New Angles on Groomed Observables

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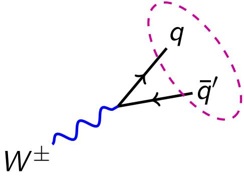
Based on upcoming works with Lina Necib and Jesse Thaler
and with Andrew Larkoski and Duff Neill

Motivation

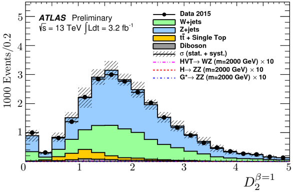
- A major focus of jet substructure has been on observables for boosted object tagging:

$$\tau_{2,1}^{(\beta)},\ \tau_{3,2}^{(\beta)},\ C_2^{(\beta)},\ D_2^{(\beta)},\ \ldots$$



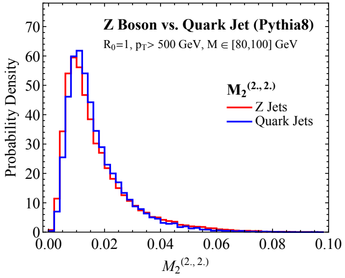
- Large push towards groomed observables

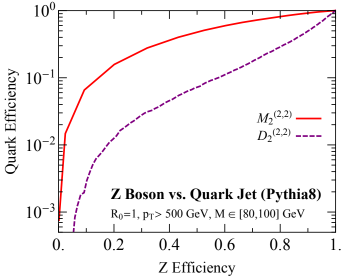
$$\tau_{2,1}^{(\beta)} \to \tau_{2,1}^{(\beta),\text{groomed}}$$
$$\tau_{3,2}^{(\beta)} \to \tau_{3,2}^{(\beta),\text{groomed}}$$
$$D_2^{(\beta)} \to D_2^{(\beta),\text{groomed}}$$



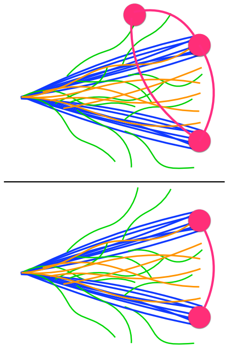
- For ungroomed substructure, many improvements came from more detailed theoretical understanding, and explicit calculations.
  - Recoil free observables, ECFs, WTA axes, $D_2^{(\beta)}, \cdots$

An Interesting Example: M_2

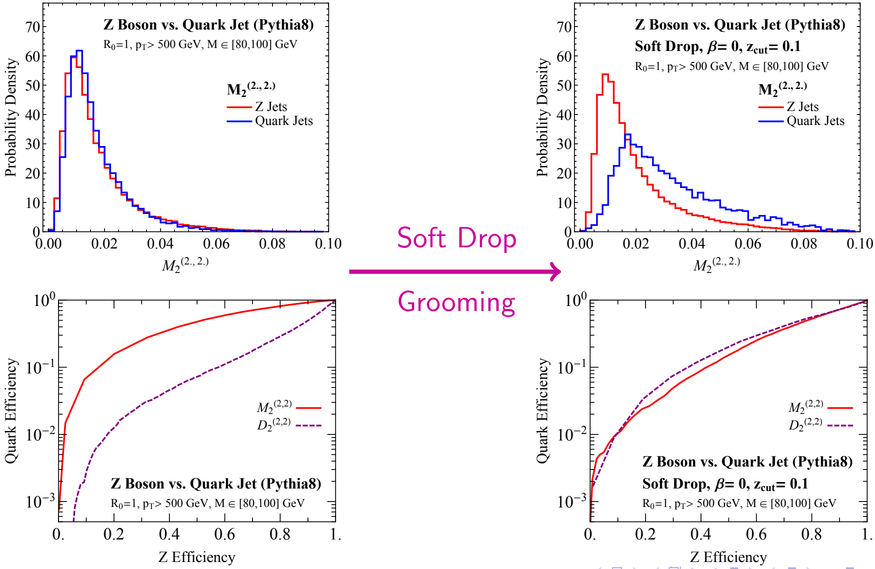




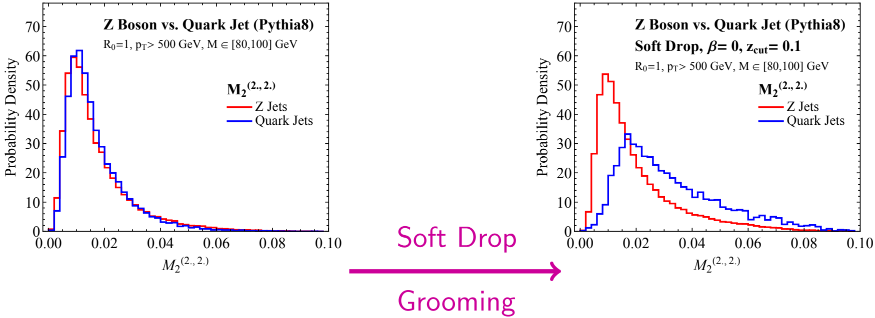
$$M_2^{(\beta)} = \frac{{}_1e_3^{(\beta)}}{e_2^{(\beta)}} =$$



An Interesting Example: M_2



An Interesting Example: M_2

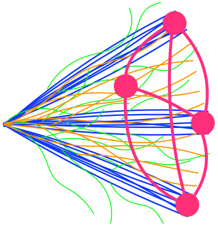


- Grooming has highly non-trivial impact on radiation pattern!
- Are the observables we are currently using optimal?
- It is essential to understand properties of groomed multi-prong observables from first principles QCD.

