QCD RESUMMATION FOR BOOSTED TOP PRODUCTION

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My talk is about precision QCD calculations for boosted top production ($m_t \ll M_{t\bar{t}}$ or $m_t \ll p_T^t$)

[Broggio, Ferroglia, Marzani, BP, Scott, Yang, Wang: arXiv:1205.3662, arXiv:1207.4798, arXiv:1306.1537, arXiv:1310.3836, arXiv:1409.5294, arXiv:1601.07020 (PRL)]

Such calculations involve a combination of

- finesse (effective field theory to deal with multiple scales)
- brute force (NNLO+NNLL calculations for precision)

Factorization for inclusive production

Factorization for $h_1 h_2 \to t\bar{t}X$:

$$d\sigma^{t\bar{t}X}_{h_1,h_2} = \sum_{i,j=q,\bar{q},g} \int dx_1 dx_2 f_i^{h_1}(x_1,\mu_F) f_j^{h_2}(x_2,\mu_F) d\hat{\sigma}_{ij}^{t\bar{t}\hat{X}}\left(\hat{s}, m_t, \ldots, \alpha_s(\mu_R), \mu_F, \mu_R\right) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_t}\right)$$

$$s = \left(p_{h_1} + p_{h_2}\right)^2 \,,\; \hat{s} = x_1 x_2 s$$

- partonic cross sections $d\hat{\sigma}_{ij}$ from perturbation theory
- fixed-order perturbation theory is default approach

NNLO calculations

$$d\hat{\sigma}^{\mathrm{NNLO}}_{t\bar{t}+X} = d\hat{\sigma}^{\mathrm{VV}} + d\hat{\sigma}^{\mathrm{RV}} + d\hat{\sigma}^{\mathrm{RR}}$$

- the total $pp \to t\bar{t}X$ cross section is now known to NNLO
  [Czakon, Fiedler, Mitov '13]
- the FB asymmetry in $p\bar{p} \to t\bar{t}X$ cross section is now known to NNLO
  [Czakon, Fiedler, Mitov '14]
- arbitrary differential cross sections now known at NNLO
  [Czakon, Heymes, Mitov '15, '16]

NNLO is now the baseline for QCD in top-pair production

But not necessarily the end of the story...

Consider very large pair invariant mass where $\tau = M_{t\bar{t}}^2/s \to 1$

$$\frac{d\sigma}{dM_{t\bar{t}}} = \sum_{i,j} \int_{\tau}^{1} \frac{dz}{z}\, ff_{ij}(\tau/z, \mu_f)\, \frac{d\hat{\sigma}_{ij}}{dM_{t\bar{t}}}(z, m_t, M_{t\bar{t}}, \mu_f)$$

$$ff_{ij}(y, \mu_f) = \int_{y}^{1} \frac{dx}{x}\, f_{i/h_1}(x, \mu_f)\, f_{j/h_2}(y/x, \mu_f)$$

Two kinds of large logarithms in $d\hat{\sigma}$:

- soft logs: $[\ln^n(1-z)/(1-z)]_+ \quad (z \equiv M_{t\bar{t}}^2/\hat{s})$
- small-mass (collinear) logs: $\ln(m_t/M_{t\bar{t}})$

Goal: develop a formalism to resum both types of logs simultaneously

Resummation for differential cross sections

Many differential observables involve two hard scales

$$Q \gg M \gg \Lambda_{\rm QCD}$$

- if $\alpha_s \ln \frac{Q}{M} \sim 1$, must "resum" logarithmic corrections to get a sensible answer for $d\hat{\sigma}$
- "resummation" = re-organization of large logarithms in perturbative expansion. Defining $L = \ln\left(\frac{Q}{M}\right)$:

$$d\hat{\sigma} = \overbrace{\overbrace{1+\alpha_s(L^2+L+1)}^{NLO}+\alpha_s^2(L^4+L^3+L^2+L+1)}^{NNLO}+\mathcal{O}(\alpha_s^3)$$

$$= \exp\Big(\underbrace{\underbrace{Lg_1(\alpha_s L)}_{LL}+g_2(\alpha_s L)}_{NLL}+\alpha_s g_3(\alpha_s L)+\cdots\Big)\underbrace{C(\alpha_s)}_{\text{constants}}+\mathcal{O}\left(\frac{M}{Q}\right)$$

