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Motivation:

New Era of High Precision QFT

• Improved Lattice Techniques

• Higher Order Loop Calculations in Perturbative 1PI QFT

• Improved Solutions to Schwinger–Dyson Equations

• On-Shell Amplitude Techniques

• Geometric S-Matrix Approach in $N = 4$ SYM Theories

• Other Analytical Non-Perturbative Approaches?
Outline:

- The Standard Theory of Electroweak Symmetry Breaking: SM
- The 2PI Effective Action and Field-Theoretic Problems
- Symmetry-Improved 2PI Formalism
- \( \overline{\text{MS}} \) Renormalization in 2PI
- Finite-Width Effects within Quantum Loops
- Symmetry-Improved Effective Higgs Potential in 2PI
- Conclusions and Future Directions
**The Standard Theory of Electroweak Symmetry Breaking**

**Higgs Mechanism in SM:** \( SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_{em} \)

[P. W. Higgs '64; F. Englert, R. Brout '64; G. S. Guralnik, C. R. Hagen, T. W. B. Kibble '64]

Higgs potential \( V(\phi) \)

\[
V(\phi) = -m^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2.
\]

Ground state:

\[
\langle \phi \rangle = \sqrt{\frac{m^2}{2\lambda}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

carries weak charge, but **no electric charge** and **colour**.
The Standard Theory of Electroweak Symmetry Breaking

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\]

carries weak charge, but no electric charge and colour.

Custodial Symmetry of the SM with \( g' = Y_f = 0 \) and \( V(\phi) \):

[P. Sikivie, L. Susskind, M. B. Voloshin, V. I. Zakharov '80.]

\[
\Phi \equiv (\phi , i\sigma^2 \phi^*) \quad \mapsto \quad \Phi' \equiv U_L \Phi U_C ,
\]

with \( U_L \in SU(2)_L \), \( U_C \in SU(2)_C \), and \( SU(2)_L \otimes SU(2)_C / \mathbb{Z}_2 \cong SO(4) \)
Higgs Boson @ LHC: Signal Strength for Decay Modes

Signal strength: $\mu = \frac{\sigma_{\text{observed}}}{\sigma_{\text{SM}}}$

- Results consistent with SM
The Coleman-Weinberg Effective Potential [S. Coleman and E. Weinberg, '73]

Loopwise $\hbar$ Expansion of the Scalar Potential:

$$V_{\text{eff}}(\phi) = V(\phi) - \frac{i\hbar}{2} \int \frac{d^4k}{(2\pi)^4} \ln \left( \frac{k^2 - V''(\phi)}{k^2} \right) + O(\hbar^2)$$

The Coleman–Weinberg Effective Potential

• is based on the One-Particle-Irreducible (1PI) formalism

• breaks Classical Scale Symmetries $\Rightarrow$ Dimensional Transmutation
The Coleman-Weinberg Effective Potential  [S. Coleman and E. Weinberg, ’73]

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- is based on the One-Particle-Irreducible (1PI) formalism
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The SM Coleman-Weinberg Effective Potential at NNLO


Higgs Potential versus Variations in Top Mass $M_t$ by 0.1 MeV

[Analysis includes the Multi-Critical scenario: D.L. Bennett, H.B. Nielsen, IJMA9 (1994) 5155]
Metastability of the SM Vacuum


But, Planckian physics may modify stability predictions by many orders!

[V. Branchina, E. Messina, PRL111 (2013) 241801]
• Improved Loopwise Convergence in the 2PI Formalism

Connected Generating Functional of 2PI Effective Action:

\[ W[J, K] = -i \ln \int D\phi \exp \left[ i \left( S[\phi] + J^i \phi_x^i + \frac{1}{2} K^i_{xy} \phi_x^i \phi_y^j \right) \right], \]

where \( S[\phi] = \int_x L[\phi] \) is the classical action of a \( \mathbb{O}(N) \) theory.
The 2PI Effective Action and Field-Theoretic Problems


Connected Generating Functional of 2PI Effective Action:

\[
W[J, K] = -i \ln \int \mathcal{D}\phi^i \exp \left[ i \left( S[\phi] + J^i_x \phi^i_x + \frac{1}{2} K^{ij}_{xy} \phi^i_x \phi^j_y \right) \right],
\]

where \( S[\phi] = \int_x \mathcal{L}[\phi] \) is the classical action of a \( \mathcal{O}(N) \) theory.

