Beyond the Coleman–Weinberg Effective Potential

Apostolos Pilaftsis

School of Physics and Astronomy, University of Manchester,
Manchester M13 9PL, United Kingdom

CERN, 19 August 2015

Based on A.P. and D. Teresi, Nucl. Phys. B **874** (2013) 594
arXiv:1502.07986 and in preparation

Motivation:

New Era of High Precision QFT

- **Improved Lattice Techniques**
- **Higher Order Loop Calculations in Perturbative 1PI QFT**
- **Improved Solutions to Schwinger–Dyson Equations**
- **On-Shell Amplitude Techniques**
- **Geometric S-Matrix Approach in $N = 4$ SYM Theories**

$\vdots$

- **Other Analytical Non-Perturbative Approaches?**

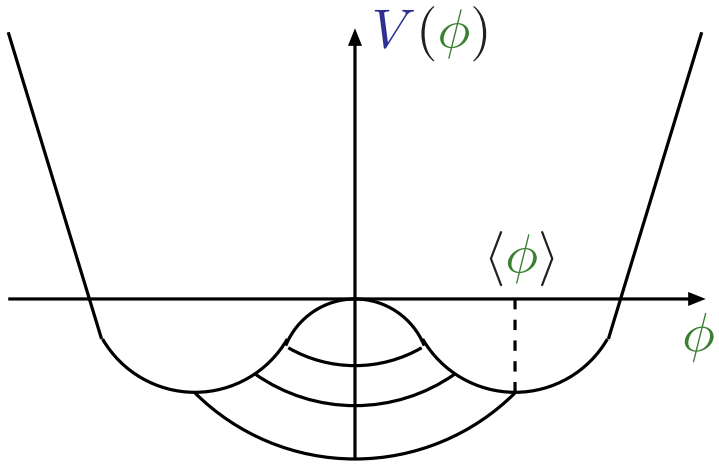
Outline:

- The Standard Theory of Electroweak Symmetry Breaking: SM
- The 2PI Effective Action and Field-Theoretic Problems
- Symmetry-Improved 2PI Formalism
- $\overline{\mathrm{MS}}$ Renormalization in 2PI
- Finite-Width Effects within Quantum Loops
- Symmetry-Improved Effective Higgs Potential in 2PI
- Conclusions and Future Directions

• The Standard Theory of Electroweak Symmetry Breaking

Higgs Mechanism in SM: $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \to \mathrm{SU}(3)_c \otimes \mathrm{U}(1)_{\mathrm{em}}$

[P. W. Higgs '64; F. Englert, R. Brout '64; G. S. Guralnik, C. R. Hagen, T. W. B. Kibble '64]



Higgs potential $V(\phi)$

$$V(\phi) \;=\; -m^2\phi^\dagger\phi \;+\; \lambda(\phi^\dagger\phi)^2 \,.$$

Ground state:

$$\langle\phi\rangle \;=\; \sqrt{\frac{m^2}{2\lambda}}\begin{pmatrix}0\\1\end{pmatrix}$$

carries weak charge, but no electric charge and colour.

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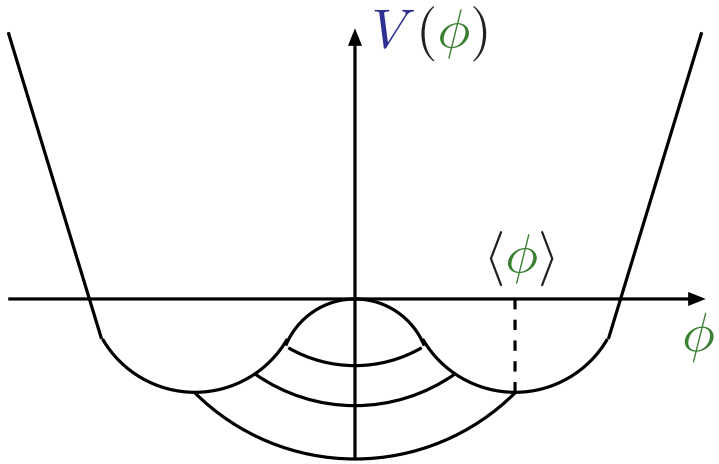

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Custodial Symmetry of the SM with $g' = Y_f = 0$ and $V(\phi)$:

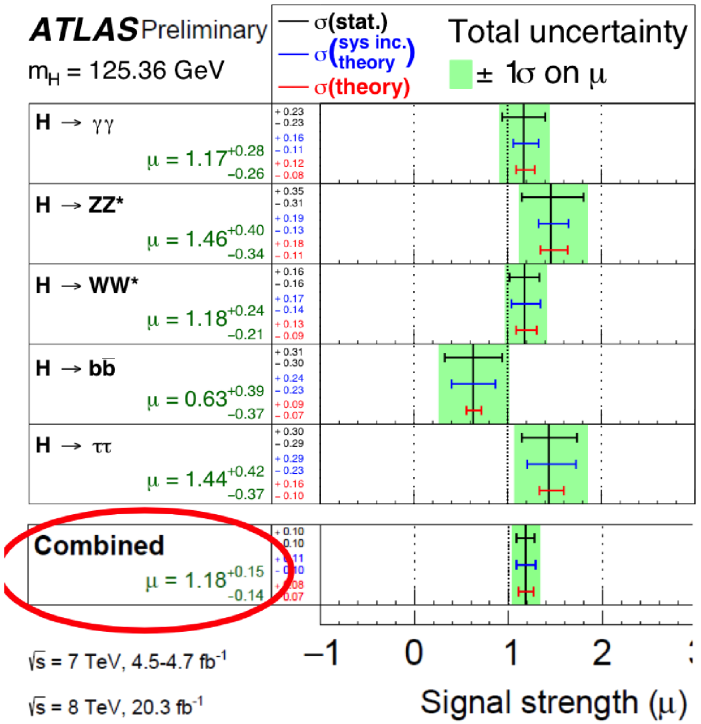
[P. Sikivie, L. Susskind, M. B. Voloshin, V. I. Zakharov '80.]

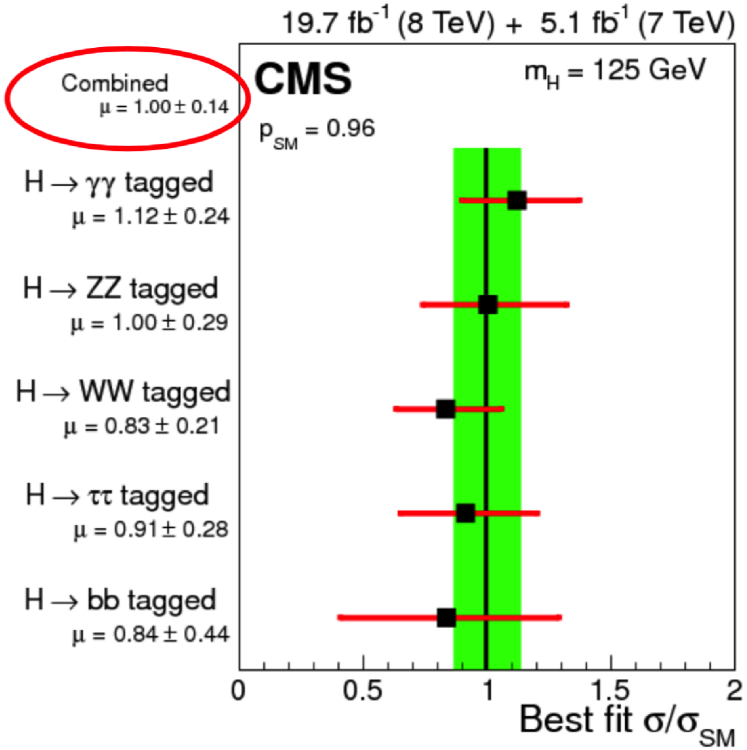
$$\mathbf{\Phi} \;\equiv\; \left(\phi\,,\; i\sigma^2\phi^*\right) \quad\mapsto\quad \mathbf{\Phi}' \;\equiv\; U_L\,\mathbf{\Phi}\,U_{\mathrm{C}}\,,$$

with $U_L \in \mathrm{SU}(2)_L$, $U_{\mathrm{C}} \in \mathrm{SU}(2)_{\mathrm{C}}$, and $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_{\mathrm{C}}/\mathbb{Z}_2 \simeq \mathrm{S}\mathbb{O}(4)$

Higgs Boson @ LHC: Signal Strength for Decay Modes

Signal strength: $\mu = \sigma_{observed}/\sigma_{SM}$





- Results consistent with SM

• The Coleman-Weinberg Effective Potential [S. Coleman and E. Weinberg, '73]

Loopwise $\hbar$ Expansion of the Scalar Potential:

$$V_{\rm eff}(\phi) \;=\; V(\phi) \;-\; \frac{i\hbar}{2}\int \frac{d^4k}{(2\pi)^4}\;\ln\left(\frac{k^2 - V''(\phi)}{k^2}\right) \;+\; \mathcal{O}(\hbar^2)$$

The Coleman–Weinberg Effective Potential

- is based on the One-Particle-Irreducible (1PI) formalism
- breaks Classical Scale Symmetries $\Longrightarrow$ Dimensional Transmutation