- to derive resummation requires factorization in certain limits

Two limits of partonic cross sections

$$\text{Soft Limit:} \quad \hat{s}, t_1, m_t^2 \gg \hat{s}(1-z)^2$$

$$\text{Boosted Soft Limit:} \quad \hat{s}, t_1 \gg m_t^2 \gg \hat{s}(1-z)^2 \gg m_t^2(1-z)^2$$

- Resummation in soft limit for differential cross sections has been studied extensively , starting with [Sterman, Kidonakis '97], leading to NNLL calculations in [Ahrens et. al '10, Kidonakis '11]
- Resummation for boosted soft limit has been studied only recently: [Ferroglia, BP, Marzani, Yang '12, '13], [BP, Scott, Wang, Yang ,'16]

EFT approach: Factorize $d\hat{\sigma}$ into one-scale functions, then use RG equations to resum logs

Factorization in the soft limit ($M \equiv M_{t\bar{t}}$)

Soft limit:

$$\hat{s}, \hat{t}_1, m_t^2 \gg \hat{s}(1-z)^2$$

Partonic cross section factorizes [Kidonakis, Sterman '97]

$$d\hat{\sigma}_{ij} = \mathrm{Tr}\left[\mathbf{H}_{ij}^m(M, m_t, \cos\theta, \mu_f)\, \mathbf{S}_{ij}^m(\sqrt{\hat{s}}(1-z), M, m_t, \cos\theta, \mu_f)\right] + \mathcal{O}(1-z)$$

- $\mathbf{H}_{ij}^m$ are color-space matrices related to virtual corrections
- $\mathbf{S}_{ij}^m$ are color-space matrices related to real emission in soft limit

  Soft corrections involve $\delta(1-z)$ or

$$\alpha_s^n \left[\frac{\ln^m(1-z)}{1-z}\right]_+ ; \quad m = 0, \cdots, 2n-1$$

Mellin transforms

- Under Mellin transforms

$$\tilde{f}(N) = \mathcal{M}[f](N) = \int_0^1 dx\, x^{N-1} f(x); \mathcal{M}^{-1}[\tilde{f}](x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\, x^{-N} f(N)$$

$$d\widetilde{\sigma}(N) = \sum_{ij} \widetilde{\mathcal{L}}_{ij}(N,\mu_f)_{ij}\, d\widetilde{\hat{\sigma}}_{ij}(N, M, m_t, \cos\theta, \mu_f)$$

- Factorization in Mellin space soft limit: $M, m_t \gg M/N$:

$$d\widetilde{\hat{\sigma}}_{ij} = \mathrm{Tr}\left[\mathbf{H}^m_{ij}(M, m_t, \cos\theta, \mu_f)\widetilde{\mathbf{s}}^m_{ij}\left(\ln\frac{M^2}{\bar{N}^2\mu_f^2}, M, m_t, \cos\theta, \mu_f\right)\right] + \mathcal{O}\left(\frac{1}{N}\right)$$

- Mellin-space soft function has simple RG equation

$$\frac{d}{d\ln\mu}\widetilde{\mathbf{s}}^m_{ij} = -\left[\Gamma_{\mathrm{cusp}}\ln\frac{M^2}{\bar{N}^2\mu^2} - \gamma^{m,h\dagger}_{ij}\right]\widetilde{\mathbf{s}}^m_{ij} - \widetilde{\mathbf{s}}^m_{ij}\left[\Gamma_{\mathrm{cusp}}\ln\frac{M^2}{\bar{N}^2\mu^2} - \gamma^{m,h}_{ij}\right]$$

- can solve with standard RG techniques to get resummed partonic cross section in Mellin space

