A Geometrical View of Higgs Effective Theory

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Outline

- SM and its $\mathcal{G}/\mathcal{H}$ symmetry breaking structure
- SMEFT
- Linear vs nonlinear transformations
- HEFT
- Curvature
- Experimental probes
- Radiative Corrections

Talk based on: Rodrigo Alonso, Elizabeth Jenkins, AM arXiv:1511.00724

Input

Spontaneously Broken Gauge Theory

An $SU(2) \times U(1)$ gauge theory spontaneously broken to $U(1)_{\rm em}$ at a scale $v = 246$ GeV.

- Different methods of breaking the gauge symmetry
- A particle has been seen with a mass $M_h \sim 126$ GeV; 0^+ quantum numbers favored
- No evidence for any new particles in the few hundred GeV range.

Standard Model – A fundamental scalar doublet

A field H which transforms as $\mathbf{2}_{1/2}$

$$H = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} \qquad\qquad V(H) = -\lambda \left(H^\dagger H - \frac{v^2}{2} \right)$$

Expanding about the minimum,

$$H = \begin{bmatrix} \phi^+ \\ \frac{1}{\sqrt{2}} \left(v + h + i\eta \right) \end{bmatrix}$$

h is the neutral Higgs with mass

$$m_h^2 = 2\lambda v^2$$

with mass of order the electroweak scale. ϕ^+ and η are the eaten Goldstone bosons that give mass to the W and Z.

Standard Model

H transforms linearly under the gauge symmetry $\mathcal{G} = SU(2) \times U(1)$

$$H \to U\, H$$

The minimum $\langle H \rangle$ is invariant under $\mathcal{H} = U(1)_{\text{em}}$:

$$e^{iQ\alpha} \left[\begin{array}{c} 0 \\ \frac{1}{\sqrt{2}} v \end{array} \right] = \left[\begin{array}{c} 0 \\ \frac{1}{\sqrt{2}} v \end{array} \right]$$

There is a $\mathcal{G} = SU(2) \times U(1)$ invariant point $H = 0$.

Standard Model – Custodial Symmetry

$$H = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} i\varphi_1 + \varphi_2 \\ \varphi_4 - i\varphi_3 \end{bmatrix}$$

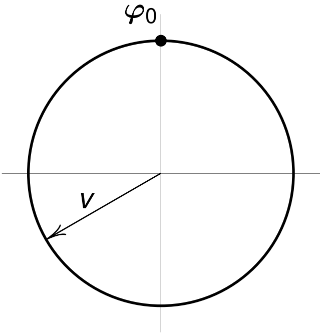
$$\boldsymbol{\varphi} = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{bmatrix} \qquad H^\dagger H = \frac{1}{2}\boldsymbol{\varphi} \cdot \boldsymbol{\varphi} = \frac{1}{2}\left(\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2\right)$$

$$V(H) = -\lambda \left(H^\dagger H - \frac{v^2}{2}\right) = -\frac{\lambda}{4}\left(\boldsymbol{\varphi} \cdot \boldsymbol{\varphi} - v^2\right)^2$$

Scalar Potential

$$V = -\frac{\lambda}{4}\left(\varphi \cdot \varphi - v^2\right)^2$$

$$\varphi_0 = (0, 0, 0, v)^T$$



Pick φ_0 to be the North pole of S^3.

$\mathcal{H} = O(3)$ leaves φ_0 invariant

$\mathcal{G} = O(4)$ maps φ_0 to points on S^3.

Vacuum manifold is $\mathcal{G}/\mathcal{H} = O(4)/O(3)$

Goldstone bosons — fluctuations on S^3.

Custodial $SU(2)$

$$SU(2)_L : H \to UH \qquad O(4) : \varphi \to O\varphi \qquad SU(2)_L \in O(4)$$

Unbroken symmetry is $O(3)$ instead of $U(1)$, which implies that W_1, W_2 and W_3 are related, so that $W^\pm$ and Z are related.

$$M_W = M_Z \cos\theta_W$$

$$O(4) \sim SU(2)_L \times SU(2)_R \qquad O(3) \sim SU(2)_V$$

$O(4) \sim SU(2) \times SU(2)$

$$\delta\varphi = (iT)\varphi$$

$O(4)$ matrices are antisymmetric

$$i\,M^{[12]} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M^{[ab]} \qquad 1 \le a < b \le 4$$

$$\mathbf{J} = \left(M^{[23]}, M^{[31]}, M^{[12]}\right) \qquad \mathbf{K} = \left(M^{[14]}, M^{[24]}, M^{[34]}\right)$$

