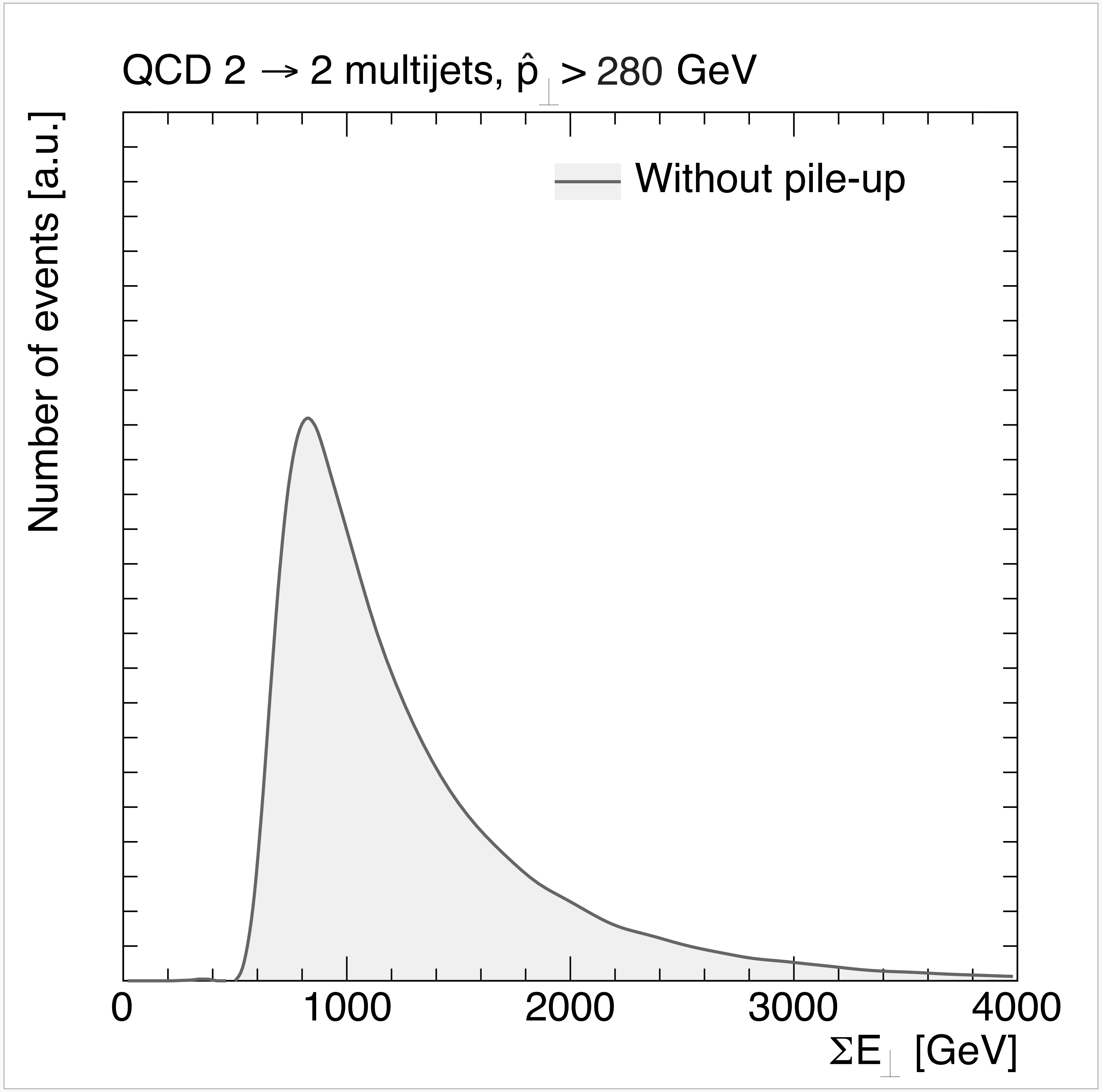
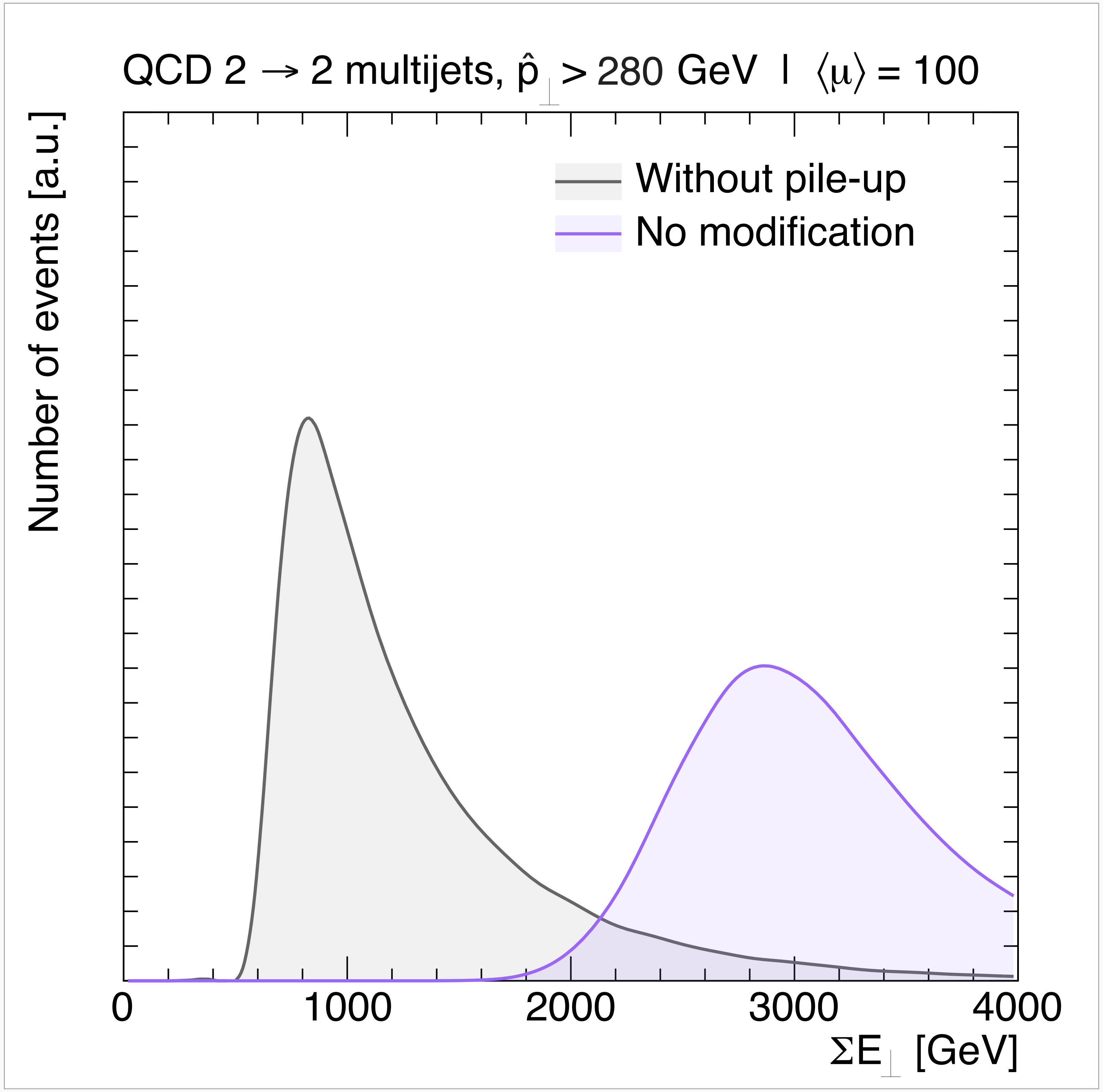
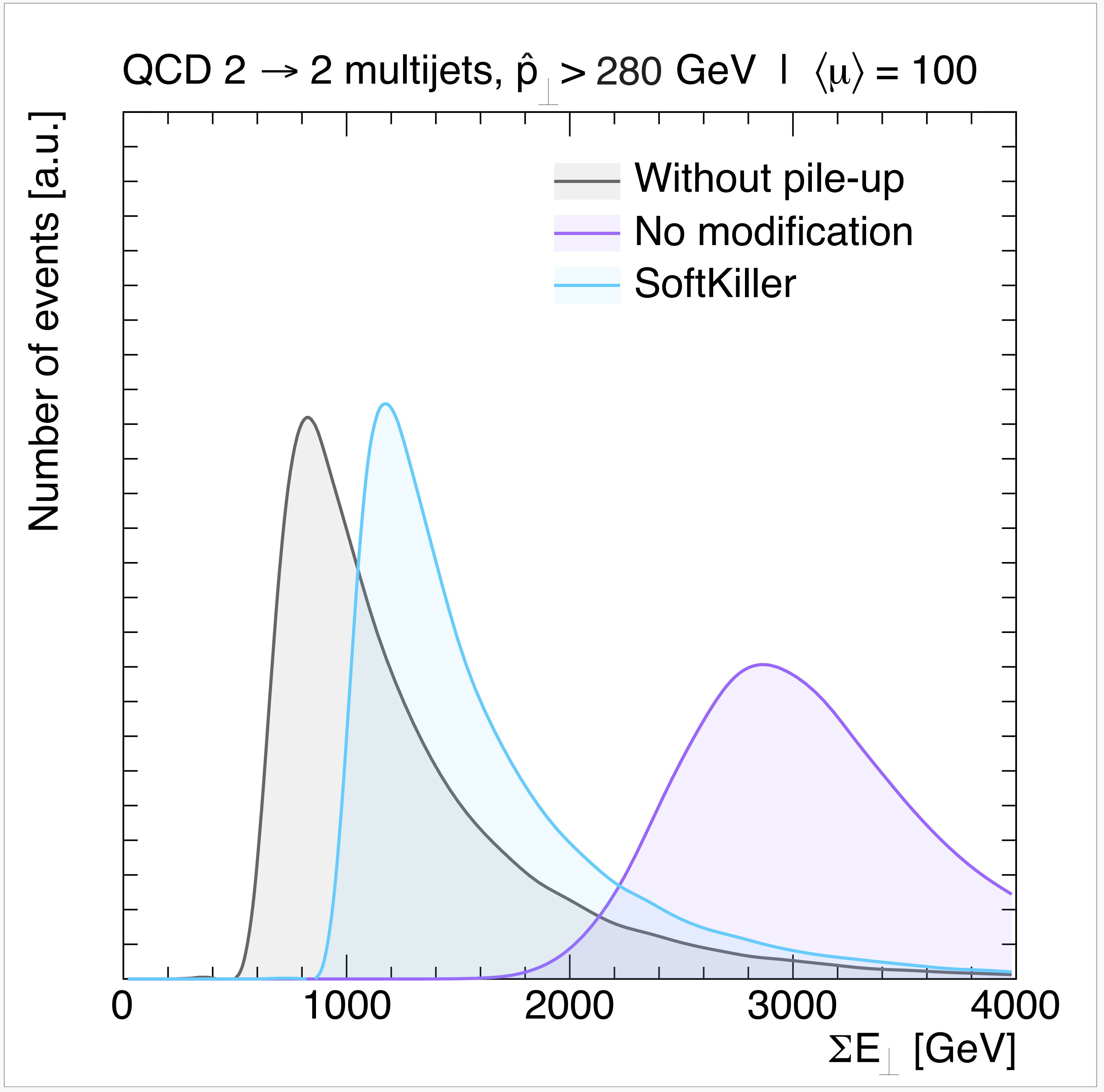

QCD 2 → 2 multijets, p̂⊥ > 280 GeV
Without pile-up
Number of events [a.u.]
0
1000
2000
3000
4000
ΣE⊥ [GeV]



QCD 2 → 2 multijets, p̂⊥ > 280 GeV | ⟨μ⟩ = 100
Without pile-up
No modification
Number of events [a.u.]
0
1000
2000
3000
4000
ΣE⊥ [GeV]



QCD 2 → 2 multijets, p̂⊥ > 280 GeV | ⟨μ⟩ = 100
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Without pile-up
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SoftKiller
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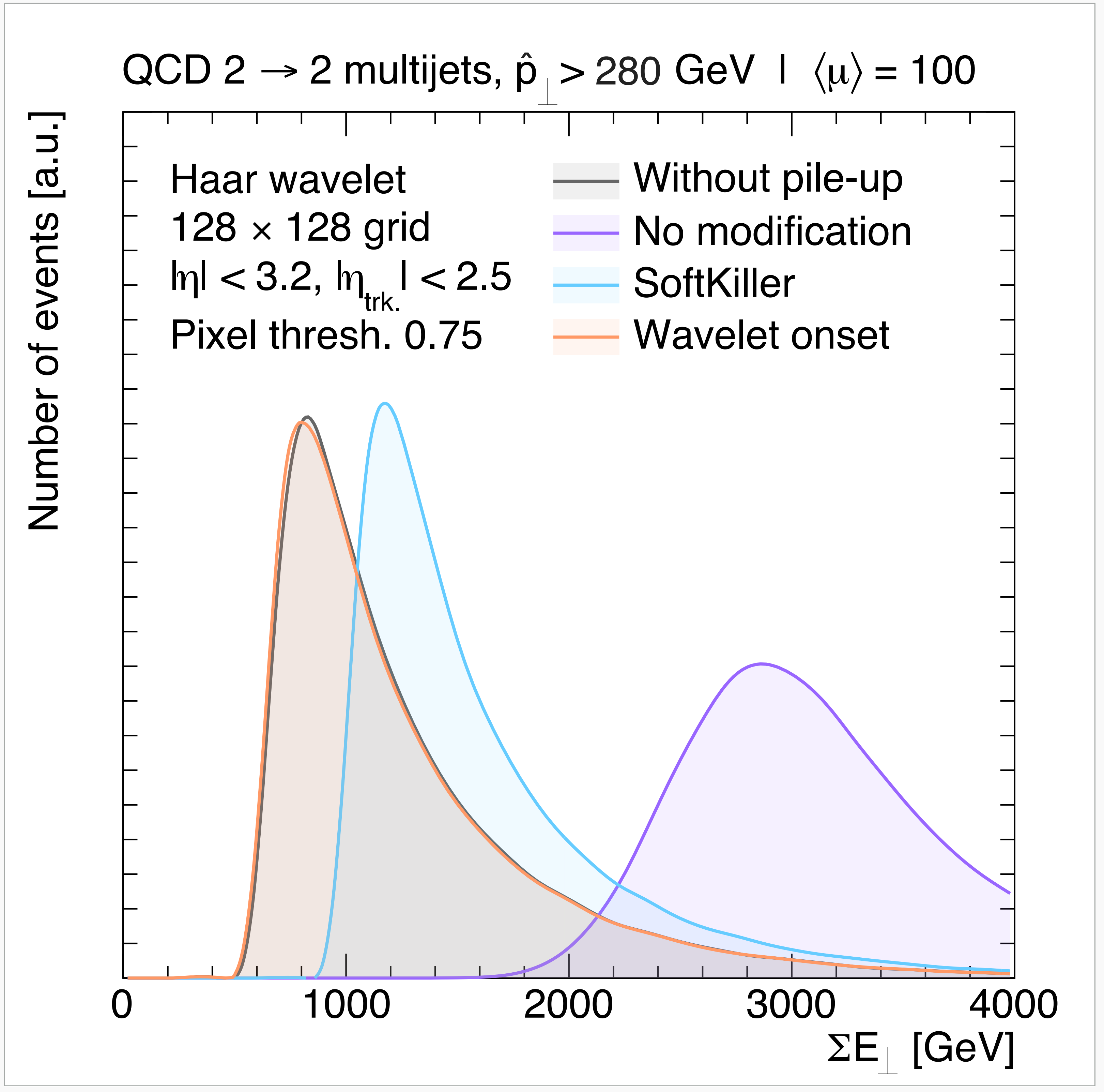

QCD 2 → 2 multijets, p̂⊥ > 280 GeV | ⟨μ⟩ = 100
Number of events [a.u.]
Haar wavelet
128 × 128 grid
|η| < 3.2, |ηtrk.| < 2.5
Pixel thresh. 0.75
Without pile-up
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SoftKiller
Wavelet onset
0
1000
2000
3000
4000
ΣE⊥ [GeV]

Using wavelets for pile-up mitigation

James Monk · Troels Petersen · **Andreas Søgaard**

University of Copenhagen · University of Edinburgh

BOOST Conference · Zürich
20 July 2016

Outline

1. Wavelet fundamentals
2. Missing- and sum E_T studies
3. Boosted jet studies
4. Summary and outlook

- *Bonus: Learning optimal bases*

Wavelet fundamentals

- Basis functions encoding both *frequency* and *position*
  - “Localised Fourier series”

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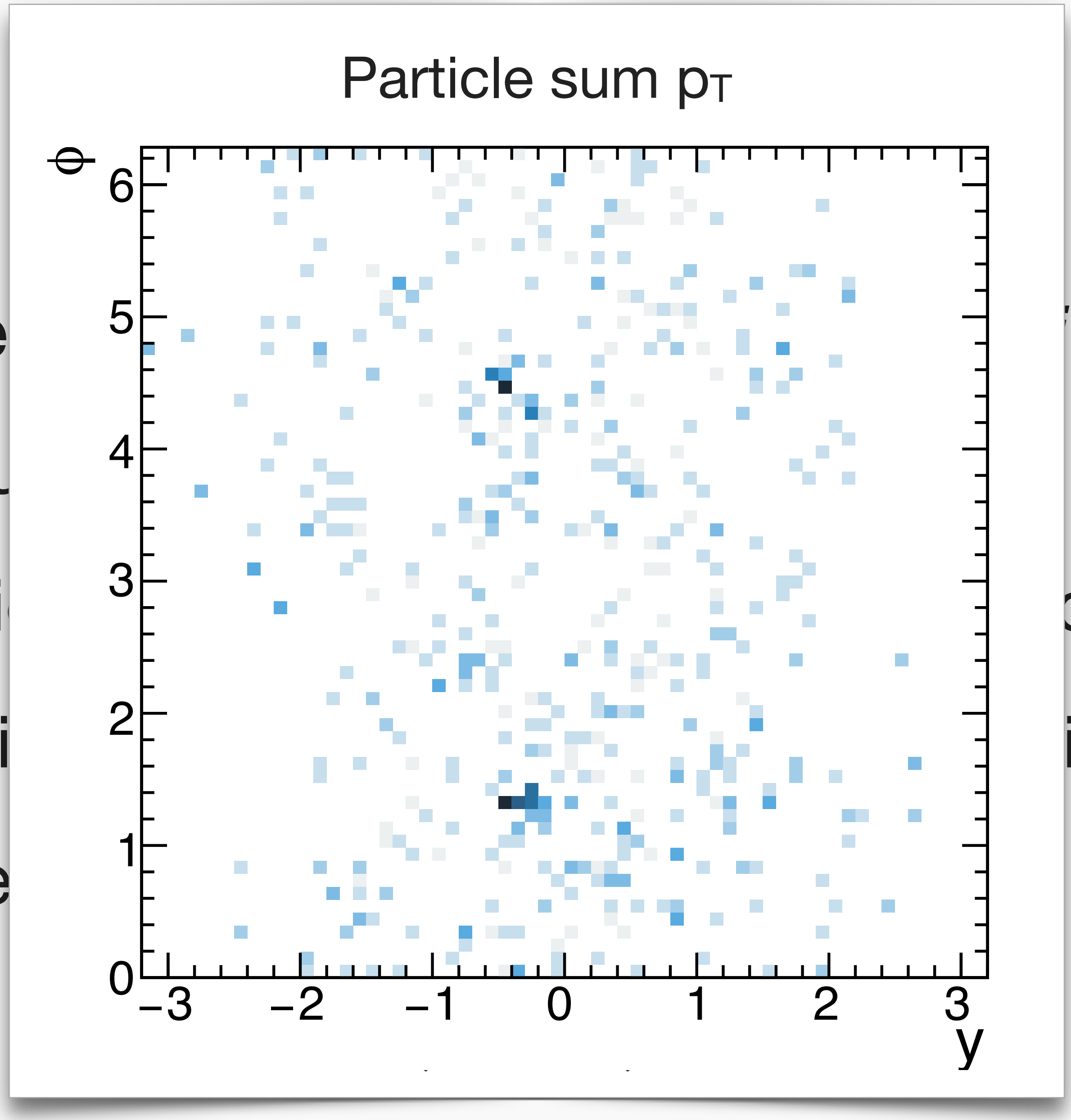
- For HEP purposes: Discrete 2D (y,ϕ)-input

Wavelet fundamentals



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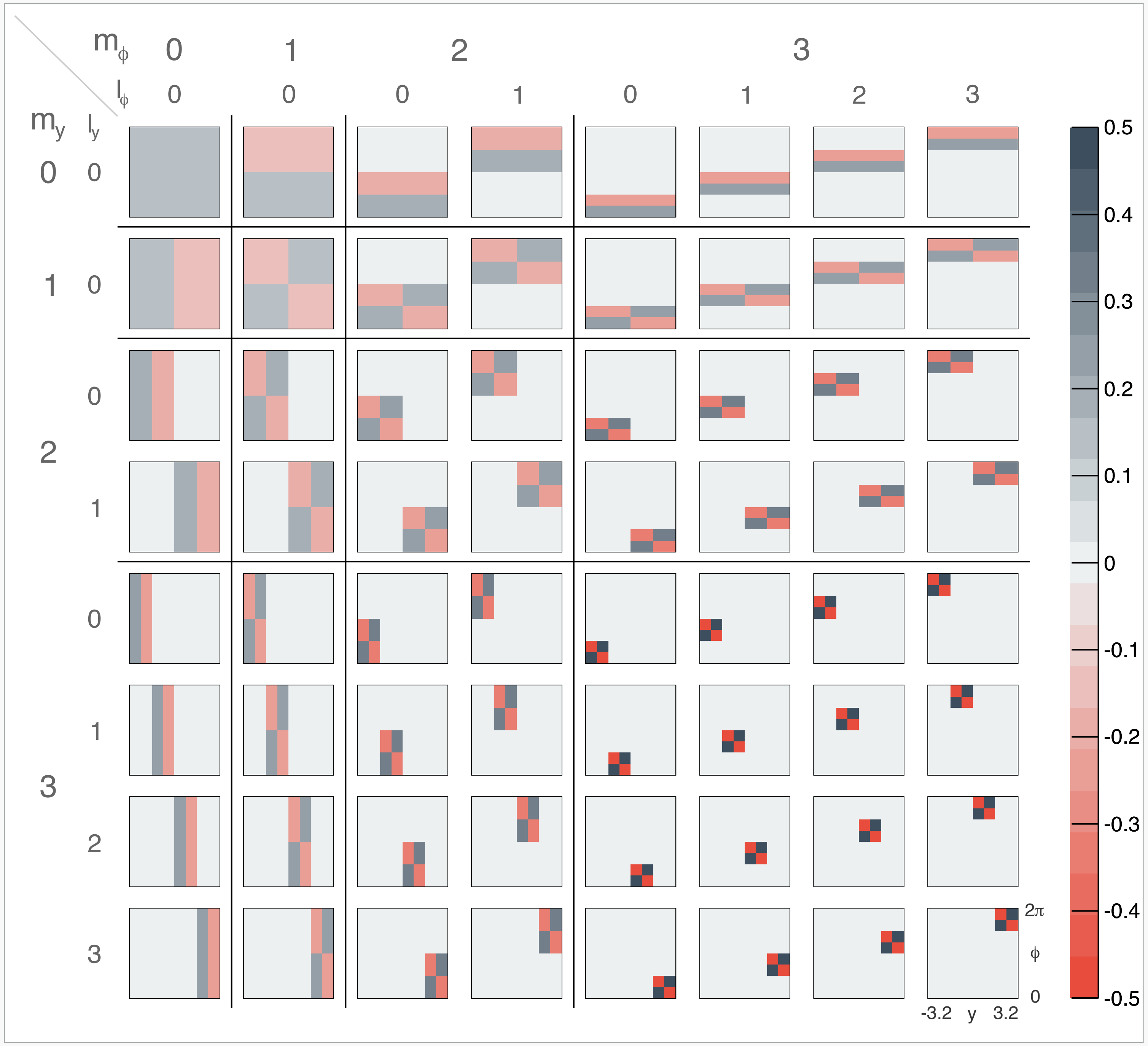
Basis functions · Haar



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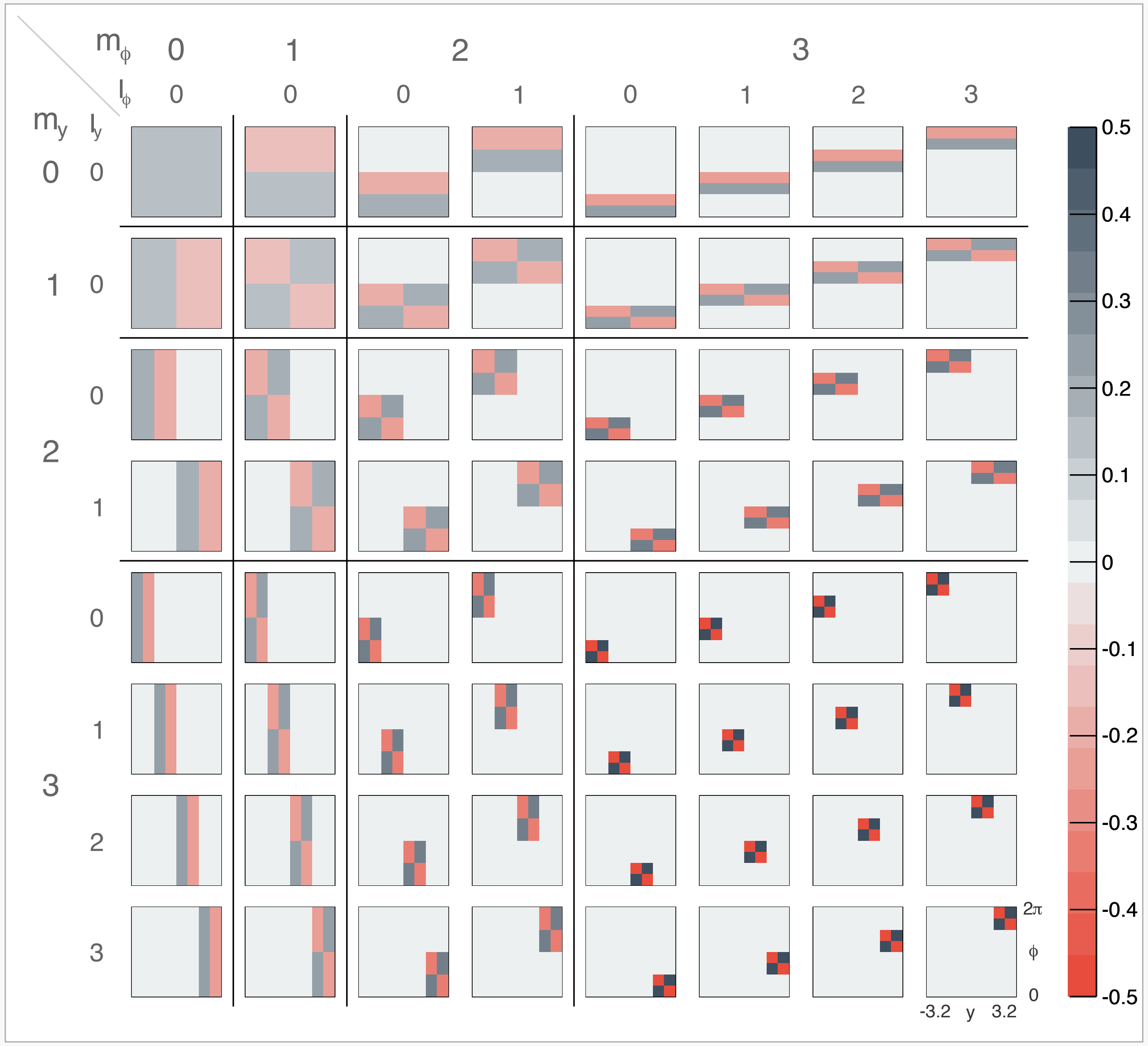

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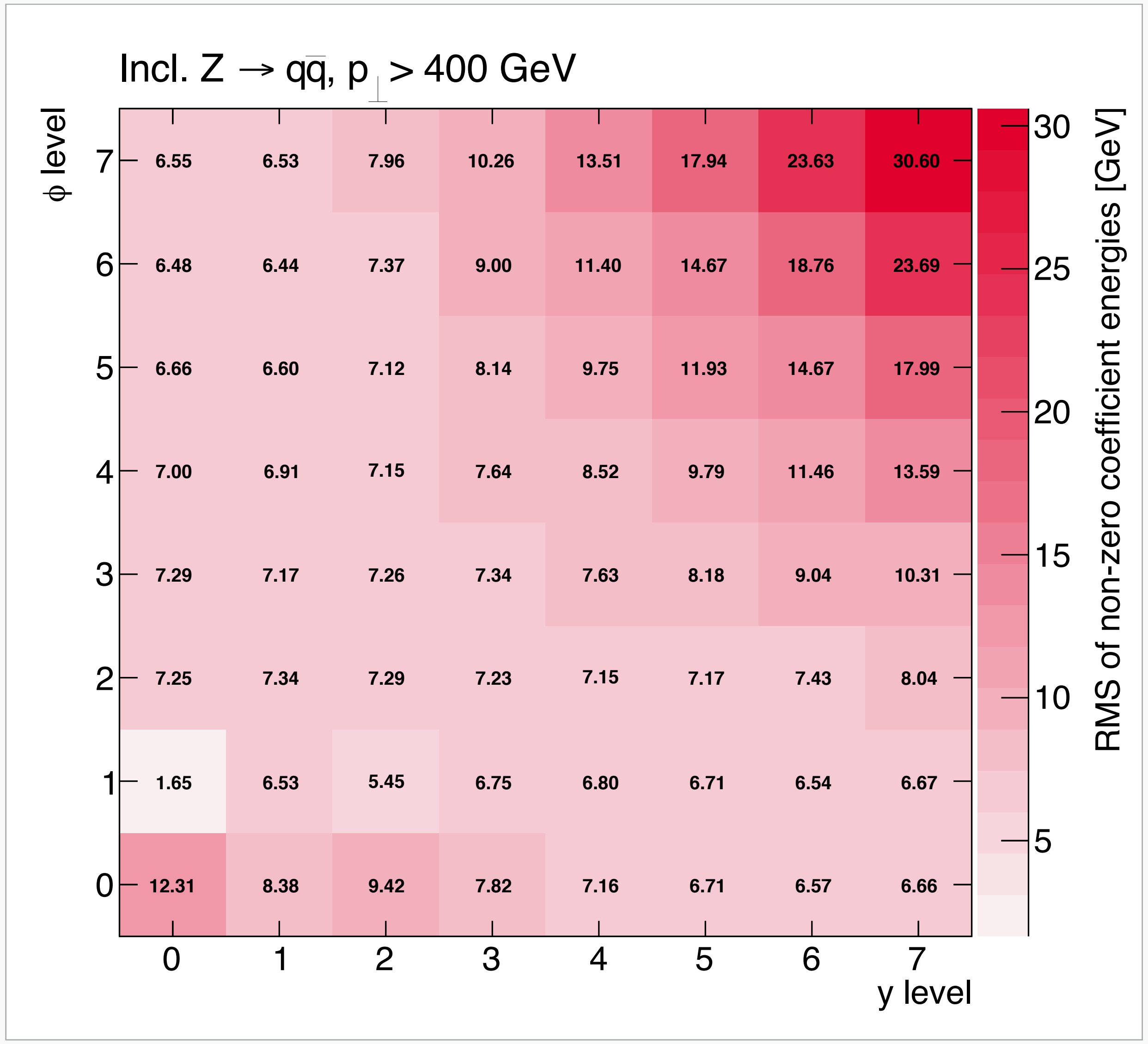