Resummed partonic cross section

$$d\widetilde{\hat{\sigma}}_{ij}(\mu_f) = \exp\left[\frac{4\pi}{\alpha_s(\mu_h)} g_1^m(\lambda,\lambda_f) + g_2^m(\lambda,\lambda_f) + \frac{\alpha_s(\mu_h)}{4\pi} g_3^m(\lambda,\lambda_f) + \dots\right]$$
$$\times \mathrm{Tr}\left[\widetilde{\mathbf{u}}_{ij}^m(\mu_h,\mu_s)\mathbf{H}_{ij}^m(M,m_t,\cos\theta,\mu_h)\widetilde{\mathbf{u}}_{ij}^{m\dagger}(\mu_h,\mu_s)\widetilde{\mathbf{s}}_{ij}^m\left(\ln\frac{M^2}{\bar{N}^2\mu_s^2},M,m_t,\cos\theta,\mu_s\right)\right]$$

$$\lambda \equiv \beta_0\frac{\alpha_s(\mu_h)}{2\pi}\ln(\mu_h/\mu_s), \qquad \lambda_f \equiv \beta_0\frac{\alpha_s(\mu_h)}{2\pi}\ln(\mu_h/\mu_f)$$

$$\widetilde{\mathbf{u}}_{ij}^m(\mu_h,\mu_s) \equiv \mathcal{P}\exp\left\{\int_{\alpha_s(\mu_h)}^{\alpha_s(\mu_s)}\frac{d\alpha}{\beta(\alpha)}\gamma_{ij}^{m,h}(M,m_t,\cos\theta,\alpha)\right\}$$

- choosing $\mu_h \sim M$, $\mu_s \sim M/N$ resums all logs into exponentials
- inverse Mellin transforming (using Minimal prescription [Catani, Mangano, Nason, Trentadue '96]) gives resummed cross section

The boosted soft limit

Mellin space boosted soft limit: $M \gg m_t \gg \frac{M}{N} \gg \frac{m_t}{N}$

$$d\widetilde{\hat{\sigma}}_{ij} = \mathrm{Tr}\left[\mathbf{H}_{ij}^m(M, m_t, \cos\theta, \mu_f)\widetilde{\mathbf{s}}_{ij}^m\left(\ln\frac{M^2}{\bar{N}^2\mu_f^2}, M, m_t, \cos\theta, \mu_f\right)\right] + \mathcal{O}\left(\frac{1}{N}\right)$$

Can no longer use formula derived in soft limit, because both $\mathbf{H}_{ij}^m$ and $\widetilde{\mathbf{s}}_{ij}^m$ contain large logs of form $\ln(m_t/M)$ in the boosted soft limit

However, these small-mass logs are of collinear origin: can understand and factorize them

Factorization in the boosted soft limit

- Factorization of hard function [Ferroglia, BP, Yang '12], based on [Mitov/Moch '06] relation between massless and small-mass amplitudes

$$\mathbf{H}_{ij}^{m}(M, m_t, \cos\theta, \mu) = C_D^2(m_t, \mu)\mathbf{H}_{ij}(M, \cos\theta, \mu) + \mathcal{O}\left(\frac{m_t^2}{M^2}\right)$$

  - $\mathbf{H}_{ij}$ related to virtual corrections to massless $gg, q\bar{q} \to \bar{Q}Q$ scattering
  - C_D related to virtual corrections to heavy-quark fragmentation function

- Factorization of soft-function relies on analysis of phase-space integrals using method of regions [Ferroglia, BP, Marzani, Yang '12,'13]

$$\widetilde{\mathbf{s}}_{ij}^{m}\left(\ln\frac{M^2}{\bar{N}^2\mu^2}, M, m_t, \cos\theta, \mu\right) = \widetilde{\mathbf{s}}_{ij}\left(\ln\frac{M^2}{\bar{N}^2\mu^2}, M, \cos\theta, \mu\right)\tilde{s}_D^2\left(\ln\frac{m_t}{\bar{N}\mu}, \mu\right) + \mathcal{O}\left(\frac{m_t^2}{M^2}\right)$$