$O(4) \sim SU(2) \times SU(2)$

$$\mathbf{T}_L = \frac{1}{2}(\mathbf{J} + \mathbf{K}) \qquad \mathbf{T}_R = \frac{1}{2}(\mathbf{J} - \mathbf{K})$$

$$iA_L \cdot T_L = \frac{1}{2}\begin{bmatrix} 0 & A_L^3 & -A_L^2 & A_L^1 \\ -A_L^3 & 0 & A_L^1 & A_L^2 \\ A_L^2 & -A_L^1 & 0 & A_L^3 \\ -A_L^1 & -A_L^2 & -A_L^3 & 0 \end{bmatrix}$$

$$iA_R \cdot T_R = \frac{1}{2}\begin{bmatrix} 0 & A_R^3 & -A_R^2 & -A_R^1 \\ -A_R^3 & 0 & A_R^1 & -A_R^2 \\ A_R^2 & -A_R^1 & 0 & -A_R^3 \\ A_R^1 & A_R^2 & A_R^3 & 0 \end{bmatrix}$$

SM

φ transforms linearly under $\mathcal{G} = O(4)$:

$$\delta\varphi = i\, T\, \varphi$$

The minimum φ_0 is invariant under $\mathcal{H} = O(3)$

There is a $\mathcal{G} = O(4)$ invariant point $\varphi = 0$.

The gauge group $SU(2) \times U(1) \in O(4)$

[When you add fermions, you have to have an additional $U(1)_X$ where Q_L has charge $1/6$, etc.]

SMEFT

An effective theory with the same field content as the SM. The scalar Lagrangian can have higher dimension operators

$$\mathcal{L} = \partial_\mu H^\dagger \partial^\mu H - \lambda \left(H^\dagger H - \frac{v^2}{2} \right) + \frac{c_H}{\Lambda^2} \left(H^\dagger H \right)^3 - \frac{c_\Box}{\Lambda^2} \partial_\mu \left(H^\dagger H \right) \partial^\mu \left(H^\dagger H \right) + \ldots$$

Will use the custodial $SU(2)$ invariant form

$$\mathcal{L} = \frac{1}{2} f(\varphi \cdot \varphi)\, \partial_\mu \varphi \cdot \partial^\mu \varphi + \frac{1}{2} g(\varphi \cdot \varphi)\, (\varphi \cdot \partial_\mu \varphi)(\varphi \cdot \partial^\mu \varphi) + \ldots$$

and

$$f(\varphi \cdot \varphi) = 1 + \frac{c_1}{\Lambda^2} \varphi \cdot \varphi + \ldots$$

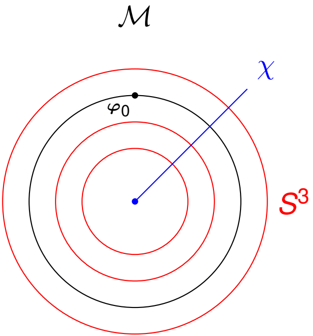
SMEFT

In both SM and SMEFT:

- φ transforms linearly under $\mathcal{G} = O(4)$:
- The minimum φ_0 is invariant under $\mathcal{H} = O(3)$
- There is a $\mathcal{G} = O(4)$ invariant point $\varphi = 0$.

$$\text{SM} \subsetneq \text{SMEFT}$$

Scalar Manifold $\mathcal{M}$



Decompose $\mathbb{R}^4$ into a radial direction χ and S^3.

The $O(4)$ symmetry acts on S^3, and χ is a singlet.

Vacuum manifold: black sphere

Vacuum: black dot φ_0

All you need for EW symmetry breaking is the black sphere

Terminology

Goldstone boson manifold is $\mathcal{G}/\mathcal{H} = S^3$ with symmetry $\mathcal{G} = O(4)$

Radial direction χ — any direction which is invariant under $O(4)$.