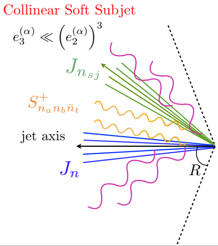
For progress for groomed masses, see: [Dasgupta, Fregoso, Marzani, Salam], Andrew Larkoski's Talk Later Today

Outline

- New Observables for Groomed Jet Substructure
  - Generalizing the energy correlation functions
  - The N_i series of observables and M_2



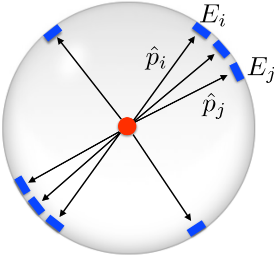
- Analytic Boosted Boson Discrimination with Grooming
  - Factorization for groomed multi-prong observables
  - Universality of groomed D_2 distribution



New Observables for Groomed Jet Substructure

Back to Basics

- What is an observable?



$$F_N(P) = \sum E_{i_1} \cdots E_{i_N} f_N(\hat{p}_{i_1}, \cdots, \hat{p}_{i_N})$$

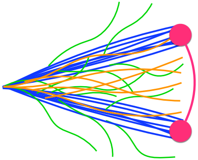
- Linear in the energies by IRC safety.
- f_N is symmetric, and $f_N \to 0$ if $\hat{p}_i || \hat{p}_j$

- Known that from this one can reconstruct any IRC safe observable in the QFT (i.e. the energy momentum tensor). [Tkachov]
- Is this useful for jet substructure?

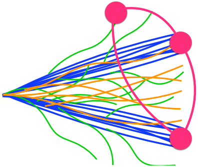
Example: Energy Correlation Functions

- Energy Correlation Functions: $f_N(\hat{p}_{i_1}, \cdots, \hat{p}_{i_N}) = \prod R_{ij}^{\beta}$
  [Larkoski, Salam, Thaler]

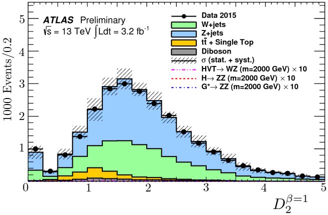
$$e_2^{(\beta)} = \frac{1}{p_{TJ}^2} \sum_{i<j\in J} p_{Ti} p_{Tj} (R_{ij})^{\beta}$$



$$e_3^{(\beta)} = \frac{1}{p_{TJ}^3} \sum_{i<j<k\in J} p_{Ti} p_{Tj} p_{Tk} (R_{ij} R_{jk} R_{ik})^{\beta}$$



- Powerful discriminant $D_2^{(\beta)} = \frac{e_3^{(\beta)}}{\left(e_2^{(\beta)}\right)^3}$
  [Larkoski, Moult, Neill]
- Rigid structure.



New Angles on the Energy Correlation Functions

- To tailor jet substructure observables, need more freedom.

$$F_N(P) = \sum E_{i_1} \cdots E_{i_N} f_N(\hat{p}_{i_1}, \cdots, \hat{p}_{i_N})$$

- Energy dependence of general IRC safe observable is fixed.
- Angular dependence, $f_N(\hat{p}_{i_1}, \cdots, \hat{p}_{i_N})$, more flexible.
- A convenient set of functions for identifying hierarchical structures is

$$f_N(\hat{p}_{i_1}, \cdots, \hat{p}_{i_N}) = \min \left(\prod_{s,t}^{i} R_{st}^{\beta} \right)$$

General Energy Correlation Functions

$$ {}_i e_j^{(\beta)} = \frac{1}{p_{TJ}^j} \sum_{1 \leq n_1 < \cdots < n_j \leq n} p_{Tn_1} p_{Tn_2} \ldots p_{Tn_j} \min \left(\prod_{s,t}^{i} R_{st}^{\beta} \right)$$

Generalized Energy Correlation Functions

General Energy Correlation Functions

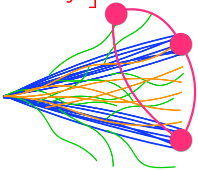
$$ {}_i e_j^{(\beta)} = \frac{1}{p_{TJ}^j} \sum_{1\le n_1 < \cdots < n_j \le n} p_{T n_1} p_{T n_2} \ldots p_{T n_j} \min \left(\prod_{s,t}^{i} R_{st}^{\beta} \right) $$