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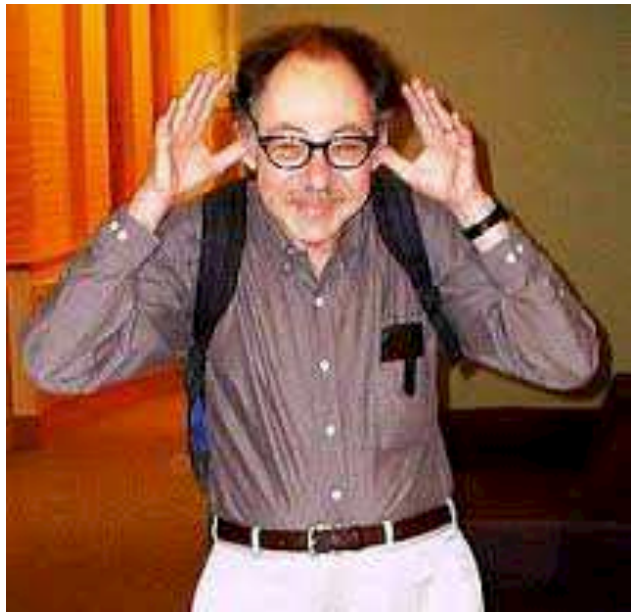
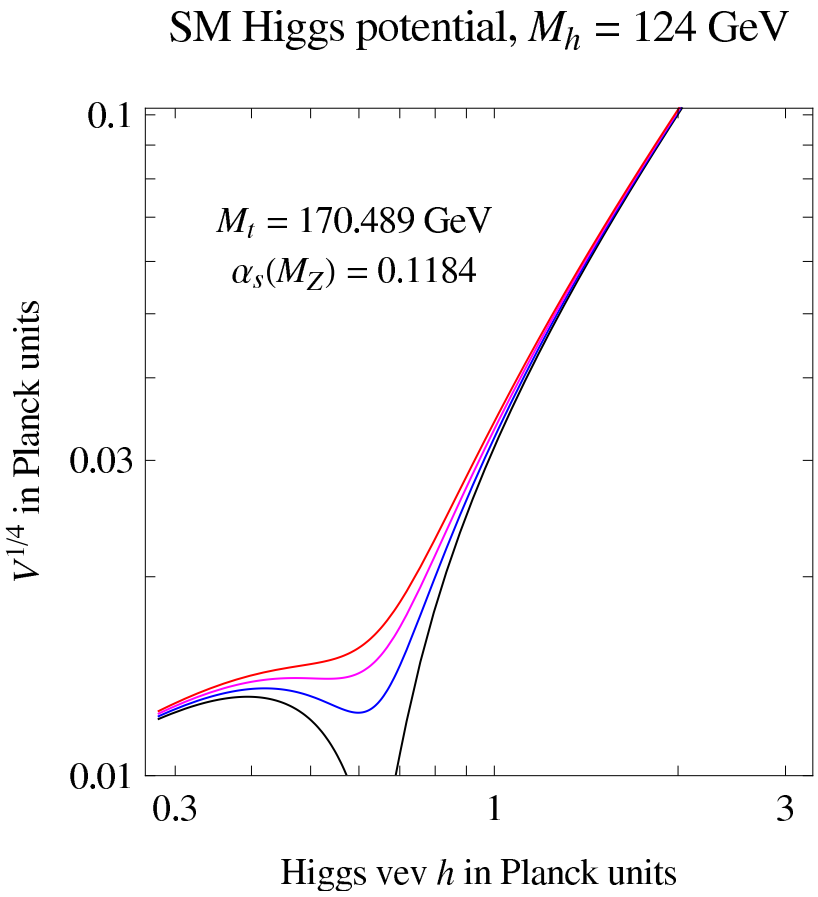
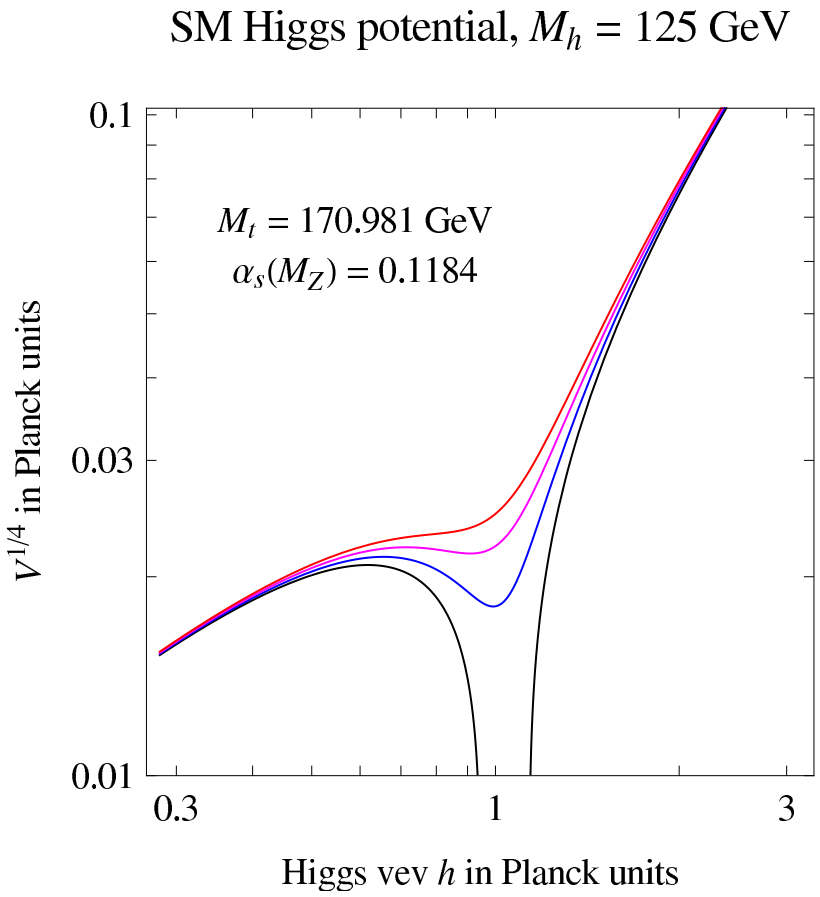


• The SM Coleman-Weinberg Effective Potential at NNLO

[G. Degrassi, S. Di Vita, J. Elias-Miro, J.R. Espinosa, G.F. Giudice, G. Isidori, A. Strumia, JHEP1208 (2012) 098;
F. Bezrukov, M. Y. Kalmykov, B.A. Kniehl, M. Shaposhnikov, JHEP1210 (2012) 140.]



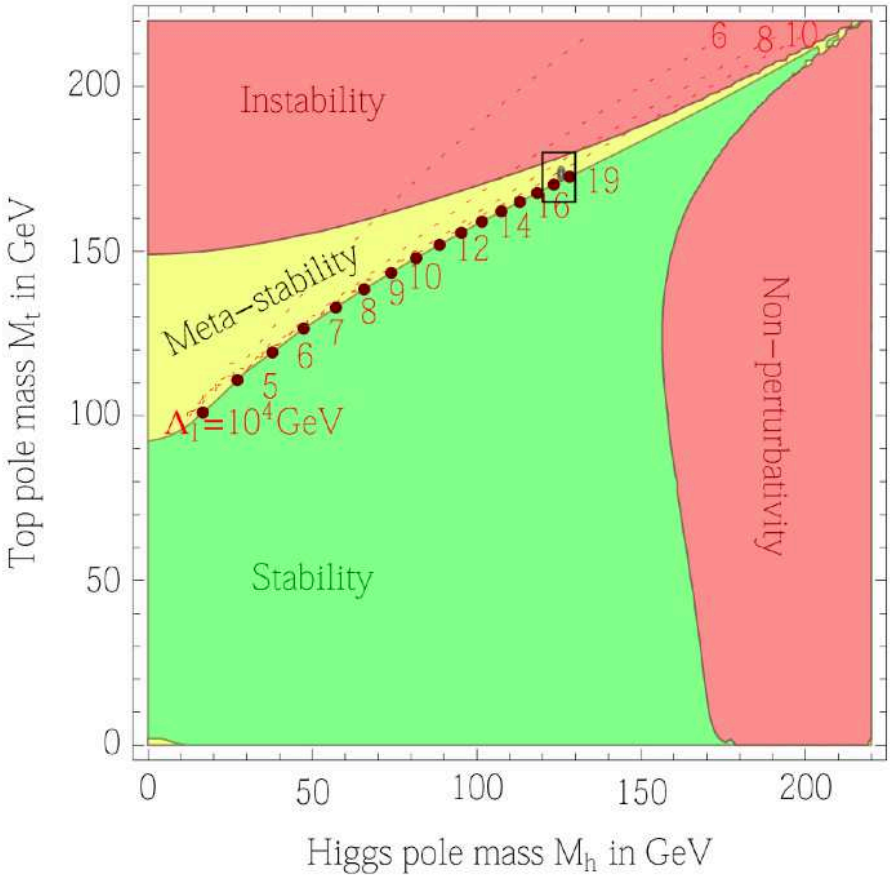


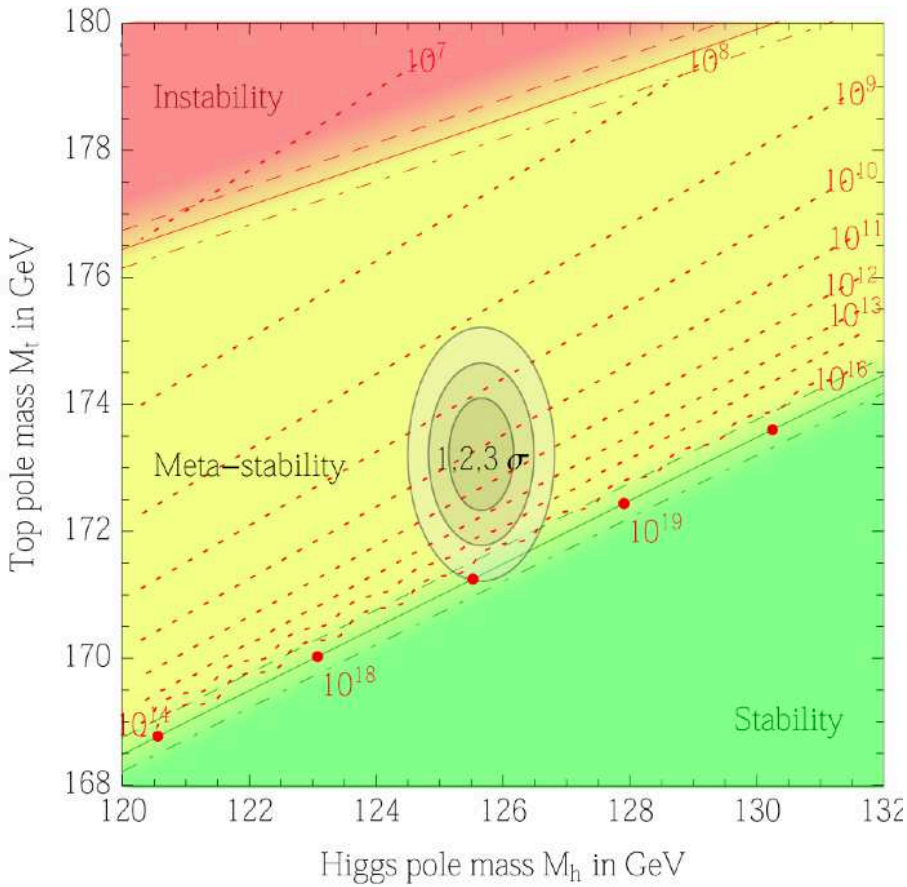
Higgs Potential versus Variations in Top Mass M_t by **0.1 MeV**

[Analysis includes the Multi-Critical scenario: D.L. Bennett, H.B. Nielsen, IJMA9 (1994) 5155]

• Metastability of the SM Vacuum

[D. Buttazzo, G. Degrassi, P.P. Giardino, G.F. Giudice, F. Sala, A. Salvio, and A. Strumia, JHEP1312 (2013) 089]