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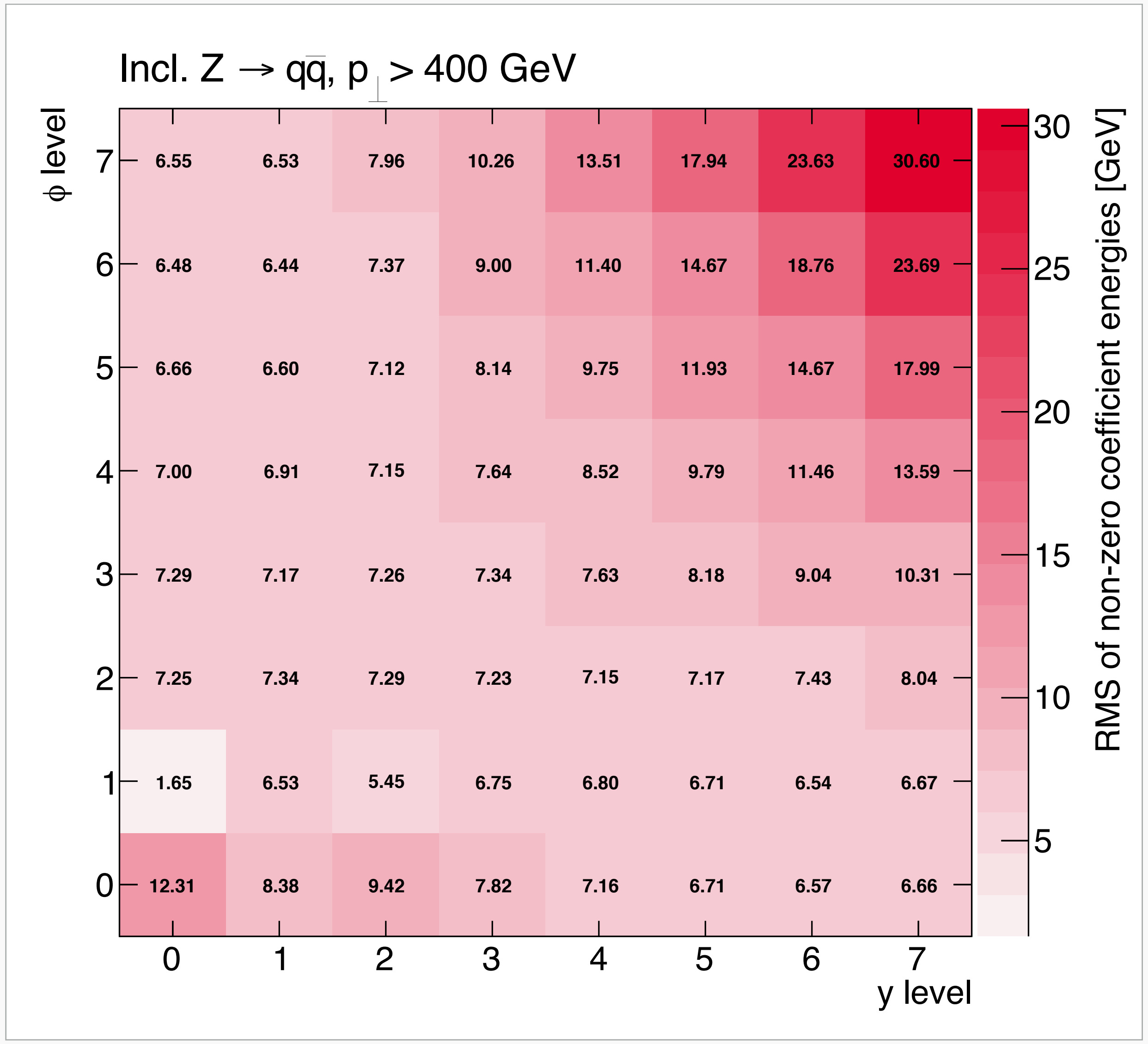

High frequency / small angles →

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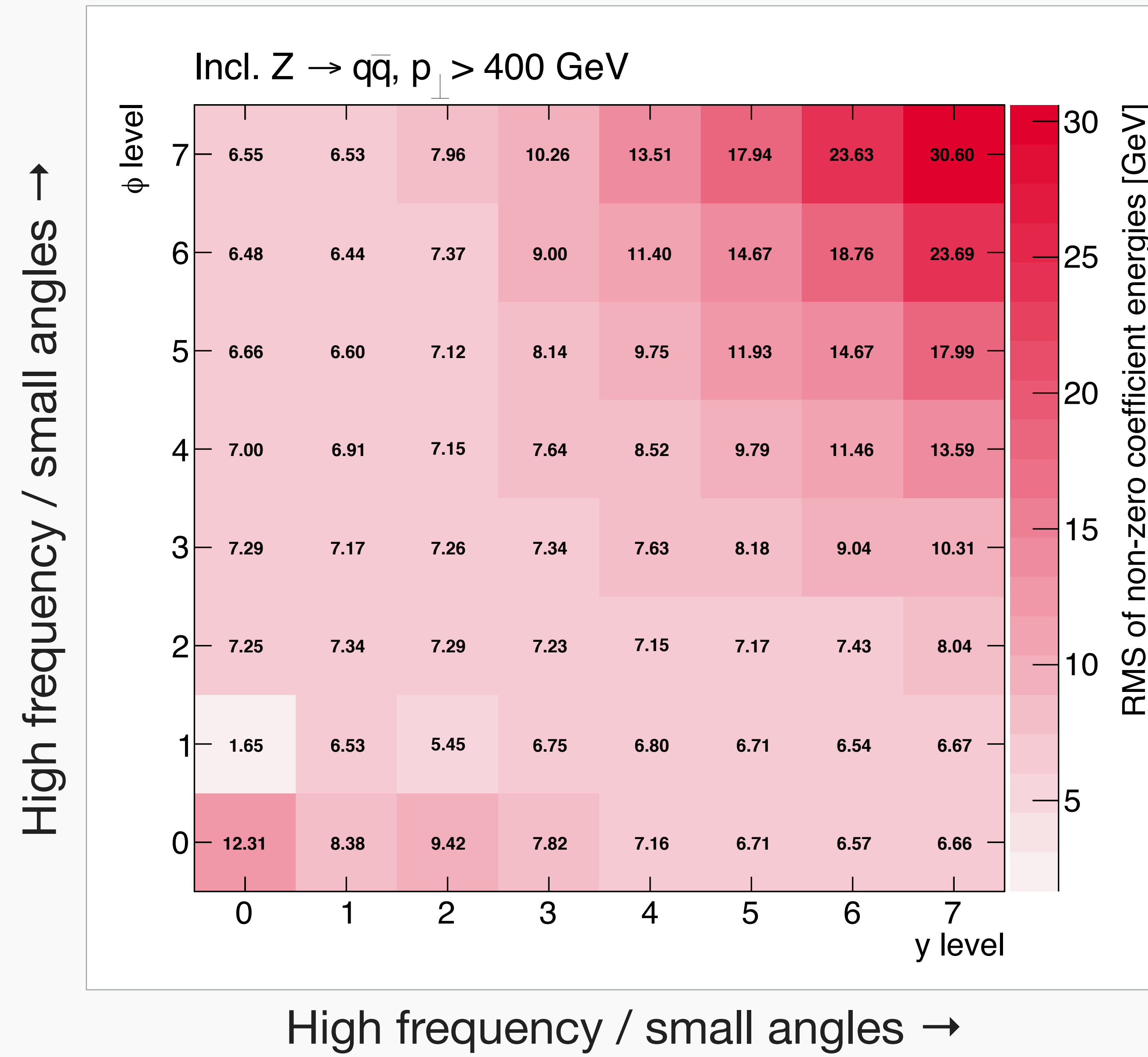
Structure of coefficient energies

- Hard scatter events:
  - Jets characterised by parton showering
  - Should be dominated by small-angle activity
- Pile-up:
  - “White noise”
  - No angular structure: constant activity across frequency bands



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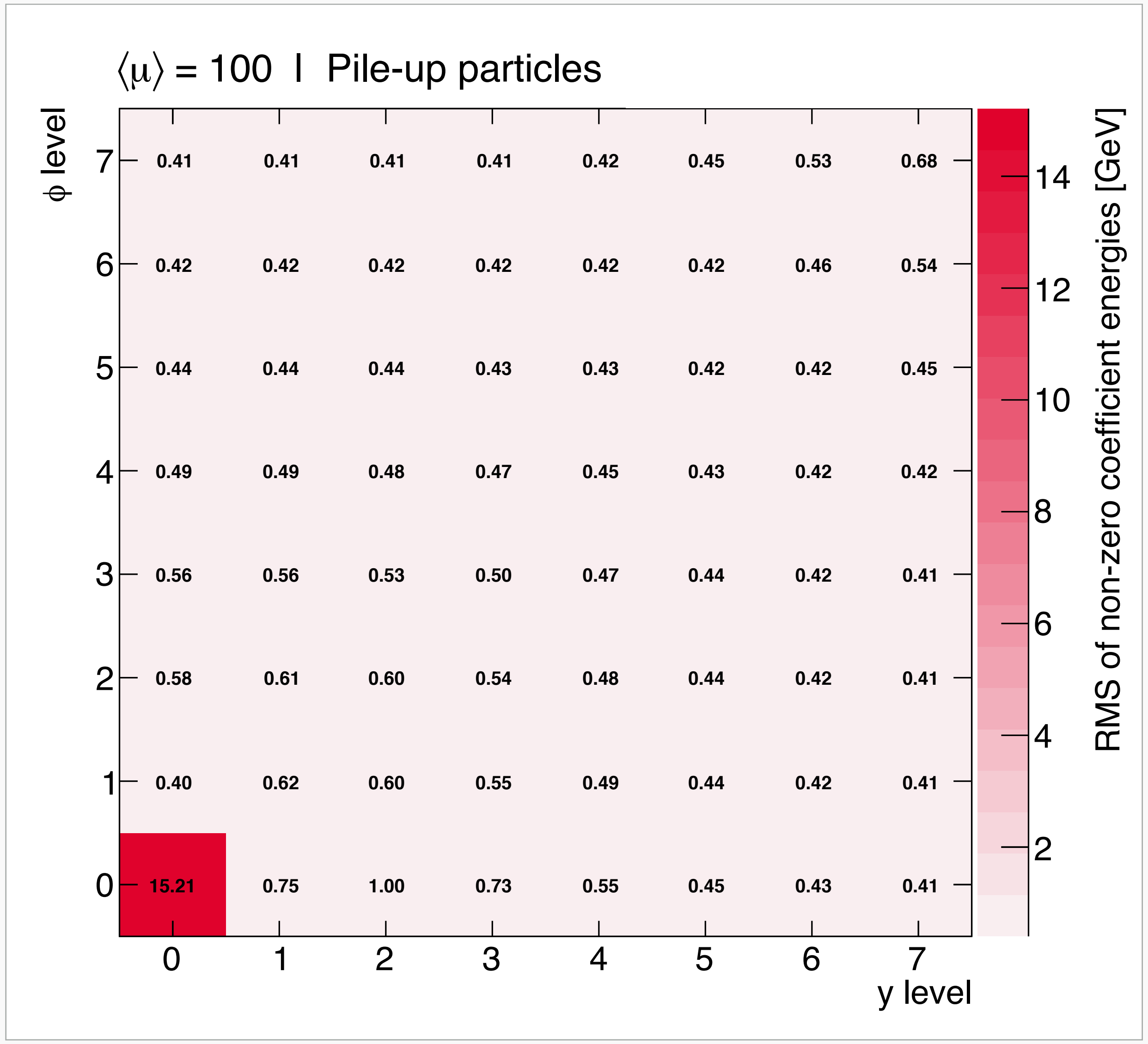

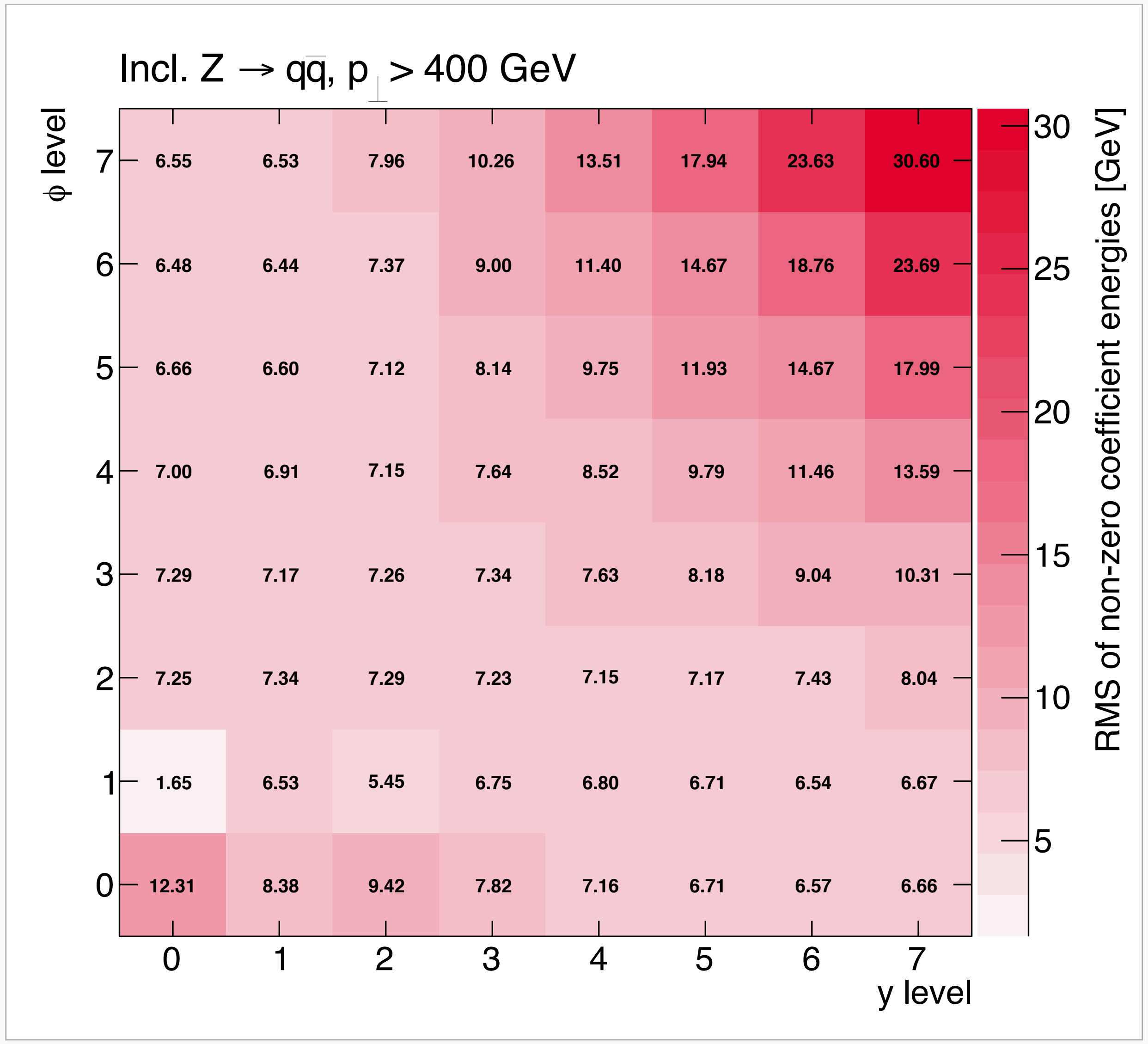

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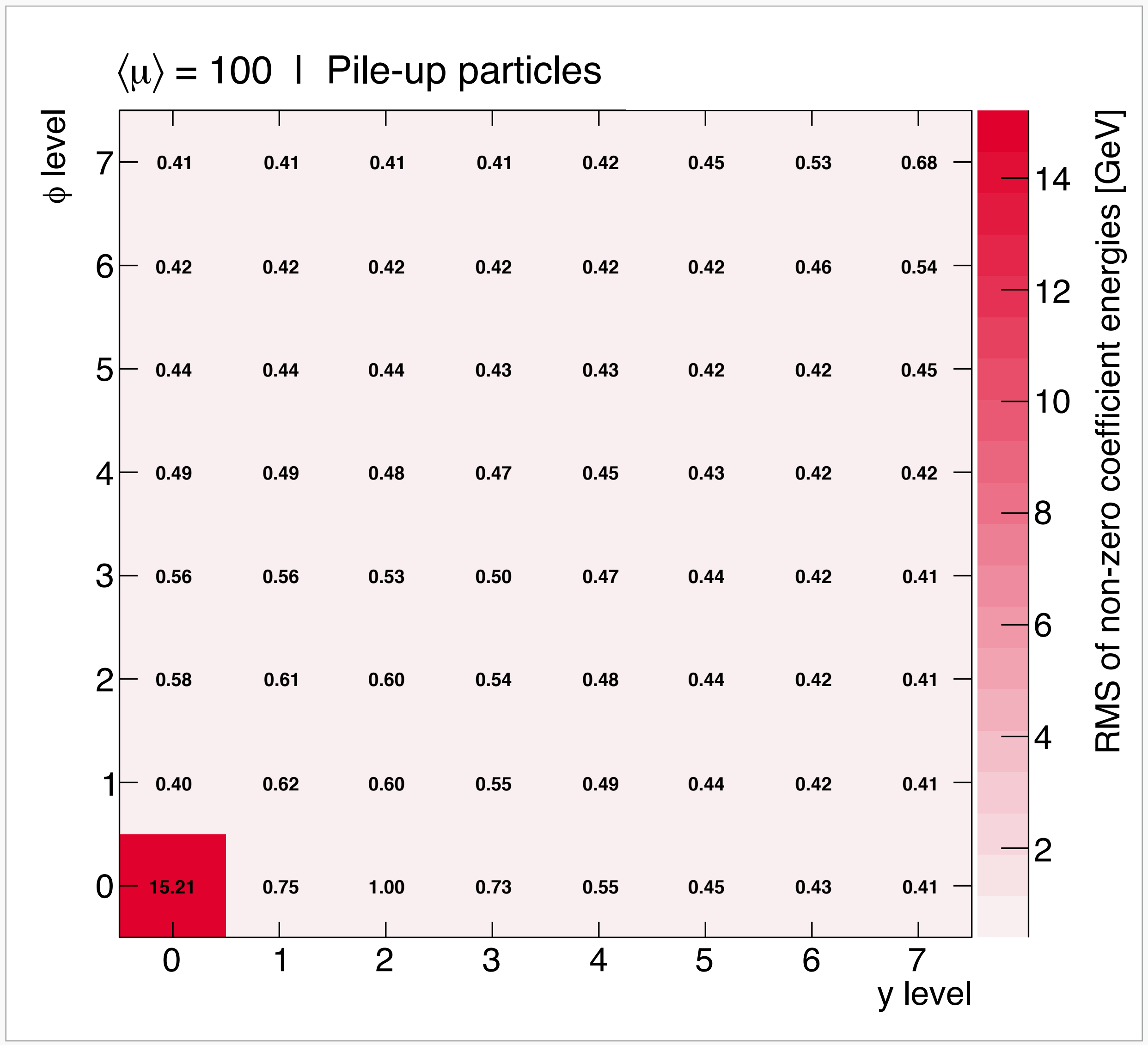
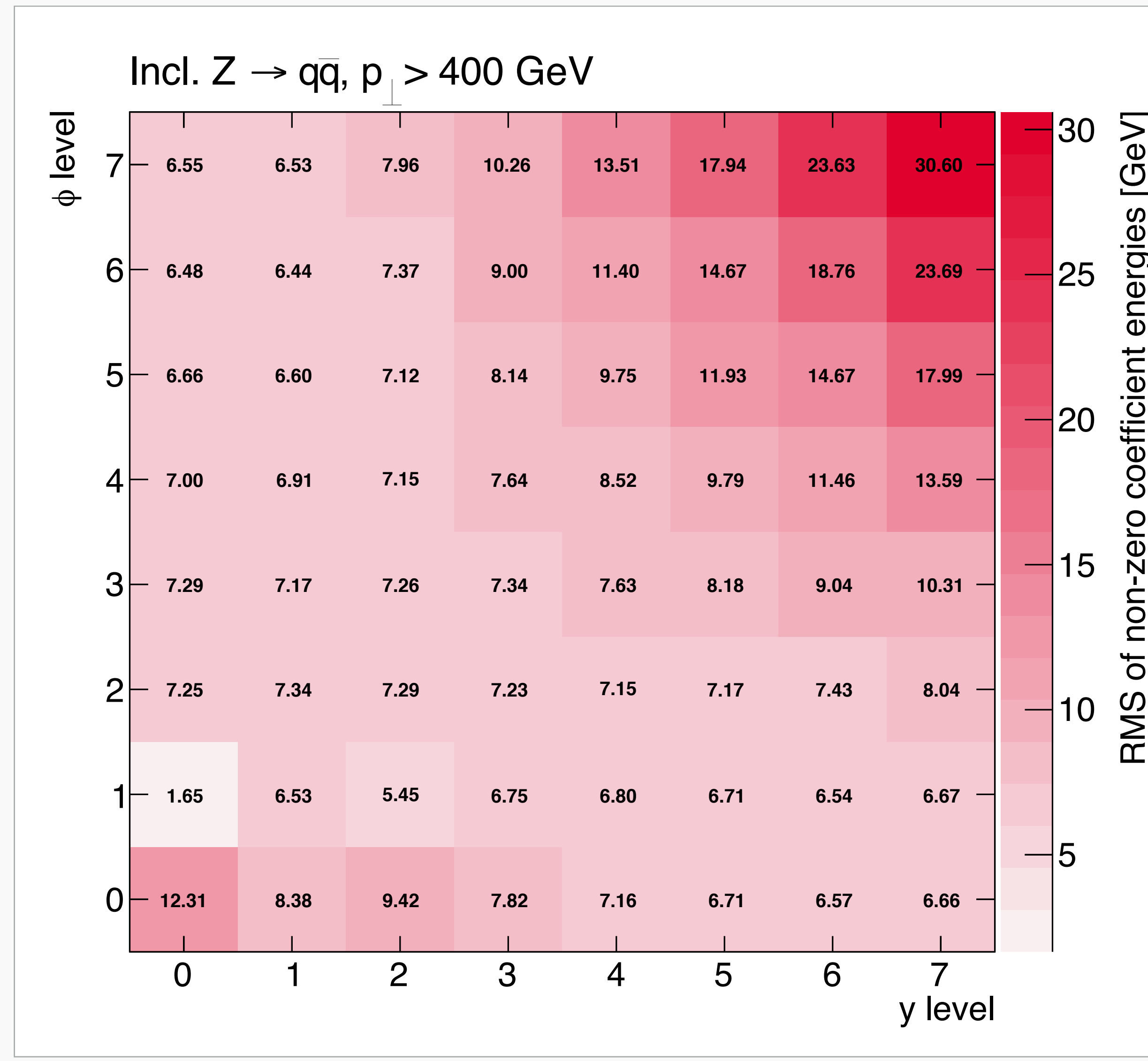
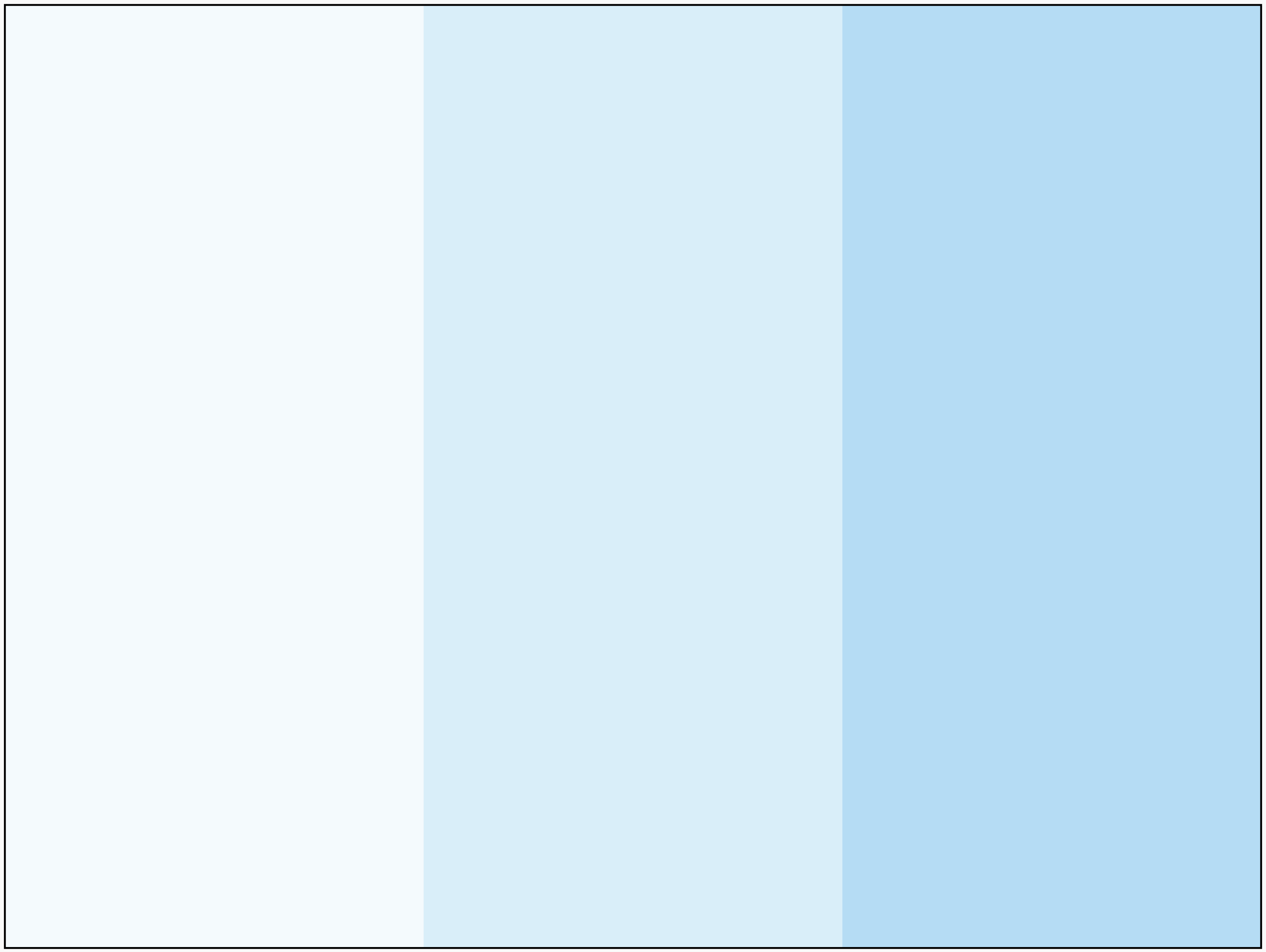




This difference may allow for good separation

Wavelet analysis



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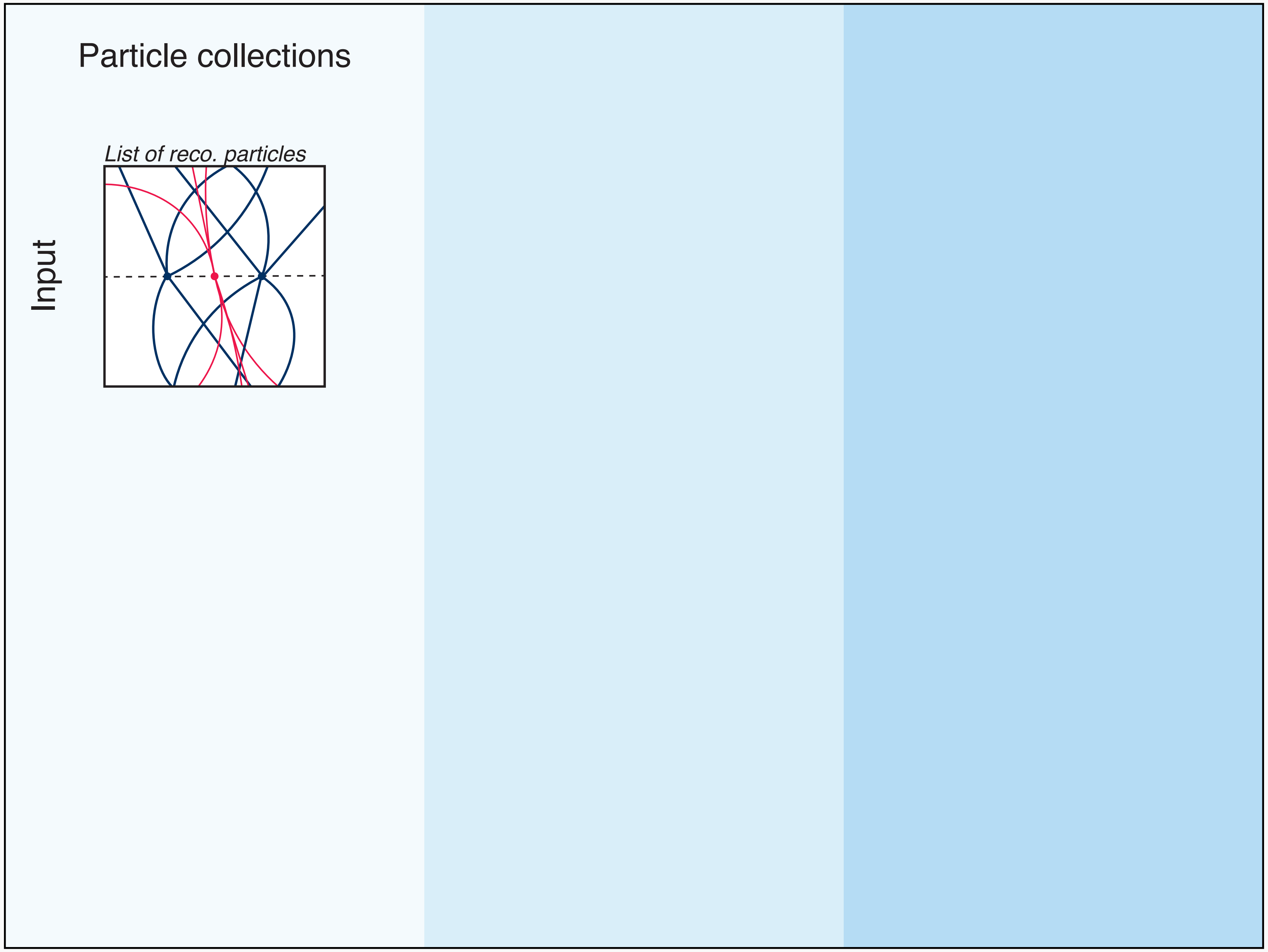