Symmetry of ground state φ_0 is $O(3)$

SM in spherical polar coordinates

$$L = \frac{1}{2} D_\mu \varphi \cdot D^\mu \varphi - \frac{\lambda}{4} \left(\varphi \cdot \varphi - v^2 \right)^2$$

Switch to spherical polar coordinates

$$\varphi = \chi \, \boldsymbol{u} \qquad \boldsymbol{u} \cdot \boldsymbol{u} = 1 \qquad \boldsymbol{u} = (u_1, u_2, u_3, u_4)$$

Components of $\boldsymbol{u}$ are not independent.
Under $O(4)$,

$$\chi \to \chi \qquad \boldsymbol{u} \to O \, \boldsymbol{u}$$

$$\boldsymbol{u} = \begin{bmatrix} \boldsymbol{\pi} \\ \sqrt{1 - \boldsymbol{\pi} \cdot \boldsymbol{\pi}} \end{bmatrix} \qquad \boldsymbol{\pi} = (\pi_1, \pi_2, \pi_3)$$

$$\begin{bmatrix} \delta\boldsymbol{\pi} \\ \delta\sqrt{1-\boldsymbol{\pi}\cdot\boldsymbol{\pi}} \end{bmatrix} = \left[\begin{array}{c|c} C & B \\ \hline -B^T & 0 \end{array}\right] \begin{bmatrix} \boldsymbol{\pi} \\ \sqrt{1-\boldsymbol{\pi}\cdot\boldsymbol{\pi}} \end{bmatrix}$$

$$\delta\boldsymbol{\pi} = C\boldsymbol{\pi} + B\sqrt{1-\boldsymbol{\pi}\cdot\boldsymbol{\pi}}$$

$\boldsymbol{\pi}$ transforms linearly under $O(3)$, and nonlinearly under the broken $O(4)$ generators.

$$\boldsymbol{\pi}\cdot\delta\boldsymbol{\pi} = \boldsymbol{\pi}^T C\boldsymbol{\pi} + \boldsymbol{\pi}^T B\sqrt{1-\boldsymbol{\pi}\cdot\boldsymbol{\pi}} = \boldsymbol{\pi}^T B\sqrt{1-\boldsymbol{\pi}\cdot\boldsymbol{\pi}}$$

$$\frac{\boldsymbol{\pi}\cdot\delta\boldsymbol{\pi}}{\sqrt{1-\boldsymbol{\pi}\cdot\boldsymbol{\pi}}} = -B^T\boldsymbol{\pi}\sqrt{1-\boldsymbol{\pi}\cdot\boldsymbol{\pi}}$$

Covariant Derivative

$$\varphi = \chi \boldsymbol{u} \qquad \boldsymbol{u} \cdot \boldsymbol{u} = 1 \qquad \boldsymbol{u} = (u_1, u_2, u_3, u_4)$$

$$\partial_\mu \varphi = \partial_\mu \chi \, \boldsymbol{u} + \chi \partial_\mu \, \boldsymbol{u} \qquad D_\mu \varphi = \partial_\mu \chi \, \boldsymbol{u} + \chi \, D_\mu \boldsymbol{u}$$

$$D_\mu \boldsymbol{u} = \partial_\mu \boldsymbol{u} + i \, g \, A_\mu \, \boldsymbol{u}$$

$$L = \frac{1}{2} (\partial_\mu \chi)^2 + \frac{1}{2} \chi^2 \, D_\mu \boldsymbol{u} \cdot D^\mu \boldsymbol{u} - \frac{\lambda}{4} \left(\chi^2 - v^2 \right)^2$$

Finally, let

$$\chi = v + h$$

HEFT

$$L = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}(v+h)^2\, D_\mu \boldsymbol{u} \cdot D^\mu \boldsymbol{u} - \frac{\lambda}{4}\left(2vh + h^2\right)^2 \qquad \text{SM}$$

HEFT

$O(4) \to O(3)$ symmetry breaking

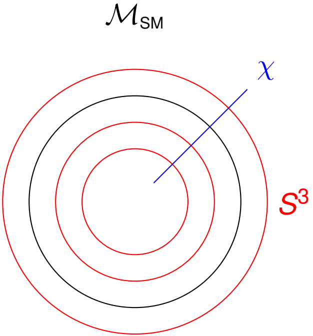
Goldstone boson manifold is $G/H = S^3$.

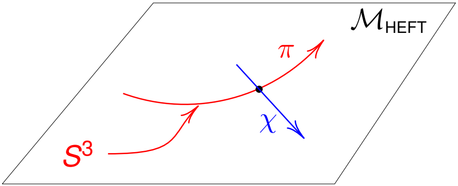
$$L = \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}v^2 F(h)^2\, D_\mu \boldsymbol{u} \cdot D^\mu \boldsymbol{u} - V(h)$$

A general radial function $F(h)$, with

$$F(0) = 1$$

$v \sim 246$ GeV fixed by W, Z masses.