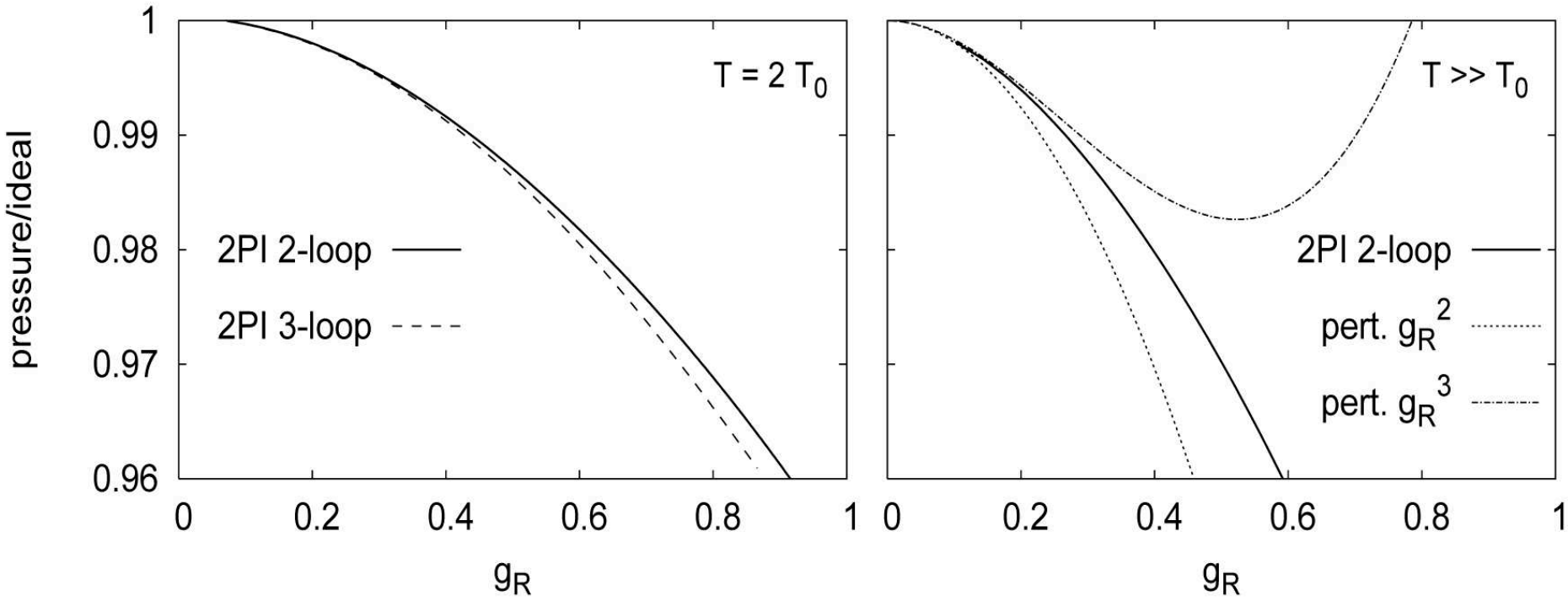


But, Planckian physics may modify stability predictions by many orders!

[V. Branchina, E. Messina, PRL111 (2013) 241801]

• Improved Loopwise Convergence in the 2PI Formalism

[J. Berges, S. Borsanyi, U. Reinosa, J. Serreau, PRD71 (2005) 105004]



• The 2PI Effective Action and Field-Theoretic Problems

[J.M. Cornwall, R. Jackiw, E. Tomboulis, PRD10 (1974) 2428]

Connected Generating Functional of 2PI Effective Action:

$$W[J,K] \;=\; -i\,\ln\int \mathcal{D}\phi^i\,\exp\left[i\left(S[\phi] \,+\, J^i_x\,\phi^i_x \,+\, \frac{1}{2}\,K^{ij}_{xy}\,\phi^i_x\,\phi^j_y\right)\right] \;,$$

where $S[\phi] = \int_x \mathcal{L}[\phi]$ is the classical action of a $\mathbb{O}(N)$ theory.

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Legendre transform of $W[J,K]$ with respect to J and K:

$$\frac{\delta W[J,K]}{\delta J^i_x} \;\equiv\; \phi^i_x \;, \qquad \frac{\delta W[J,K]}{\delta K^{ij}_{xy}} \;=\; \frac{1}{2}\left(i\Delta^{ij}_{xy} \;+\; \phi^i_x\,\phi^j_y\right) \;,$$

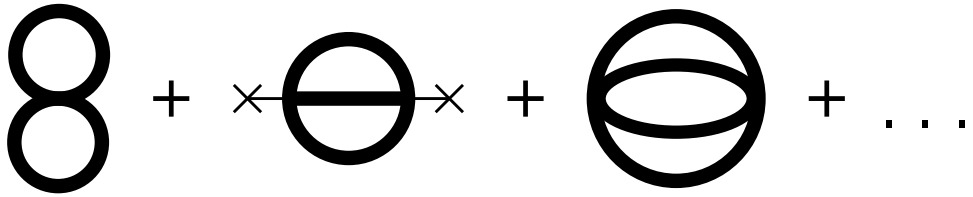
to get the 2PI effective action

$$\Gamma[\phi,\Delta] \;=\; W[J,K] \;-\; J^i_x\,\phi^i_x \;-\; \frac{1}{2}\,K^{ij}_{xy}\,\left(i\Delta^{ij}_{xy} \;+\; \phi^i_x\phi^j_y\right) \;.$$

The 2PI Effective Action:

$$\Gamma[\phi,\Delta] \;=\; S[\phi] \;+\; \tfrac{i}{2}\,\mathrm{Tr}\,\ln\Delta^{-1} \;+\; \tfrac{i}{2}\,\mathrm{Tr}\,(\Delta^{0^{-1}}\,\Delta) \;-\; i\,\Gamma^{(2)}_{2\mathrm{PI}}[\phi,\Delta]\;,$$

where $\Gamma^{(2)}_{2\mathrm{PI}}[\phi,\Delta] \;=$



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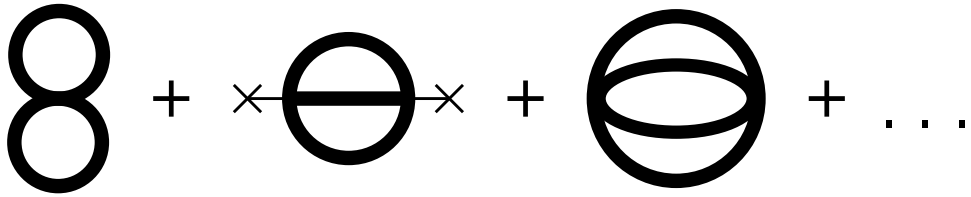
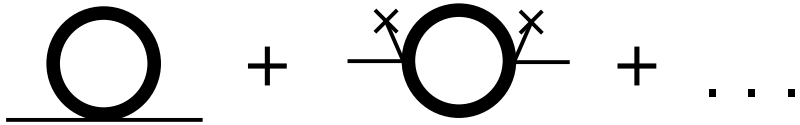


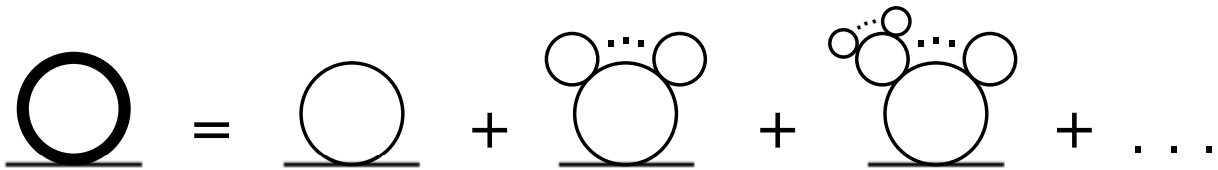
Equations of Motion:

- $$\frac{\delta\Gamma[\phi,\Delta]}{\delta\phi} \;=\; 0$$

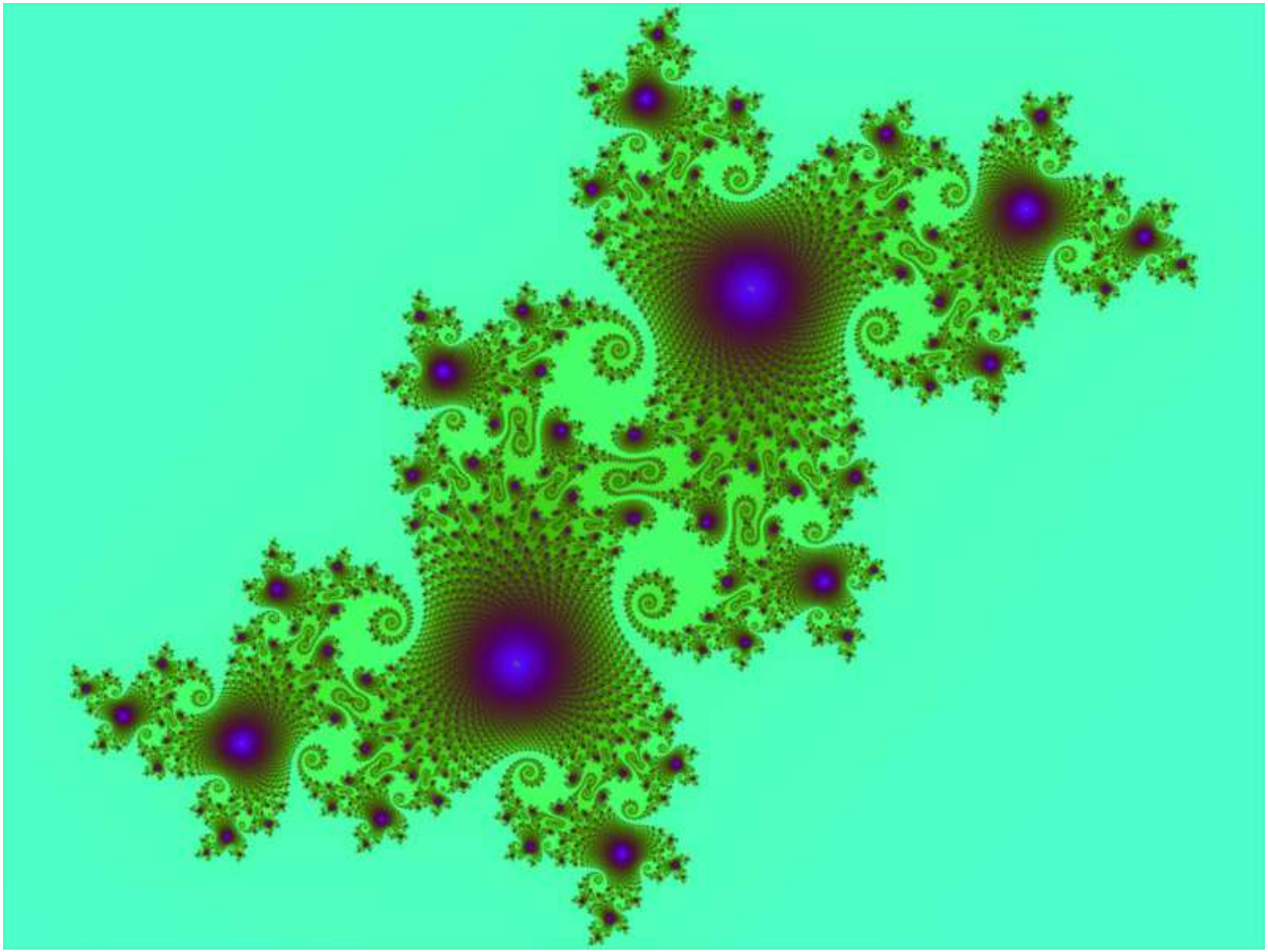
- $$\frac{\delta\Gamma[\phi,\Delta]}{\delta\Delta} \;=\; 0 \;\Rightarrow\; \Delta^{-1} \;=\; \Delta^{0^{-1}} \;+$$



Hartree-Fock:



Artist's impression of the infinite HF-term!