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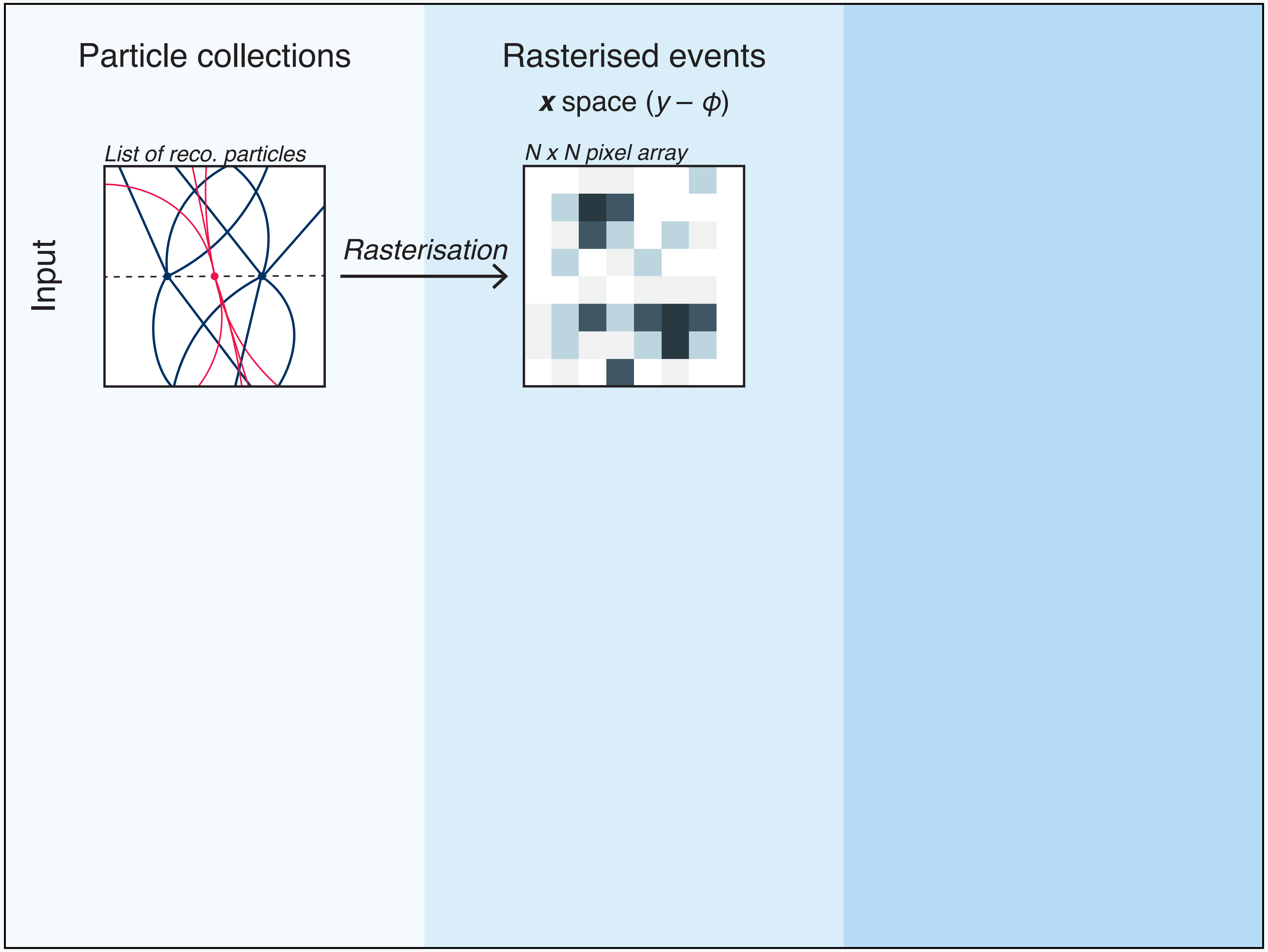

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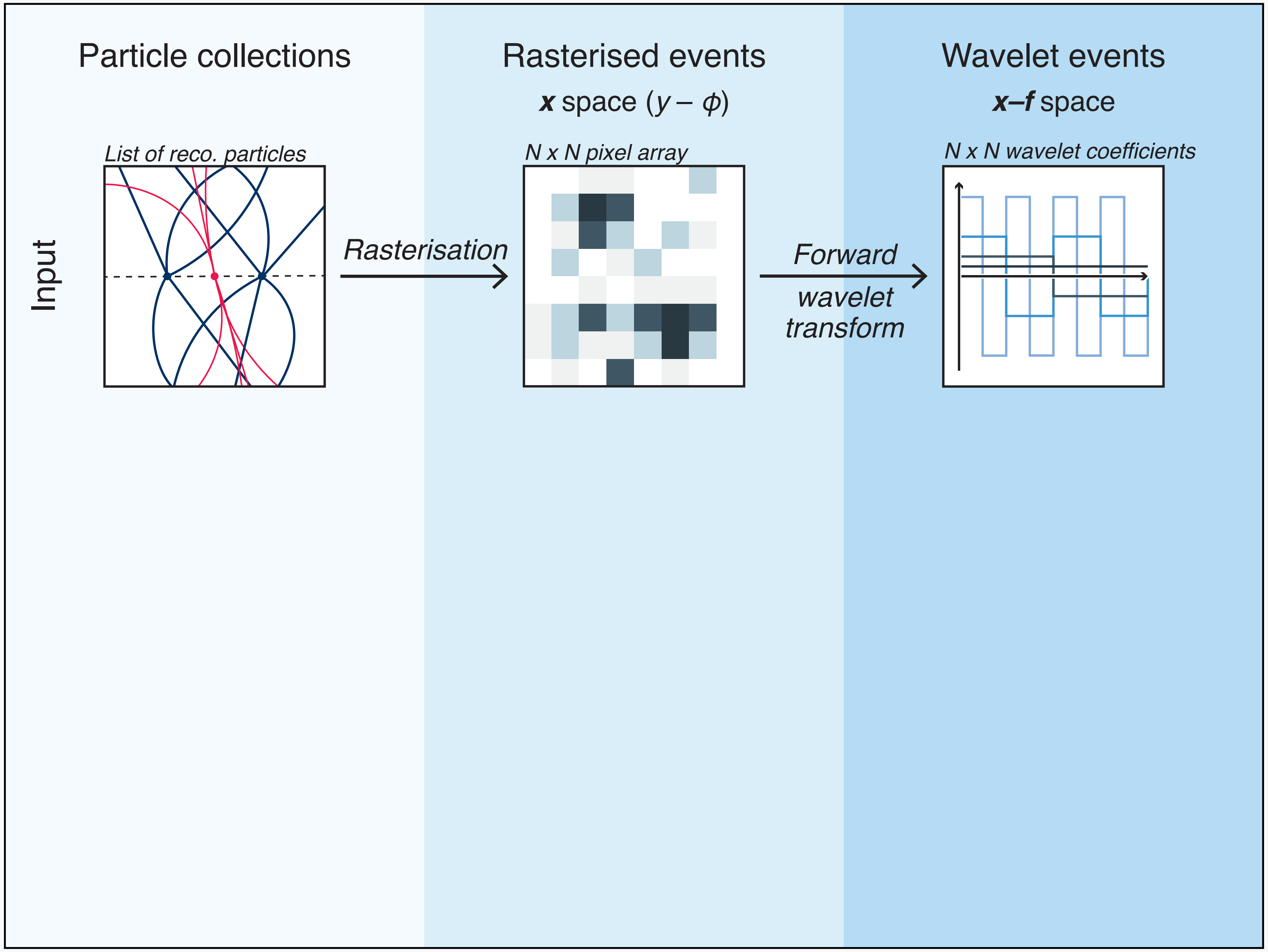

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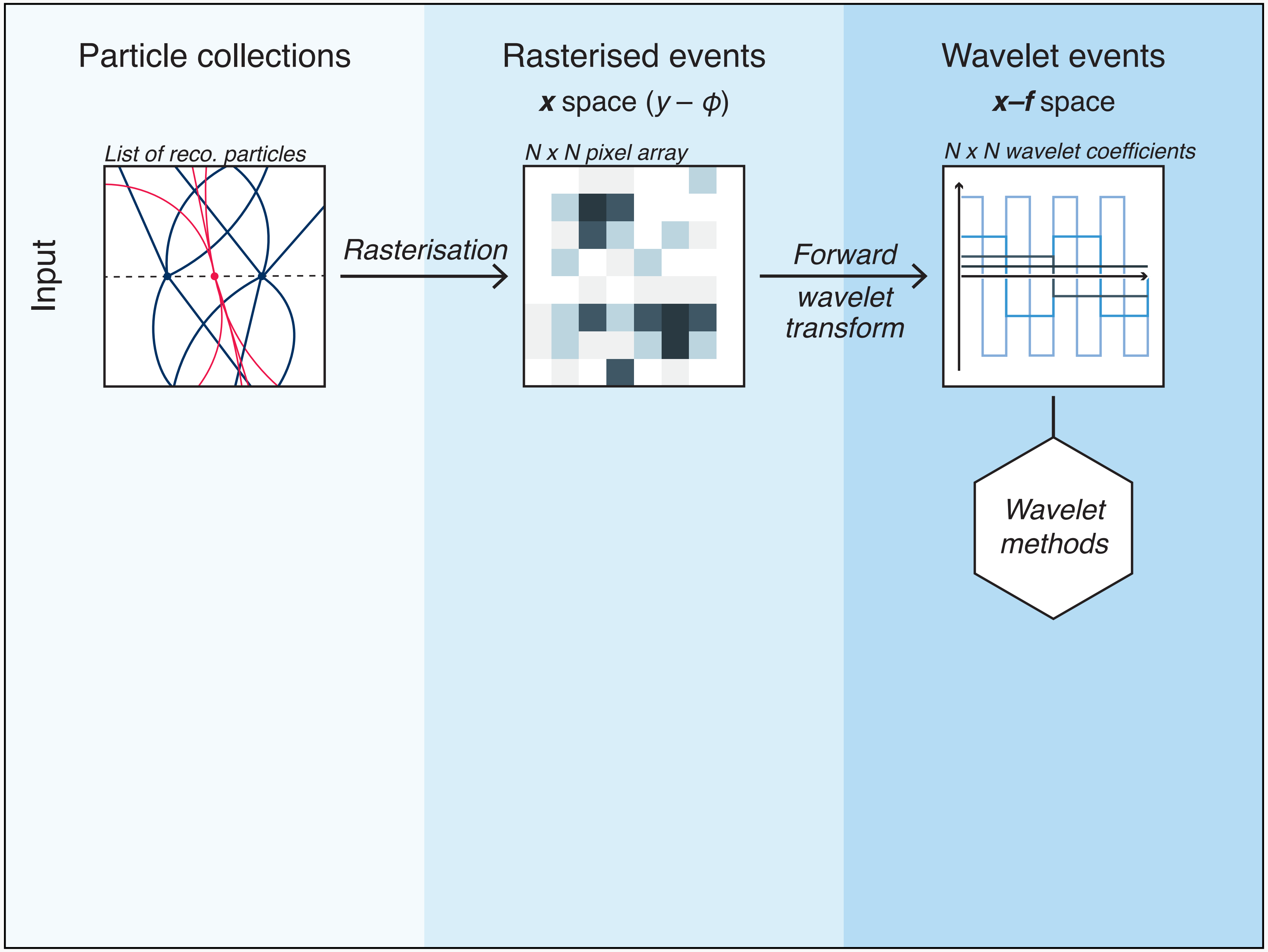

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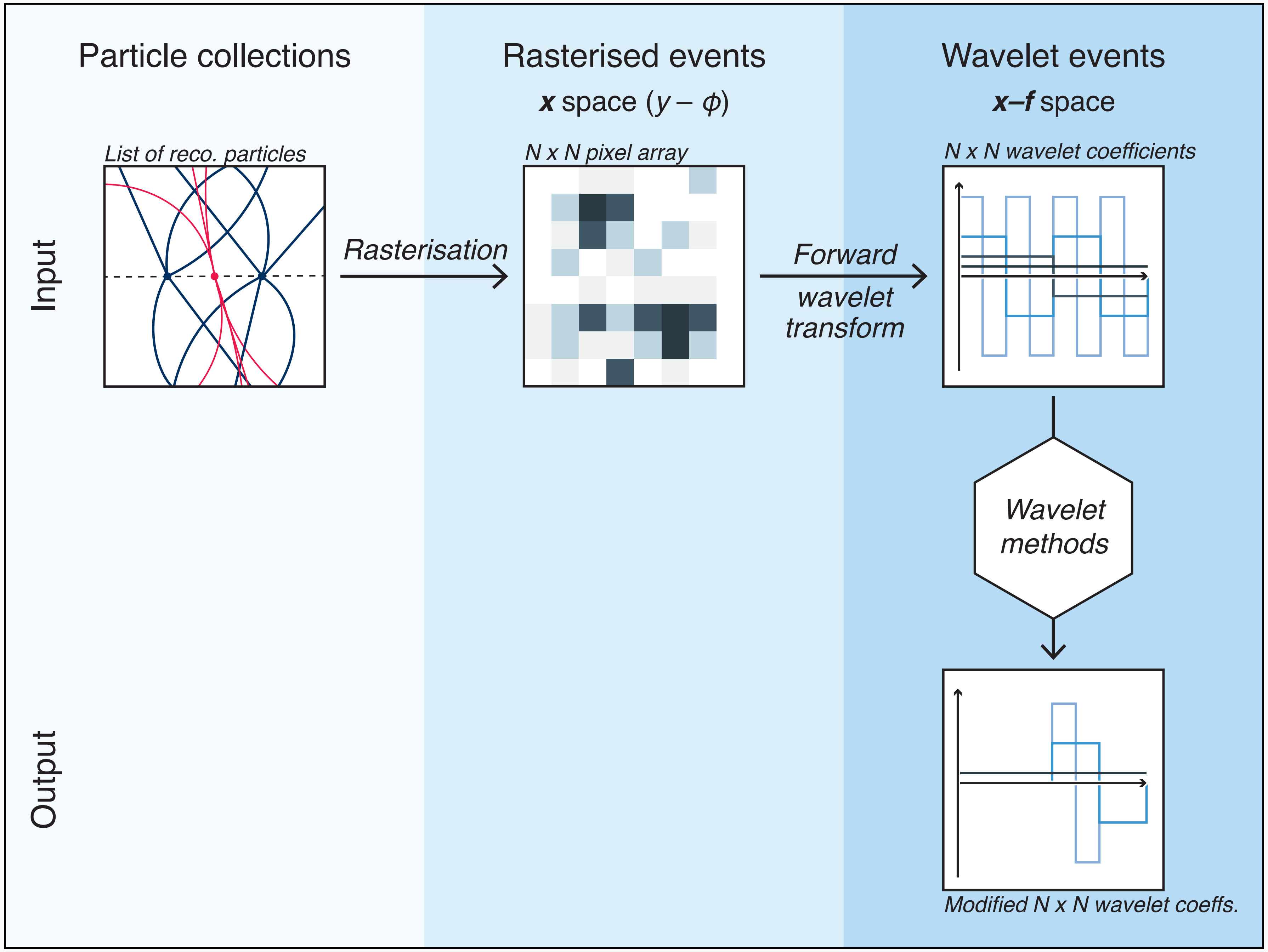

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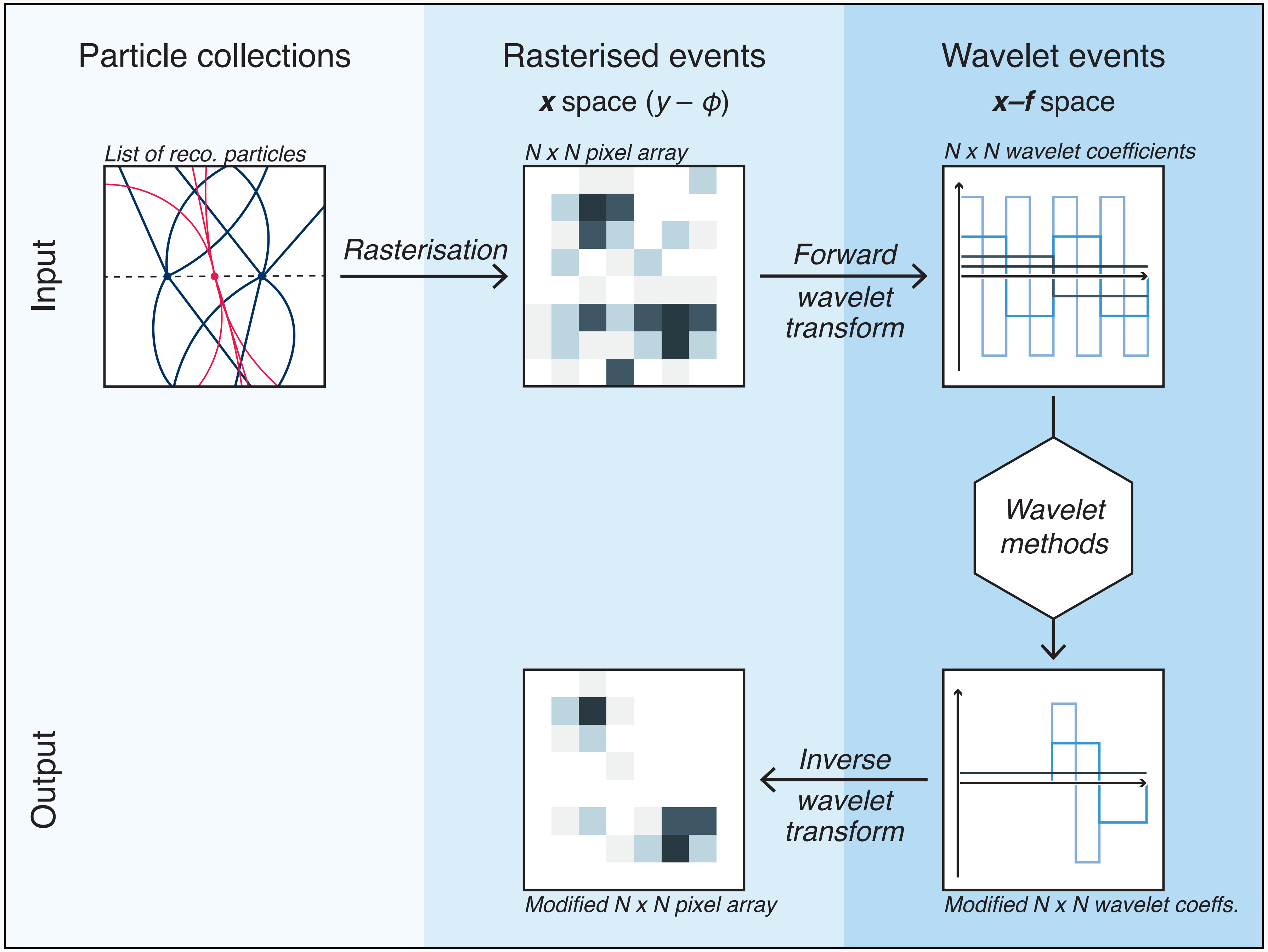

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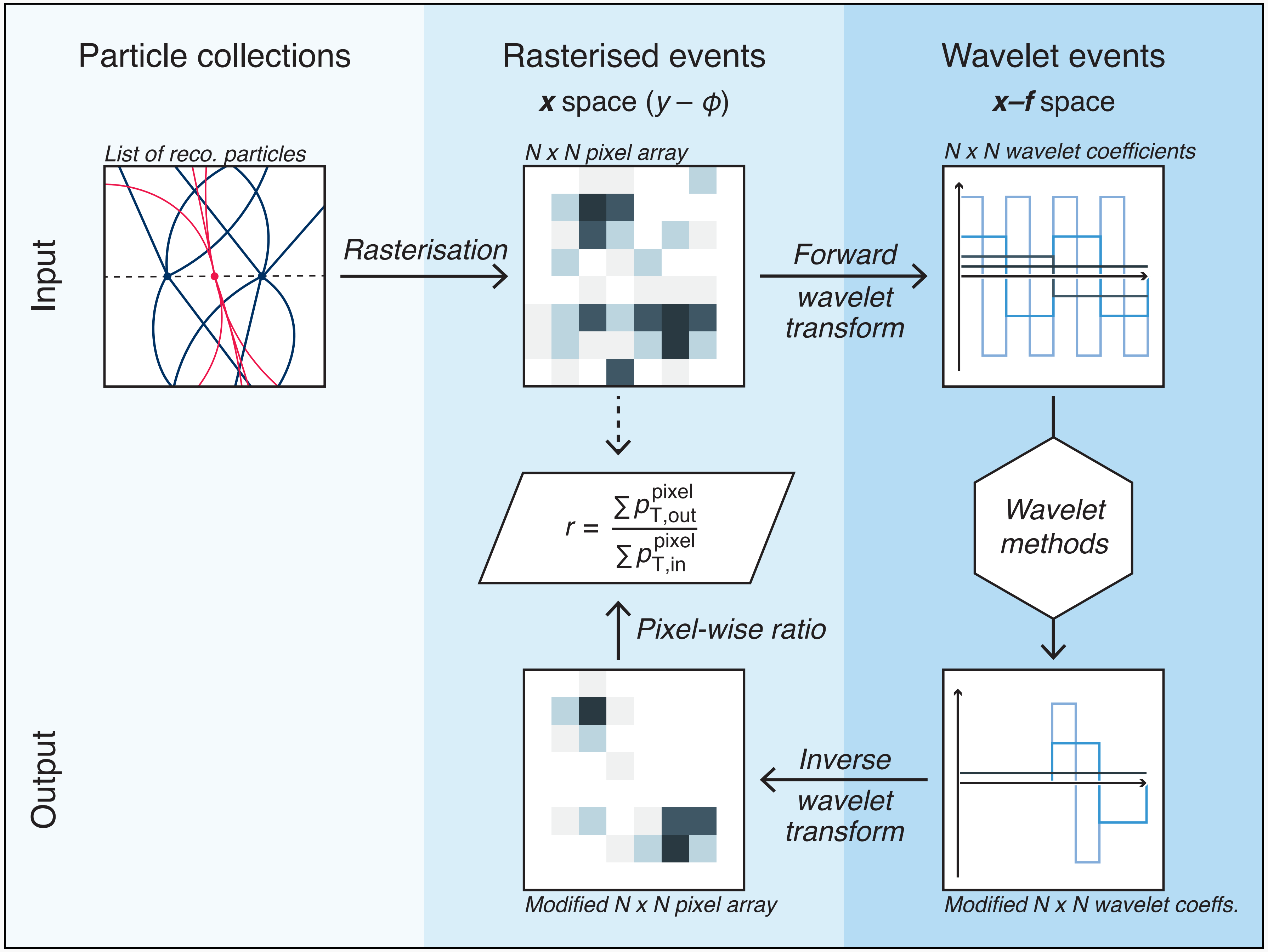

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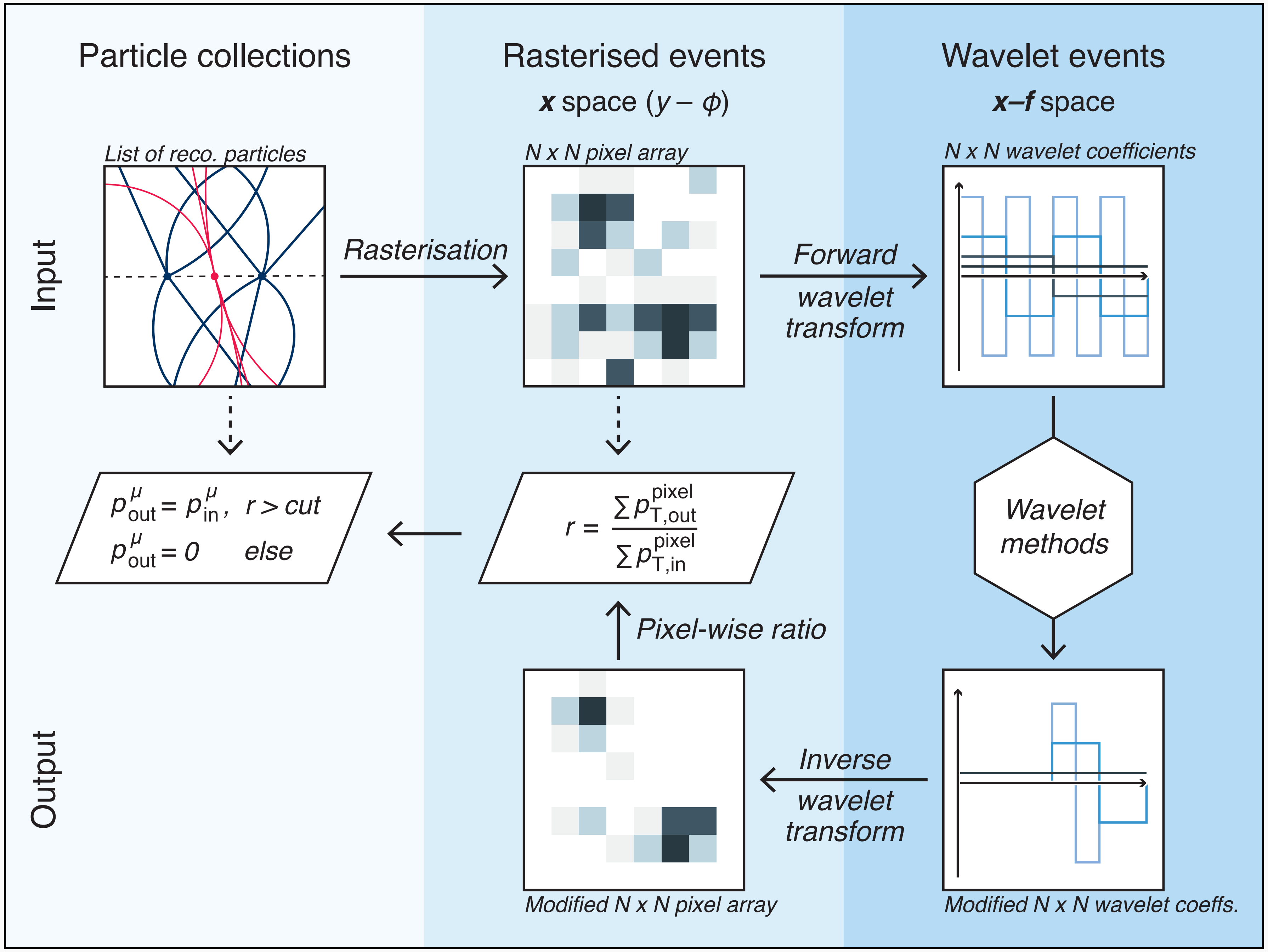

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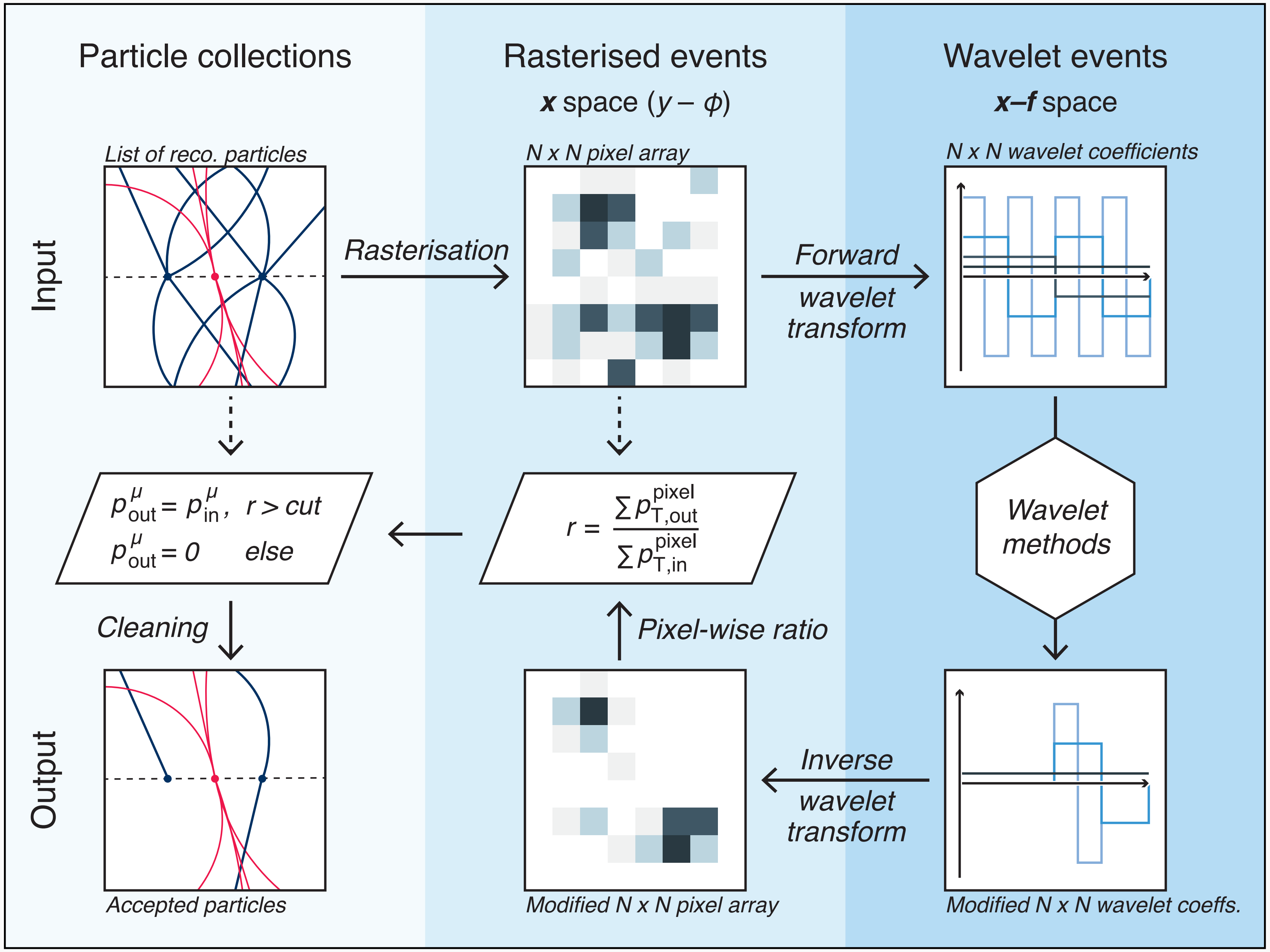

Methods for pile-up mitigation

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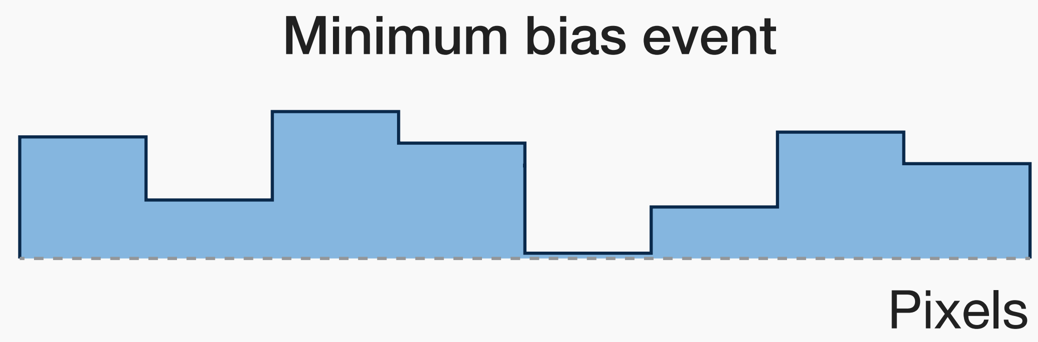