**Legendre transform** of \( W[J, K] \) with respect to \( J \) and \( K \):

\[
\frac{\delta W[J, K]}{\delta J^i_x} \equiv \phi^i_x, \quad \frac{\delta W[J, K]}{\delta K^{ij}_{xy}} = \frac{1}{2} \left( i \Delta^{ij}_{xy} + \phi^i_x \phi^j_y \right),
\]

to get the 2PI effective action

\[
\Gamma[\phi, \Delta] = W[J, K] - J^i_x \phi^i_x - \frac{1}{2} K^{ij}_{xy} \left( i \Delta^{ij}_{xy} + \phi^i_x \phi^j_y \right).
\]
The 2PI Effective Action:

\[ \Gamma[\phi, \Delta] = S[\phi] + \frac{i}{2} \text{Tr} \ln \Delta^{-1} + \frac{i}{2} \text{Tr} (\Delta^{0-1} \Delta) - i \Gamma_{2\text{PI}}^{(2)}[\phi, \Delta], \]

where \( \Gamma_{2\text{PI}}^{(2)}[\phi, \Delta] = \) \( \text{graph} \) + \( \text{graph} \) + \( \text{graph} \) + \ldots \)
The 2PI Effective Action:

\[
\Gamma[\phi, \Delta] = S[\phi] + \frac{i}{2} \text{Tr} \ln \Delta^{-1} + \frac{i}{2} \text{Tr} (\Delta^{0-1} \Delta) - \frac{i}{2} \Gamma^{(2)}_{2\text{PI}}[\phi, \Delta],
\]

where \( \Gamma^{(2)}_{2\text{PI}}[\phi, \Delta] = \)

Equations of Motion:

1. \( \frac{\delta \Gamma[\phi, \Delta]}{\delta \phi} = 0 \)

2. \( \frac{\delta \Gamma[\phi, \Delta]}{\delta \Delta} = 0 \Rightarrow \Delta^{-1} = \Delta^{0-1} + \)

Hartree-Fock:

\[
\%
\]

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Beyond the Coleman–Weinberg Effective Potential

A. Pilaftsis
Artist’s impression of the infinite HF-term!
Field-Theoretic Problems addressed with 2PI

- Systematic formal resummation of high-order graphs:
  - Rigorous Derivation of Schwinger–Dyson Equations
  - Thermal Masses in the high-$T$ Regime
  - Finite-Width Effects within Quantum Loops
  - The IR Problem of the Coleman-Weinberg Effective Potential

- Non-Equilibrium QFT, through Kadanoff–Baym equations.
  [For instance, P. Millington, AP, PRD88 (2013) 085009]
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  [For instance, P. Millington, AP, PRD88 (2013) 085009]

BUT

• Truncations of 2PI lead to residual violations of symmetries,
  e.g. global or local symmetries.
  → Erroneous First-Order Phase Transition in $\mathcal{O}(N)$ Theories
  → Goldstone Bosons become Massive
  → Erroneous Thresholds for the Resummed Higgs-Boson Propagator.
  → …
QCD Phase Transition in the Standard 2PI Formalism

Pertinent Literature to the Goldstone-Symmetry Problem


Symmetry-Improved 2PI Formalism

[AP, D. Teresi, NPB874 (2013) 594]
Symmetry-Improved 2PI Formalism

Equivalence between 1PI and 2PI Effective Actions to All Orders:

$$\Gamma^{1\text{PI}}[\phi] = \Gamma[\phi, \Delta(\phi)] , \quad \text{with} \quad \frac{\delta \Gamma[\phi, \Delta(\phi)]}{\delta \Delta} = 0 .$$
Equivalence between 1PI and 2PI Effective Actions to All Orders:

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1PI Ward Identity (e.g. for \( \mathbb{O}(2) \)):

\[ \frac{\delta \Gamma^{1\text{PI}}[\phi]}{\delta \phi_x^i} T_{ij}^{\alpha} \phi_x^j = 0 \quad \Longrightarrow \quad v \int_x \frac{\delta^2 \Gamma^{1\text{PI}}[\phi]}{\delta G_y \delta G_x} = \frac{\delta \Gamma^{1\text{PI}}[\phi]}{\delta H} \rightarrow 0. \]
Equivalence between 1PI and 2PI Effective Actions to All Orders:

\[ \Gamma^{1\text{PI}}[\phi] = \Gamma[\phi, \Delta(\phi)] , \quad \text{with} \quad \frac{\delta \Gamma[\phi, \Delta(\phi)]}{\delta \Delta} = 0 . \]

1PI Ward Identity (e.g. for \( \mathcal{O}(2) \)):

\[ \frac{\delta \Gamma^{1\text{PI}}[\phi]}{\delta \phi^i_x} T^a_{ij} \phi^j_x = 0 \implies v \int_x \frac{\delta^2 \Gamma^{1\text{PI}}[\phi]}{\delta G_y \delta G_x} = \frac{\delta \Gamma^{1\text{PI}}[\phi]}{\delta H} \to 0 . \]