– Field-Theoretic Problems addressed with 2PI

- Systematic formal resummation of high-order graphs:
  - Rigorous Derivation of Schwinger–Dyson Equations
  - Thermal Masses in the high-T Regime
  - Finite-Width Effects within Quantum Loops
  - The IR Problem of the Coleman-Weinberg Effective Potential

- Non-Equilibrium QFT, through Kadanoff–Baym equations.

  [For instance, P. Millington, AP, PRD88 (2013) 085009]

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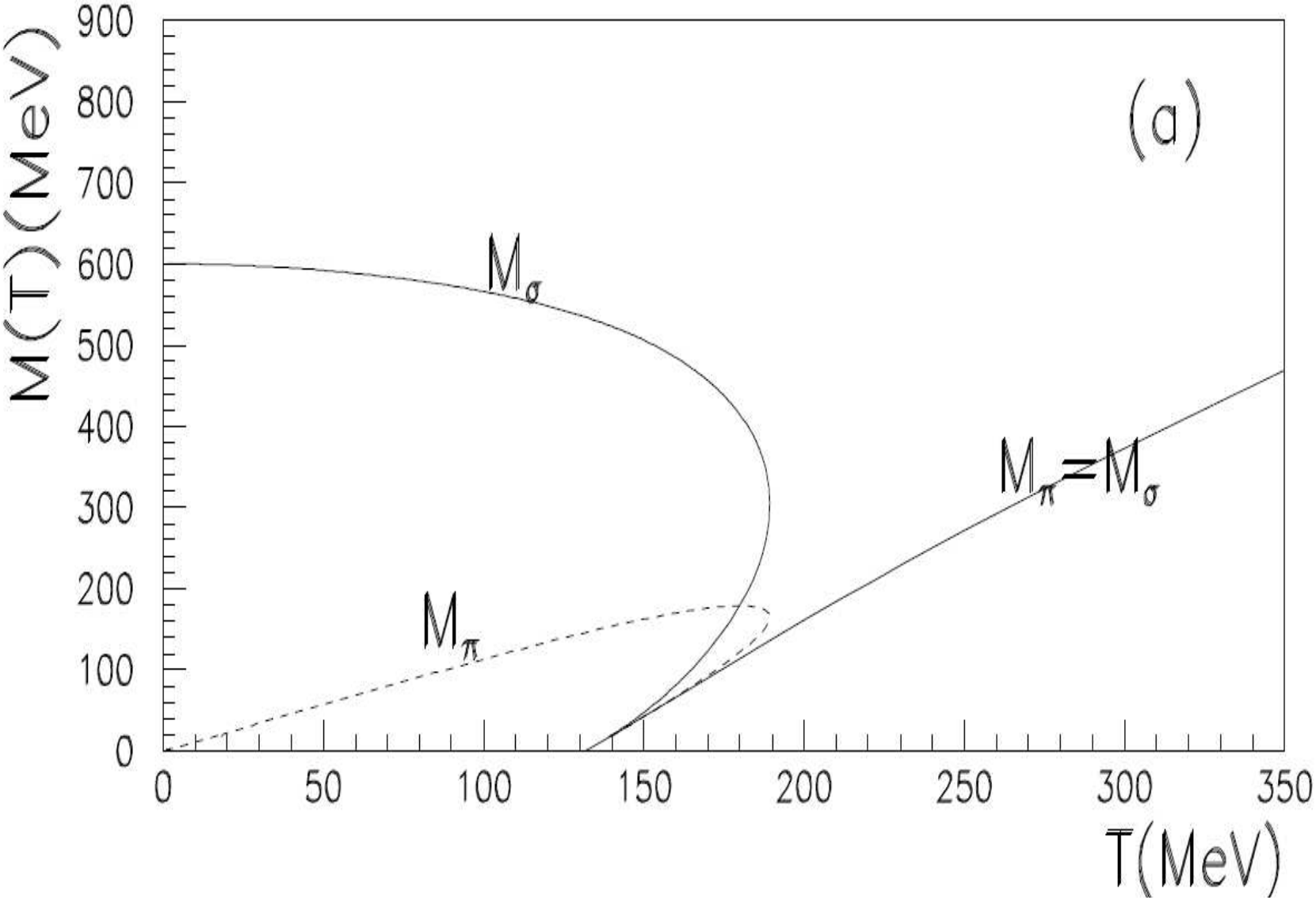
BUT

- Truncations of 2PI lead to residual violations of symmetries, e.g. global or local symmetries.

$\rightarrow$ Erroneous First-Order Phase Transition in $\mathbb{O}(N)$ Theories
$\rightarrow$ Goldstone Bosons become Massive
$\rightarrow$ Erroneous Thresholds for the Resummed Higgs-Boson Propagator.
$\rightarrow$ $\cdots$

• QCD Phase Transition in the Standard 2PI Formalism

[N. Petropoulos, J. Phys. G 25 (1999) 2225]



Pertinent Literature to the Goldstone-Symmetry Problem

- G. Baym, G. Grinstein, Phys. Rev. D **15** (1977) 2897.

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- G. Amelino-Camelia, Phys. Lett. B **407** (1997) 268.
- N. Petropoulos, J. Phys. G **25** (1999) 2225.
- Y. Nemoto, K. Naito, M. Oka, Eur. Phys. J. A **9** (2000) 245.
- J. T. Lenaghan, D. H. Rischke, J. Phys. G **26** (2000) 431.
- H. van Hees, J. Knoll, Phys. Rev. D **66** (2002) 025028.
- J. Baacke, S. Michalski, Phys. Rev. D **67** (2003) 085006.
- Y. Ivanov, F. Riek, H. van Hees, J. Knoll, Phys. Rev. D **72** (2005) 036008.
- E. Seel, S. Struber, F. Giacosa, D. H. Rischke, Phys. Rev. D **86** (2012) 125010.
- G. Markó, U. Reinosa, Z. Szép, Phys. Rev. D **87** (2013) 105001.

• Symmetry-Improved 2PI Formalism

[AP, D. Teresi, NPB874 (2013) 594]

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Equivalence between 1PI and 2PI Effective Actions to All Orders:

$$\Gamma^{1\mathrm{PI}}[\phi] \;=\; \Gamma[\phi\,,\,\Delta(\phi)]\;, \qquad \text{with} \quad \frac{\delta\Gamma[\phi\,,\,\Delta(\phi)]}{\delta\Delta} \;=\; 0\;.$$

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1PI Ward Identity (e.g. for $\mathbb{O}(2)$):

$$\frac{\delta\Gamma^{1\mathrm{PI}}[\phi]}{\delta\phi^i_x}\,T^a_{ij}\,\phi^j_x \;=\; 0 \quad\Longrightarrow\quad v\int_x \frac{\delta^2\Gamma^{1\mathrm{PI}}[\phi]}{\delta G_y\,\delta G_x} \;=\; \frac{\delta\Gamma^{1\mathrm{PI}}[\phi]}{\delta H} \;\to\; 0\;.$$

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Replace:

$$\frac{\delta^2\Gamma^{1\mathrm{PI}}[\phi]}{\delta G_y\,\delta G_x} \;=\; \Delta^{-1,G}_{xy}\;,$$

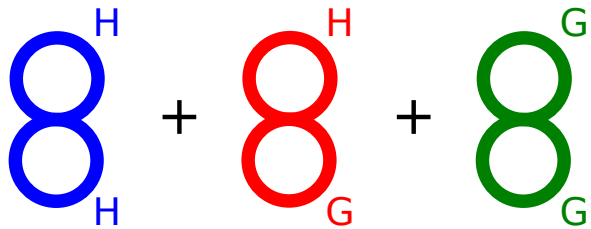
to obtain the Symmetry-Improved Equations of Motion:

$$\frac{\delta\Gamma[v\,,\,\Delta]}{\delta\Delta_{H/G}} \;=\; 0\;,$$

$$v\,\Delta^{-1}_G(k=0,v) \;=\; 0\;.$$

– $\mathbb{O}(2)$ Hartree–Fock Equations of Motion:

HF Approximation: $\Gamma^{(2)}_{\rm HF}[\Delta_H\,,\,\Delta_G] \;=$



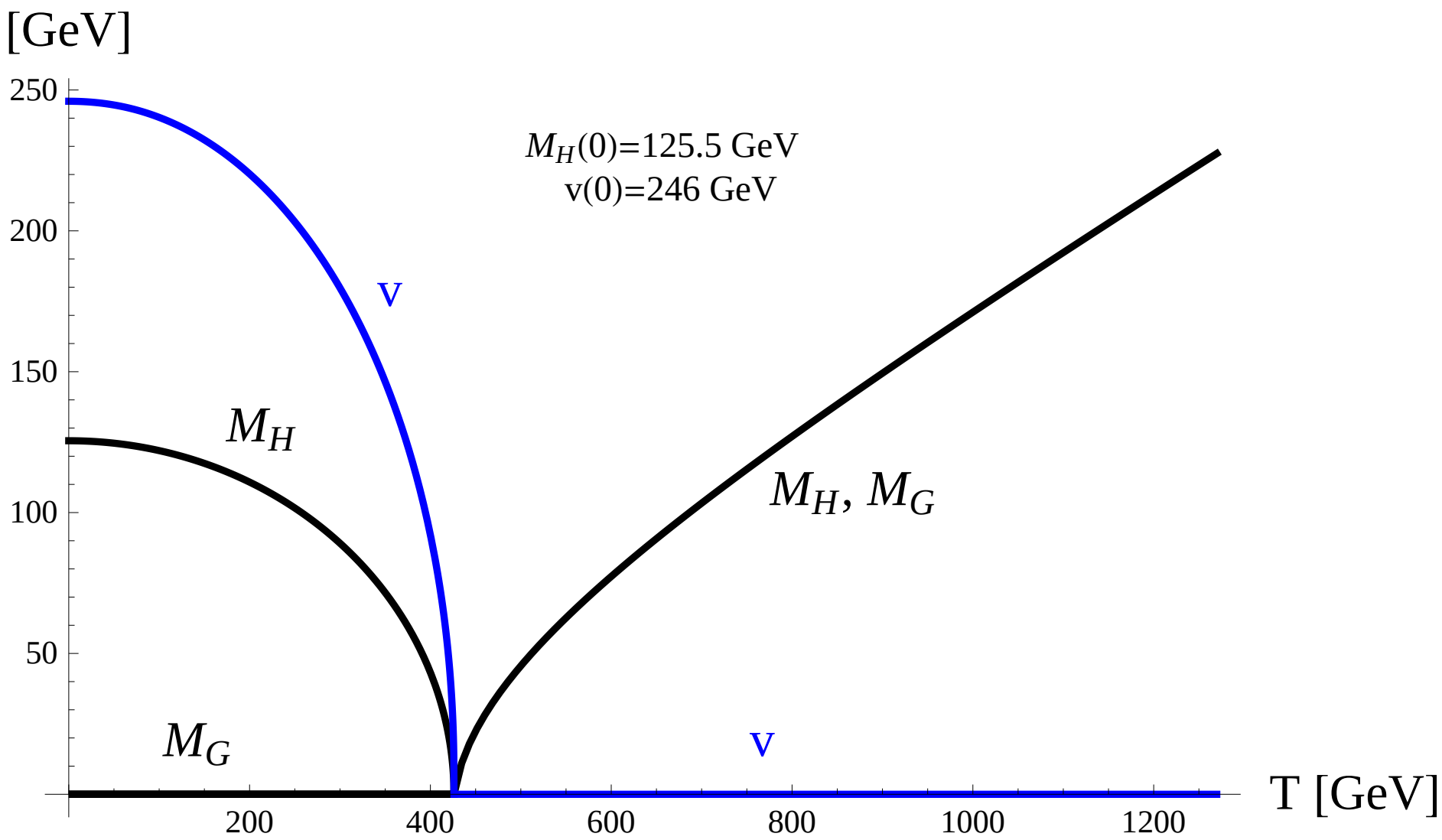
Ansatz: $\Delta^{-1}_{H/G}(k) \;=\; k^2 \;-\; M^2_{H/G} \;+\; i\varepsilon$