  - $\widetilde{\mathbf{s}}_{ij}$ related to wide-angle soft emission to massless $gg, q\bar{q} \to \bar{Q}Q$
  - $\widetilde{s}_D$ related to soft emission collinear to top or anti-top (soft part of heavy-quark fragmentation function)

Resummation in the boosted limit

Can derive and solve RG equations to get joint resummation formula
[BP, Scott, Wang, Yang '16]

$$
\begin{aligned}
d\widetilde{\hat{\sigma}}_{ij}(\mu_f) = \mathrm{Tr}\Bigg[&\widetilde{\mathbf{U}}_{ij}(\mu_f,\mu_h,\mu_s)\mathbf{H}_{ij}(M,\cos\theta,\mu_h)\widetilde{\mathbf{U}}^{\dagger}_{ij}(\mu_f,\mu_h,\mu_s) \\
&\times \widetilde{\boldsymbol{s}}_{ij}\left(\ln\frac{M^2}{\bar{N}^2\mu_s^2}, M, \cos\theta, \mu_s\right)\Bigg] \times \widetilde{U}_D^2(\mu_f,\mu_{dh},\mu_{ds})\, C_D^2(m_t,\mu_{dh})\, \tilde{s}_D^2\left(\ln\frac{m_t}{\bar{N}\mu_{ds}}, \mu_{ds}\right) \\
&+\mathcal{O}\left(\frac{1}{N}\right) + \mathcal{O}\left(\frac{m_t^2}{M^2}\right)
\end{aligned}
$$

- remove large logs in matching functions through choices
  $\mu_h \sim M, \quad \mu_s \sim M/N, \quad \mu_{dh} \sim m_t \quad \mu_{ds} \sim m_t/N$
- $\tilde{U}_{ij}$ and $\widetilde{U}_D$ are RG factors which resum logs
- can estimate perturbative uncertainties by varying all scales independently

Logarithmic accuracy: the frontier

Resummation involves anomalous dimensions Γ, γ and matching functions

	$\Gamma^i_{\rm cusp}$	γ_i	$\boldsymbol{H}, \widetilde{\boldsymbol{s}}, c_D, \widetilde{s}_D$
NLL	2-loop	1-loop	0-loop
NNLL	3-loop	2-loop	1-loop
NNLL'	3-loop	2-loop	2-loop

Analytic frontier (differential cross sections):

- soft limit: NNLL [Ahrens, Ferroglia, Neubert, BP, Yang, Kidonakis]
- boosted soft limit: NNLL': [Ferroglia, BP, Yang]

Numerical implementations of resummation (not fixed-order approximations!)

- soft limit: NLL in Mellin space for A_{FB} [Almeida,Sterman,Vogelsang '08]
- soft limit: NNLL in momentum space [Ahrens, Ferroglia, Neubert, BP, Yang '10,'11]
- soft limit to NNLL and boosted soft limit to NNLL' in Mellin space [BP, Scott, Wang, Yang '16]

Matching across kinematic limits and with fixed order

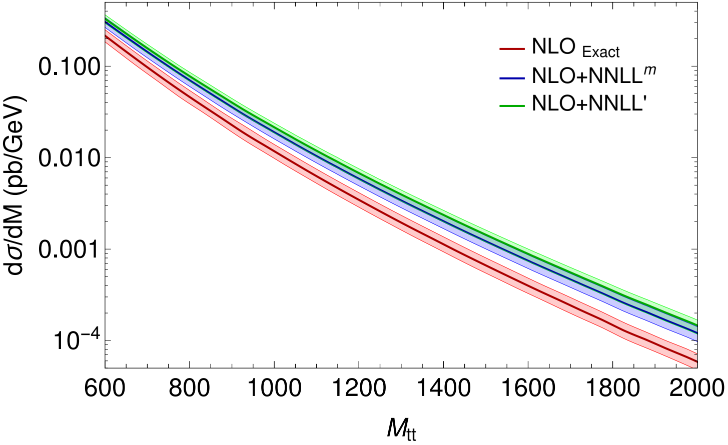
Best prediction matches NNLL'^{b} (boosted) with NNLL^{m} (soft) with NLO