- Example: Three different ways to probe three particle correlations.

$$ {}_1 e_3^{(\beta)} = \frac{1}{p_{TJ}^3} \sum_{1\le i<j<k\le n_J} p_{Ti} p_{Tj} p_{Tk} \min \left[R_{ij}^{\beta}, R_{ik}^{\beta}, R_{jk}^{\beta} \right], $$

$$ {}_2 e_3^{(\beta)} = \frac{1}{p_{TJ}^3} \sum_{1\le i<j<k\le n_J} p_{Ti} p_{Tj} p_{Tk} \min \left[R_{ij}^{\beta} R_{ik}^{\beta}, R_{ij}^{\beta} R_{jk}^{\beta}, R_{ik}^{\beta} R_{jk}^{\beta} \right], $$

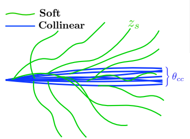
$$ {}_3 e_3^{(\beta)} = \frac{1}{p_{TJ}^3} \sum_{1\le i<j<k\le n_J} p_{Ti} p_{Tj} p_{Tk} R_{ij}^{\beta} R_{ik}^{\beta} R_{jk}^{\beta} = e_3^{(\beta)} $$



- Flexible basis for substructure observables.

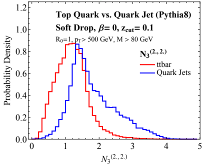
Applications

- Observables for particular applications constructed using "power counting".
  [Larkoski, Moult, Neill], Larkoski Boost 2014, Moult Boost 2015



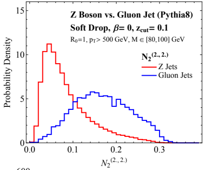
$\Longrightarrow$ Improving boosted top tagging:

$$N_3^{(\beta)} = \frac{{}_2e_4^{(\beta)}}{({}_1e_3^{(\beta)})^2}$$



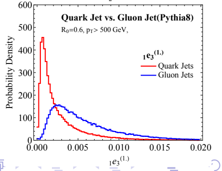
$\Longrightarrow$ New observables for $W/Z/H$ tagging:

$$M_2^{(\beta)} = \frac{{}_1e_3^{(\beta)}}{e_2^{(\beta)}}, \quad N_2^{(\beta)} = \frac{{}_2e_3^{(\beta)}}{(e_2^{(\beta)})^2}, \quad D_2^{(\alpha,\beta)} = \frac{{}_3e_3^{(\alpha)}}{(e_2^{(\beta)})^{3\alpha/\beta}}$$



$\Longrightarrow$ Improving Quark vs. Gluon discrimination:

$${}_1e_3^{(\beta)}$$



fjcontrib: Energy correlator tag 1.2.0-alpha0

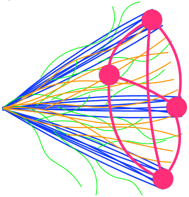
Boosted Top Tagging with N_3

- Appropriate observable for boosted top tagging

$$N_3^{(\beta)} = \frac{{}_2e_4^{(\beta)}}{({}_1e_3^{(\beta)})^2}$$

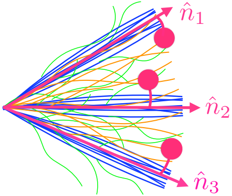
- It can be shown that in the limit of groomed, well resolved subjets, four point correlations factorize as

$$N_3^{(\beta)} \sim \frac{\tau_3^{(\beta)} \cdot \tau_2^{(\beta)}}{(\tau_2^{(\beta)})^2} = \frac{\tau_3^{(\beta)}}{\tau_2^{(\beta)}}$$



$N_3^{(\beta)}$

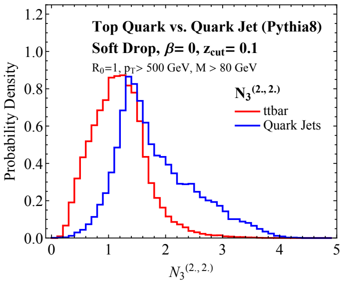
- N_3 is defined as a sum of four particle correlations.
- No axes, reclustering into subjets, etc.
- Smooth transition from resolved to unresolved regime.

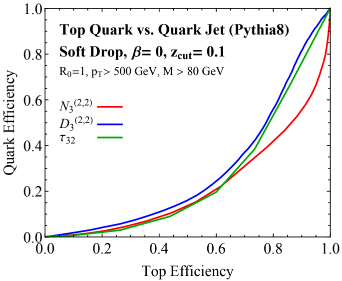


$\tau_{3,2}^{(\beta)}$

Boosted Top Tagging with N_3

- $\tau_N^{(\beta)}$ behave well in the limit of well resolved substructure, $\tau_N^{(\beta)} \ll 1$.
- Can have pathological or undesired behavior in the $\tau_3^{(\beta)}/\tau_2^{(\beta)} \to 1$ limit. Precisely where used for high efficiency taggers!
- N_3 well behaved at high efficiency. Leads to significantly improved performance.