Equations of Motion:

$$
\begin{aligned}
M_H^2 \;&=\; 3\lambda v^2 - m^2 \;+\; (\delta\lambda_1^A + 2\delta\lambda_1^B)v^2 \;-\; \delta m_1^2 \\
&\quad + (3\lambda + \delta\lambda_2^A + 2\delta\lambda_2^B)\int_k i\Delta_H(k) \;+\; (\lambda + \delta\lambda_2^A)\int_k i\Delta_G(k)\;, \\
M_G^2 \;&=\; \lambda v^2 - m^2 \;+\; \delta\lambda_1^A v^2 \;-\; \delta m_1^2 \\
&\quad + (\lambda + \delta\lambda_2^A)\int_k i\Delta_H(k) \;+\; (3\lambda + \delta\lambda_2^A + 2\delta\lambda_2^B)\int_k i\Delta_G(k)\;, \\
v\,M_G^2 \;&=\; 0\;.
\end{aligned}
$$

– Second-Order Phase Transition in the HF Approximation

[AP, D. Teresi, NPB874 (2013) 594]



• $\overline{\mathrm{MS}}$ Renormalization in 2PI

Naive Renormalization:

 $= \int_k i\Delta(k) \sim M^2 \frac{1}{\epsilon}$, Naive CT: $\delta m^2 \stackrel{?}{=} M^2 \frac{1}{\epsilon}$.

But, $M^2 = M^2(T) \implies T$emperature-dependent CT?!

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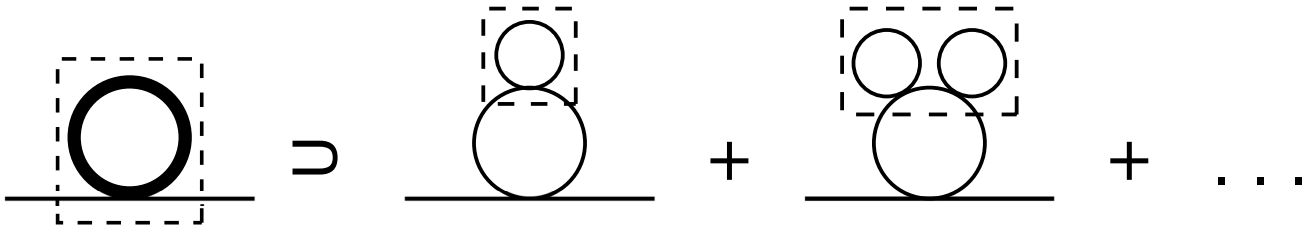
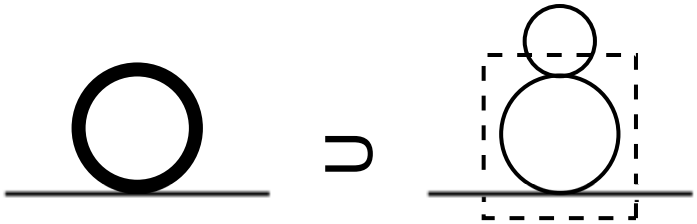
What has gone wrong?

[J.-P. Blaizot, E. Iancu, U. Reinosa, NPA736 (2004) 149;
J. Berges *et al*, AP320 (2005) 344]

$\delta m^2 \sim$ 

 $\sim \delta\lambda$

Is there any systematic renormalization, e.g. in the $\overline{\mathrm{MS}}$ scheme?

[W.A. Bardeen, A. Buras, D. Duke, T. Muta, PRD18 (1978) 3998]

– $\overline{\mathrm{MS}}$ Renormalization in the 2PI Formalism

Procedure:

- Isolate UV infinities in EoMs, e.g. by Dimensional Regularization.

- Require that the UV-finite part of EoMs **be** UV finite:

$$(\ldots)_{\mathrm{UV}}\,\mathcal{T}_H^{\mathrm{fin}} \;+\; (\ldots)_{\mathrm{UV}}\,\mathcal{T}_G^{\mathrm{fin}} \;+\; (\ldots)_{\mathrm{UV}}\,v^2 \;+\; (\ldots)_{\mathrm{UV}}\,1 \;\overset{!}{=}\; 0$$

- Cancel seperately the UV infinities $\propto\; \mathcal{T}_H^{\mathrm{fin}}(T),\;\; \mathcal{T}_G^{\mathrm{fin}}(T),\;\; v^2(T),\;\; 1.$

- Check UV consistency:

$$4 \;\times\; 2 \;=\; 8 \text{ Constraints}, \quad \text{for } 5 \text{ CTs}: \;\; \delta m_1^2\,,\; \delta\lambda_1^A\,,\; \delta\lambda_1^B\,,\; \delta\lambda_2^A\,,\; \delta\lambda_2^B\,.$$

This is a non-trivial check!

– T-independent Resummed Counterterms in the HF approximation:

[AP, D. Teresi, NPB874 (2013) 594]

$$\delta\lambda_1^A = \delta\lambda_2^A = \frac{2\lambda^2}{16\pi^2\epsilon}\,\frac{3-\dfrac{4\lambda}{16\pi^2\epsilon}}{1-\dfrac{6\lambda}{16\pi^2\epsilon}+\dfrac{8\lambda^2}{(16\pi^2\epsilon)^2}}$$

$$= -\lambda + \frac{(16\pi^2\epsilon)^2}{8\lambda} + O(\epsilon^3)\,,$$

$$\delta\lambda_1^B = \delta\lambda_2^B = \frac{2\lambda^2}{16\pi^2\epsilon}\,\frac{1}{1-\dfrac{2\lambda}{16\pi^2\epsilon}}$$

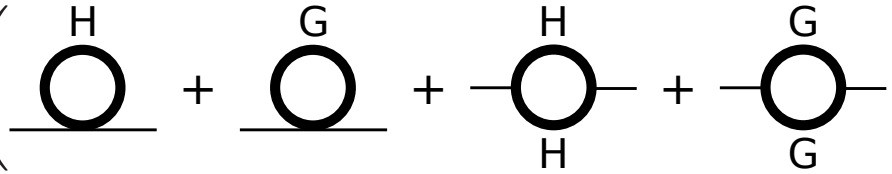
$$= -\lambda - \frac{16\pi^2\epsilon}{2} - \frac{(16\pi^2\epsilon)^2}{4\lambda} + O(\epsilon^3)\,,$$

$$\delta m_1^2 = \frac{4\lambda m^2}{16\pi^2\epsilon}\,\frac{1}{1-\dfrac{4\lambda}{16\pi^2\epsilon}} = -m^2 - m^2\frac{16\pi^2\epsilon}{4\lambda} + O(\epsilon^2)\,.$$

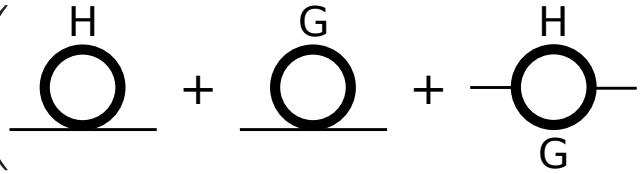
• Finite-Width Effects within Quantum Loops

Equations of Motion including Sunset Diagrams:

- $$\Delta_H^{-1}(p) \;=\; p^2 \;-\; (3\lambda + \delta\lambda_1^A + 2\delta\lambda_1^B)v^2 \;+\; m^2 \;+\; \delta m_1^2$$

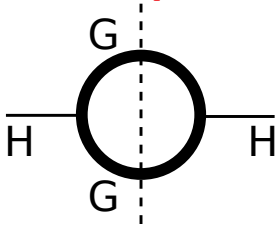
  $-i\,\Big($  $\Big)\,,$

- $$\Delta_G^{-1}(p) \;=\; p^2 \;-\; (\lambda + \delta\lambda_1^A)v^2 \;+\; m^2 \;+\; \delta m_1^2$$

  $-i\,\Big($  $\Big)\,,$

- $$v\,\Delta_G^{-1}(0) \;=\; 0\,.$$

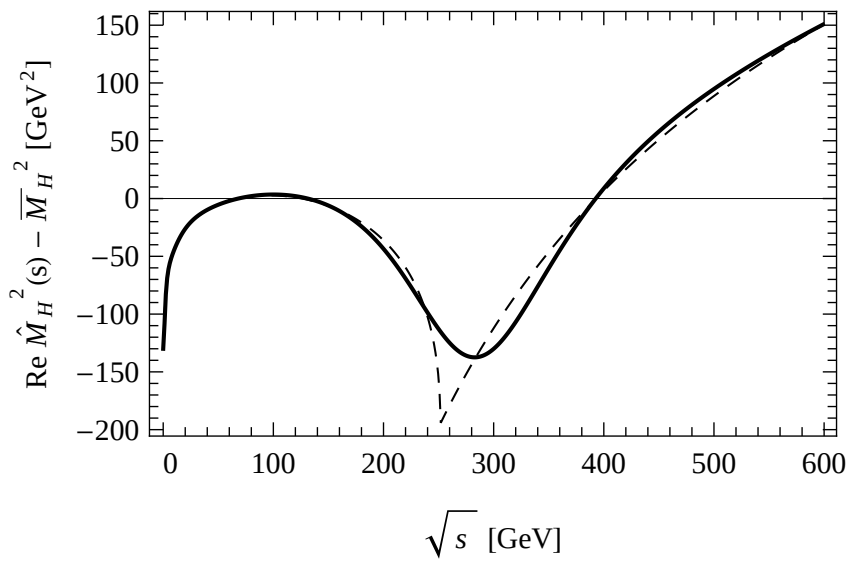
Absorptive Effects:

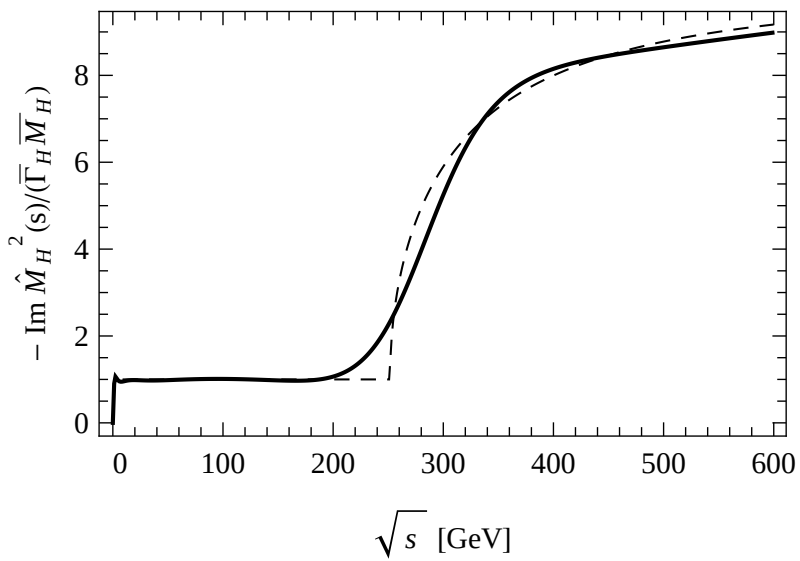


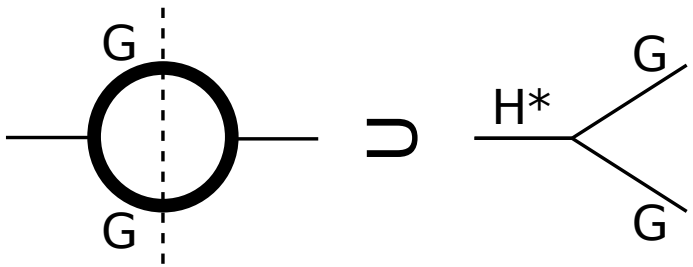
G consistently massless $\Longleftrightarrow$ threshold at $s \equiv p^2 = 0$

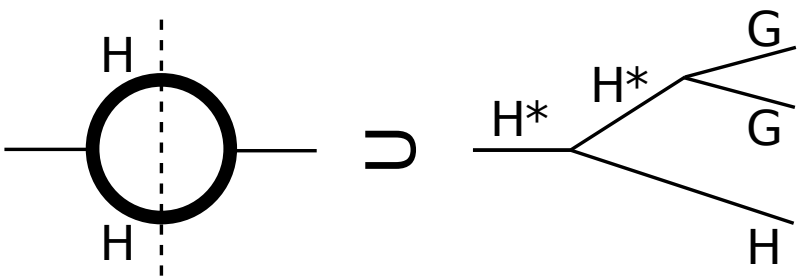
– Higgs Selfenergy in 2PI

[AP, D. Teresi, NPB874 (2013) 594]



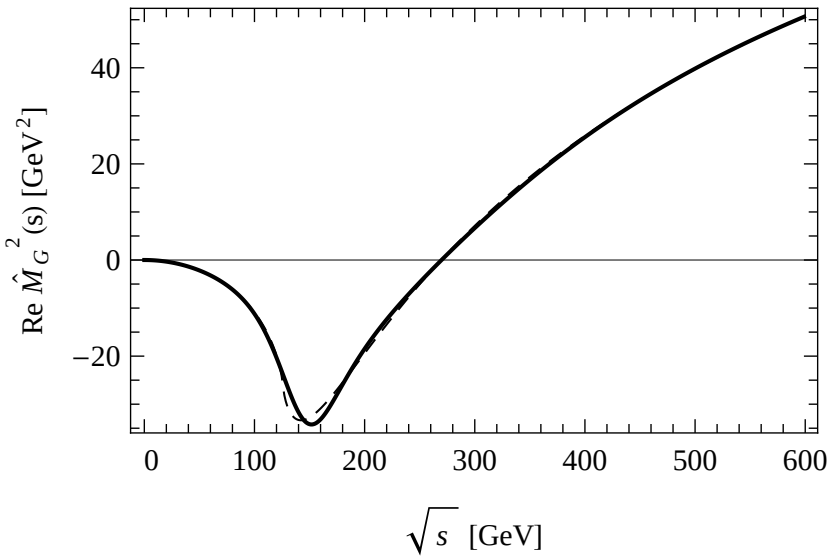


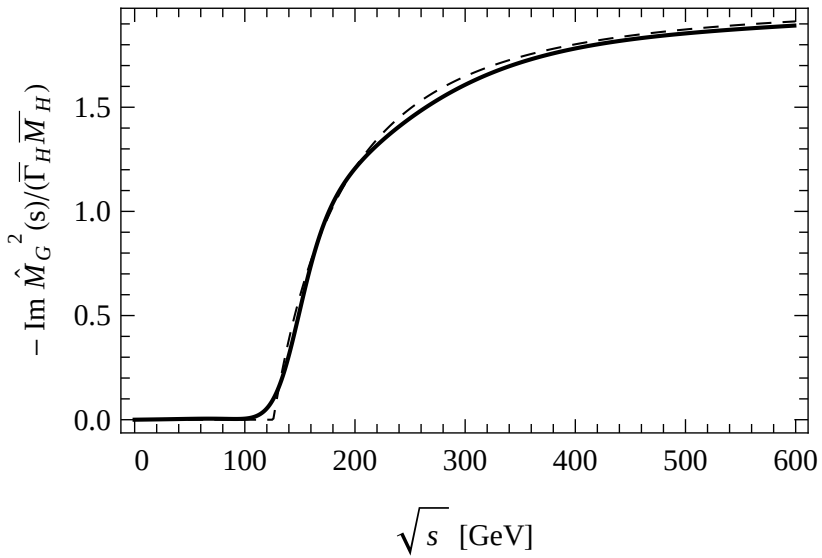


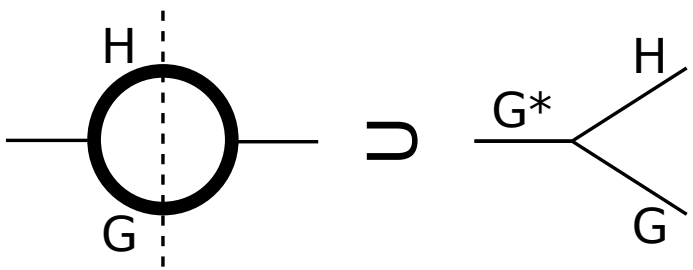


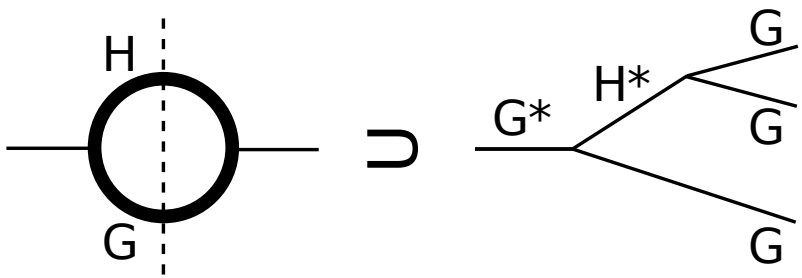
– Goldstone Selfenergy in 2PI

[AP, D. Teresi, NPB874 (2013) 594]









• Symmetry-Improved Effective Higgs Potential in 2PI

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Symmetry-Improved Effective Potential $\widetilde{V}_{\rm eff}(\phi)$ from 1PI Ward Identity:

$$\phi\,\Delta_G^{-1}(k=0;\phi) \;=\; -\,\frac{d\widetilde{V}_{\rm eff}(\phi)}{d\phi}\,.$$

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Solution:

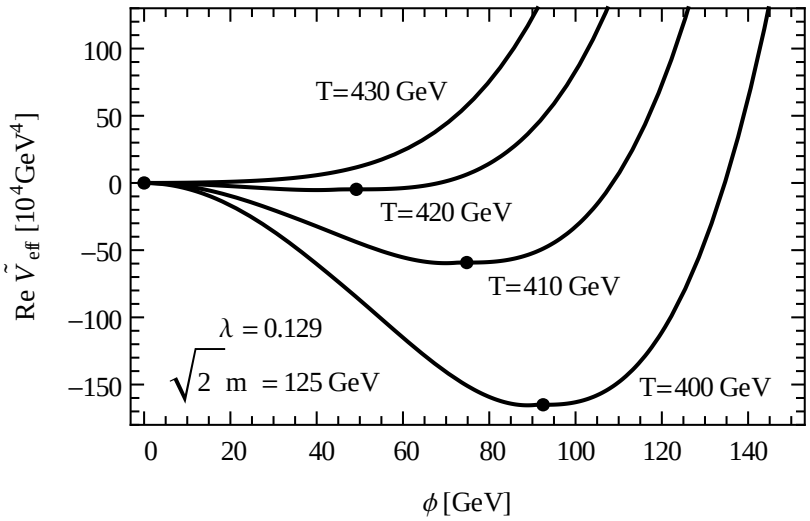
$$\begin{aligned}\widetilde{V}_{\rm eff}(\phi) \;&=\; -\int_0^{\phi} d\phi\,\phi\,\Delta_G^{-1}(k=0;\phi) \;+\; \widetilde{V}_{\rm eff}(\phi=0)\\ &=\; -\int_v^{\phi} d\phi\,\phi\,\Delta_G^{-1}(k=0;\phi) \;+\; P(T,\mu)\;,\end{aligned}$$

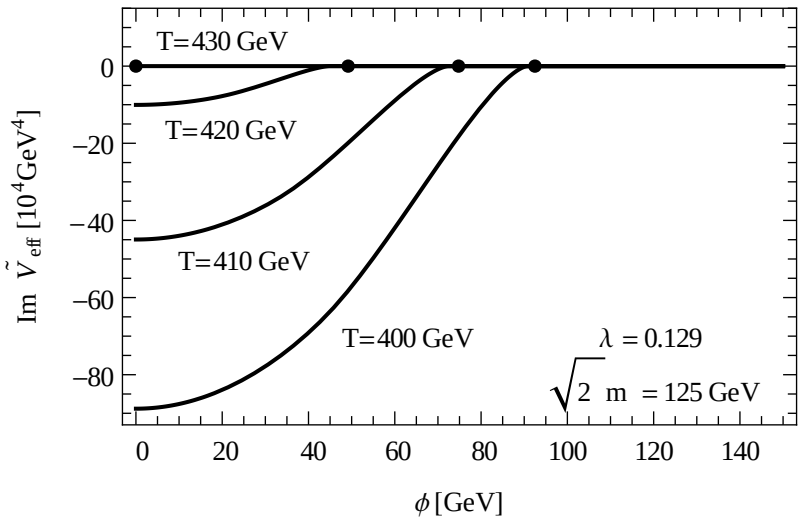
where $P(T,\mu)$ is the thermodynamic pressure $=$ hydrostatic pressure, i.e. it satisfies Baym's thermodynamic consistency.

[G. Baym, PR127 (1962) 1391]

– 2PI Effective Higgs Potential

[AP, D. Teresi, NPB874 (2013) 594]





Note: $\mathrm{Im}\,\widetilde{V}_{\mathrm{eff}}(\phi) < 0, \quad$ for $0 < \phi < v$

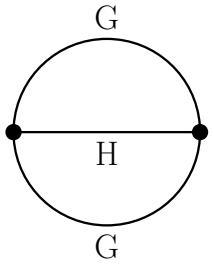
$\Longrightarrow$ Vacuum instability for the concave part of the potential.