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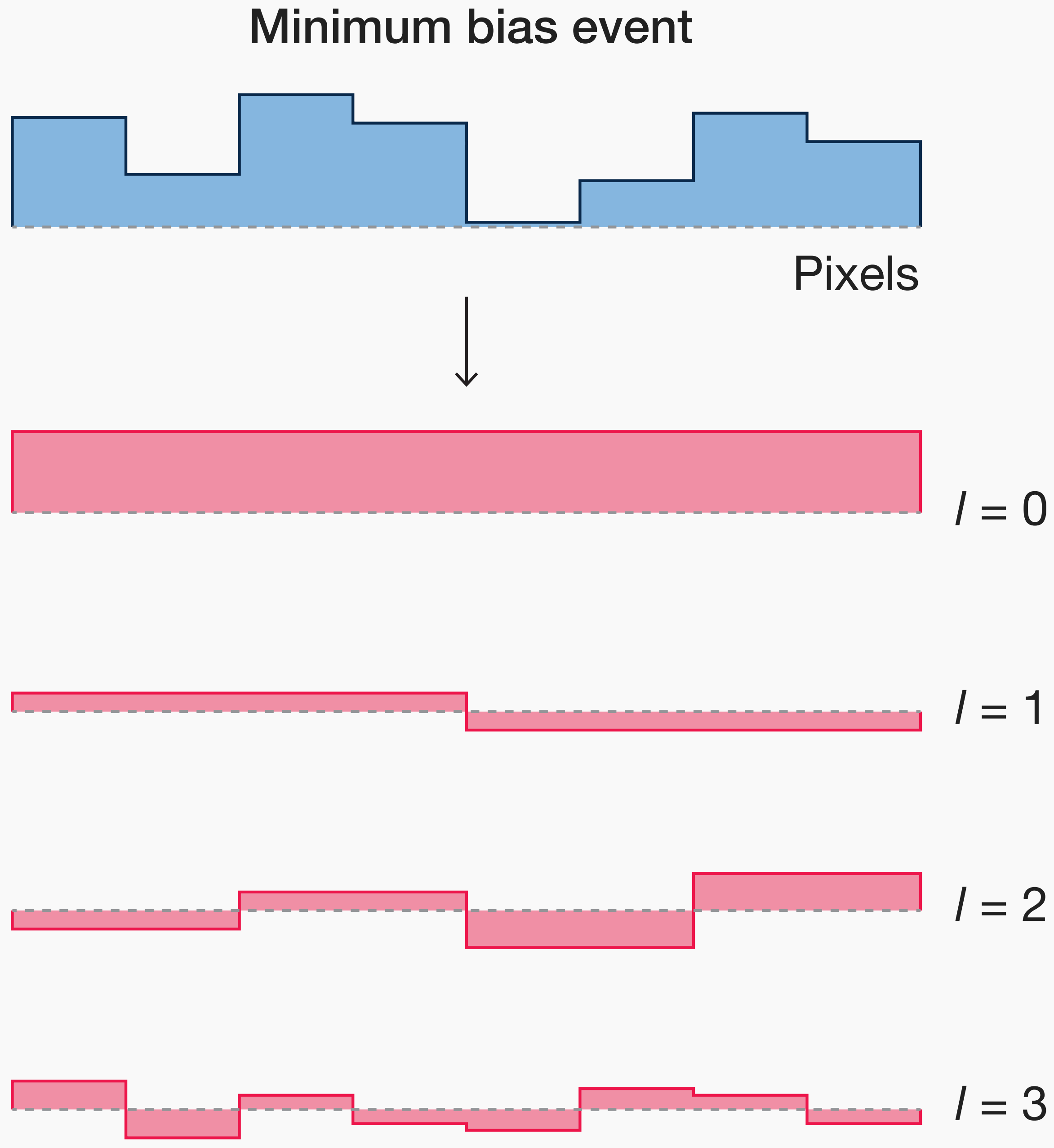

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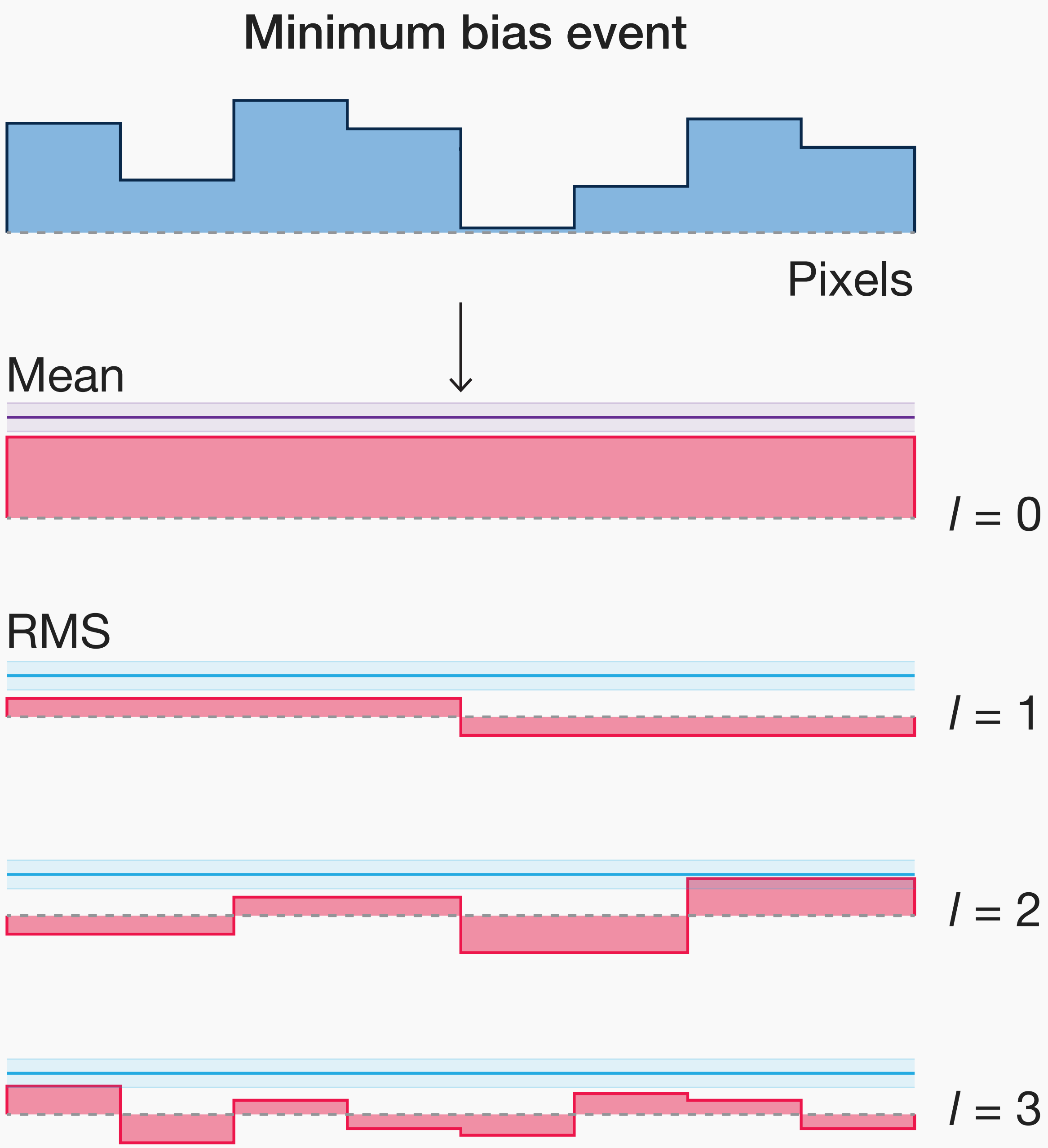

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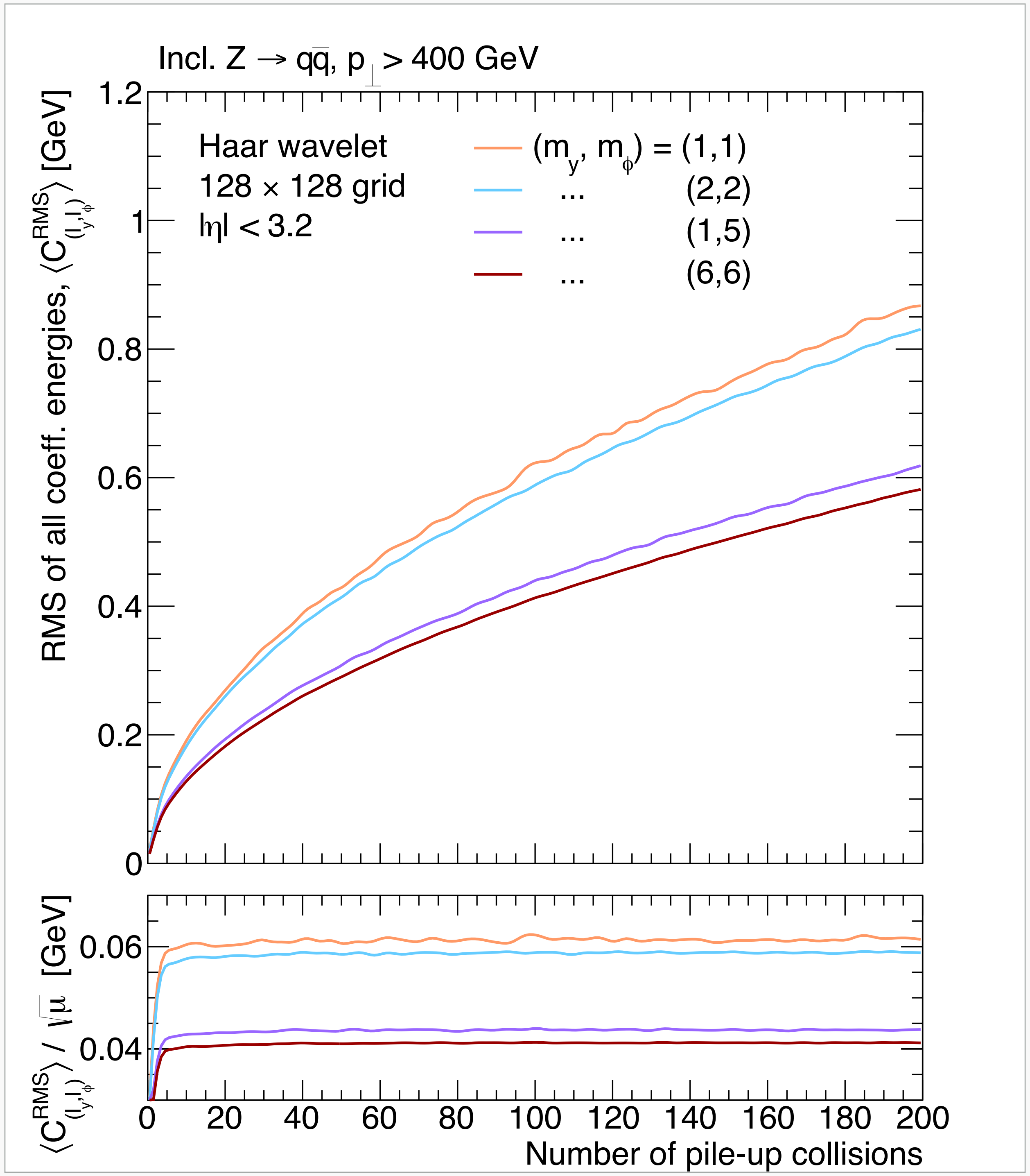

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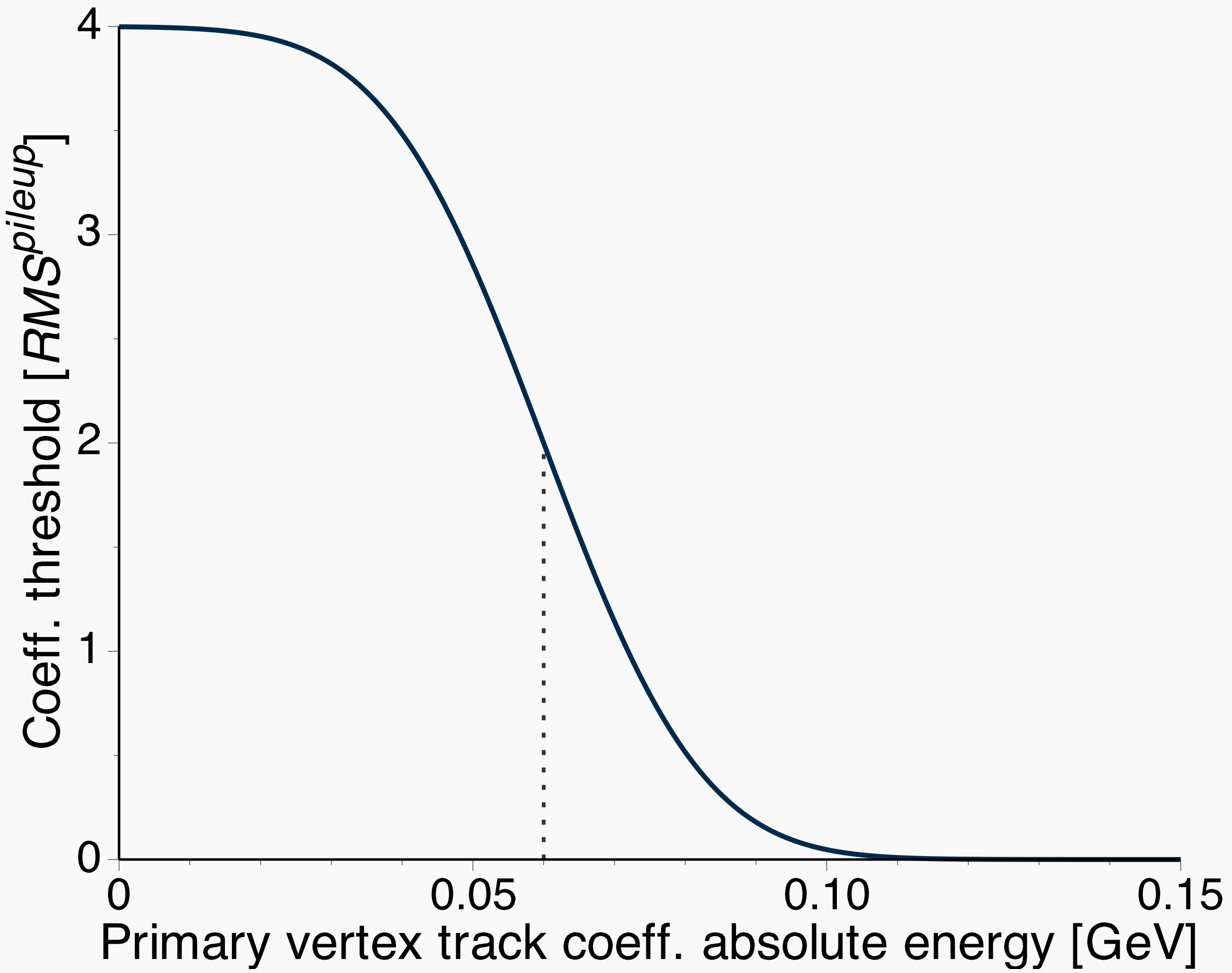
Improvement:

- Use track information



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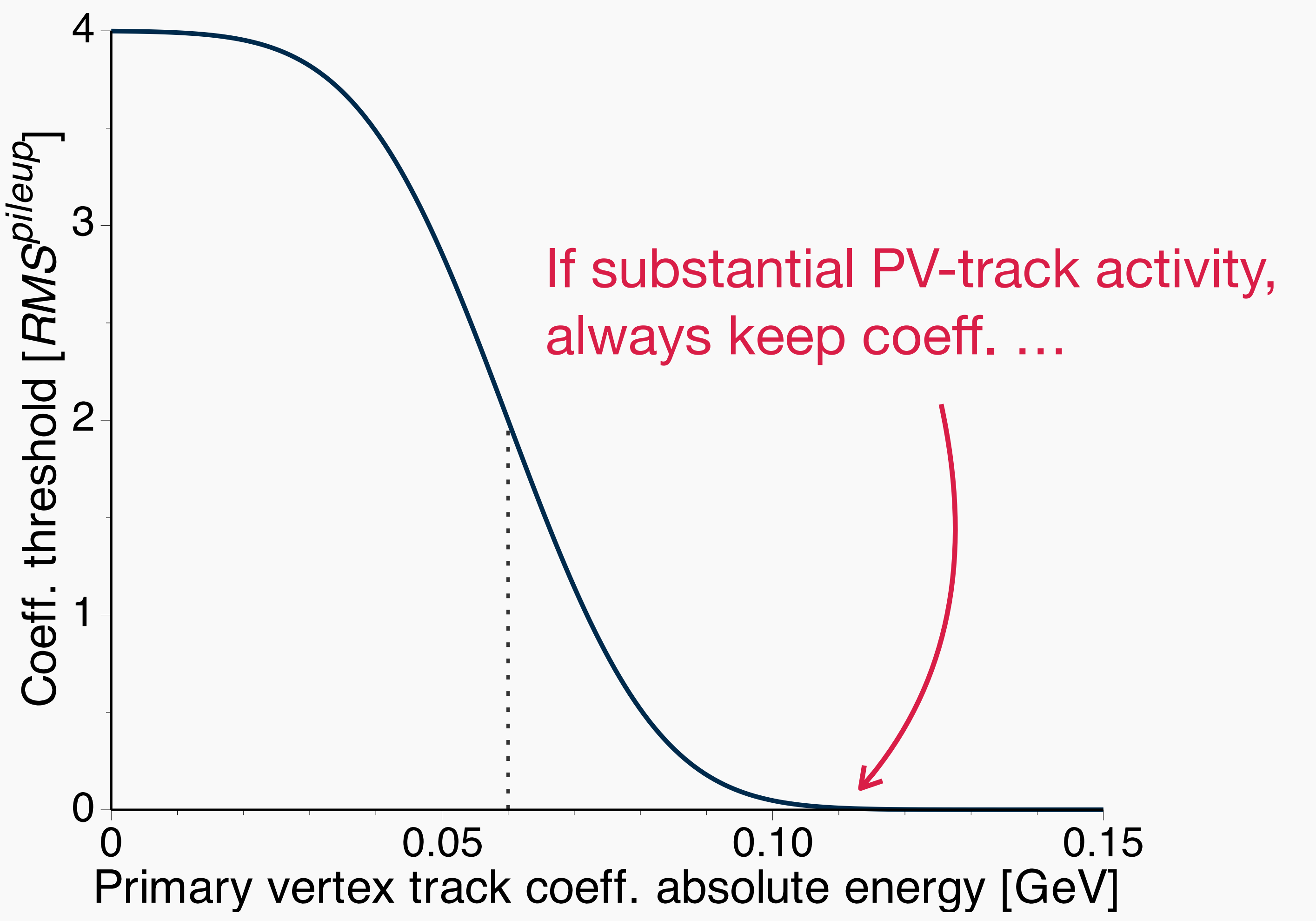

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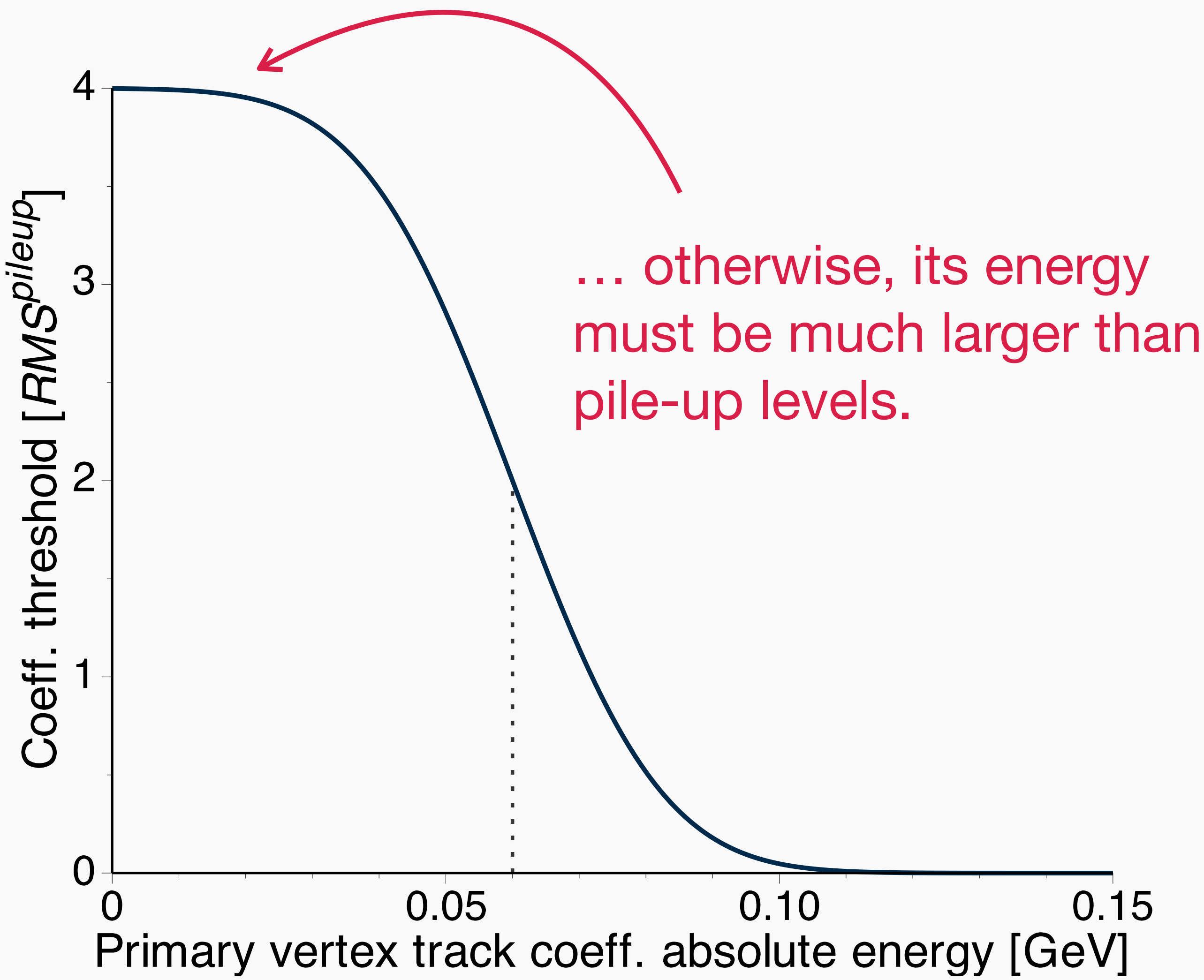

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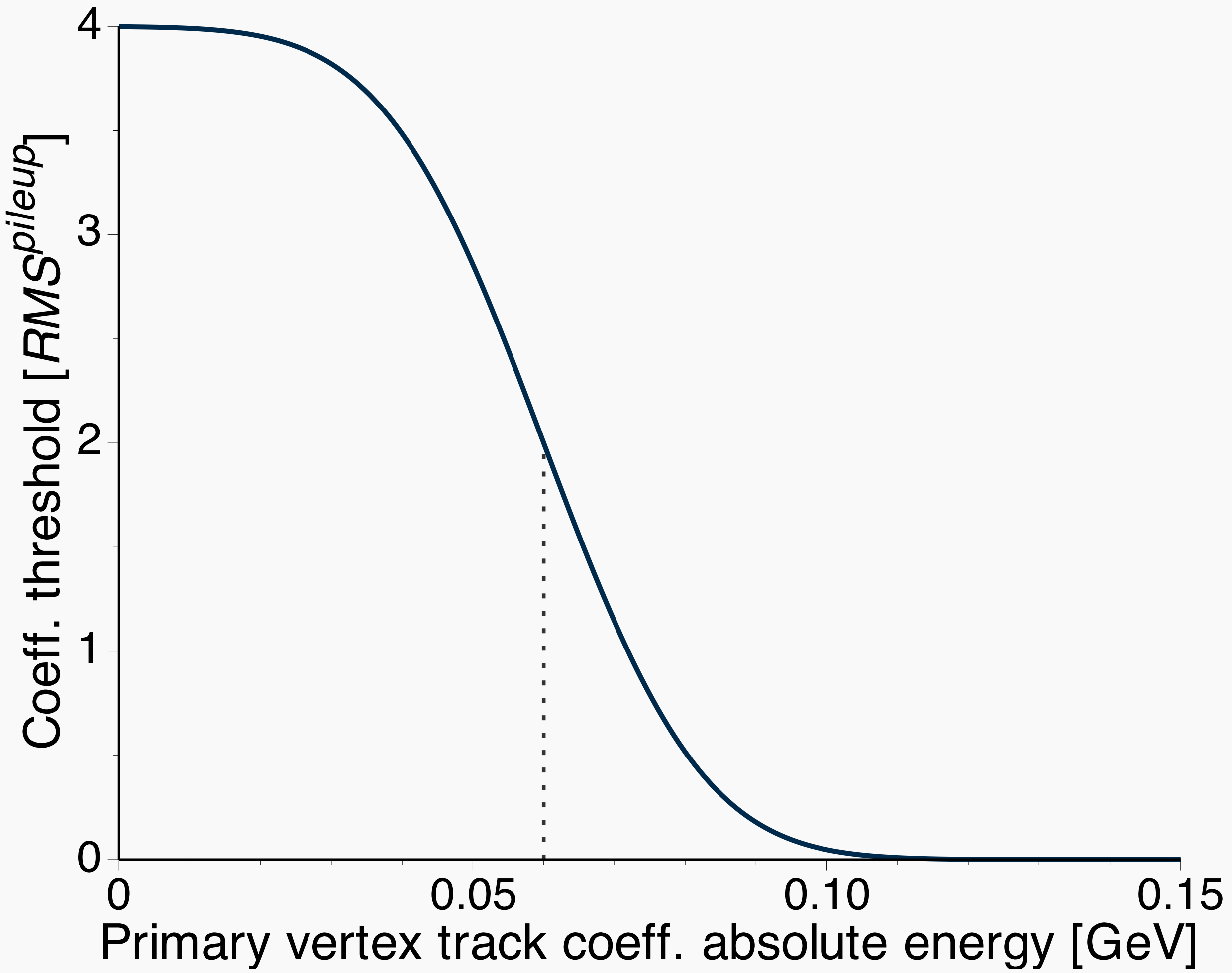
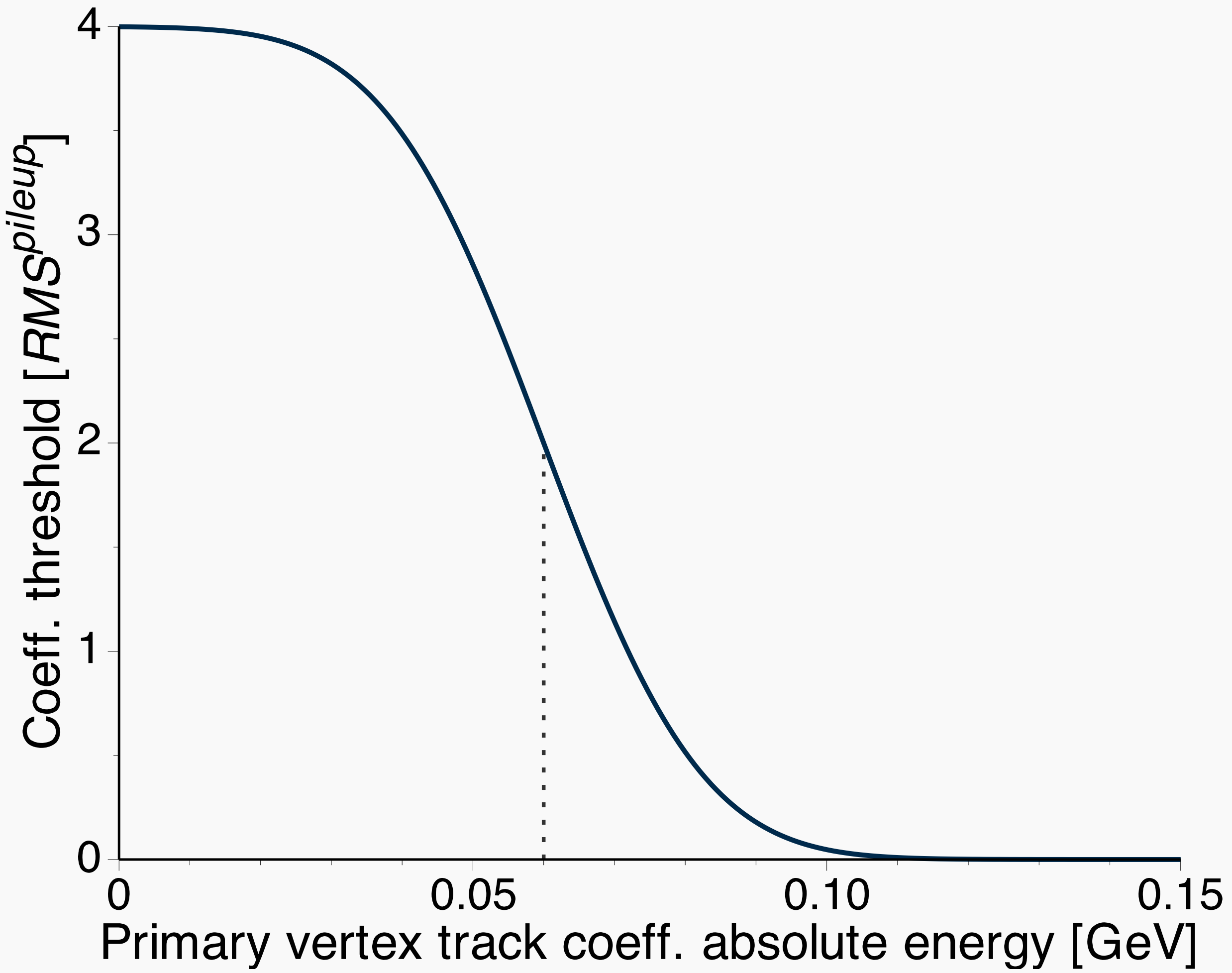
Improvement:

- Use track information
- 'Wavelet onset'



Wavelet setup

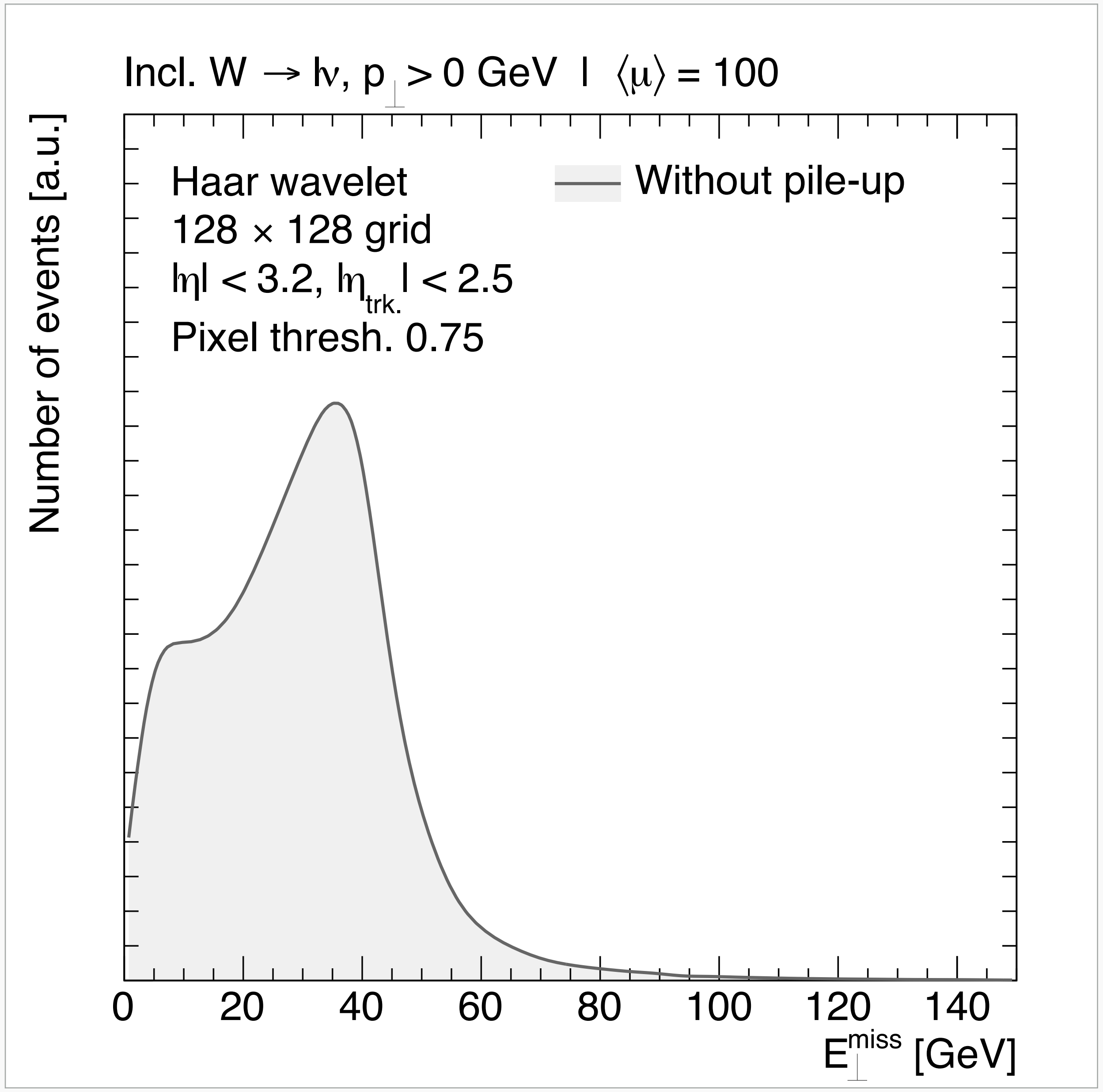
- 128 x 128 pixel grid
- |y| < 3.2 (square)
- Haar wavelet (simplest)
- ‘Wavelet onset’ method with max. of 4 x pileup RMS
- Keep pixels with final-to-initial ratio > 0.75