$$d\sigma^{\mathrm{NLO+NNLL}'} = d\sigma^{\mathrm{NNLL}'^{b}} + \left(d\sigma^{\mathrm{NNLL}^{m}} - d\sigma^{\mathrm{NNLL}^{b}}\Big|_{\substack{\mu_{\mathrm{ds}}=\mu_s \\ \mu_{\mathrm{dh}=\mu_h}}} \right) + \left(d\sigma^{\mathrm{NLO}} - d\sigma^{\mathrm{NNLL}^{m}}\Big|_{\substack{\mu_{\mathrm{s}}=\mu_f \\ \mu_{\mathrm{h}=\mu_f}}} \right)$$

- first parenthesis vanishes as $m_t \to 0$, matches boosted soft and soft
- second line vanishes as $z \to 1$, matches resummed to NLO (can easily be modified to NNLO)
- NLO+NNLL' includes all possible perturbative improvements across phase space

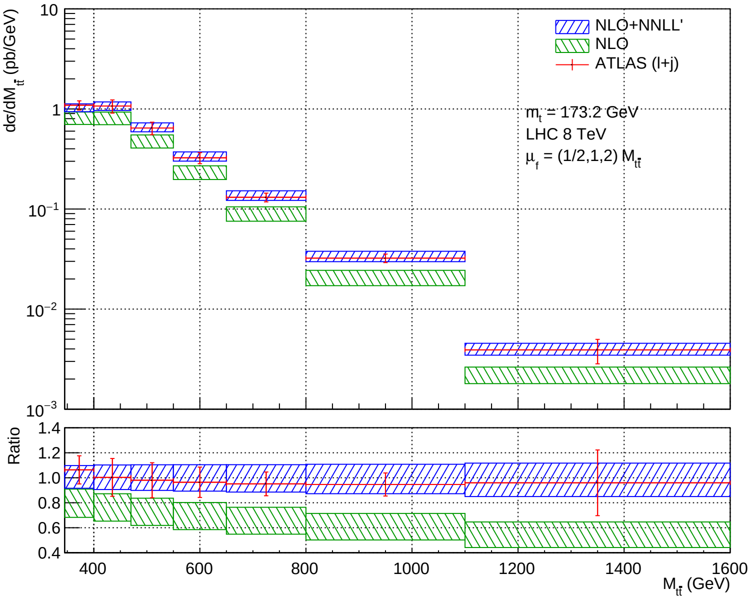
Results at high M_{tt} at LHC with $\sqrt{s} = 8$ TeV

MSTW2008NNLO PDFs, $m_t = 173.2$ GeV, $\mu_f \in [M_{tt}/2, 2M_{tt}]$

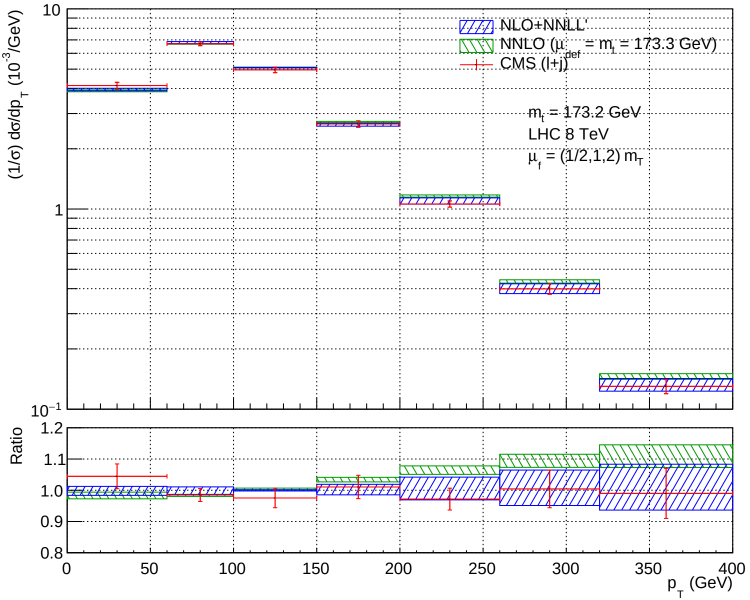


- bands from scale variations of μ_f, and μ_h, μ_s (NNLL^m), and μ_h, μ_s, μ_{dh}, μ_{ds} (NNLL$'$)
- NNLL^m resummation in soft limit is large effect compared to NLO (at $\mu_f = M$)
- boosted resummation included in NNLL$'$ produces mild, further enhancement