Bunch of spheres at each point of the radial direction h, but the radii do not have to be χ^2.

- $\boldsymbol{u}$ transforms linearly under $\mathcal{G} = O(4)$, but $\boldsymbol{\pi}$ transforms nonlinearly
- The vacuum is invariant under $\mathcal{H} = O(3)$
- There is a $\mathcal{G} = O(4)$ invariant point iff $F = 0$, so S^3 has zero radius
- SMEFT $\subsetneq$ HEFT, and a HEFT is a SMEFT iff $\exists$ an $O(4)$ invariant point on $\mathcal{M}$.

CCWZ formulation

Callan, Coleman, Wess, Zumino (1969)

If $\mathcal{G} \to \mathcal{H}$, then Goldstone bosons live on $\mathcal{G}/\mathcal{H}$.

Fields transform linearly under the unbroken group $\mathcal{H}$ and nonlinearly under the broken generators.

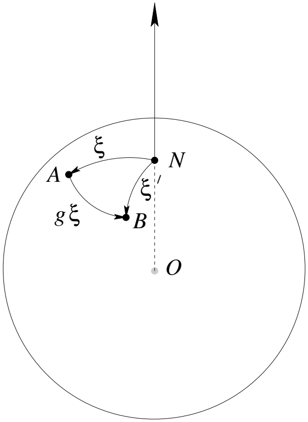
Can always make a change of variables to put field transformations in a standard form.

$$\xi(x) = e^{i\boldsymbol{X}\cdot\boldsymbol{\theta}(x)} \qquad\qquad \xi(x) \to g\,\xi(x)\,h^{-1}(x)$$

The SM has $\mathcal{G}/\mathcal{H} = S^3$, construction done explicitly in terms of $\boldsymbol{u}$.

CCWZ

CCWZ: an alternate coordinate choice using a geodesic



Need $h(x)$ because $\mathcal{M}$ is curved.

Field Redefinitions

In a QFT, the S matrix is unchanged under field redefinitions.

Change of variables in an integral:

$$\int D\phi\; e^{i\,S(\phi)+i\,J\,\phi} = \int D\phi'\;\det\left[\frac{\delta f(\phi')}{\delta\phi'}\right] e^{i\,S(f(\phi'))+i\,J\,f(\phi')}$$
$$= \int D\phi'\; e^{i\,S'(\phi')+i\,J\,f(\phi')} \qquad \text{(caution: anomalies)}$$

Green's functions of S with source ϕ are the same as Green's functions of S' with source $f(\phi')$

Computing after field redefinitions:

$$\int D\phi'\; e^{i\,S'(\phi')+i\,J\,\phi'}$$

Green's functions of S' with source ϕ'

LSZ Reduction Formula

Greens' functions change under field redefinitions

$$G = \langle 0|\phi(x_1)\ldots\phi(x_n)|0\rangle \neq \langle 0|\phi'(x_1)\ldots\phi'(x_n)|0\rangle = G'$$

LSZ reduction formula — you can use any field Φ to compute the S-matrix as long as

$$\langle 0|\Phi|p\rangle \neq 0$$

So as long as ϕ and ϕ' create particles, the two Green's functions give the same S-matrix.

$$G \implies S = S' \impliedby G'$$

Geometry

Scalar fields live on some manifold $\mathcal{M}$

The S matrix is independent of field redefinitions

Field redefinitions $=$ changing coordinates on $\mathcal{M}$.

Observables only depend on the geometry of $\mathcal{M}$:

S matrix $\longleftrightarrow$ Geometry

$$L_{\text{linear}} = \frac{1}{2} D_\mu \varphi \cdot D^\mu \varphi - \frac{\lambda}{4}\left(\varphi \cdot \varphi - v^2\right)^2$$

$$L_{\text{nonlinear}} = \frac{1}{2}\left(\partial_\mu h\right)^2 + \frac{1}{2}(v+h)^2\, D_\mu \boldsymbol{u} \cdot D^\mu \boldsymbol{u} - \frac{\lambda}{4}\left(2vh + h^2\right)^2$$

SM in linear sigma model and nonlinear sigma model form are completely equivalent.