[E.J. Weinberg, A.-Q. Wu, PRD36 (1987) 2474]

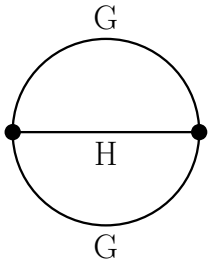
• The IR Problem in the Coleman–Weinberg Effective Potential

[S.P. Martin, PRD89 (2014) 013003]

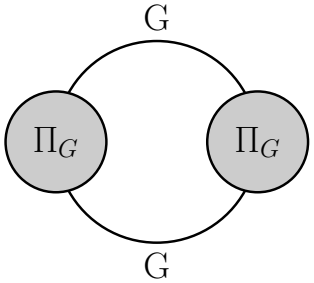
Contributions to the SM effective potential:



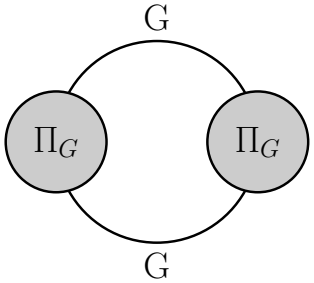
$\sim \quad m_G^2 \log m_G^2$ ✓

$\frac{d}{d\phi}$ 

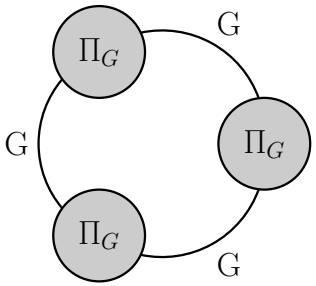
$\sim \quad \log m_G^2$ ✗



$\sim \quad \log m_G^2$ ✗

$\frac{d}{d\phi}$ 

$\sim \quad \frac{1}{m_G^2}$ ✗



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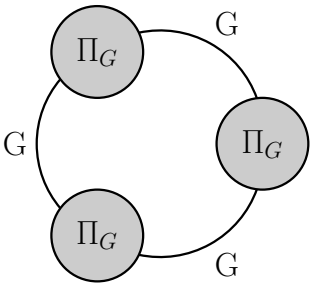
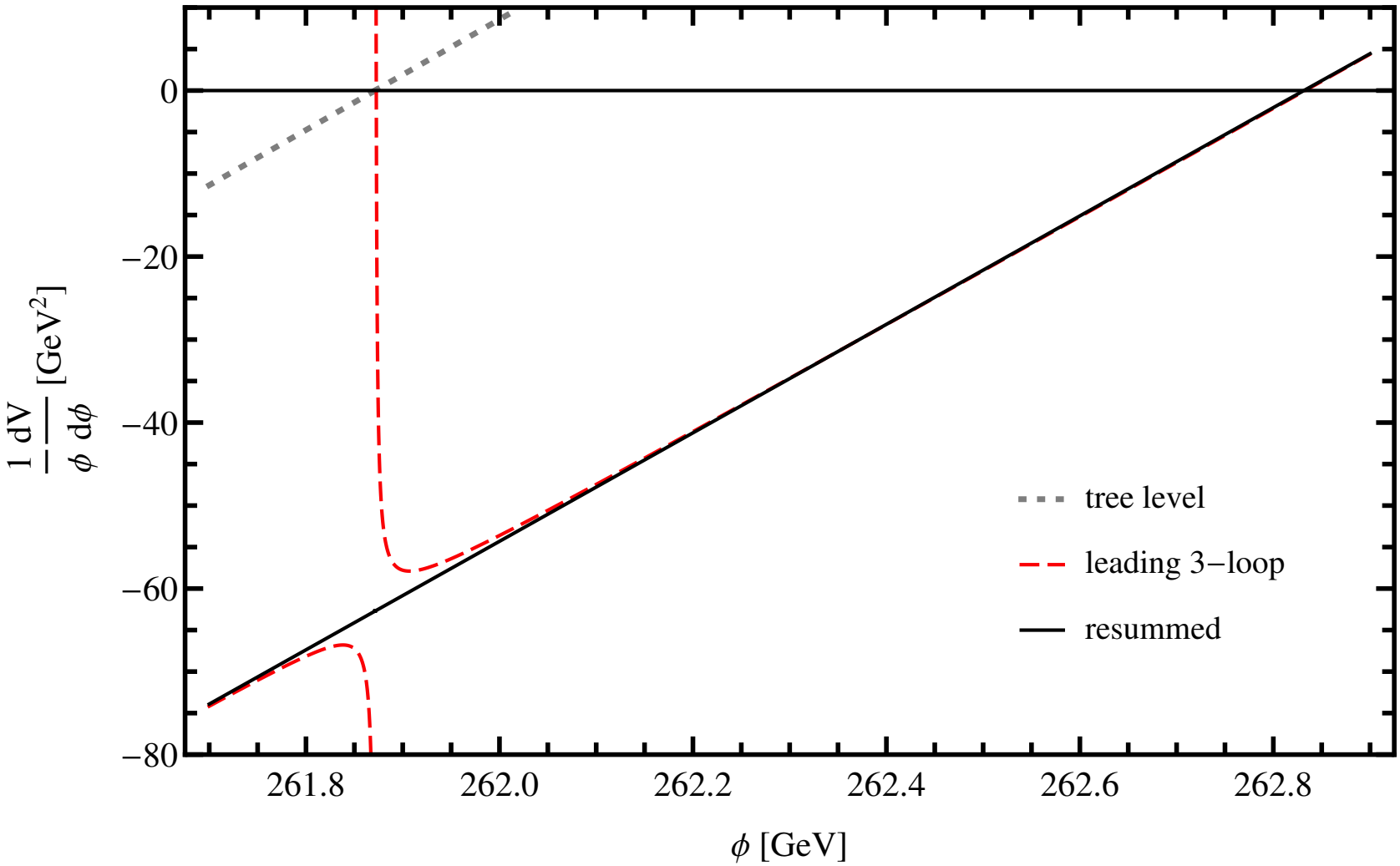
$\frac{d}{d\phi}$ 

$\sim \quad \left(\frac{1}{m_G^2}\right)^2$ ✗

$m_G^2 = \lambda\phi^2 - m^2$ $\qquad$ $M_G^2 = m_G^2 + \Pi_G^{(1)}|_{k=0} + \ldots = 0$ at $\phi = v$

• IR Divergence in the Coleman–Weinberg Potential



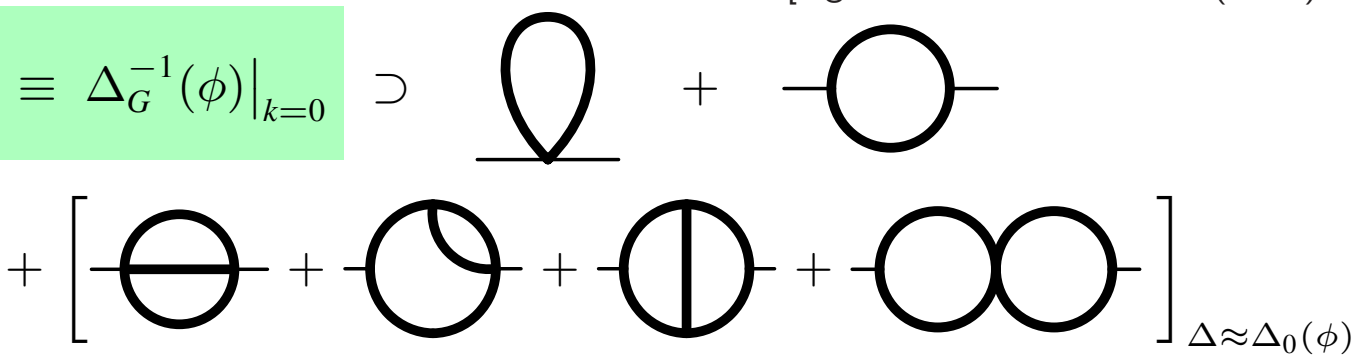
• Symmetry-Improved 2PI Approach to the IR Divergence

[A.P., D. Teresi, arXiv:1502.07986 and in preparation.]

- More complete resummation:
  - first-principle approach
  - it takes into account the momentum-dependence of self-energy insertions
  - more topologies
  - no ad-hoc subtraction of $m_G^2 \log m_G^2$ contributions in Π_G

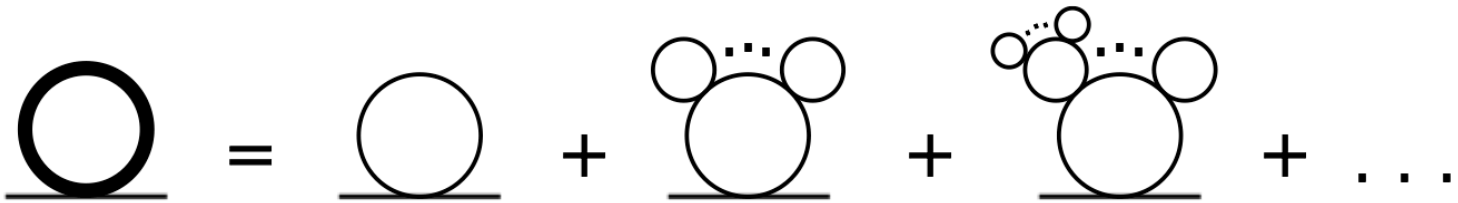
[e.g., S.P. Martin, PRD90 (2014) 016013]

- $$-\frac{1}{\phi}\frac{d\widetilde{V}_{\rm eff}}{d\phi} \equiv \Delta_G^{-1}(\phi)\big|_{k=0} \supset$$