Replace:

\[ \frac{\delta^2 \Gamma^{1\text{PI}}[\phi]}{\delta G_y \delta G_x} = \Delta_{xy}^{-1,G} , \]

to obtain the Symmetry-Improved Equations of Motion:

\[ \frac{\delta \Gamma[v, \Delta]}{\delta \Delta_{H/G}} = 0 , \]

\[ v \Delta_G^{-1}(k = 0, v) = 0 . \]
– $\mathcal{O}(2)$ Hartree–Fock Equations of Motion:

HF Approximation: $\Gamma^{(2)}_{\text{HF}}[\Delta_H, \Delta_G] = \begin{array}{c}
\text{Blue} \\
\text{Red} \\
\text{Green}
\end{array} + \begin{array}{c}
\text{Blue} \\
\text{Red} \\
\text{Green}
\end{array} + \begin{array}{c}
\text{Blue} \\
\text{Red} \\
\text{Green}
\end{array}$

Ansatz: $\Delta^{-1}_{H/G}(k) = k^2 - M^2_{H/G} + i\varepsilon$

Equations of Motion:

\[ M^2_H = 3\lambda v^2 - m^2 + (\delta\lambda^A_1 + 2\delta\lambda^B_1)v^2 - \delta m^2_1 \\
+ (3\lambda + \delta\lambda^A_2 + 2\delta\lambda^B_2) \int_k i\Delta_H(k) + (\lambda + \delta\lambda^A_1) \int_k i\Delta_G(k) , \]

\[ M^2_G = \lambda v^2 - m^2 + \delta\lambda^A_1 v^2 - \delta m^2_1 \\
+ (\lambda + \delta\lambda^A_2) \int_k i\Delta_H(k) + (3\lambda + \delta\lambda^A_2 + 2\delta\lambda^B_2) \int_k i\Delta_G(k) , \]

\[ v M^2_G = 0 . \]
Second-Order Phase Transition in the HF Approximation

$[\text{AP, D. Teresi, NPB874 (2013) 594}]$

$M_H(0)=125.5 \text{ GeV}$
$v(0)=246 \text{ GeV}$
\textbf{\underline{\textit{MS} Renormalization in 2PI}}

**Naive Renormalization:**

\[ \Box = \int_k \! i \Delta(k) \sim M^2 \frac{1}{\epsilon}, \quad \text{Naive CT: } \delta m^2 \overset{?}{=} M^2 \frac{1}{\epsilon}. \]

**But,** \( M^2 = M^2(T) \implies \text{Temperature-dependent CT?}! \)
**\( \overline{\text{MS}} \) Renormalization in 2PI**

**Naive Renormalization:**

\[
\mathcal{O} = \int k i\Delta(k) \sim M^2 \frac{1}{\epsilon}, \quad \text{Naive CT: } \delta m^2 \sim M^2 \frac{1}{\epsilon}.
\]

But, \( M^2 = M^2(T) \implies \) Temperature-dependent CT?!

**What has gone wrong?**

\[
\delta m^2 \sim \bigg( \begin{array}{c}
\mathcal{O} \\
\mathcal{O}
\end{array} \bigg) \supset \begin{array}{c}
\mathcal{O} \\
\mathcal{O}
\end{array} + \begin{array}{c}
\mathcal{O} \\
\mathcal{O}
\end{array} + \ldots
\]

\[
\mathcal{O} \supset \begin{array}{c}
\mathcal{O} \\
\mathcal{O}
\end{array} \sim \delta \lambda
\]

Is there any systematic renormalization, e.g. in the \( \overline{\text{MS}} \) scheme?


[W.A. Bardeen, A. Buras, D. Duke, T. Muta, PRD18 (1978) 3998]
– MS Renormalization in the 2PI Formalism

Procedure:

• Isolate UV infinities in EoMs, e.g. by Dimensional Regularization.