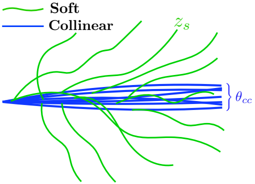
New Observables for Two-Prong Tagging

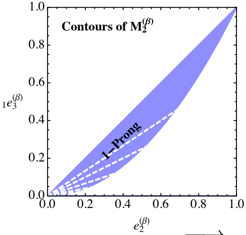
- Also allows for new two prong observables.
- Interestingly, powerful observables can be constructed from each of the ${}_i e_3^{(\beta)}$, $i = 1, 2, 3$.
- Each probes different angular correlations
  $\Longrightarrow$ Very different behavior under grooming.
- Power counting uniquely fixes the structure of the observables:

$$\boxed{M_2^{(\beta)} = \frac{{}_1 e_3^{(\beta)}}{e_2^{(\beta)}}, \qquad N_2^{(\beta)} = \frac{{}_2 e_3^{(\beta)}}{(e_2^{(\beta)})^2}, \qquad D_2^{(\alpha,\beta)} = \frac{{}_3 e_3^{(\alpha)}}{(e_2^{(\beta)})^{3\alpha/\beta}}}$$

The Power of Power Counting

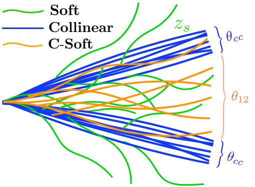
- Power counting allows us to understand $M_2^{(\beta)} = \frac{{}_1e_3^{(\beta)}}{e_2^{(\beta)}}$:

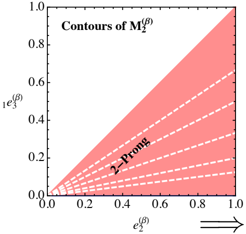




$$e_2^{(\beta)} \sim \theta_{cc}^{\beta} + z_s\,,$$
$${}_1e_3^{(\beta)} \sim \theta_{cc}^{\beta} + z_s^2$$

$\Longrightarrow$ 1-prong: $(e_2^{(\beta)})^2 \lesssim {}_1e_3^{(\beta)} \lesssim (e_2^{(\beta)})$



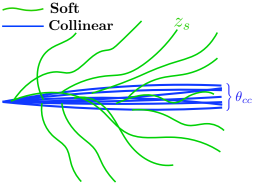


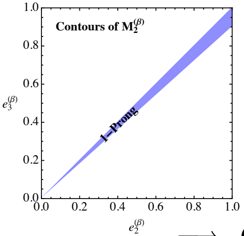
$$e_2^{(\beta)} \sim \theta_{12}^{\beta}\,,$$
$${}_1e_3^{(\beta)} \sim \theta_{12}^{\beta} z_s + \theta_{cc}^{\beta} + \theta_{12}^{\beta} z_{cs}$$

$\Longrightarrow$ 2-prong: $0 < {}_1e_3^{(\beta)} \ll e_2^{(\beta)}$

The Power of Power Counting

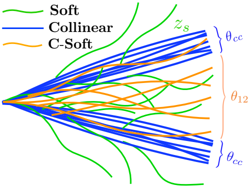
- Power counting allows us to understand $M_2^{(\beta)} = \frac{{}_1e_3^{(\beta)}}{e_2^{(\beta)}}$:

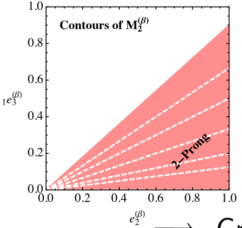




$$e_2^{(\beta)} \sim \theta_{cc}^{\beta} + \cancel{z_s},$$
$${}_1e_3^{(\beta)} \sim \theta_{cc}^{\beta} + \cancel{z_s^2}$$

$\Longrightarrow$ Groomed 1-prong: $e_2^{(\beta)} \sim {}_1e_3^{(\beta)}$!!



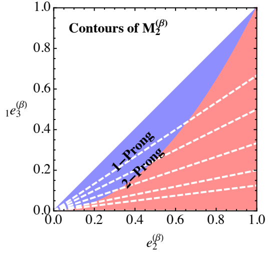


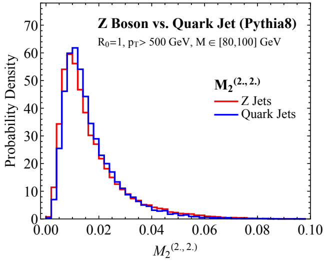
$$e_2^{(\beta)} \sim \theta_{12}^{\beta},$$
$${}_1e_3^{(\beta)} \sim \cancel{\theta_{12}^{\beta} z_s} + \theta_{cc}^{\beta} + \theta_{12}^{\beta} z_{cs}$$

$\Longrightarrow$ Groomed 2-prong: $0 < {}_1e_3^{(\beta)} \ll e_2^{(\beta)}$

Taking Advantage of Grooming

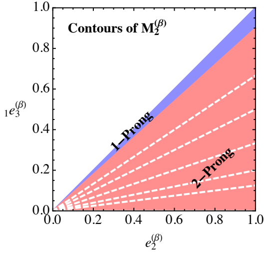
- Grooming “cleans” phase space, allowing for discrimination.