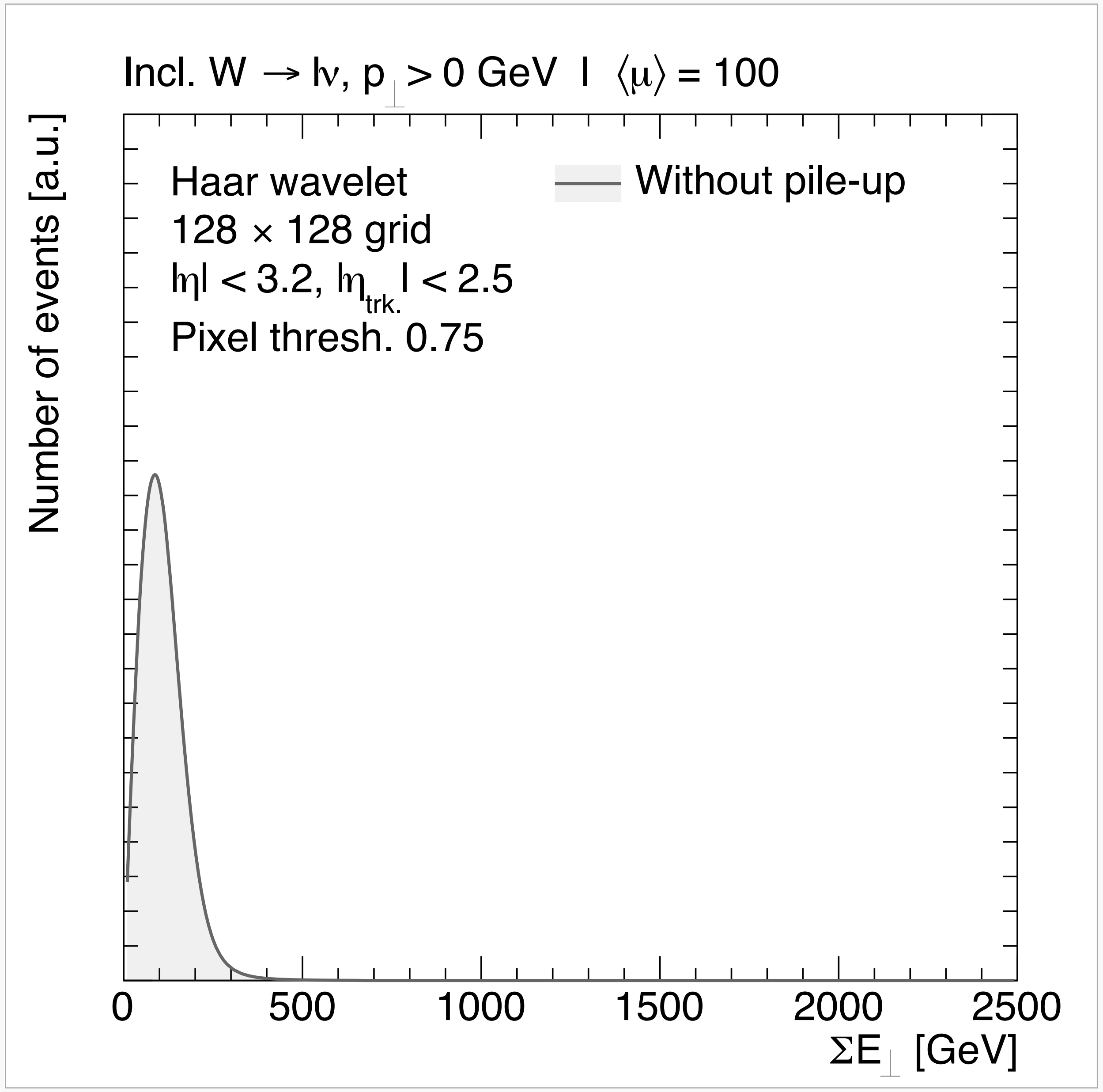


Samples and setup

- Selection:
  - Stable, visible final state particles
  - All particles $|\eta| < 3.2$, tracks $|\eta| < 2.5$
  - $p_T > 500$ MeV
  - 100% tracking and vertex matching efficiency
- Signal samples (13 TeV, PYTHIA 8.205, A14-NNPDF23LO tune):
  - $W \rightarrow l\nu$ – no gen. (reco.) level $\hat{p}_T$ (p_T) cut
  - Incl. $Z \rightarrow q\bar{q}$ – $\hat{p}_T$ (p_T) > 280 (400) GeV
  - Incl. QCD $2 \rightarrow 2$ multijets – $\hat{p}_T$ (p_T) > 280 (400) GeV
- Minimum bias samples: PYTHIA 8.205, A2-MSTW (MB) tune:
  - Overlaid using PILEMC.

Missing and sum E_T





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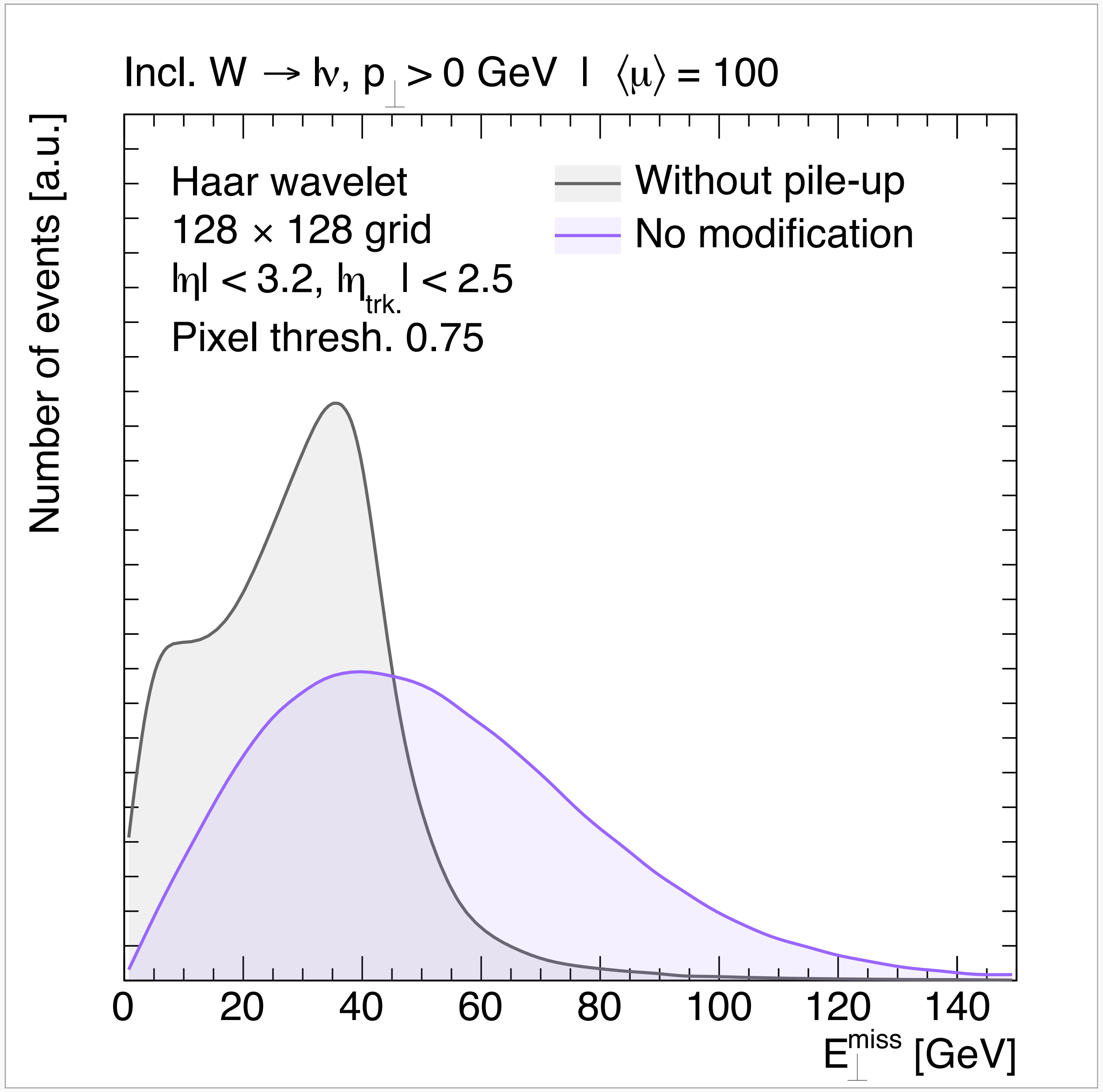

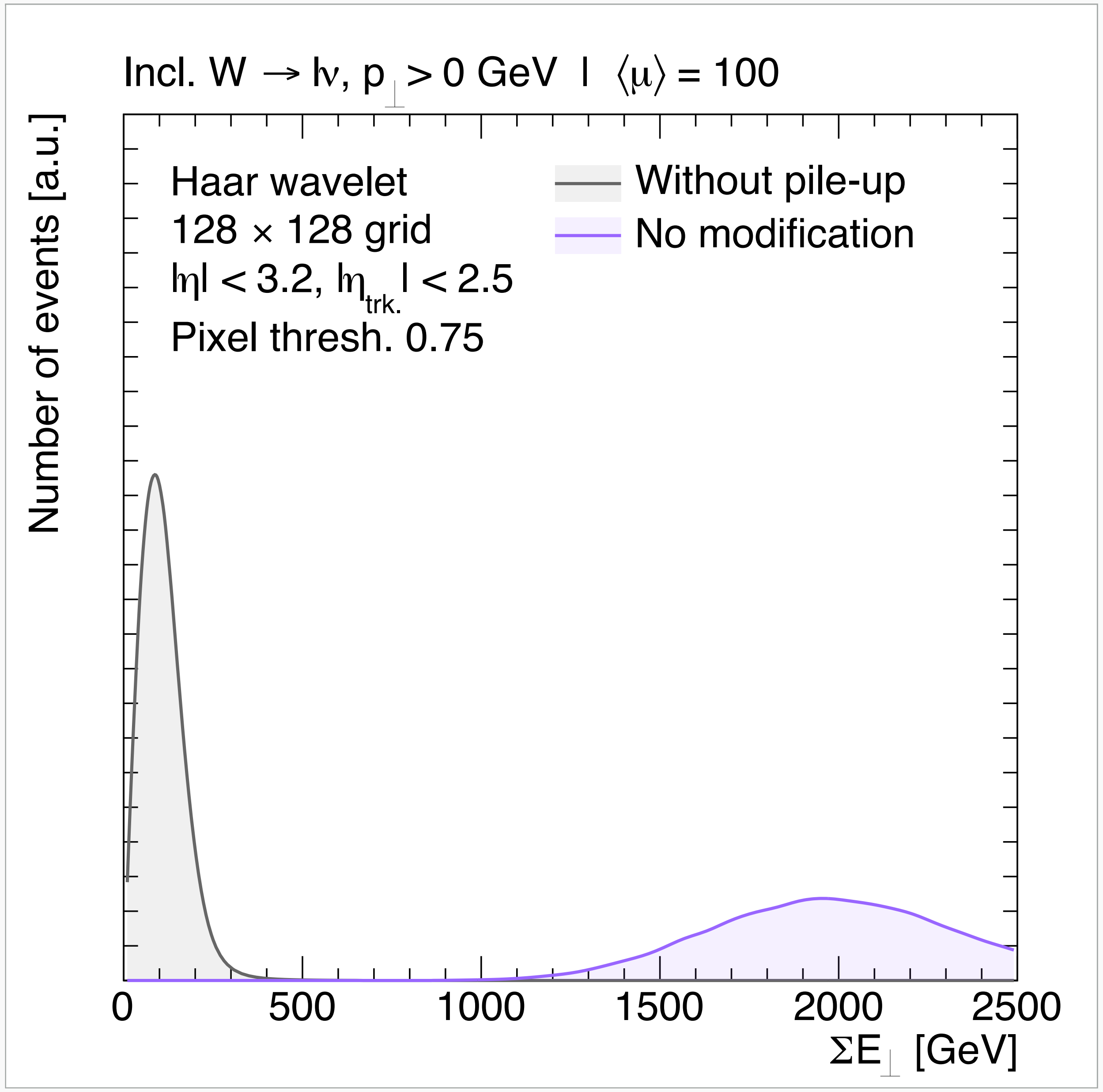

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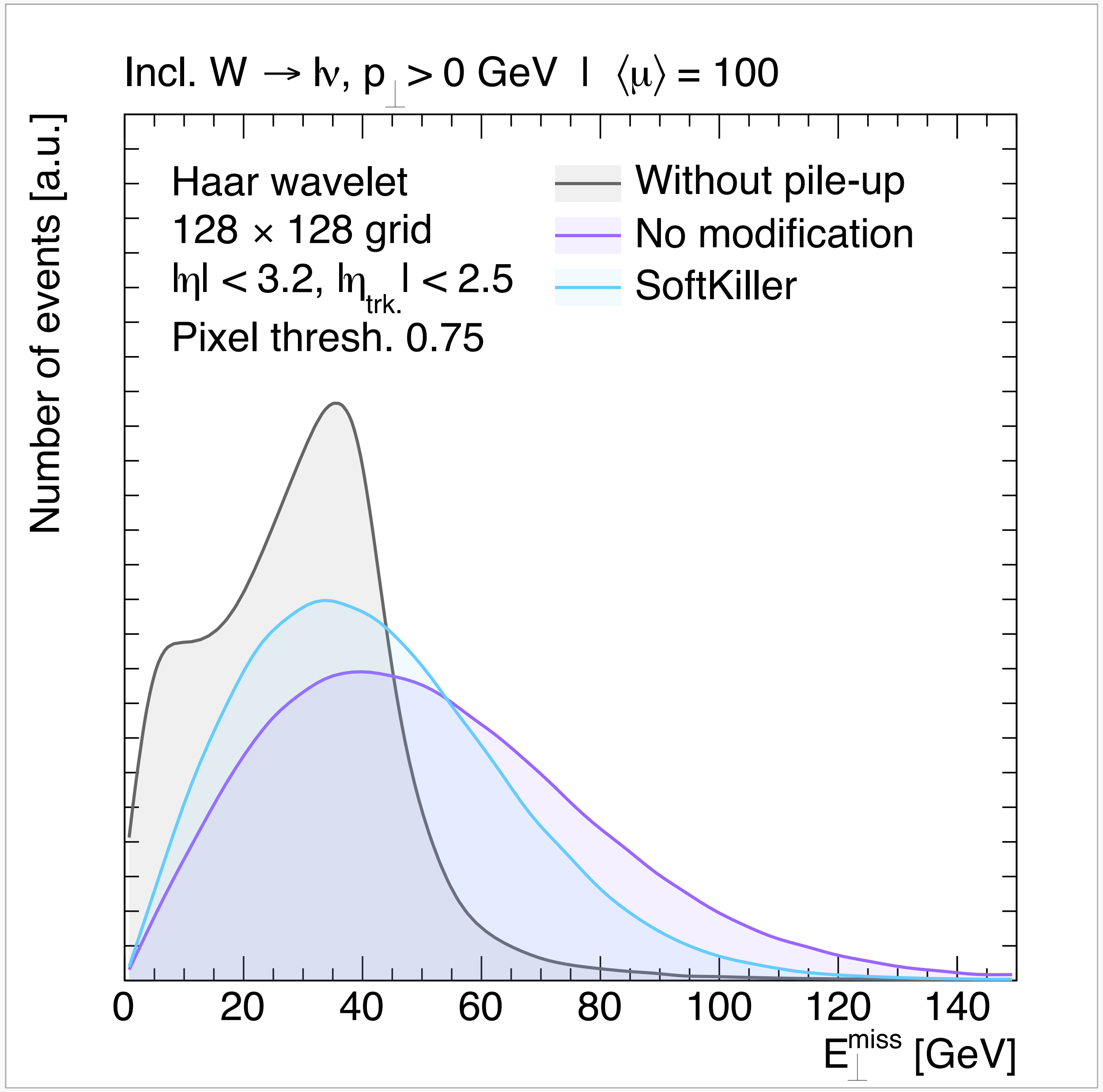

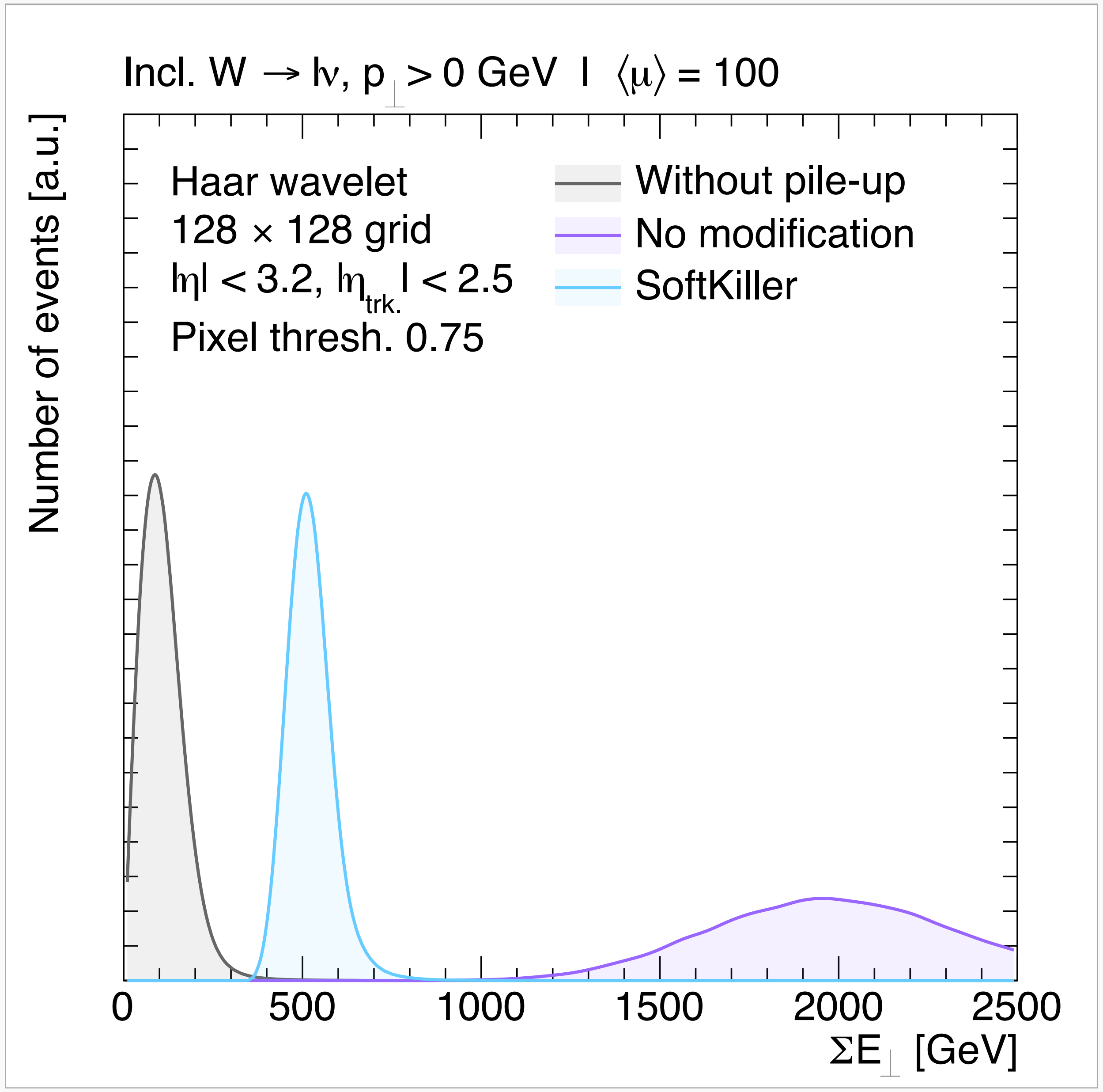

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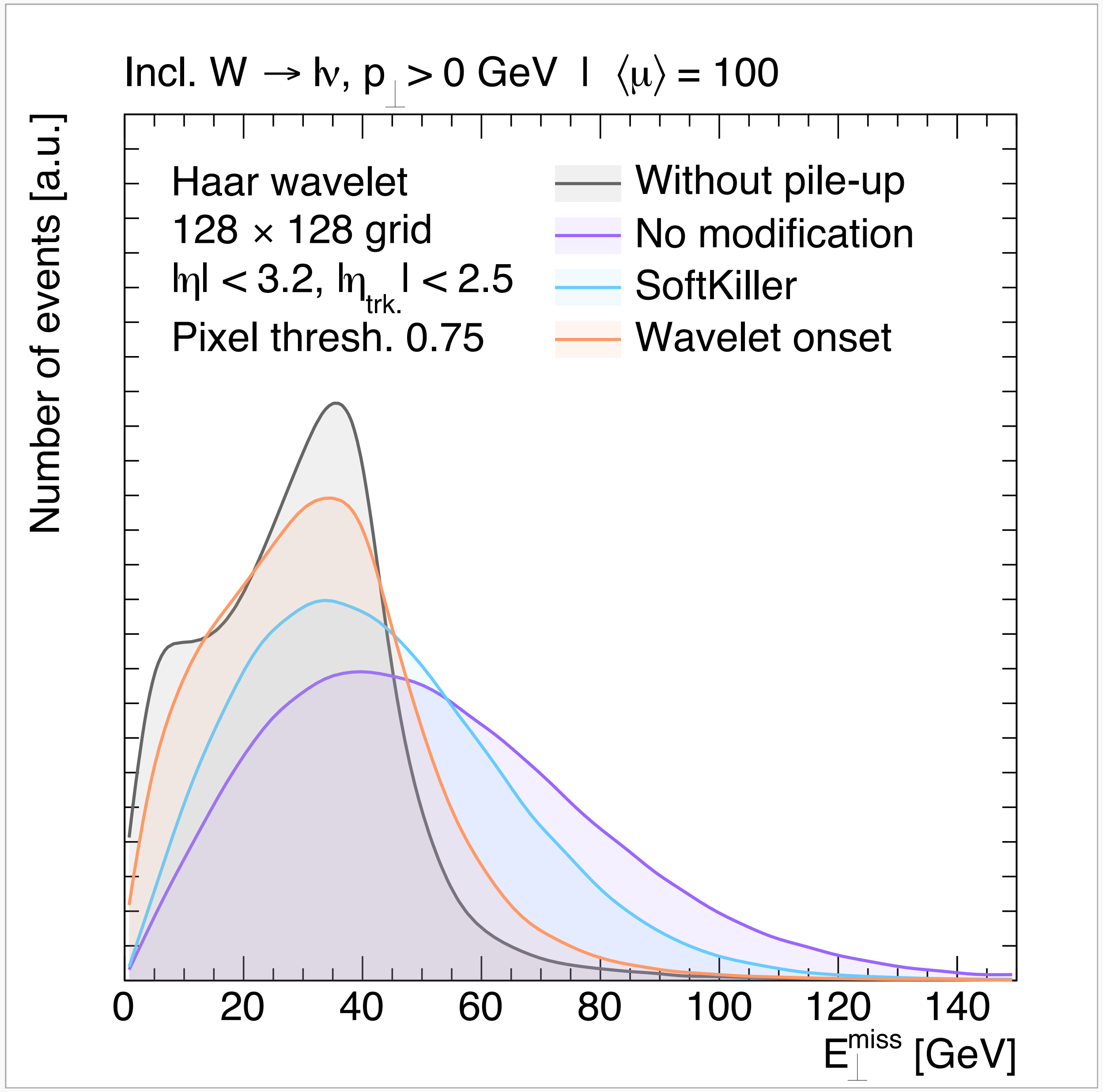
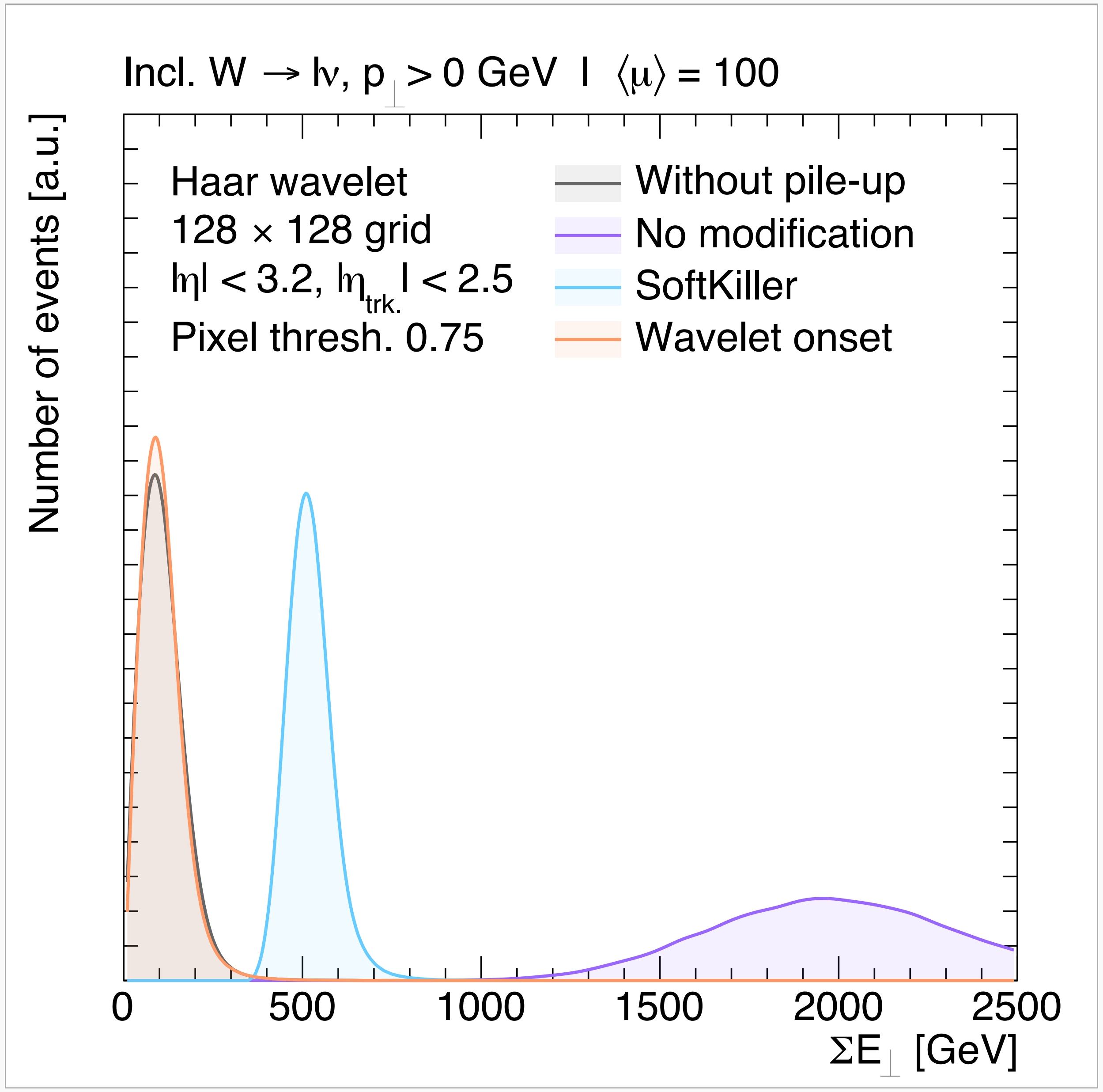
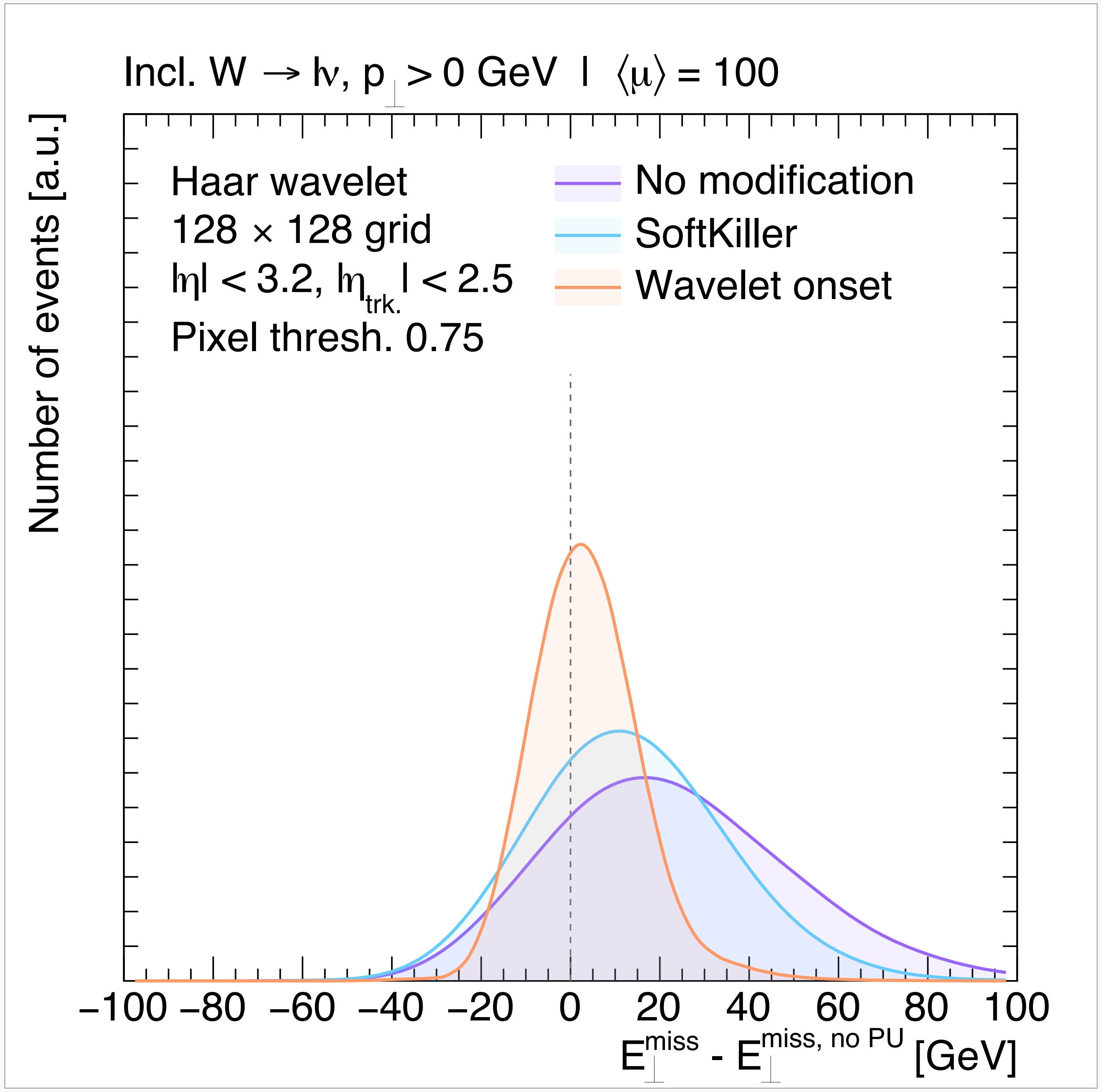
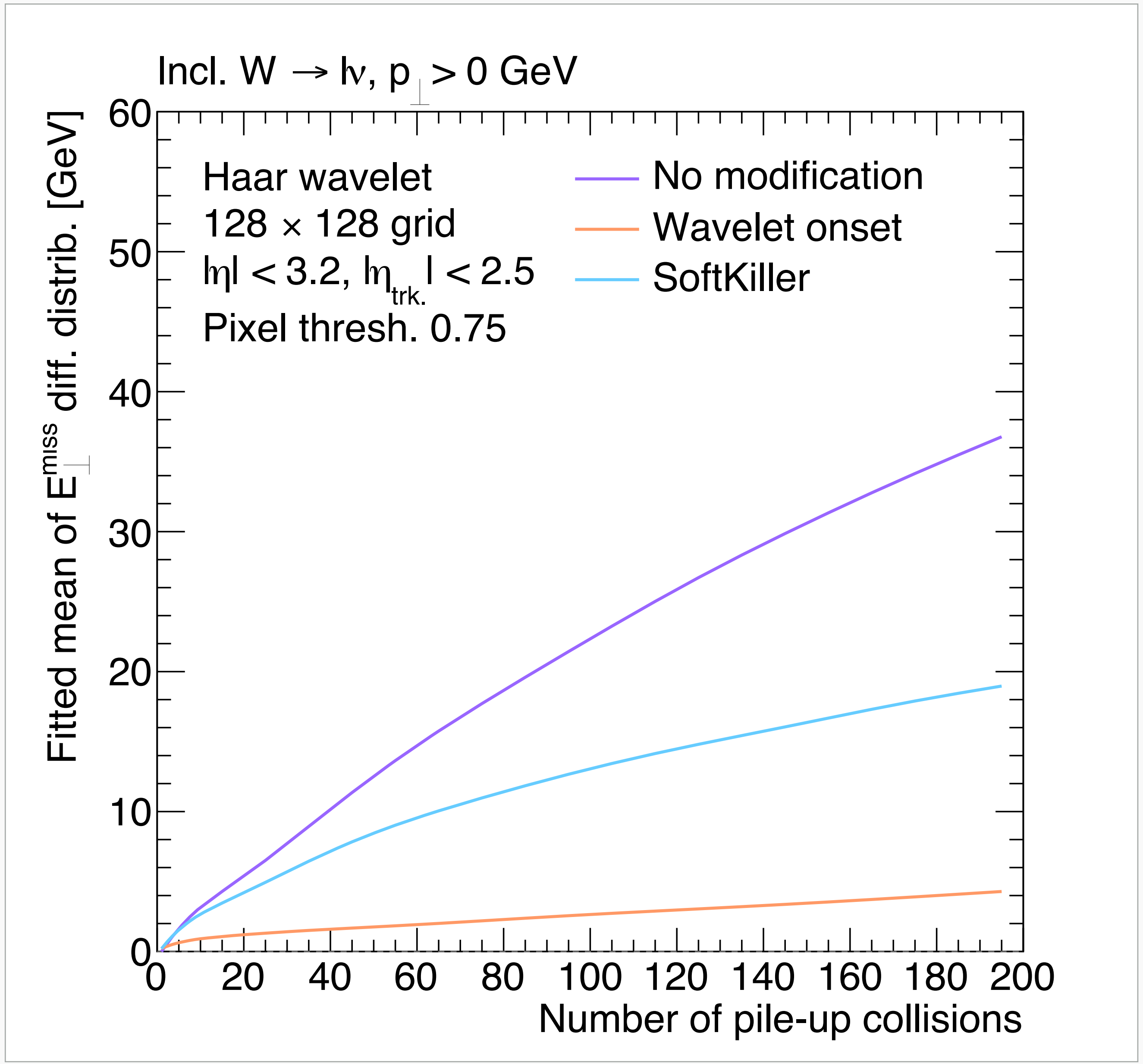