• Require that the UV-finite part of EoMs be UV finite:

\[(\ldots)_{\text{UV}} T^\text{fin}_H + (\ldots)_{\text{UV}} T^\text{fin}_G + (\ldots)_{\text{UV}} v^2 + (\ldots)_{\text{UV}} 1 \overset{!}{=} 0\]

• Cancel separately the UV infinities \(\propto T^\text{fin}_H(T), T^\text{fin}_G(T), v^2(T), 1.\)

• Check UV consistency:

\[4 \times 2 = 8 \text{ Constraints, for 5 CTs: } \delta m^2_1, \delta \lambda^A_1, \delta \lambda^B_1, \delta \lambda^A_2, \delta \lambda^B_2.\]

This is a non-trivial check!
T-independent Resummed Counterterms in the HF approximation:

[AP, D. Teresi, NPB874 (2013) 594]

\[
\begin{align*}
\delta \lambda_1^A &= \delta \lambda_2^A =\frac{2\lambda^2}{16\pi^2\varepsilon} \frac{3 - \frac{4\lambda}{16\pi^2\varepsilon}}{1 - \frac{6\lambda}{16\pi^2\varepsilon} + \frac{8\lambda^2}{(16\pi^2\varepsilon)^2}} \\
&= -\lambda + \frac{(16\pi^2\varepsilon)^2}{8\lambda} + O(\varepsilon^3),
\end{align*}
\]

\[
\begin{align*}
\delta \lambda_1^B &= \delta \lambda_2^B =\frac{2\lambda^2}{16\pi^2\varepsilon} \frac{1}{1 - \frac{2\lambda}{16\pi^2\varepsilon}} \\
&= -\lambda - \frac{16\pi^2\varepsilon}{2} - \frac{(16\pi^2\varepsilon)^2}{4\lambda} + O(\varepsilon^3),
\end{align*}
\]

\[
\delta m_1^2 = \frac{4\lambda m^2}{16\pi^2\varepsilon} \frac{1}{1 - \frac{4\lambda}{16\pi^2\varepsilon}} = -m^2 - m^2\frac{16\pi^2\varepsilon}{4\lambda} + O(\varepsilon^2).
\]
Finite-Width Effects within Quantum Loops

Equations of Motion including Sunset Diagrams:

\[ \Delta_H^{-1}(p) = p^2 - (3\lambda + \delta\lambda_1^A + 2\delta\lambda_1^B)v^2 + m^2 + \delta m_1^2 \]
\[ -i \left( \begin{array}{c} H \\ G \\ G \end{array} + \begin{array}{c} G \\ H \\ H \end{array} + \begin{array}{c} H \\ H \\ G \end{array} + \begin{array}{c} G \\ G \\ H \end{array} \right), \]

\[ \Delta_G^{-1}(p) = p^2 - (\lambda + \delta\lambda_1^A)v^2 + m^2 + \delta m_1^2 \]
\[ -i \left( \begin{array}{c} H \\ G \\ G \end{array} + \begin{array}{c} G \\ H \\ H \end{array} + \begin{array}{c} H \\ H \\ G \end{array} \right), \]

\[ v \Delta_G^{-1}(0) = 0. \]

Absorptive Effects:

\[ G \text{ consistently massless} \iff \text{threshold at } s \equiv p^2 = 0 \]
Higgs Selfenergy in 2PI

\[ \text{Re} \tilde{M}_H^2(s) - \tilde{M}_H^2 [\text{GeV}] \]

\[ \text{Im} \tilde{M}_H^2(s)/(\sqrt{s} \Gamma_H \tilde{M}_H) \]

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Goldstone Selfenergy in 2PI

\[ \text{Re} \tilde{M}_G^2(s) \Gamma_H \]

\[ \text{Im} \tilde{M}_G^2(s) \Gamma_H \]

\[
\begin{align*}
\text{Goldstone~Selfenergy~in~2PI} & \quad \text{[AP, D. Teresi, NPB874 (2013) 594]} \\
\end{align*}
\]
Symmetry-Improved Effective Higgs Potential in 2PI
Symmetry-Improved Effective Potential $\tilde{V}_{\text{eff}}(\phi)$ from 1PI Ward Identity:

$$
\phi \Delta^{-1}_G(k = 0; \phi) = -\frac{d\tilde{V}_{\text{eff}}(\phi)}{d\phi}.
$$
Symmetry-Improved Effective Higgs Potential in 2PI

Symmetry-Improved Effective Potential $\tilde{V}_{\text{eff}}(\phi)$ from 1PI Ward Identity:

$$\phi \Delta_{G}^{-1}(k = 0; \phi) = -\frac{d\tilde{V}_{\text{eff}}(\phi)}{d\phi}.$$  

Solution:

$$\tilde{V}_{\text{eff}}(\phi) = -\int_{0}^{\phi} d\phi \phi \Delta_{G}^{-1}(k = 0; \phi) + \tilde{V}_{\text{eff}}(\phi = 0)$$

$$= -\int_{v}^{\phi} d\phi \phi \Delta_{G}^{-1}(k = 0; \phi) + P(T, \mu),$$

where $P(T, \mu)$ is the thermodynamic pressure $=$ hydrostatic pressure, i.e. it satisfies Baym’s thermodynamic consistency.