Linear sigma model $\implies$ renormalizable

Nonlinear sigma model $\implies$ nonrenormalizable

False

Look at the $O(N)$ model. The SM is $N = 4$ case.

$$L = \frac{1}{2}\partial_\mu \phi \cdot \partial^\mu \phi - \frac{1}{4}\lambda \left(\phi \cdot \phi - v^2\right)^2$$

$$\phi(x) = \begin{bmatrix} \phi_1(x) \\ \vdots \\ \phi_N(x) \end{bmatrix}$$

$v^2 < 0$ unbroken phase, $v^2 > 0$ broken phase

Renormalize in unbroken phase:

$$m_\phi^2 = \lambda(-v^2)$$

$$L = \frac{1}{2} Z_\phi \, \partial_\mu \phi \cdot \partial^\mu \phi - \frac{1}{4} Z_\lambda \, \lambda \mu^{2\epsilon} \left(Z_\phi \, \phi \cdot \phi - Z_v^2 \, v^2 \, \mu^{-2\epsilon} \right)^2$$

$$Z_{m^2} = Z_\lambda Z_v^2$$

using dimensional regularization in $4 - 2\epsilon$ dimensions. At one-loop,

$$Z_i = 1 + \frac{\delta_i}{16\pi^2 \epsilon} \qquad \delta_\phi = 0 \qquad \delta_\lambda = \lambda(N + 8) \qquad \delta_v = -3\lambda$$

Symmetry breaking is an IR property, and does not affect short distance properties. The same counterterms renormalize the theory in the broken phase.

Broken phase:

$$\phi(x) = \begin{bmatrix} \varphi^1(x) \\ \vdots \\ \varphi^{n_G}(x) \\ v + h(x) \end{bmatrix}, \qquad n_G = N - 1\,.$$

$$\begin{aligned} L &= \frac{1}{2}\partial_\mu h \cdot \partial^\mu h + \frac{1}{2}\partial_\mu \boldsymbol{\varphi} \cdot \partial^\mu \boldsymbol{\varphi} \\ &\quad - \frac{1}{4}\lambda\big(h^4 + 2h^2\, \boldsymbol{\varphi} \cdot \boldsymbol{\varphi} + (\boldsymbol{\varphi} \cdot \boldsymbol{\varphi})^2 + 4vh^3 + 4vh\, \boldsymbol{\varphi} \cdot \boldsymbol{\varphi} + 4h^2 v^2\big) \\ m_\varphi^2 &= 0 \\ m_h^2 &= 2\lambda v^2 \end{aligned}$$

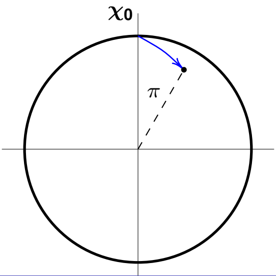
L has cubic vertices, and a different structure than the unbroken phase. Nevertheless, the same counterterms make the theory finite.

CCWZ form

$$\phi(x) = [v + h(x)]\,\xi(x)\chi_0 \qquad \chi_0 = (0, 0, \ldots, 0, 1)^T$$

where

$$\xi(x) = \exp\, \frac{1}{v} \begin{bmatrix} 0 & \ldots & 0 & \pi^1 \\ 0 & \ldots & 0 & \pi^2 \\ \vdots & & \vdots & \vdots \\ 0 & \ldots & 0 & \pi^{n_G} \\ -\pi^1 & \ldots & -\pi^{n_G} & 0 \end{bmatrix} = \exp\left(i\boldsymbol{\Pi}/v\right), \qquad \boldsymbol{\Pi} = \pi^a X^a$$

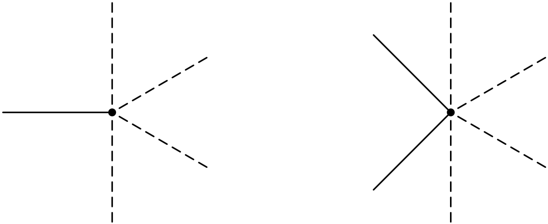


Rotate by angle $|\boldsymbol{\pi}|$

The Lagrangian is

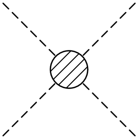
$$
\begin{aligned}
L &= \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}(h+v)^2 \boldsymbol{\chi}_0^T \partial_\mu \xi^T \partial^\mu \xi \boldsymbol{\chi}_0 - \frac{1}{4}\lambda\left(h^2+2hv\right)^2 \\
&= \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{4}\lambda\left(h^2+2hv\right)^2 + \frac{1}{2}\left(1+\frac{h}{v}\right)^2\left[\partial_\mu \boldsymbol{\pi}\cdot\partial^\mu \boldsymbol{\pi}\right] \\
&+ \frac{1}{6v^2}\left(1+\frac{h}{v}\right)^2\left[(\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi})^2 - (\boldsymbol{\pi}\cdot\boldsymbol{\pi})(\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi})\right] + \ldots
\end{aligned}
$$

Potential only depends on h, π are the Goldstone bosons.