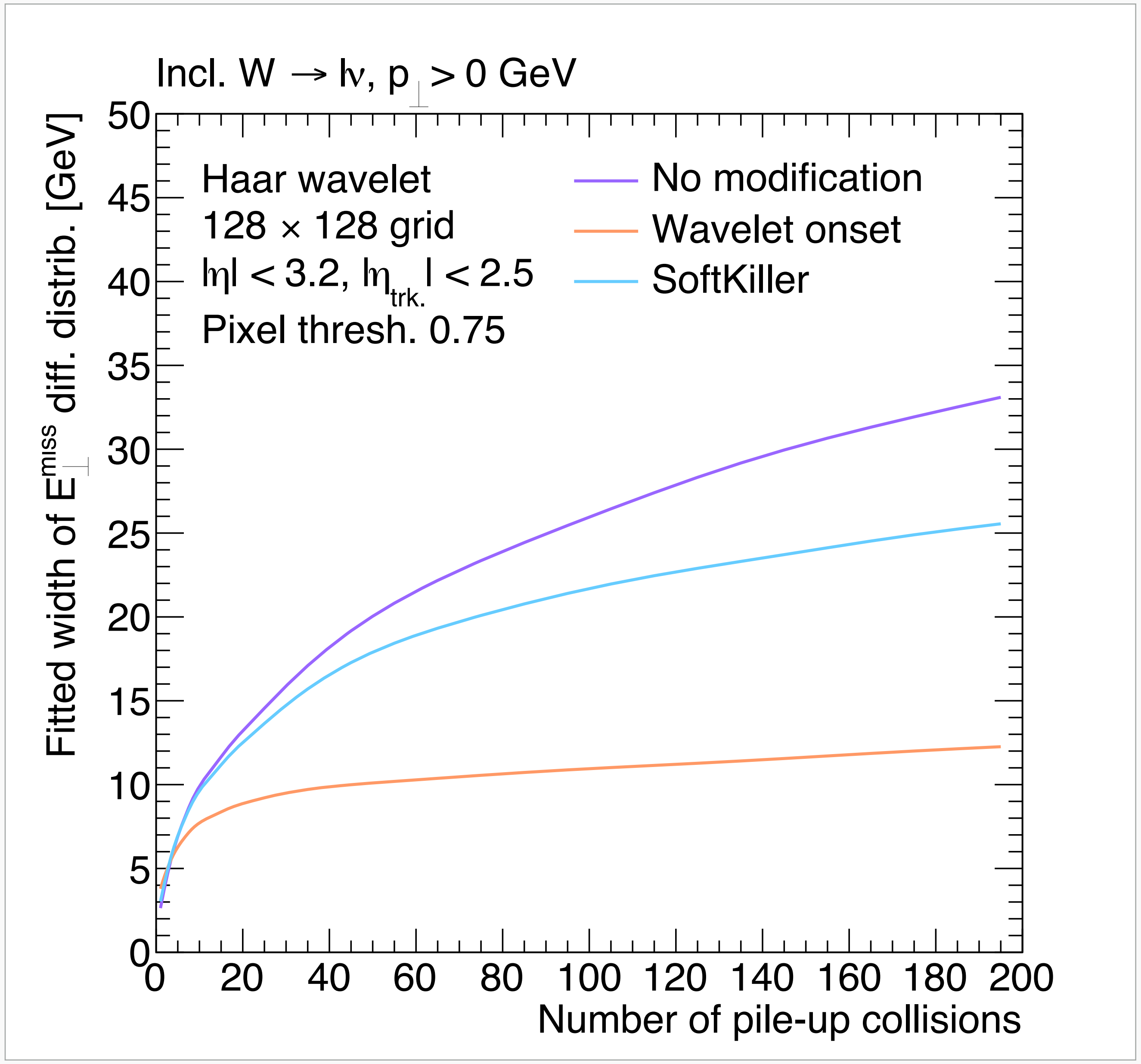


Missing E_T-resolution

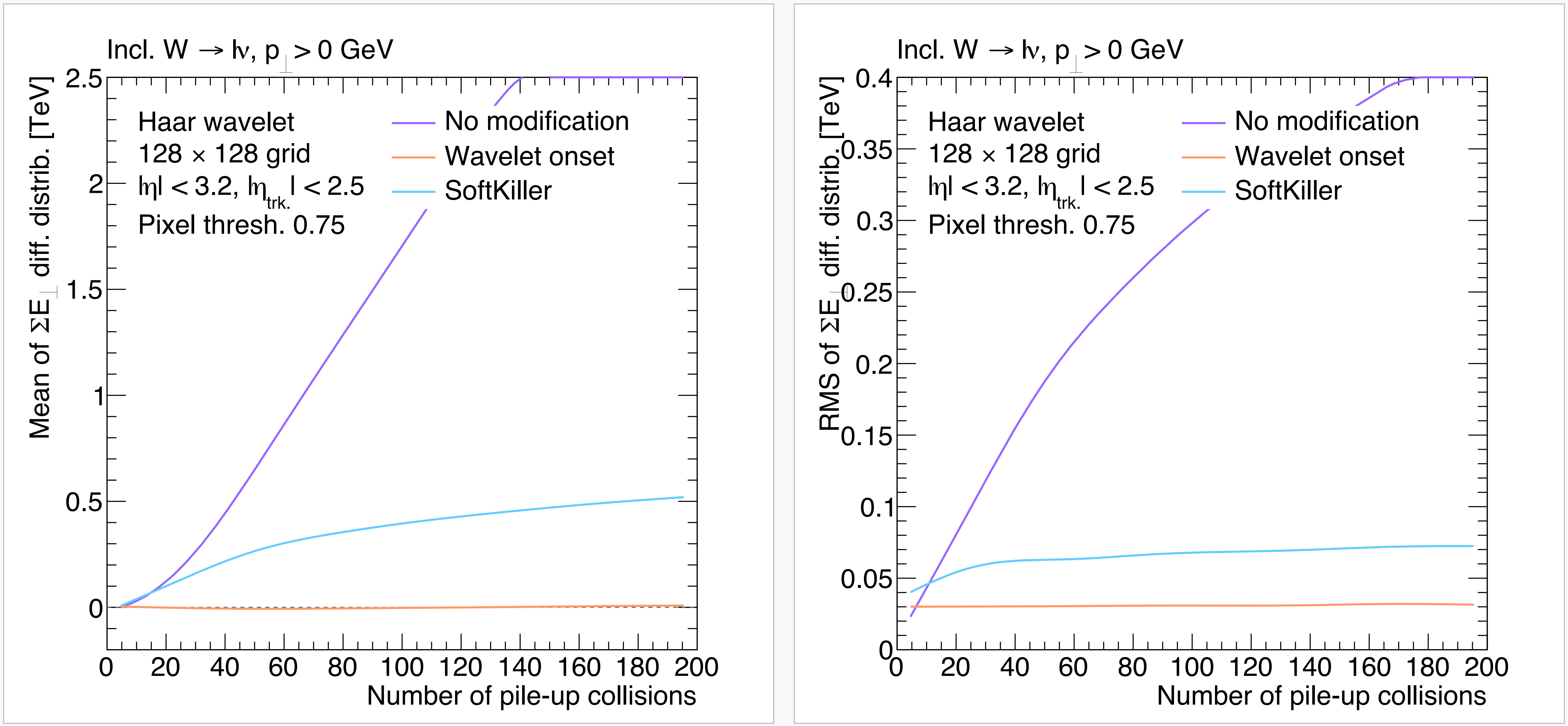


μ-dependence · Missing E_T-resolution





μ-dependence · Sum E_T-resolution



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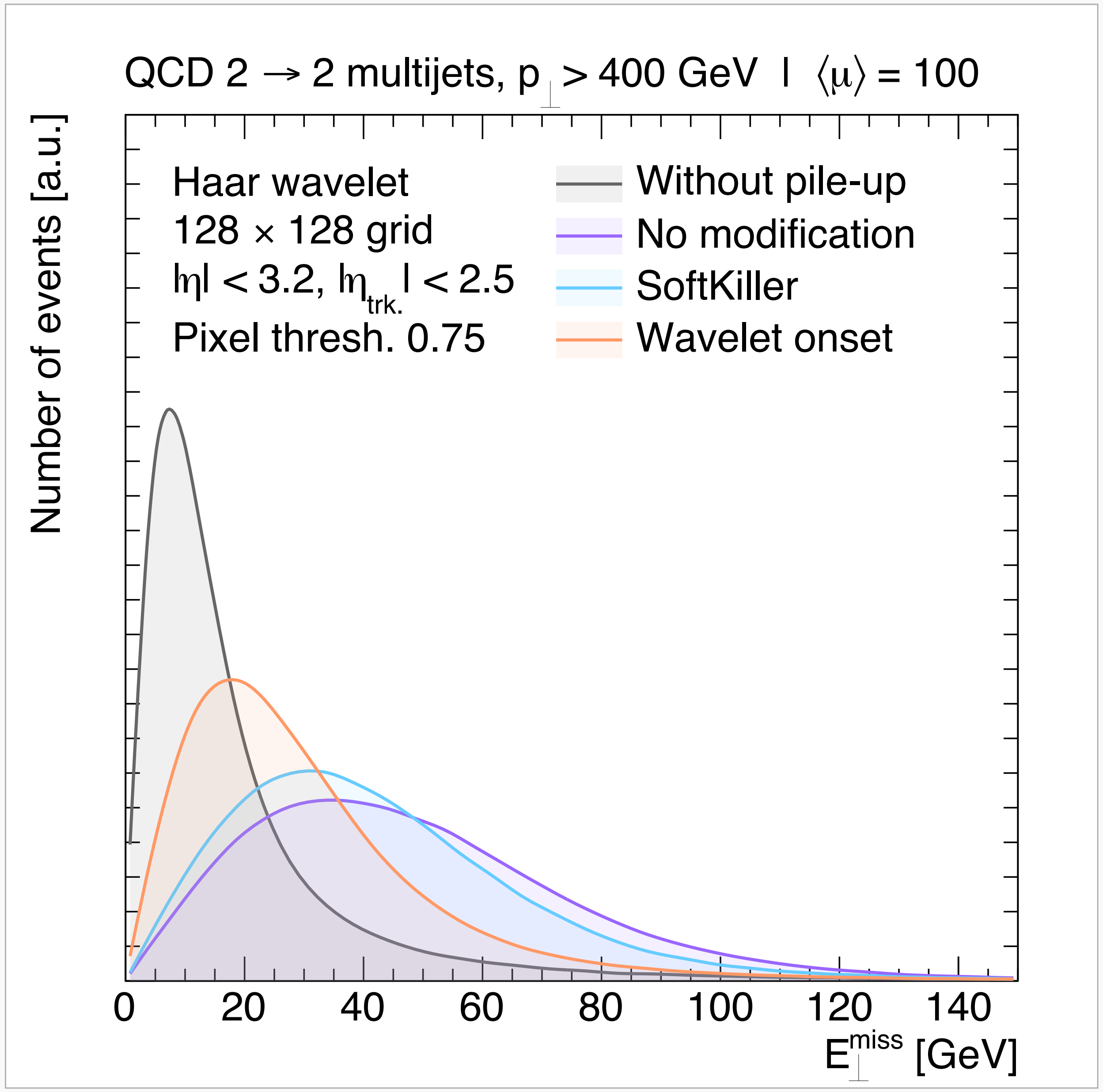
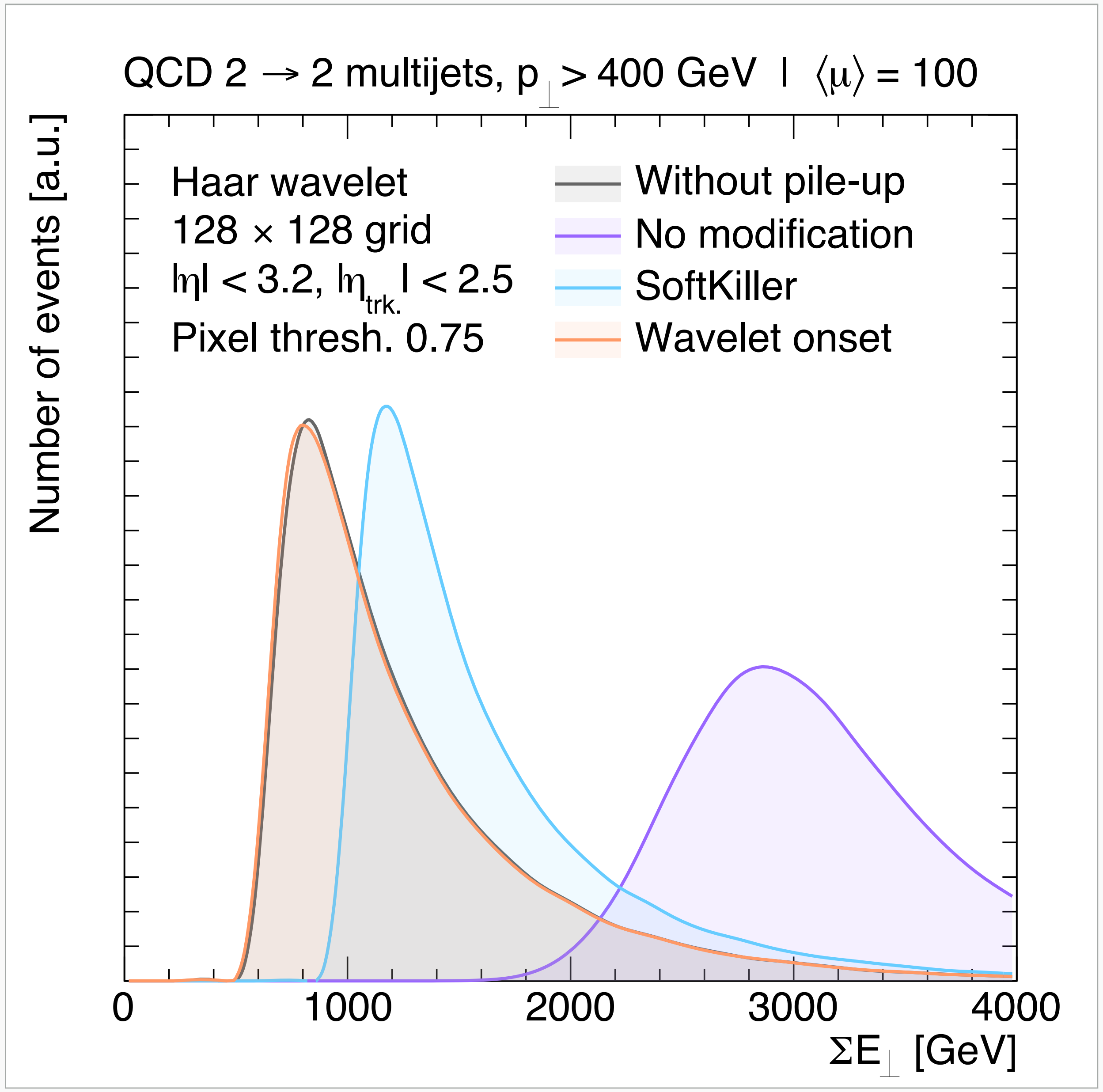
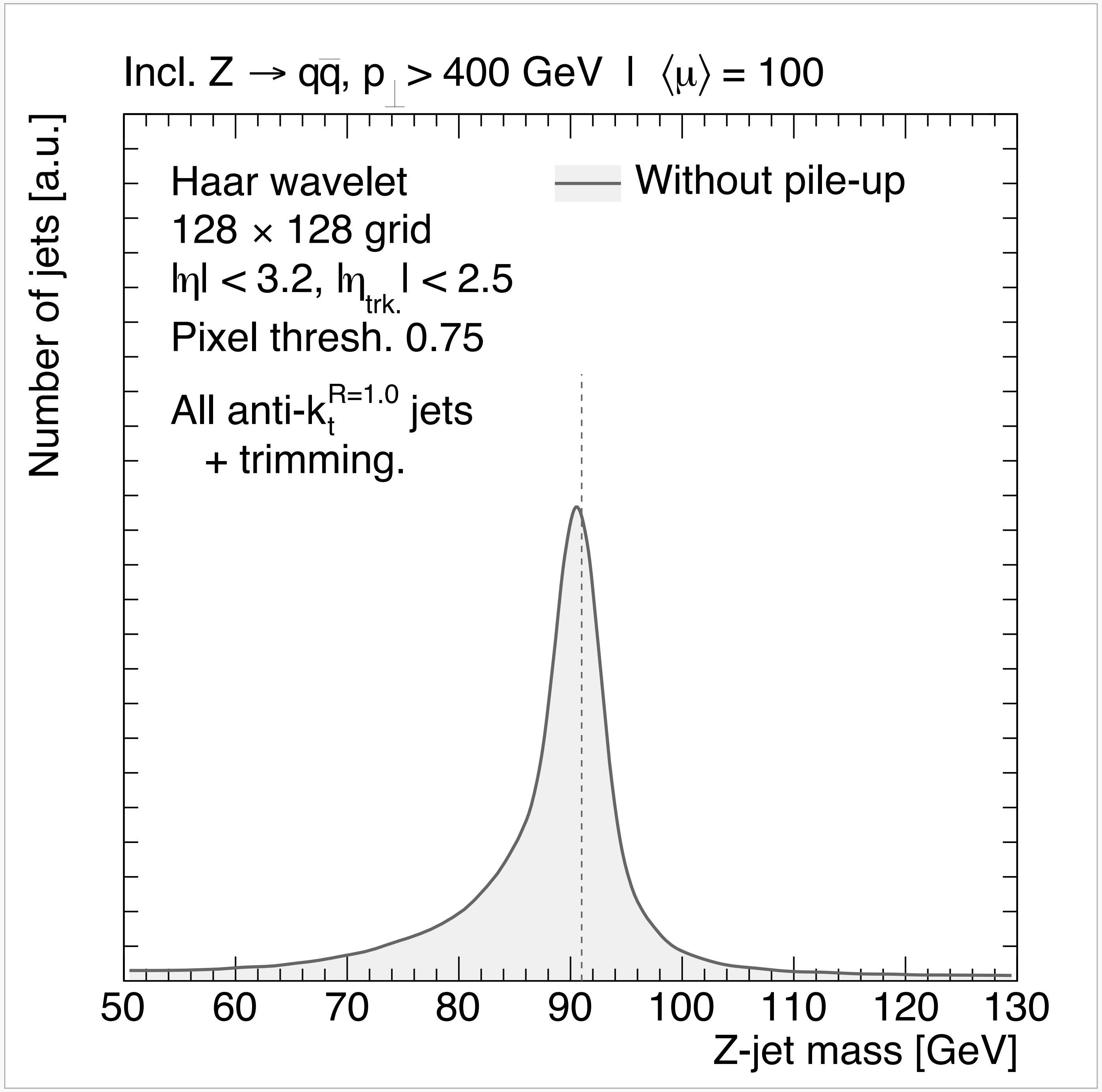




Fat jet studies

- Samples:
  - Incl. $Z \rightarrow q\bar{q}$ and QCD $2 \rightarrow 2$ multijet, $p_T > 400$ GeV
- Jets:
  - Anti-$k_T^{R\,=\,1.0}$ jets clustered with FASTJET
  - Trimming, using $k_T^{R\,=\,0.2}$ jets and p_T-fraction 0.05
  - 'Z-jet': Highest-p_T jet within dR = 0.6 of truth-level Z boson
  - 'Leading jet': Highest-p_T jet
- Comparison:
  - Area median subtraction and SoftKiller

Z-jet mass


Incl. Z → qq̄, p⊥ > 400 GeV | ⟨μ⟩ = 100
Haar wavelet
128 × 128 grid
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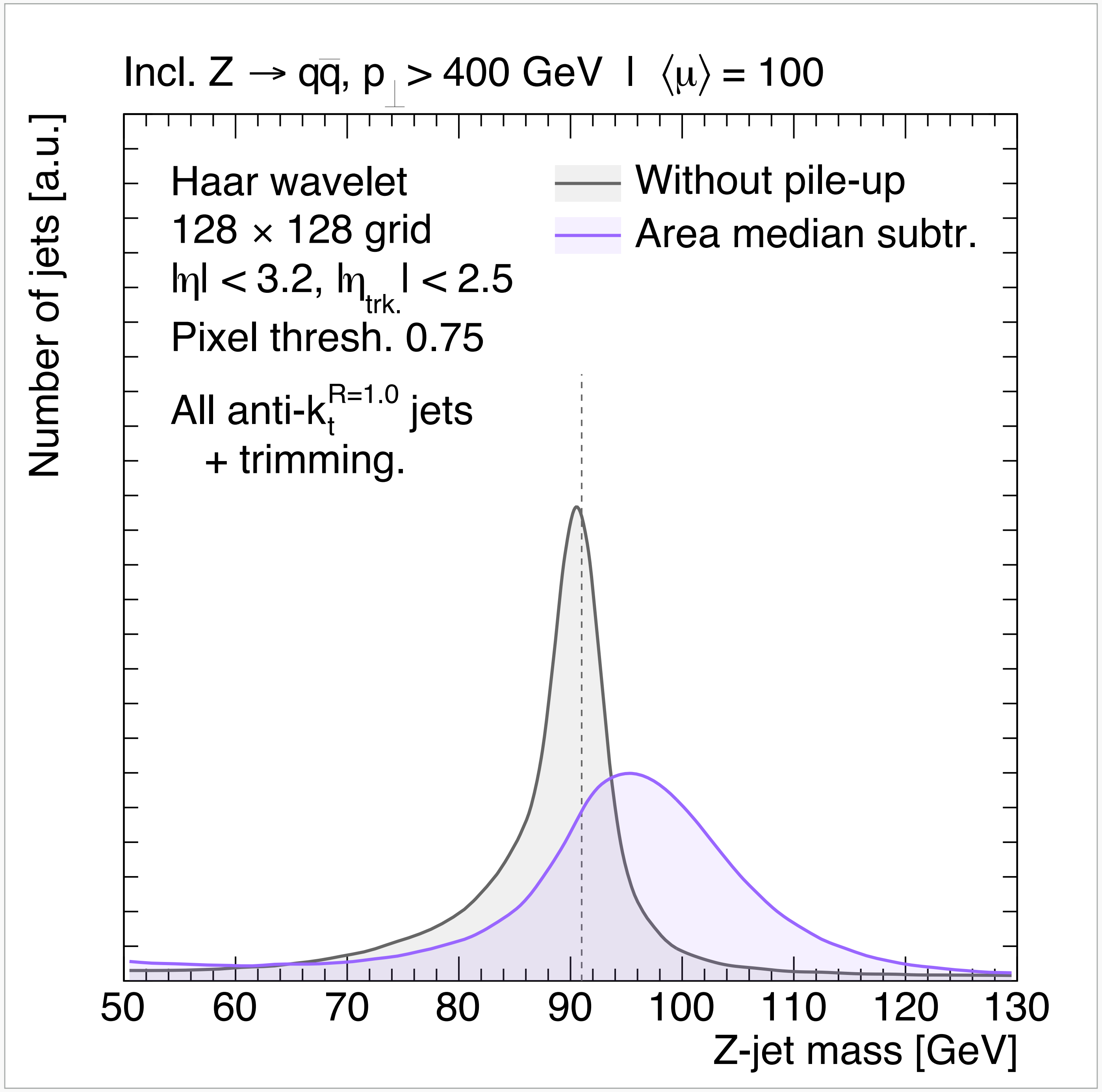
Pixel thresh. 0.75
All anti-kt R=1.0 jets
+ trimming.
Without pile-up
Number of jets [a.u.]
50
60
70
80
90
100
110
120
130
Z-jet mass [GeV]