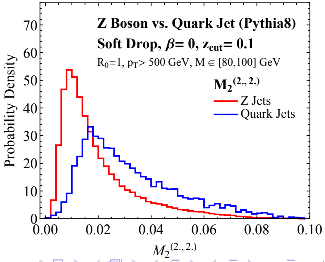




Soft Drop

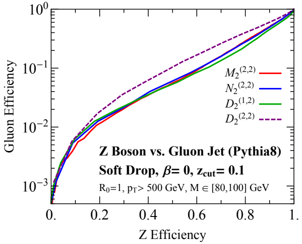
Grooming



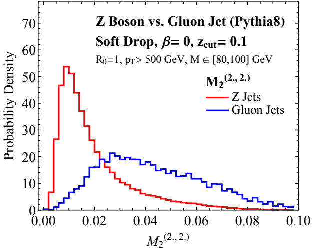


Taking Advantage of Grooming

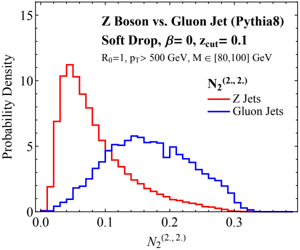
- Similar power counting used to study other observables.
- Three interesting observables for two prong discrimination on (un)groomed jets.
- Each outperforms $D_2^{(2,2)}$.
- Would be interesting to study correlations.



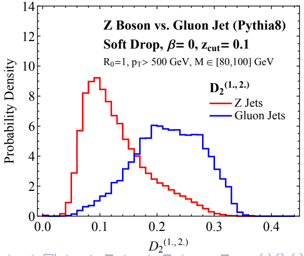
$M_2^{(\beta)}$



$N_2^{(\beta)}$



$D_2^{(1,2)}$

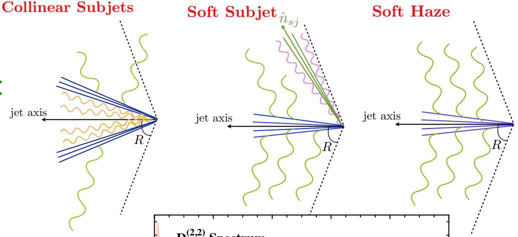


Analytic Boosted Boson Discrimination with Grooming

Analytic Boosted Boson Discrimination

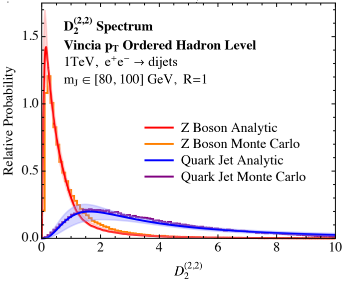
[Larkoski, Moult, Neill]

- Calculation of $D_2 = \frac{e_3^{(\beta)}}{(e_2^{(\beta)})^3}$ in e^+e^- using effective field theory.



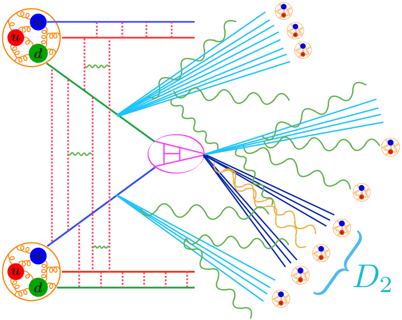
EFTs for 2-prong Substructure:

- All orders factorization theorems ✓
- Treatment of non-perturbative physics ✓
- Resummation to first non-trivial order (NLL) ✓
- Treatment of $D_2 < 1$ and $D_2 > 1$ ✓



Analytic Boosted Boson Discrimination

- Difficulties in extending to *pp*:



- Global color correlations
- Hadronization corrections
- Pile-Up
- Underlying event*

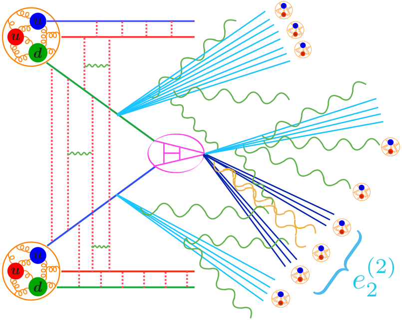
- All complications associated with soft radiation.

  $\Longrightarrow$ Groomers can make calculations simpler and more universal.