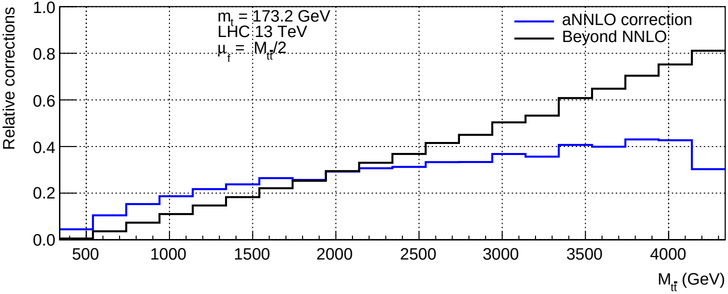
Application to the LHC: 8 TeV $M_{t\bar{t}}$ Data

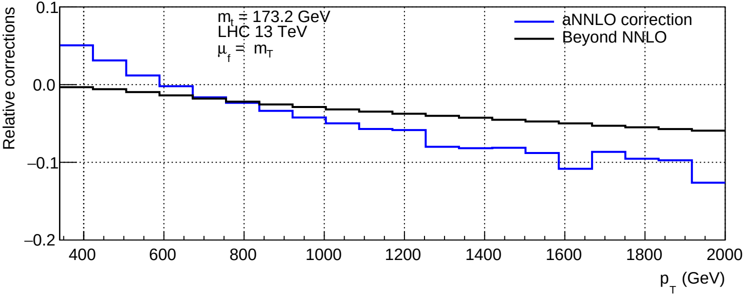


Applications to the LHC: 8 TeV p_T Data

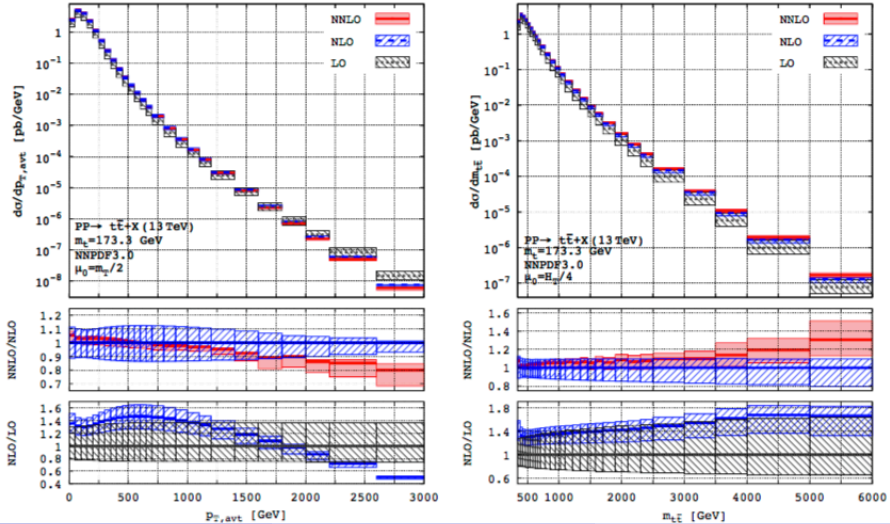


Beyond NNLO resummation effects





NNLO WITH DYNAMIC SCALES



Conclusions

Summary:

- QCD for boosted top production is a complicated multi-scale problem
- presented formalism for joint resummation of soft and small-mass logs in Mellin space
- resummation effects significant at high M_{tt}
- resummation adds information to NNLO but doesn't replace it: NNLL'+NNLO will be most interesting (preferably comparing results with several default μ_f)

To do:

- EW corrections, decays in narrow width approximation
- comparison with NLO+parton shower-based resummations