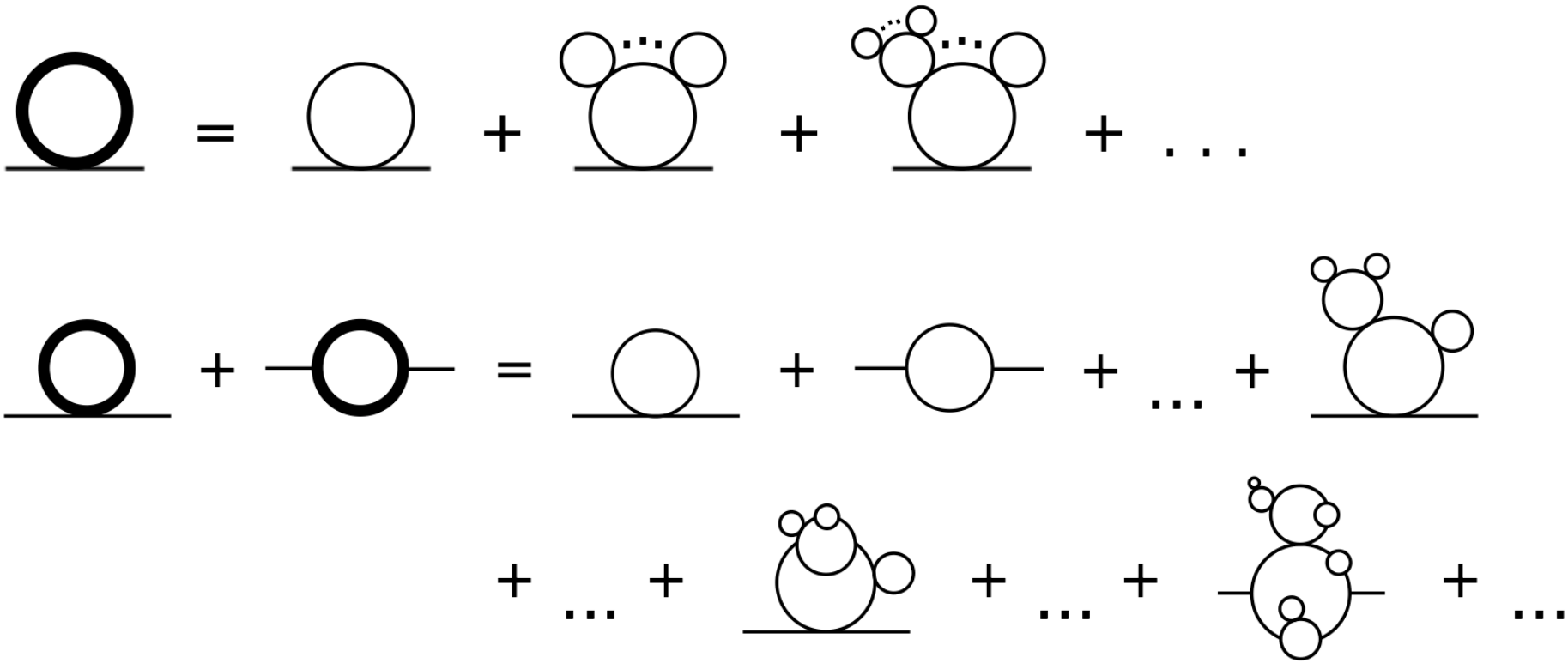


- No IR divergences: the would-be divergent self-energies are 2PR

- **2PI** *Fractal* **Resummation**



• 2PI *Fractal* Resummation



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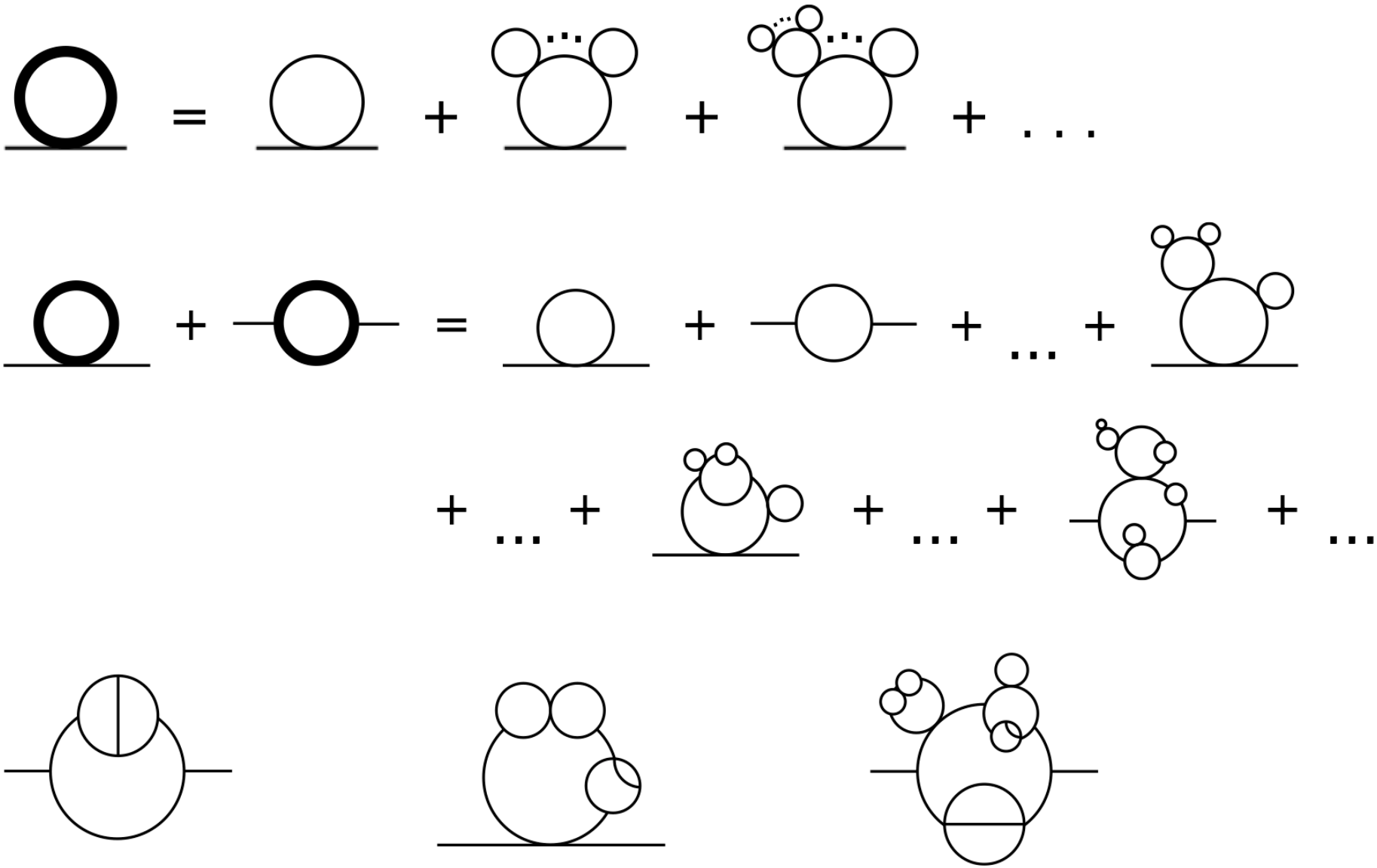
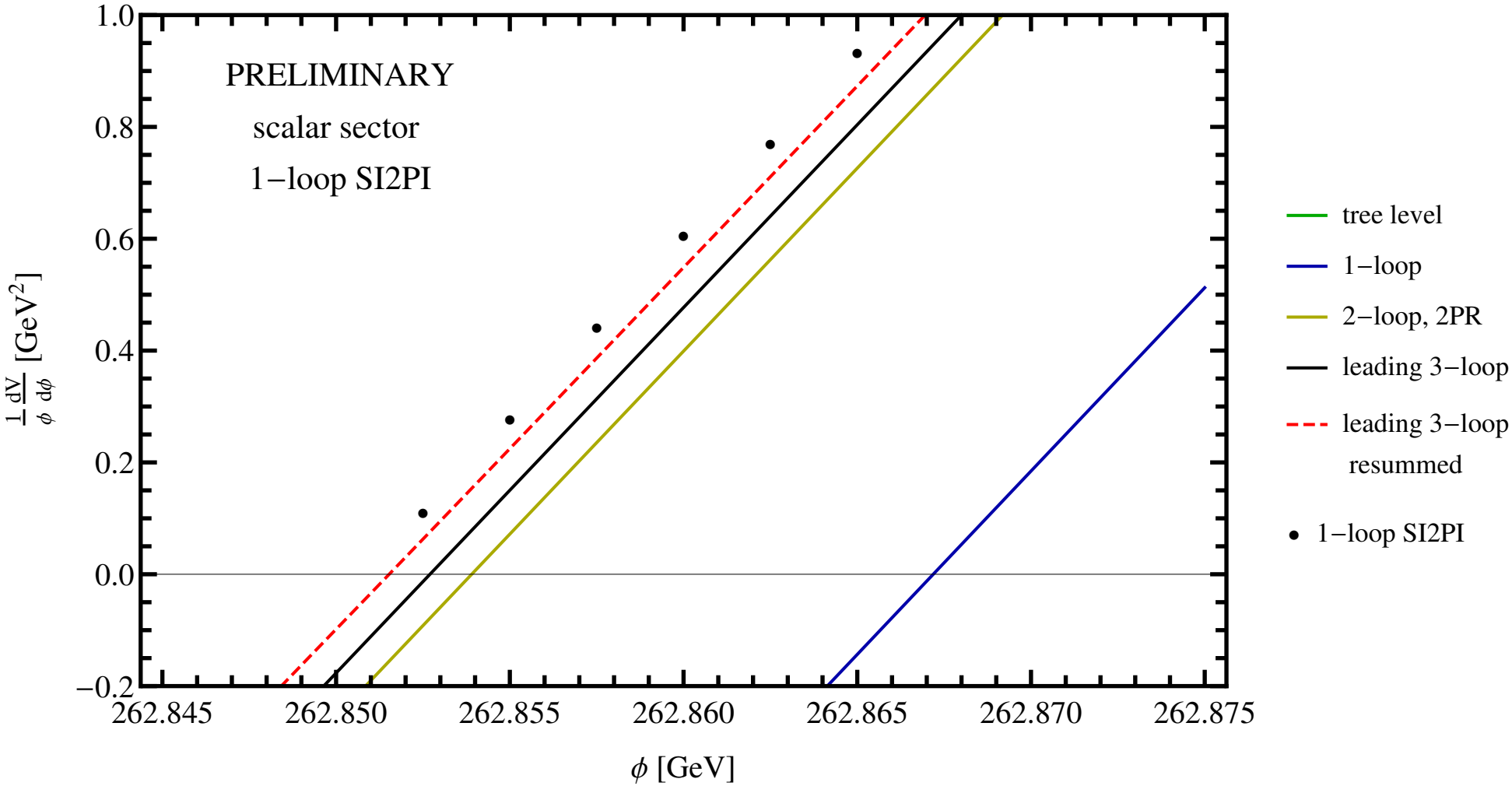


• Numerical Estimates

[A.P., D. Teresi, arXiv:1502.07986 and in preparation.]



• Conclusions

- **Maintaining symmetries in 2PI loopwise is a long-standing problem**

- **Novel Approach to Global Symmetries:**

  $\Longrightarrow$ **Symmetry-Improved 2PI Effective Action**

  $\rightarrow$ Massless Goldstone Bosons
  $\rightarrow$ 2nd-Order Phase Transition in the HF Approximation

- $\overline{\mathrm{MS}}$ **Renormalization with T-independent Resummed Counterterms**

- **Absorptive Effects properly described:**

  $\rightarrow$ Smooth thresholds consistent with massless Goldstone bosons
  $\rightarrow$ Consistent resummation within Quantum Loops.

- **Symmetry-Improved Effective Higgs Potential is unique, with proper thermodynamic properties**

• Future Directions

- **Solve the IR Problem of the SM Effective Potential**

  [A.P., D. Teresi, in preparation]

- **Extension: 2PI $\rightarrow$ nPI Effective Actions**

  [3PI: M.J. Brown and I.B. Whittingham, PRD91 (2015) 085020.]

- **Extension to Local Symmetries: $\mathbb{U}(1)$, $\mathbb{SU}(N)$**

- **Spontaneous Breaking of Local Gauge Symmetries**

- **Higher Precision Predictions in the 2PI Formalism**

$\vdots$

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New Era of Analytical Non-Perturbative QFT