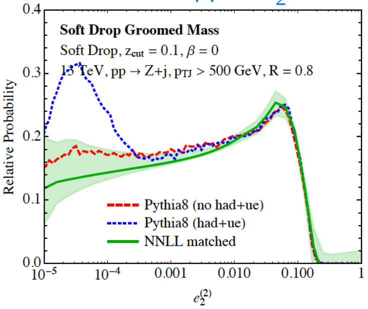
Factorization for Soft Dropped D_2

[Larkoski, Marzani, Soyez, Thaler]

- Focus on Soft Drop groomer. Involves a single additional scale z_{cut}.
- Consider first Soft Dropped $e_2^{(2)} \sim \frac{m_J^2}{p_{TJ}^2}$. [See Andrew Larkoski's Talk Later Today]



Soft Dropped $e_2^{(2)}$



[Frye, Larkoski, Schwartz, Yan]

Factorization for Soft Dropped D_2

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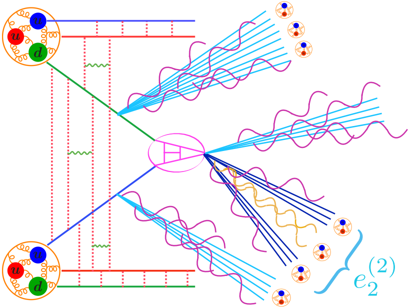
- Consider first Soft Dropped $e_2^{(2)} \sim \frac{m_J^2}{p_{TJ}^2}$. [See Andrew Larkoski's Talk Later Today]
- IF $e_2^{(2)} \ll z_{\text{cut}}$, soft radiation cannot contribute. Factorization formula in terms of collinear physics.



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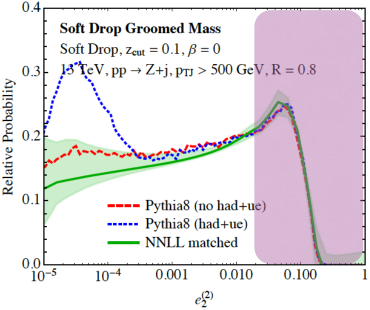

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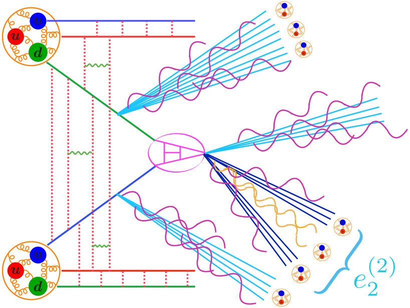
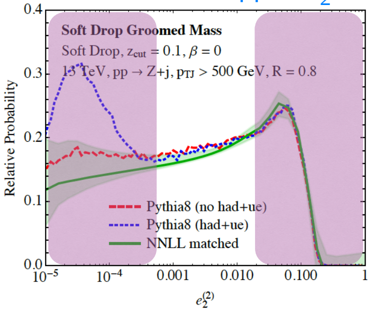
- IF $e_2^{(2)} \ll z_{\text{cut}}$, soft radiation cannot contribute. Factorization formula in terms of collinear physics.
- For substructure, apply a mass cut, e.g. $m_J \sim m_Z$.



Soft Dropped $e_2^{(2)}$



Factorization for Soft Dropped D_2

[Larkoski, Marzani, Soyez, Thaler]

- Focus on Soft Drop groomer. Involves a single additional scale z_{cut}.
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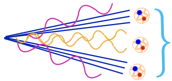
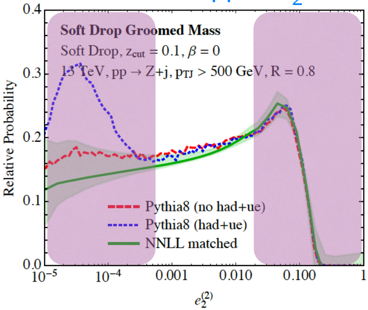
- For substructure, apply a mass cut, e.g. $m_J \sim m_Z$.
- Jet decouples!
  Can now measure D_2.



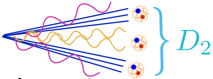
Soft Dropped $e_2^{(2)}$



- For $z_{cut} = 0.1$, need $p_{TJ} \gtrsim 500$ GeV.

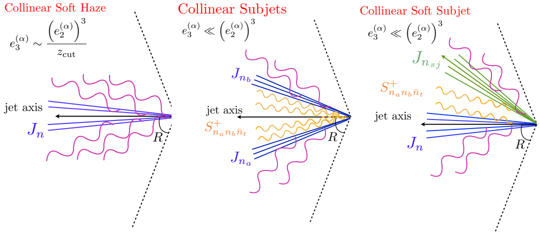
Factorization for Soft Dropped D_2: Universality

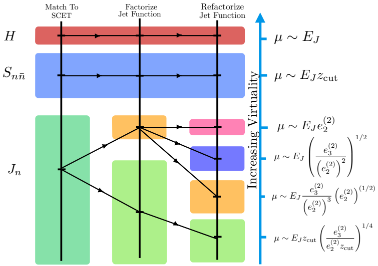
- With a mass cut jet decouples. Can measure its substructure.
  $\Longrightarrow$ Soft dropped D_2 is provably process independent!



- Possible dependence only on jet properties p_{TJ}, m_J, flavor...
- All orders (partonic) factorization formula can be proven using EFT.
- Allows for resummation using renormalization group.