Z-jet mass


Incl. Z → q$\bar{q}$, $p_\perp$ > 400 GeV | ⟨μ⟩ = 100
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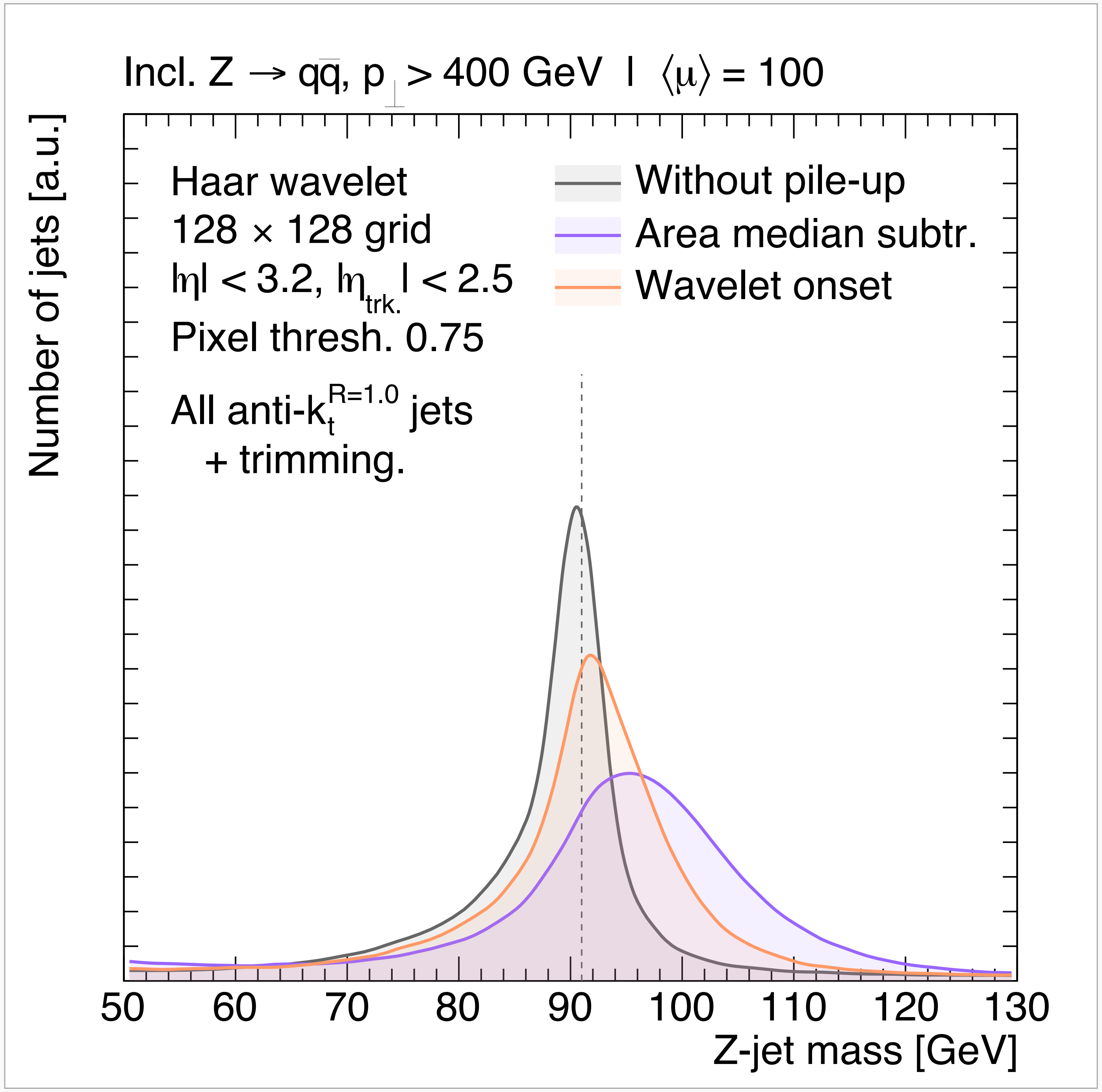
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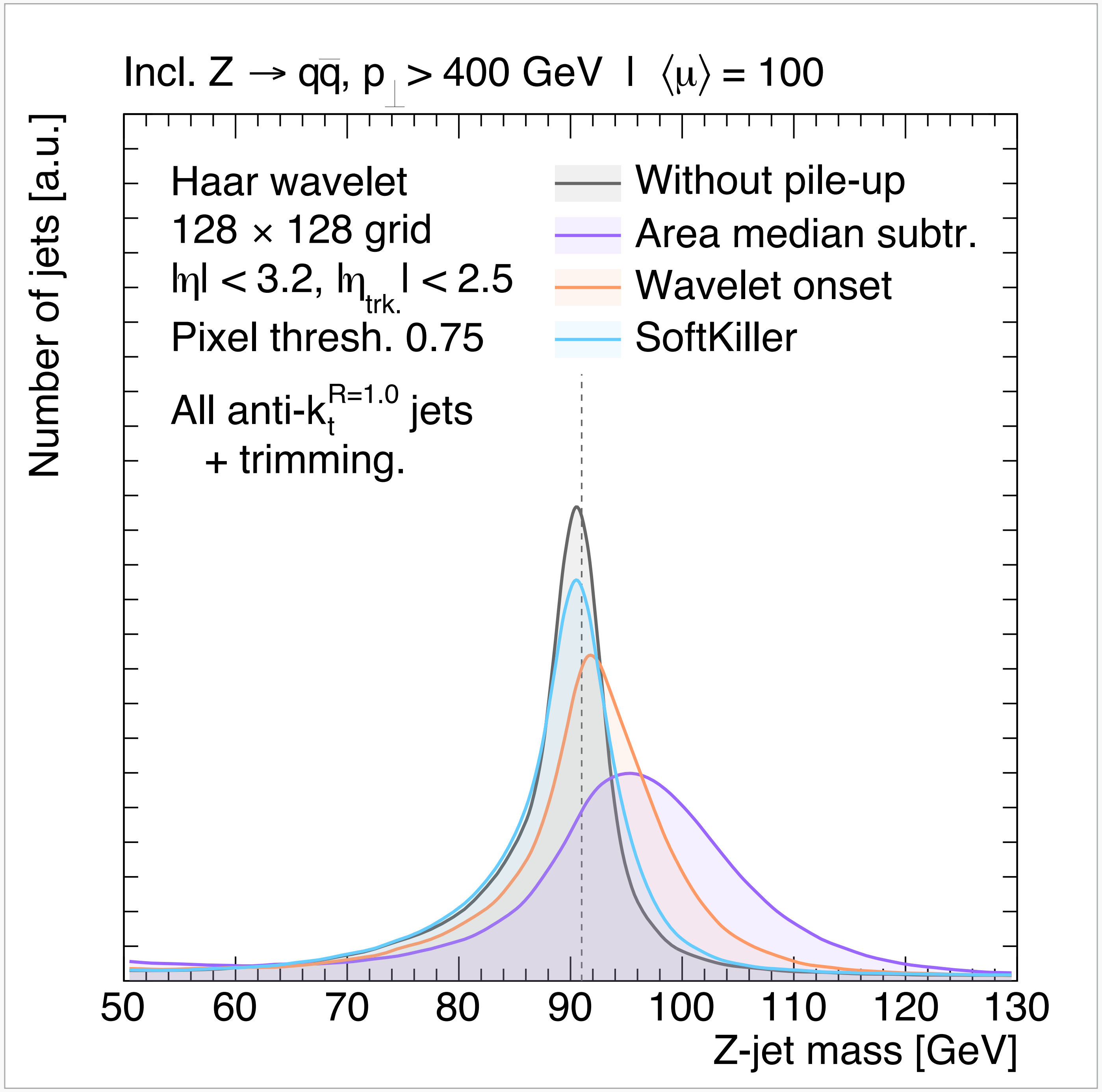
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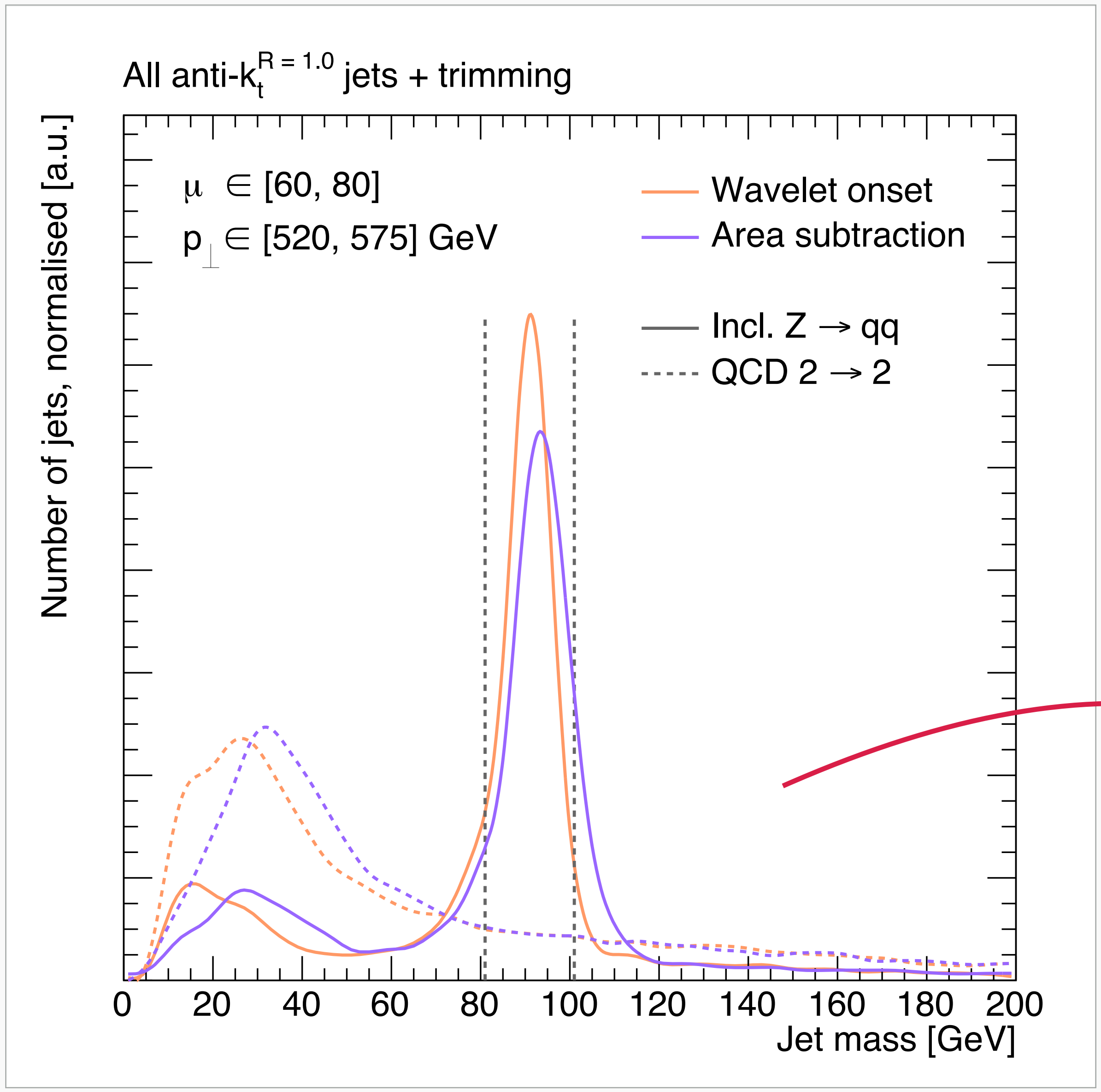
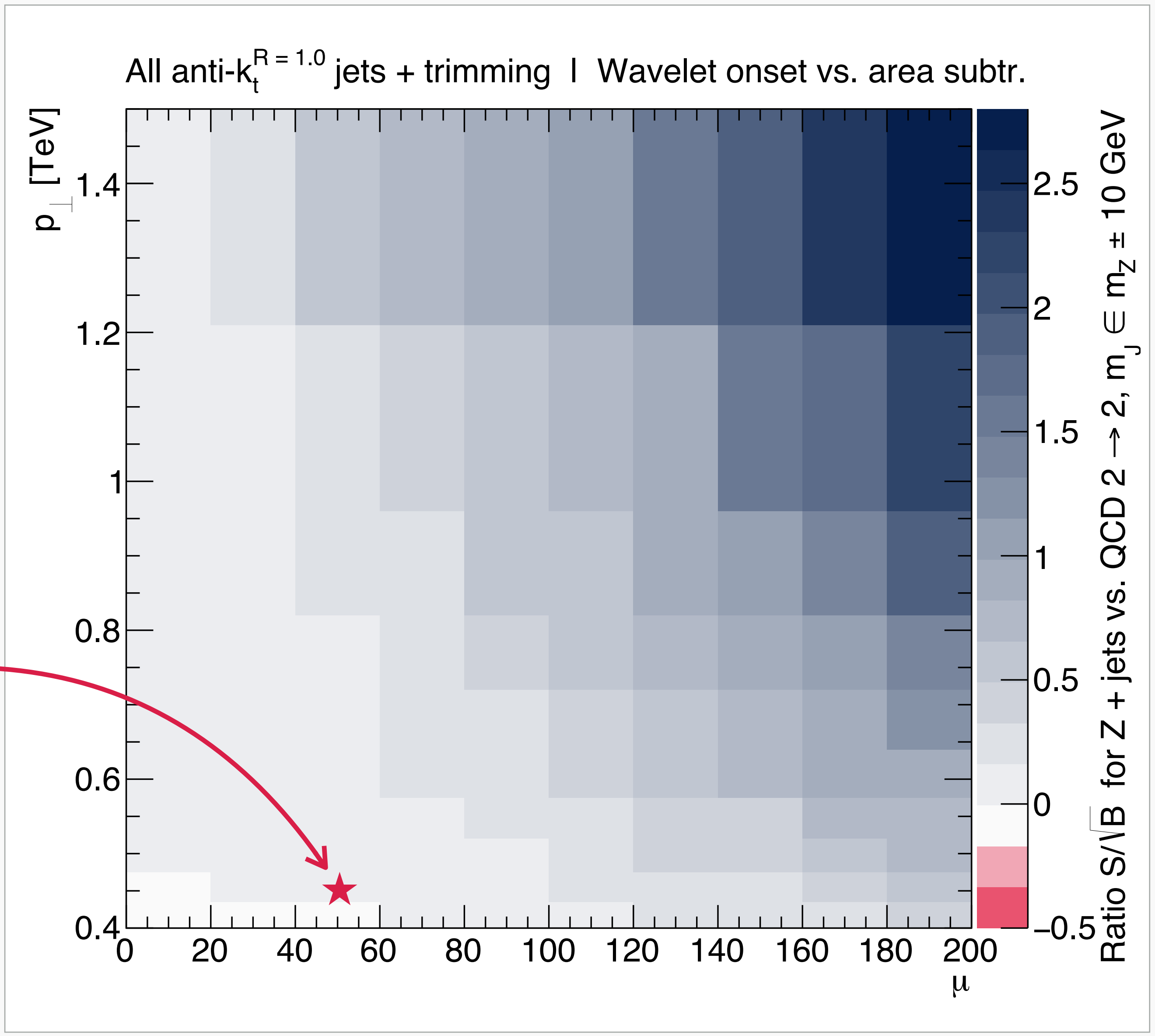
Z-jet mass [GeV]

Boson jet sensitivity improvement





Summary and outlook

- Motivated use of wavelets in HEP
- Showed the ability to naturally separate “white noise” pile-up from hard scatter events with small-angle structure
- Substantial potential is seen in improving measurement of both global and local inclusive observables
- Hope to implement similar methods in LHC analyses, ideally using a combination of the observables discussed today

Thank you.

Questions

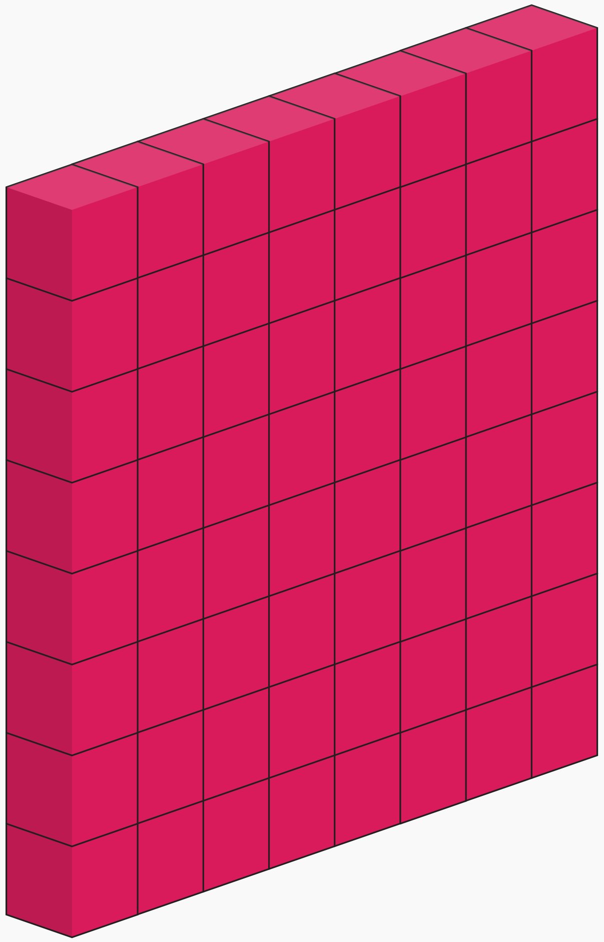
- How does it compare to PuPPI?
  - *Dunno.*
- Have you tried PuPPI?
  - *Nope.*
- Any thoughts on systematics?
  - *Nope.*

Backup

Learning optimal bases

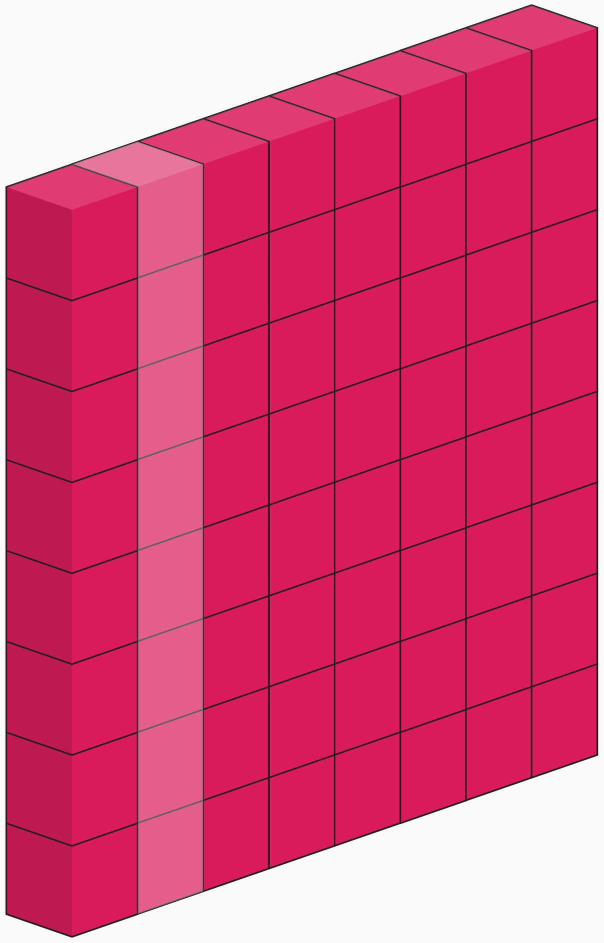
- Wavelet decomposition can be formulated as a deep neural network with a non-trivial architecture.
- Such a NN with 64 x 64 input in principle has 4.4×10^{7} weight coefficients.
- As a wavelet analysis realisation, the NN weight matrices are highly constrained.
- This means that the actual number of coefficients is N = 2, 4, 6, … i.e. the *filter coefficients* of the corresponding wavelet basis.

Wavelet/NN architecture



(NN input)

Wavelet/NN architecture



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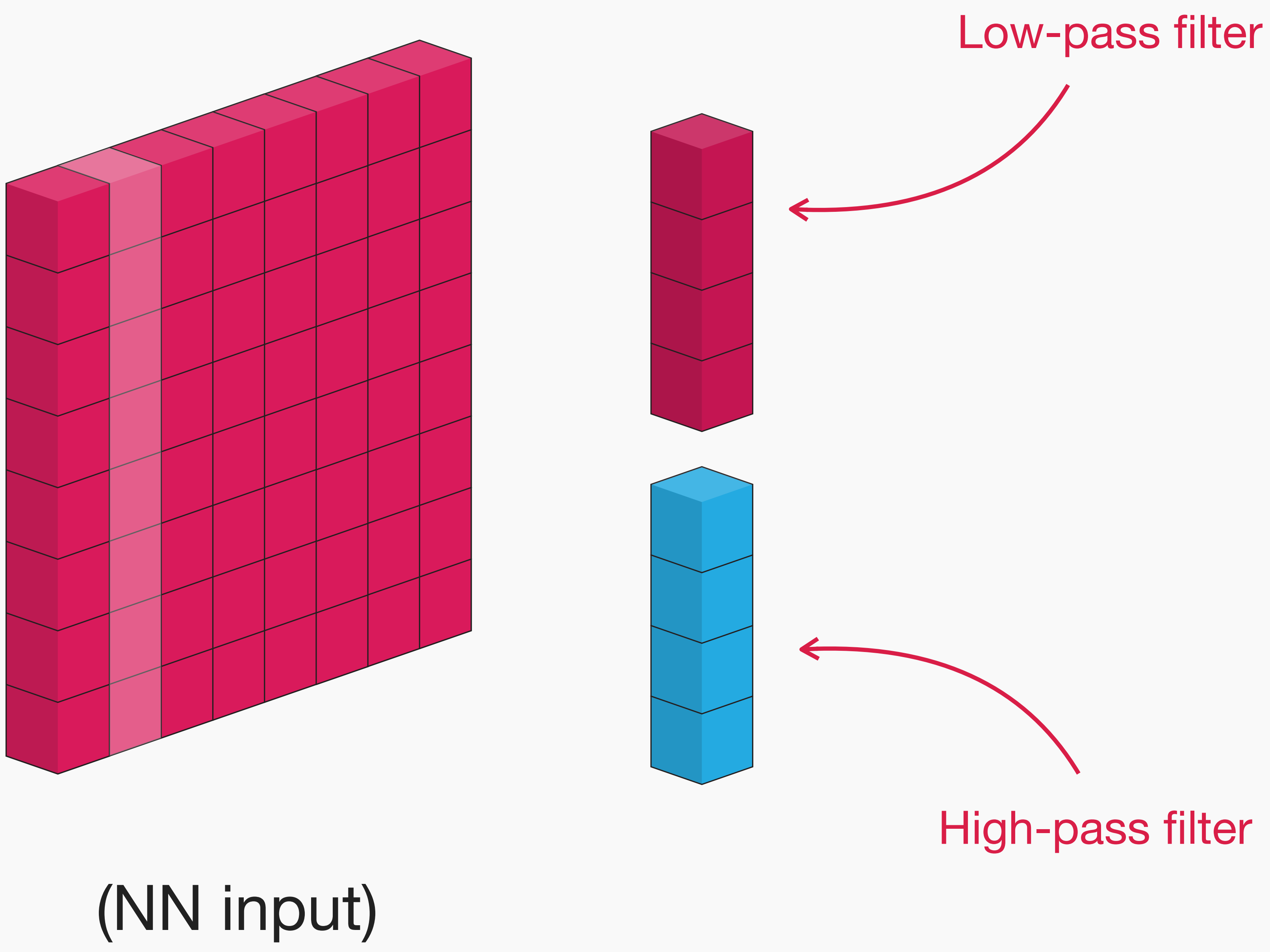

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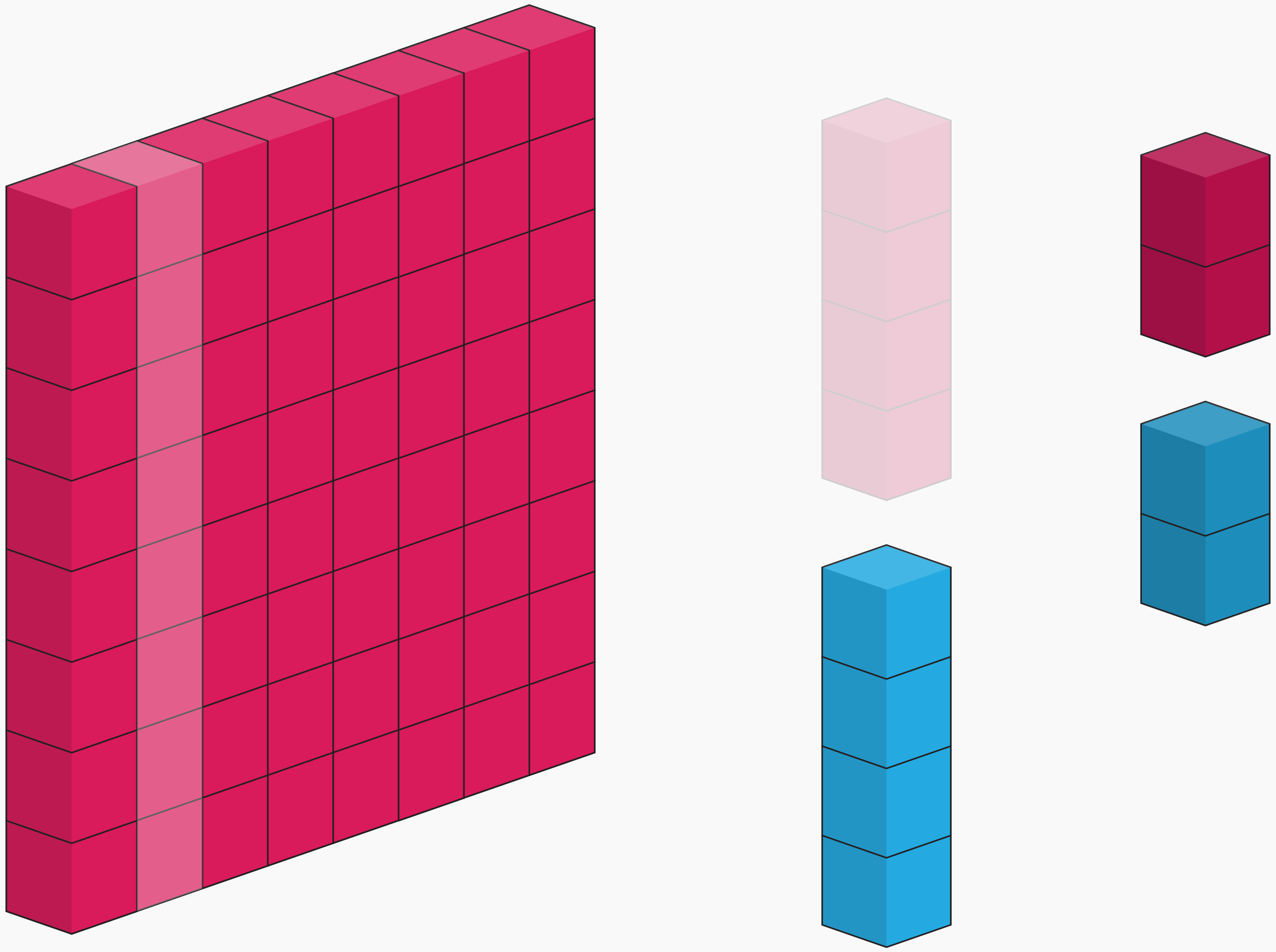

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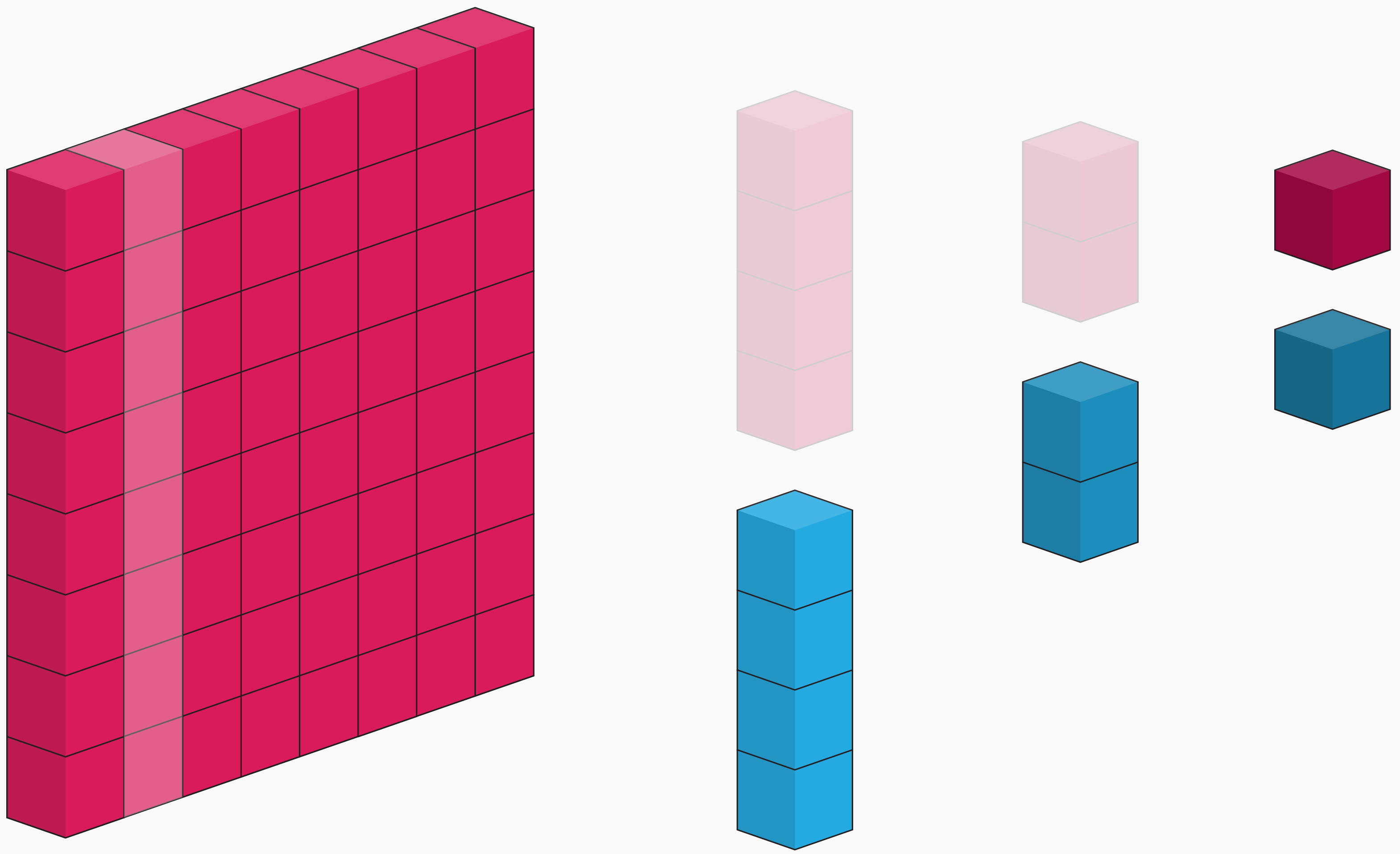

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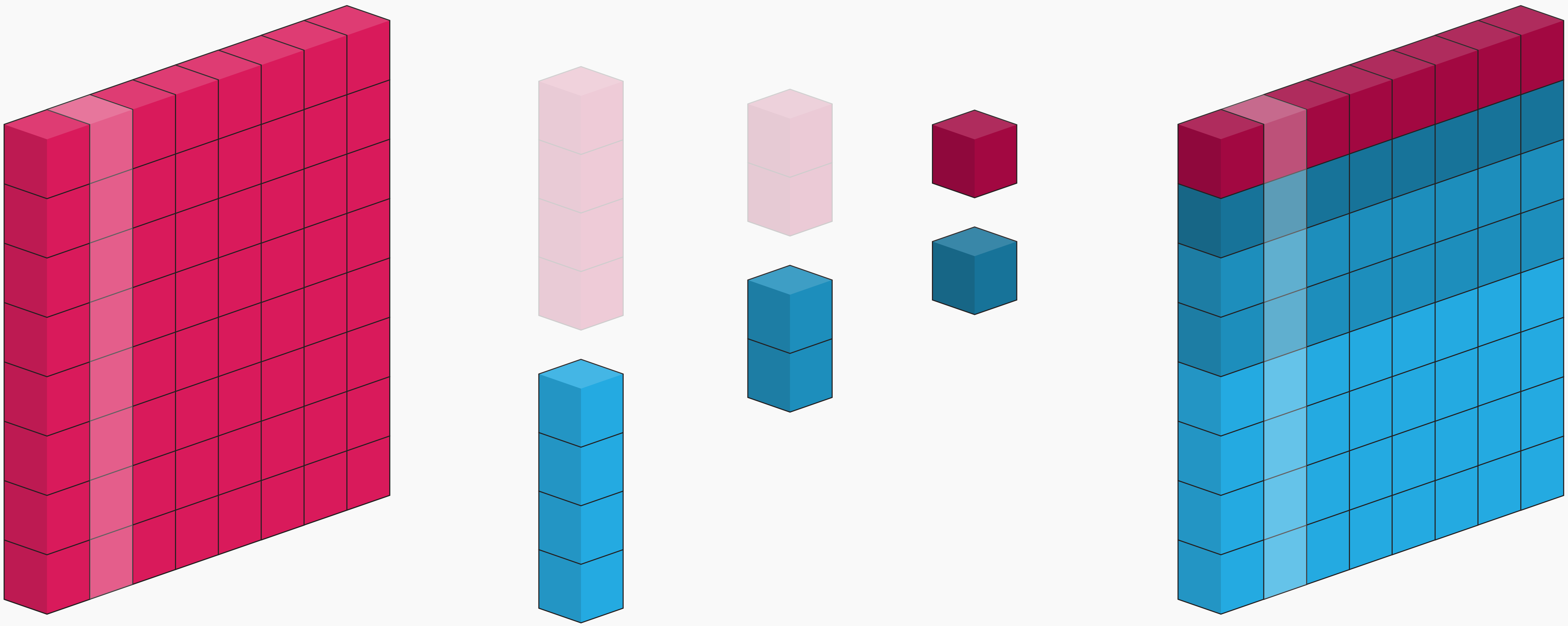

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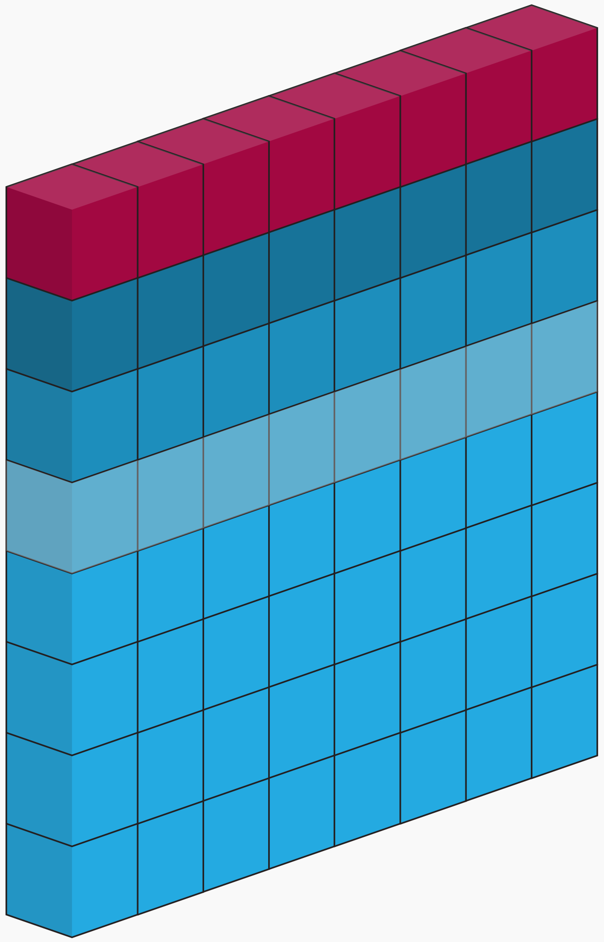