[Baym, PR127 (1962) 1391]
Note: $\text{Im} \, \tilde{V}_{\text{eff}}(\phi) < 0$, for $0 < \phi < v$

$\implies$ Vacuum instability for the concave part of the potential.

The IR Problem in the Coleman–Weinberg Effective Potential

Contributions to the SM effective potential:

\[ m_G^2 \log m_G^2 \sim \frac{d}{d\phi} \]

\[ \log m_G^2 \times \]

\[ \log m_G^2 \times \]

\[ \frac{1}{m_G^2} \times \]

\[ \left( \frac{1}{m_G^2} \right)^2 \times \]

\[ m_G^2 = \lambda \phi^2 - m^2 \]

\[ M_G^2 = m_G^2 + \Pi_G^{(1)}|_{k=0} + \ldots = 0 \text{ at } \phi = v \]
IR Divergence in the Coleman–Weinberg Potential
• Symmetry-Improved 2PI Approach to the IR Divergence


• More complete resummation:
  - first-principle approach
  - it takes into account the momentum-dependence of self-energy insertions
  - more topologies
  - no ad-hoc subtraction of $m_G^2 \log m_G^2$ contributions in $\Pi_G$

\[
- \frac{1}{\phi} \frac{d \tilde{V}_{\text{eff}}}{d \phi} \equiv \Delta^{-1}_G (\phi) \bigg|_{k=0} \supset \begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\end{array}
\end{array}
\end{array} + \begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\end{array}
\end{array}
\end{array}
\end{array}
\]

\[
+ \begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\end{array}
\end{array}
\end{array} + \begin{array}{c}
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\end{array}
\end{array} + \begin{array}{c}
\begin{array}{c}
\begin{array}{c}
\end{array}
\end{array}
\end{array} \]
\]

\[\Delta \approx \Delta_0 (\phi)\]

• No IR divergences: the would-be divergent self-energies are 2PR
2PI Fractal Resummation

\[
\begin{array}{c}
\hline
\bigcirc &= \bigcirc + \bigcirc + \bigcirc + \ldots
\end{array}
\]
\( 2 \Pi \) Fractal Resummation
\[ 2 \Pi \ Fractal \ Resummation \]

\[ \bigcirc = \bigcirc + \bigcirc + \bigcirc + \ldots \]

\[ \bigcirc + \bigcirc = \bigcirc + \bigcirc + \ldots + \bigcirc \]

\[ + \ldots + \bigcirc + \ldots + \bigcirc + \ldots \]
Numerical Estimates

[PRELIMINARY]

scalar sector

$1/\phi^2$ tree level
$1/\phi^2$ 1-loop SI2PI

$1/\phi^2$ 2-loop, 2PR

$1/\phi^2$ leading 3-loop

$1/\phi^2$ leading 3-loop resummed

$1/\phi^2$ 1-loop SI2PI


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• **Conclusions**

• **Maintaining symmetries in 2PI loopwise** is a long-standing problem

• **Novel Approach to Global Symmetries:**
  \[ \Rightarrow \text{Symmetry-Improved 2PI Effective Action} \]
  \[ \rightarrow \text{Massless Goldstone Bosons} \]
  \[ \rightarrow \text{2nd-Order Phase Transition in the HF Approximation} \]

• **\( \text{MS} \)** Renormalization with **\( T \)-independent Resummed Counterterms**

• **Absorptive Effects** properly described:
  \[ \rightarrow \text{Smooth thresholds consistent with massless Goldstone bosons} \]
  \[ \rightarrow \text{Consistent resummation within Quantum Loops.} \]

• **Symmetry-Improved Effective Higgs Potential** is **unique**, with proper **thermodynamic properties**
• **Future Directions**

• **Solve the IR Problem of the SM Effective Potential**
  [A.P., D. Teresi, in preparation]

• **Extension: 2PI $\to n$PI Effective Actions**

• **Extension to Local Symmetries**: $U(1)$, $SU(N)$

• **Spontaneous Breaking of Local Gauge Symmetries**

• **Higher Precision Predictions in the 2PI Formalism**

  :
• Solve the IR Problem of the SM Effective Potential
  [A.P., D. Teresi, in preparation]

• Extension: $2\pi \rightarrow n\pi$ Effective Actions

• Extension to Local Symmetries: $U(1)$, $SU(N)$

• Spontaneous Breaking of Local Gauge Symmetries

• Higher Precision Predictions in the 2PI Formalism

New Era of Analytical Non-Perturbative QFT
Beyond the Coleman–Weinberg Effective Potential

A. Pilaftsis