EFTs for Groomed 2-prong Substructure:

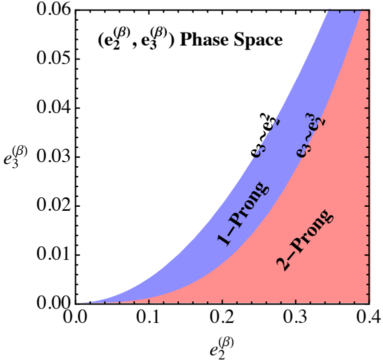


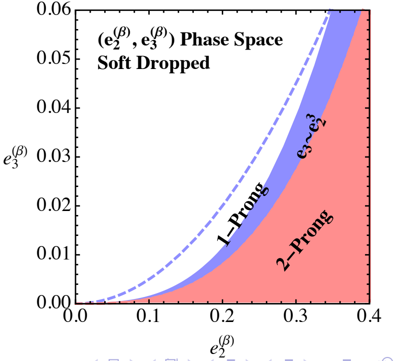


Stability

- Soft Drop modifies the $D_2 = \frac{e_3^{(\beta)}}{(e_2^{(\beta)})^3}$ phase space.

$$D_2^{\max} \sim \frac{1}{2e_2^{(\alpha)}} \sim \frac{1}{2}\frac{p_{TJ}^2}{m_J^2} \qquad D_2^{\max,\text{softdrop}} \sim \frac{1}{2z_{\text{cut}}}$$

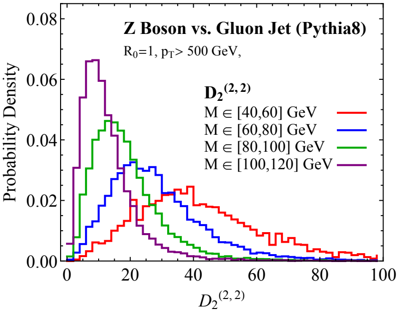




Stability

- Soft dropped D_2 distribution largely independent of
  - jet mass

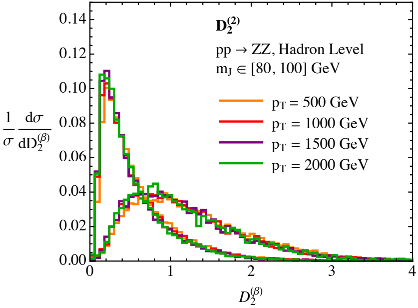
$$D_2^{\max} \sim \frac{1}{2}\frac{p_{TJ}^2}{m_J^2} \qquad D_2^{\max,\mathrm{softdrop}} \sim \frac{1}{2z_{\mathrm{cut}}}$$

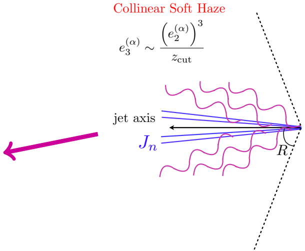


Stability

- Soft dropped D_2 distribution largely independent of
  - jet mass
  - jet p_T

$$D_2^{\text{max,softdrop}} \sim \frac{1}{2 z_{\text{cut}}}$$



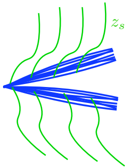


- Extremely stable discriminant!
- Expansion about small values insufficient to describe endpoint.

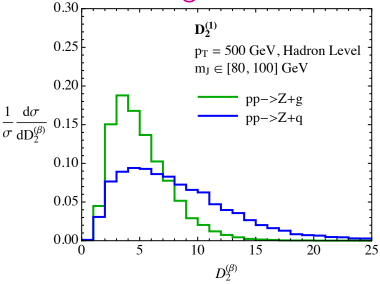
Quarks and Gluons

- Dominant dependence of D_2 on parton flavor carried by soft radiation.
- Enters ungroomed calculation as $C_i \log(z_s)$.
- Replaced in groomed calculation by $C_i \log(z_{\text{cut}})$.

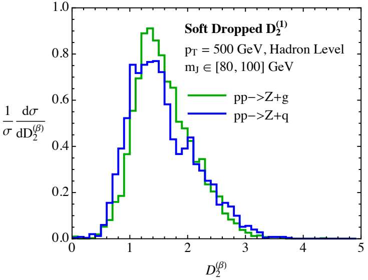
$\Longrightarrow$ Doesn't enter normalized distribution!



