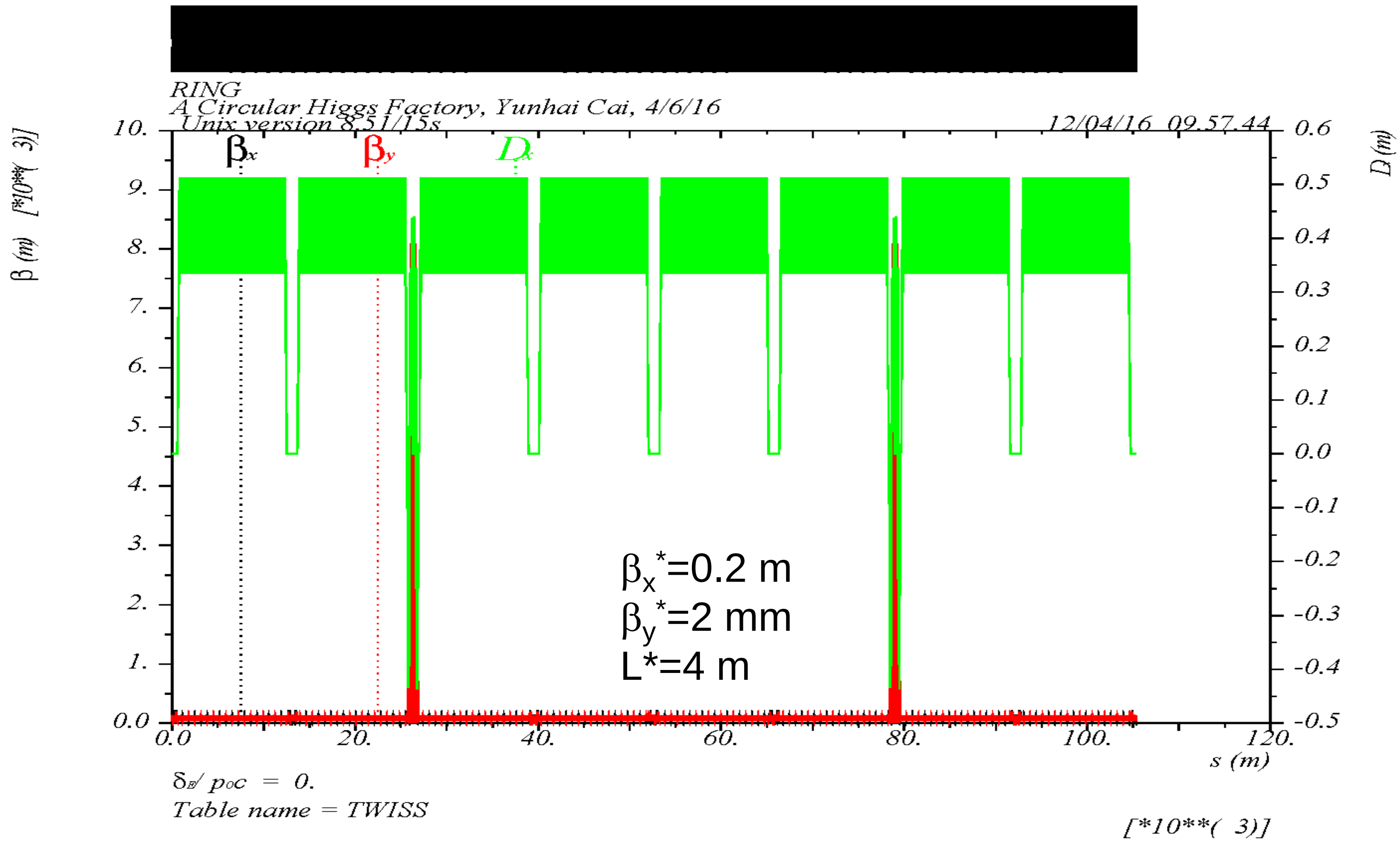


No net bending

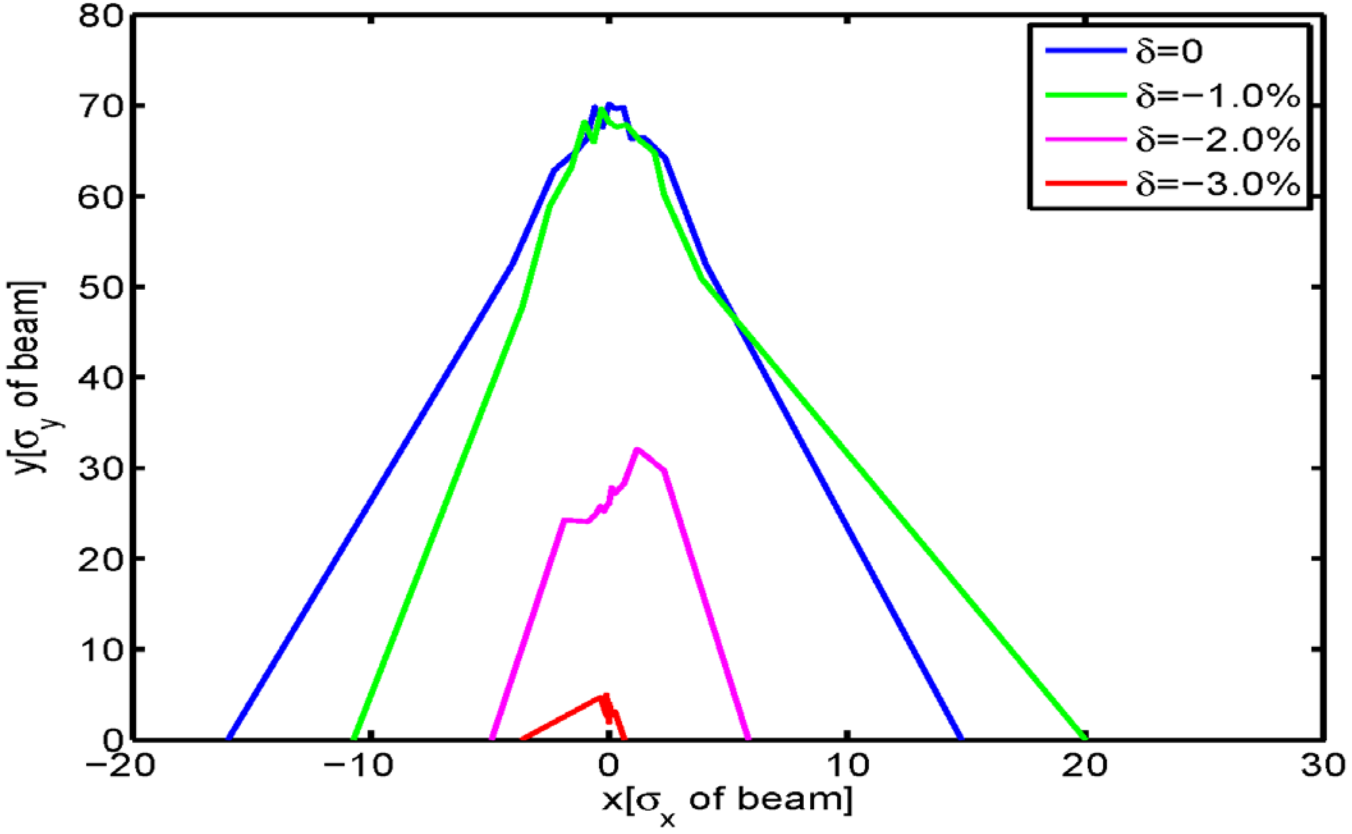
Local Chromatic Correction



Lattice of Collider Ring

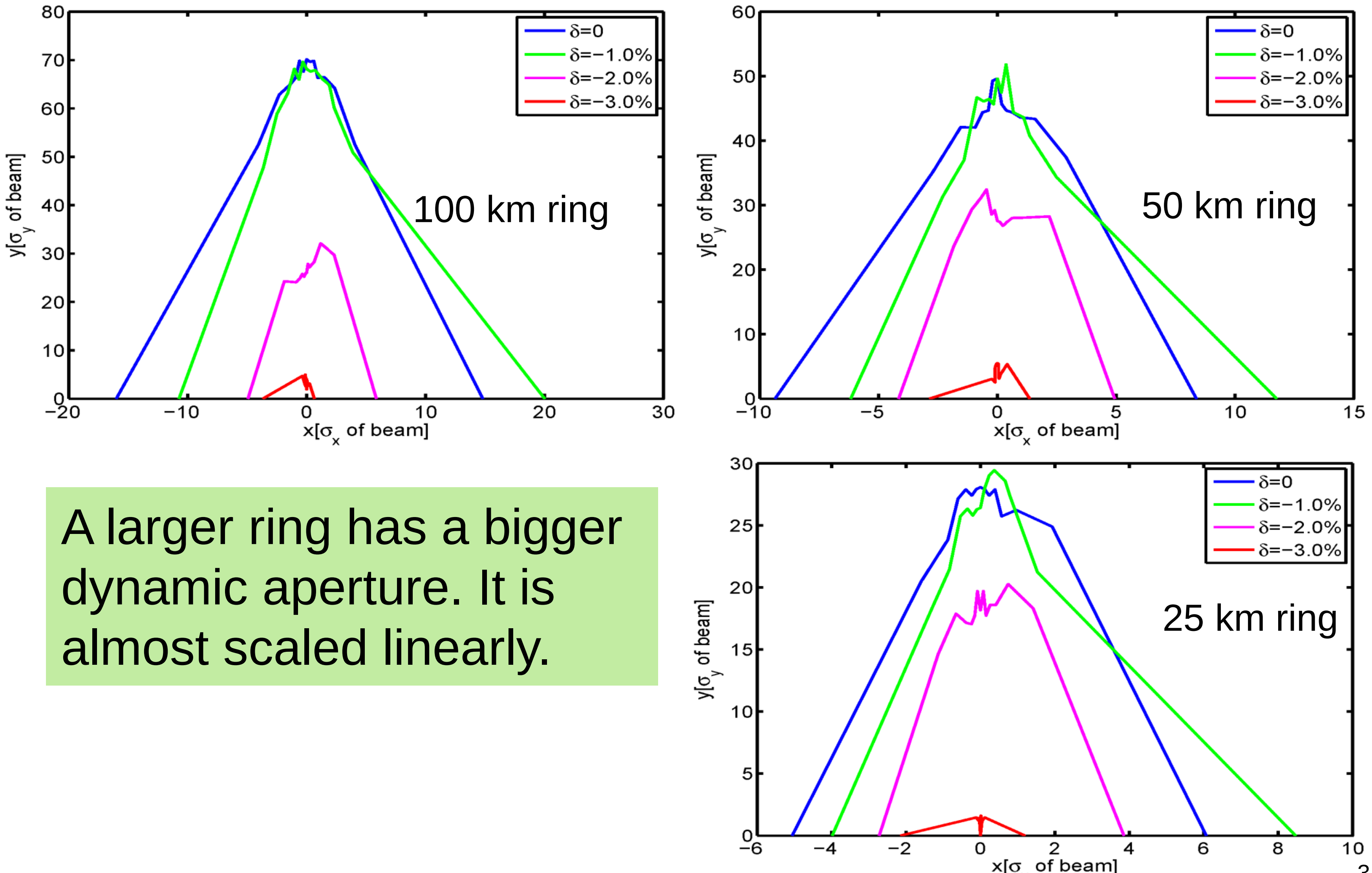


Dynamic Aperture



Tracking 1000 turns using LEGO with synchrotron oscillation. The radiation damping is on along with tapering for lattices with ε_x=4.5 nm, $\kappa=\varepsilon_y/\varepsilon_x$=0.1%.

Scaling of Dynamic Aperture



A larger ring has a bigger dynamic aperture. It is almost scaled linearly.

Conclusion

1) A collider lattice with β_y^*=2 mm and ±3% momentum aperture has been designed. Beamstrahlung lifetime is adequate. Luminosity/IP is about 2.9×10^{34} $cm^{-1}s^{-1}$. The lattice can be improved further.
2) Chromatic optics in final focus system is better understood. Chromaticity can be corrected order-by-order n δ. To achieve ±4% momentum bandwidth, we need octupoles and decapoles.
   - Simple relations between the parameters of the final focus module and the second one in the final telescope
   - In final telescope, chromatic Courant-Synder parameters are scaled according $(L^*/\beta_y^*)\delta$
3) More challenges ahead:
   - One ring possible? Harder for a larger ring
   - Radiation in interaction region?