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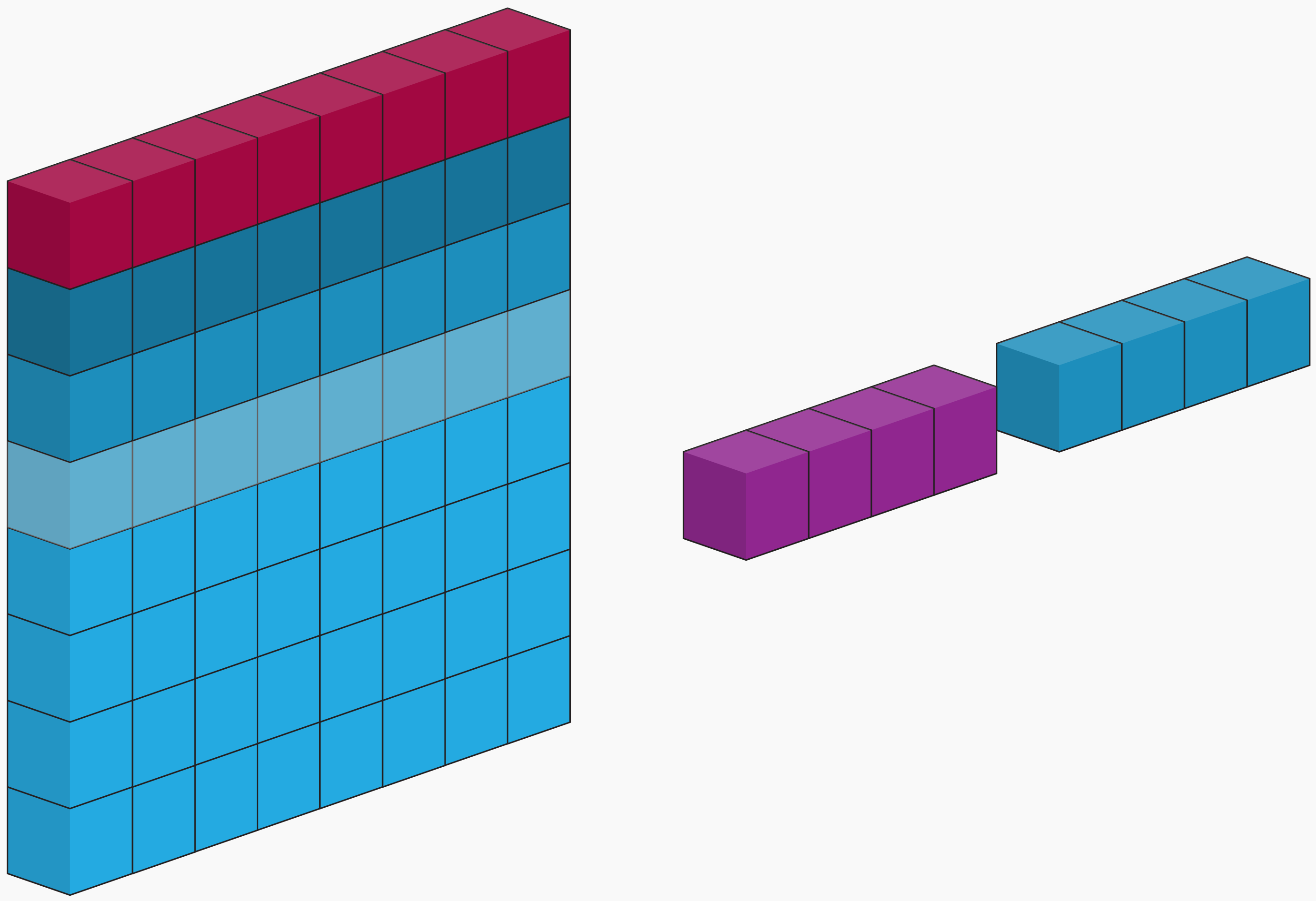

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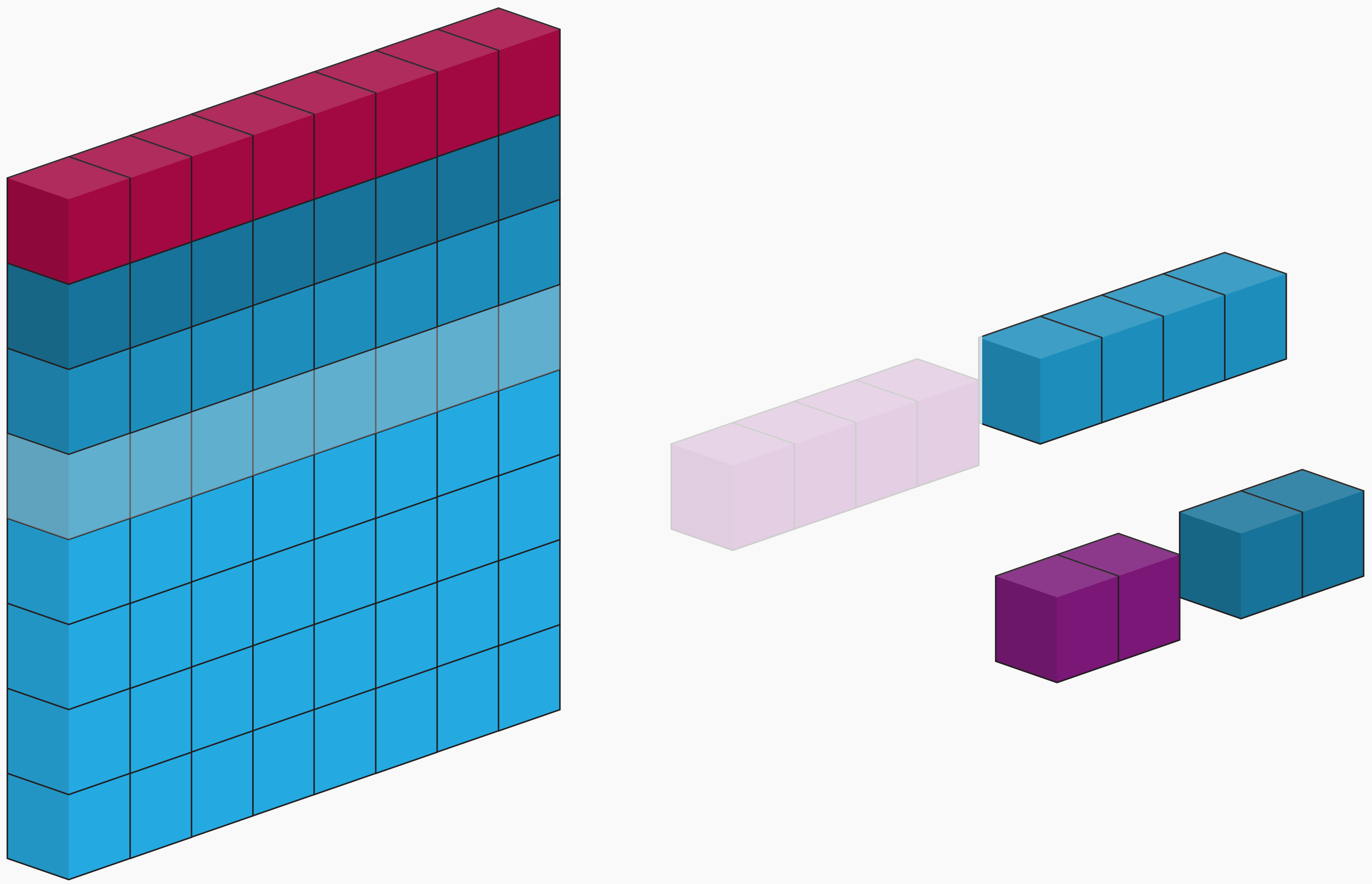

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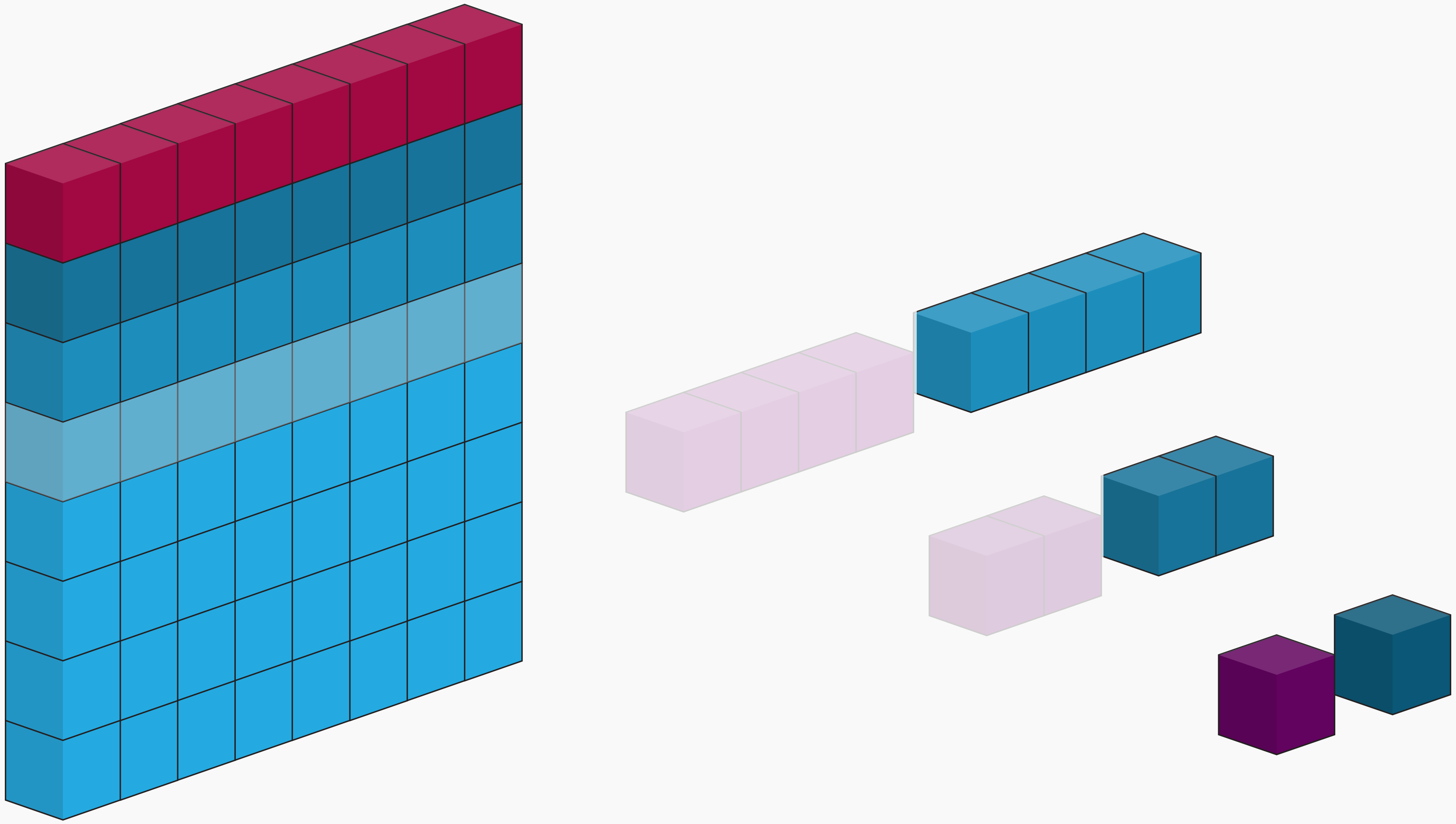

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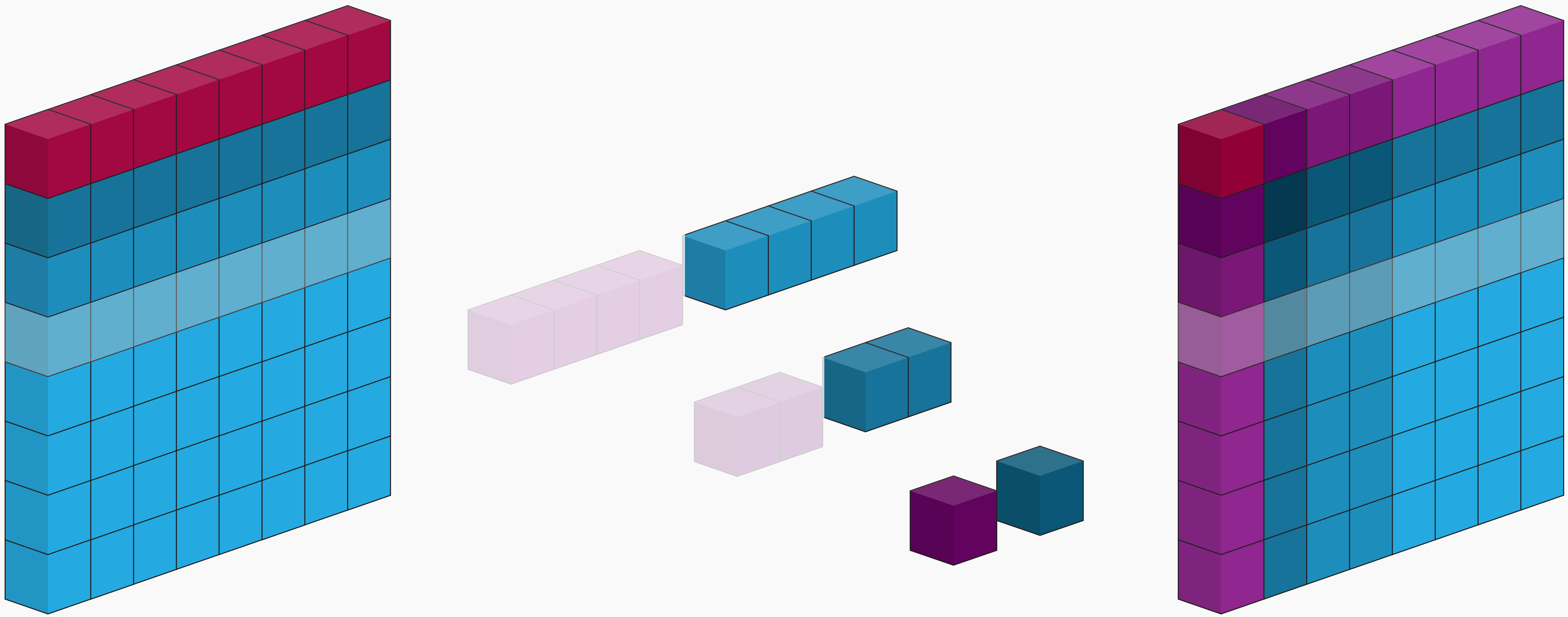

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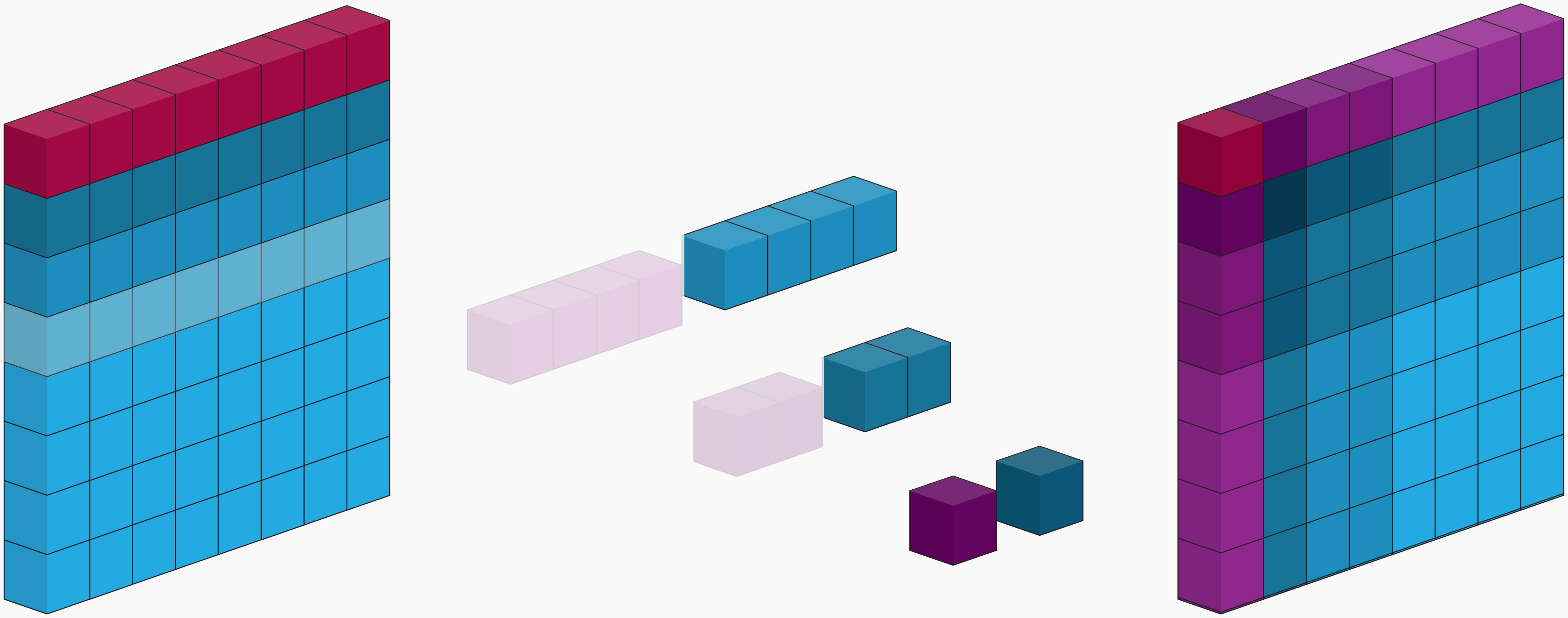


(NN output)

How to measure ‘optimality’

- One can then use gradient descend with back-propagation to learn filter coefficients which are optimal wrt. some criterion.
- One such measure is *sparsity*, i.e. the ability of a basis to efficiently encode information contained in a certain type of input, e.g. jet events.
- Sparsity can be quantified by the *Gini coefficient*
- In addition, we need to impose five constraints on the filter coefficients, to ensure that they result in an actual wavelet basis.
- These are realised as quadratic *regularisation* terms in the combined *cost function*.

Cost function

- Regularisation:

$$R_i = \left(f_i\left[\{a_k\}\right] - d_i\right)^2 \quad \Longrightarrow \quad R = \sum_{i=1}^{5} R_i$$

where a_k are the filter coefficients and d_i is some number.

- Sparsity (Gini coefficient):

$$G\left[\{c_i\}\right] = 1 - 2\sum_{i=1}^{N_c} \frac{c_i}{\|c_i\|_1} \frac{N_c - i + \frac{1}{2}}{N_c}$$

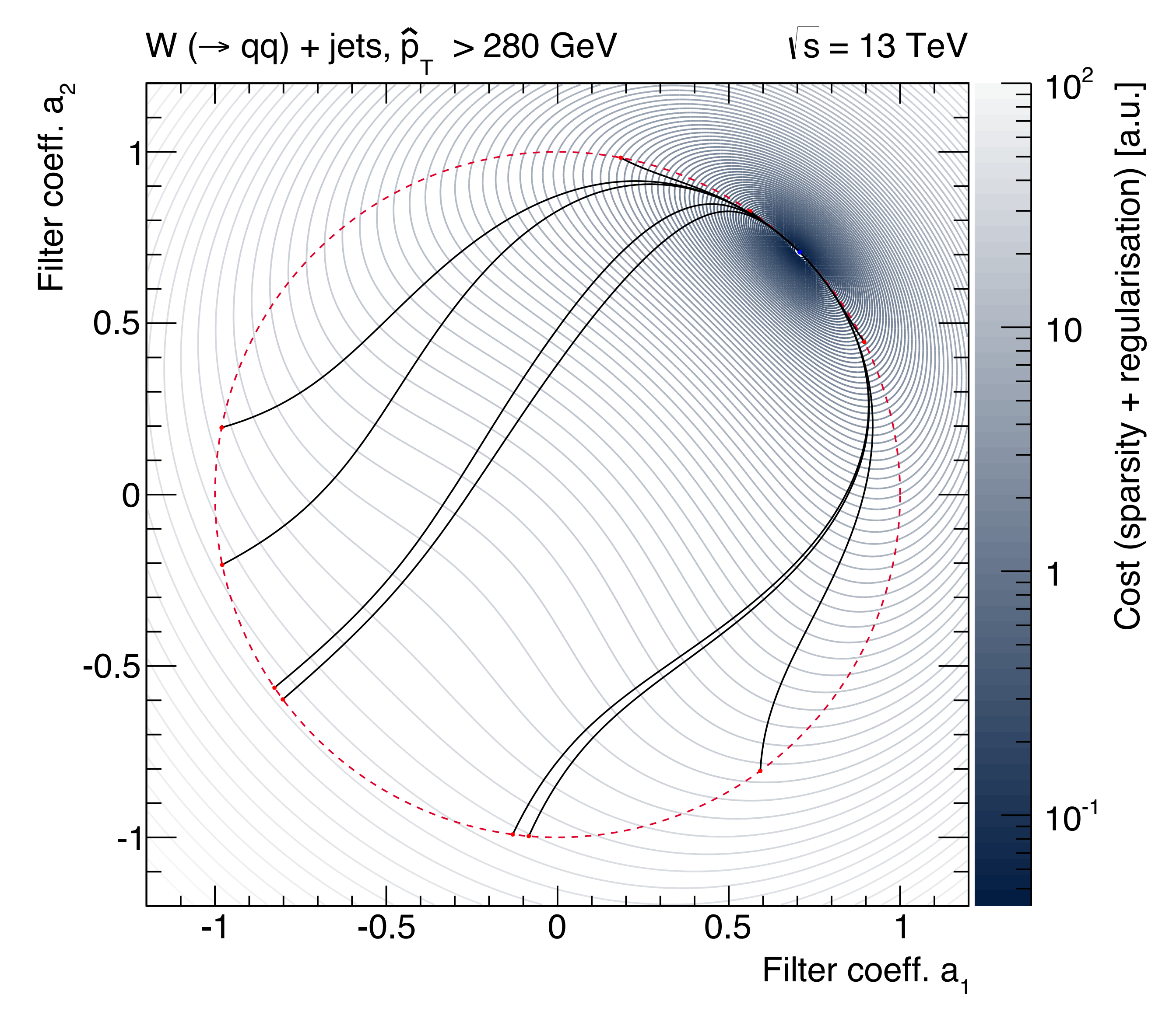
where c_i are the wavelet (not filter) coefficients for a single input, sorted such that $c_i \leq c_{i+1}$ and N_c are the number of such coefficients.

- Combined cost:

$$C = \left(G\left[\{c_i\}\right]\right)^2 + \lambda R, \qquad \lambda \in [0, \infty)$$

Cost map and gradient descend paths

- Loop over e.g. incl. W → qq events, we can optimise the combined sparsity + regularisation cost function.
- Plot shows 10 random initialisations in unit circle (red dots; one of the five constraints) and the learning paths (black lines) going towards a single, global minimum of $(1,1)/\sqrt{2}$: **Haar**
- More than one possible minimum for N > 2 filter coeffs.



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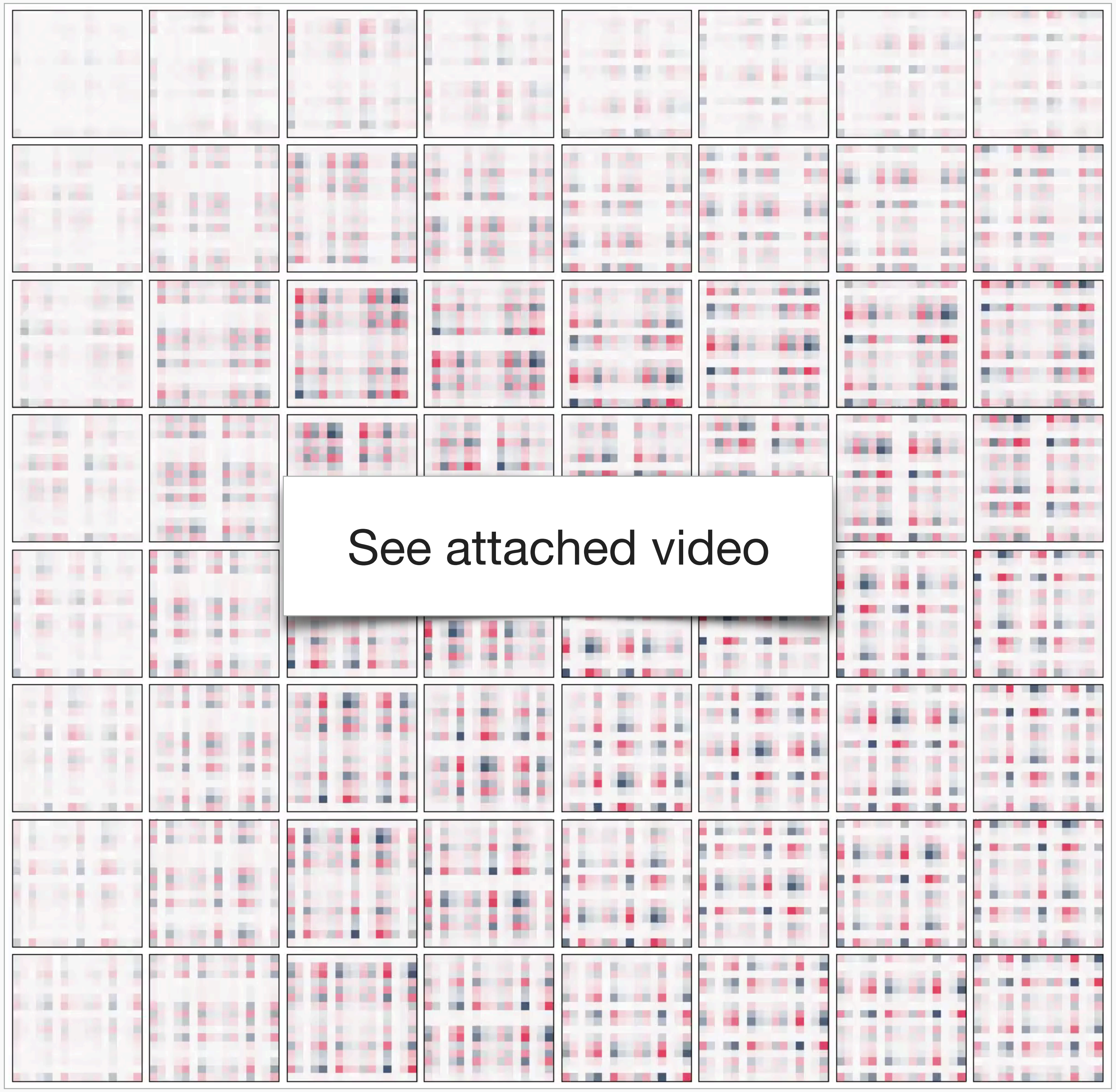
Visualising learning

- **16** filter coeffs. = 16-dim. optimisation problem
- Incl. $W \rightarrow qq$, $\hat{p}_T >$ 280 GeV, no pile-up
- 25’000 events.
- Optimisation from random initialisation

1. part: Regularisation
    - Normalisation
    - Orthogonality
    - Self-similarity
2. part: Sparsity



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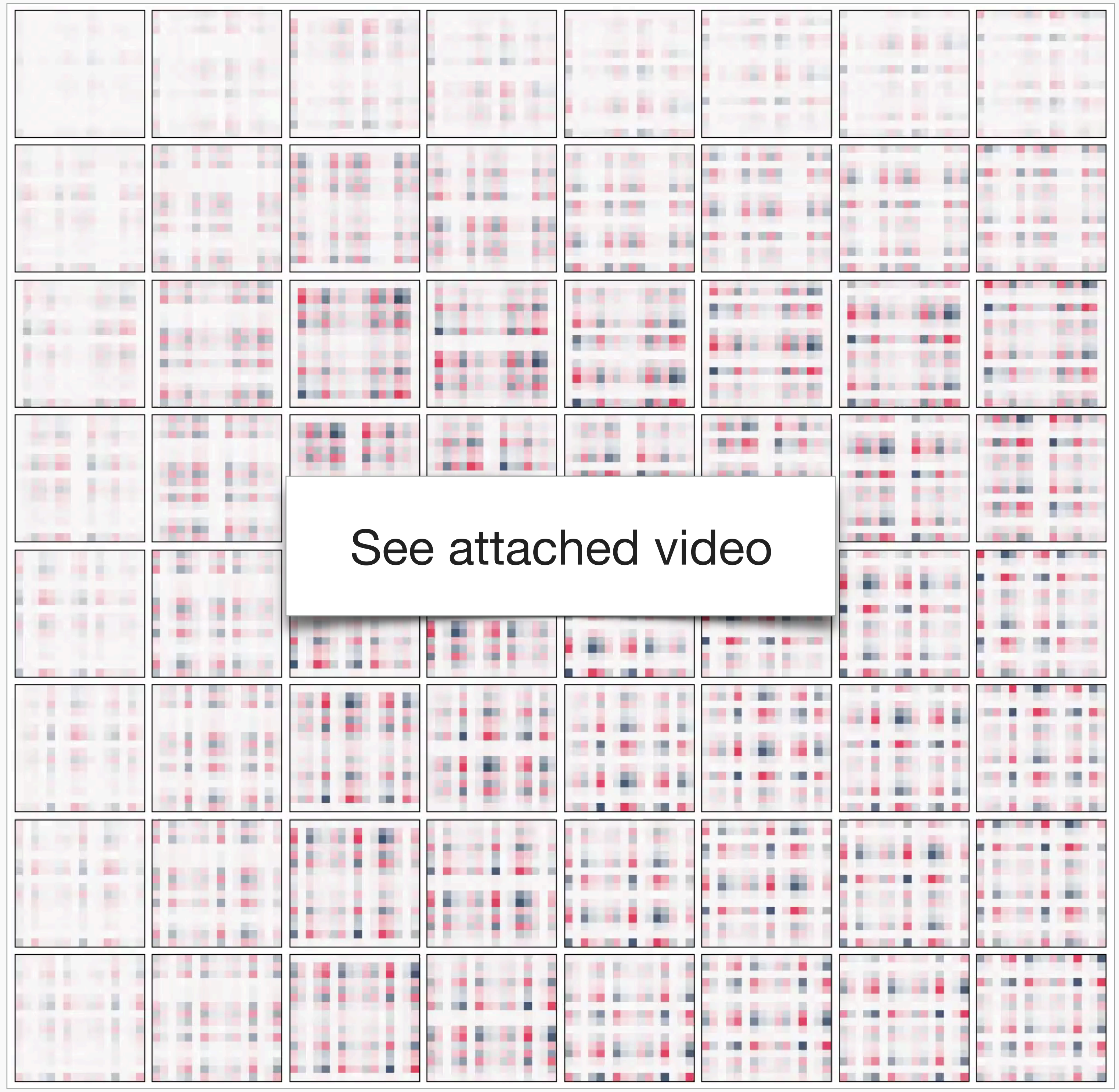

Use

1. *Learn* the best bases for various specialised tasks, like pile-up mitigation,
2. *Learn* observables for discriminating e.g. vector boson and 'QCD' jets, based on differences in their angular structure
3. Probe the splitting functions directly,
4. Possibly use these results to *learn* parton showering (and hadronisation) from data, with minimal theory input
5. Perhaps even develop a jet clustering algorithm based on this type of learning — however, this is perhaps best done using recurrent neural networks like LSTM

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