Ungroomed

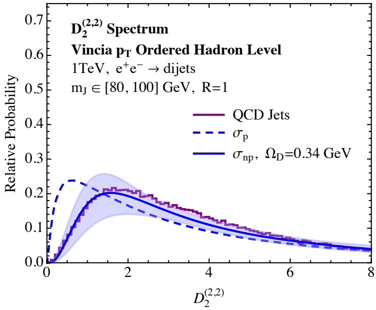


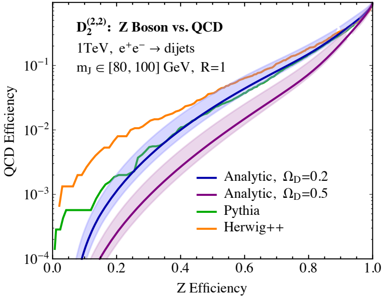
Groomed



Non-Perturbative Corrections

- Observables sensitive to low scales are sensitive to non-perturbative (hadronization) corrections.
- Can introduce non-perturbative parameters, (shift in first moment Ω_D) but leads to uncertainties.
- Discrimination power is particularly sensitive to hadronization.

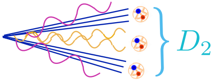




Non-Perturbative Corrections

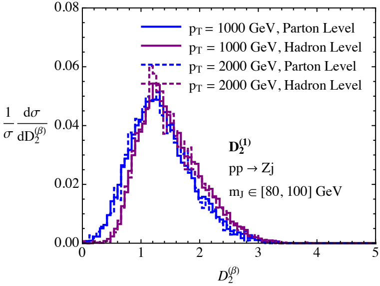
- Factorization is true to all orders, and allows us to address impact of hadronization.
- Can prove that non-perturbative corrections enter at the scale

$$D_2^{(\beta)} \simeq \left(\frac{1}{z_{\rm cut}}\right)^{2-1/\beta}\left(\frac{\Lambda_{\rm QCD}}{m_J}\right)$$

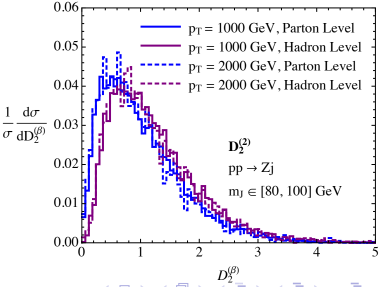


- Reduced hadronization corrections. Independent of Jet Energy !!!!

$D_2^{(\beta)}$, $\beta = 1$



$D_2^{(\beta)}$, $\beta = 2$



Conclusions

- Power counting techniques allow the design of substructure observables with specific properties.
- Introduced a versatile basis of substructure observables ${}_ie_j^{(\beta)}$, and the discriminants
  - M_2, N_2, D_2 for groomed 2-prong substructure.
  - N_3 for high efficiency top tagging.
- Gaining analytic control over soft dropped multi-prong observables. Have all orders factorization formulae.
- Soft dropped D_2 offers many advantages, that can be proven from first principles QCD.
- Numerical results for soft dropped D_2 at the LHC will be coming soon!

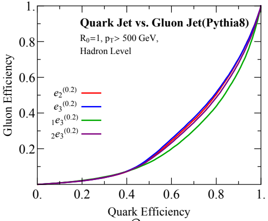
Thanks!

Backup

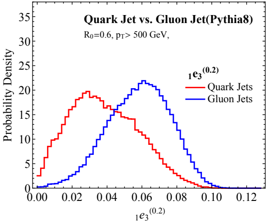
Quark vs. Gluon Discrimination with ${}_1e_3^{(\beta)}$

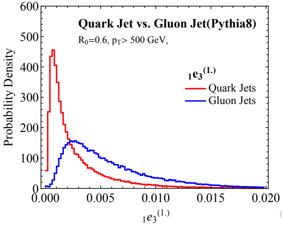
- The observable ${}_1e_3^{(\beta)}$ probes three particle correlations.
- Use of ${}_1e_3^{(\beta)}$ as compared with ${}_3e_3^{(\beta)}$ essential.

$$e_2^{(\beta)} \sim \theta_{cc}^{\beta} + z_s$$
$$e_3^{(\beta)} \sim (e_2^{(\beta)})^2$$
$${}_1e_3^{(\beta)} \sim \theta_{cc}^{\beta} + z_s^2$$



- Improved stability of discrimination power w.r.t. β.





D_2 with Grooming

- On groomed jets, $D_2^{(\alpha,\beta)}$ can be used with two distinct angular exponents.
- The choice $\alpha = 1$, $\beta = 2$ is good on groomed jets when a mass cut is used, as $\beta = 2$ corresponds to a mass in the denominator.
- The choice $\alpha = 1$, $\beta = 2$ does not work on ungroomed jets.
- Like M_2, it relies on grooming to clean the phase space

$$D_2^{(\alpha,\beta)} = \frac{_3 e_3^{(\alpha)}}{(e_2^{(\beta)})^{3\alpha/\beta}}$$

Background:

$$e_3^{(1)} \sim \theta_{cc}^3 + z_s^2 + \theta_{cc} z_s$$
$$e_2^{(2)} \sim \theta_{cc}^2 + \theta_{cc}^2 z_s$$

Signal:

$$e_3^{(1)} \lesssim (e_2^{(2)})^{3/2}$$

$$\implies (e_2^{(2)})^2 \lesssim e_3^{(1)} \lesssim (e_2^{(2)})^{3/2}$$