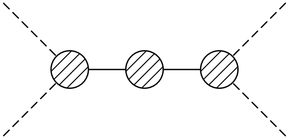


new vertices not present in linear Lagrangian

Skeleton graphs:



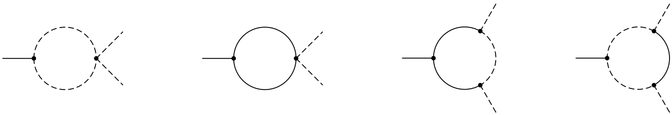
(a)



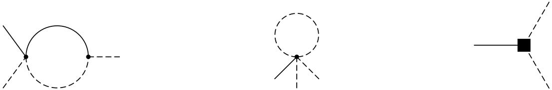
(b)

(b) missing in linear case

$h\pi\pi$

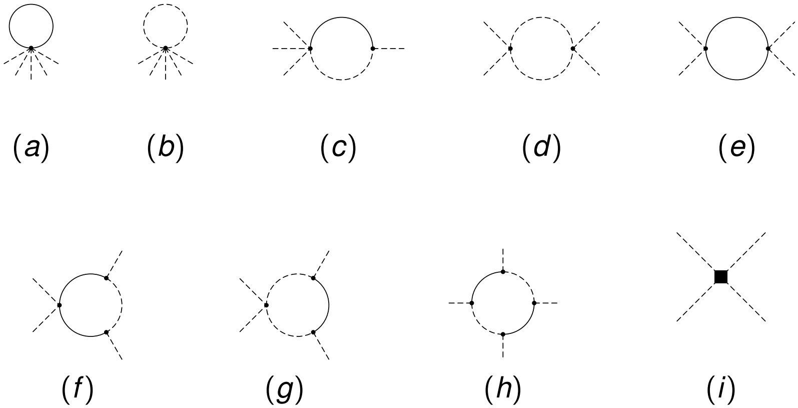


(a) (b) (c) (d)



(e) (f) (g)

π^4



only graph (d) in unbroken phase

$\mathcal{O}(p^4)$ Amplitude

divergent part

$$A_\infty = \frac{1}{16\pi^2\epsilon}\left[\frac{n_G}{3v^4}O_3 - \frac{4}{3v^4}O_4 + \frac{2n_G-3}{18v^4}O_5 + \frac{1}{18v^4}O_6\right]$$

$$O_1 = (\partial_\mu \boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi})(\partial_\nu \boldsymbol{\pi}\cdot\partial^\nu\boldsymbol{\pi}), \qquad O_2 = (\partial_\mu \boldsymbol{\pi}\cdot\partial_\nu\boldsymbol{\pi})(\partial^\mu \boldsymbol{\pi}\cdot\partial^\nu\boldsymbol{\pi}),$$
$$O_3 = \left(\partial^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}\right)(\partial_\mu \boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}), \qquad O_4 = \left(\partial^2\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}\right)(\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}),$$
$$O_5 = \left(\partial^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}\right)\left(\partial^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}\right), \qquad O_6 = \left(\partial^2\boldsymbol{\pi}\cdot\partial^2\boldsymbol{\pi}\right)(\boldsymbol{\pi}\cdot\boldsymbol{\pi}),$$

$O_{1,2}$ parts finite using unbroken phase counterterms.
A_∞ vanishes on-shell using equations of motion.
Appelquist and Bernard: Did the chiral version where $O_{1,2}$ also ∞.

S-matrix is finite and the theory is renormalizable.

Scalar Metric

$$L = \frac{1}{2} g_{ij}(\phi)\, \partial_\mu \phi^i \partial^\mu \phi^j$$

defines the metric tensor on $\mathcal{M}$.

$$L = \frac{1}{2} (\partial_\mu h)^2 + \frac{1}{2} v^2 F(h)^2\, D_\mu \boldsymbol{u} \cdot D^\mu \boldsymbol{u} - V(h)$$

$\phi = \{\boldsymbol{\pi}, h\}$ with $(u_1, u_2, u_3) = \boldsymbol{\pi}$, $u_4 = \sqrt{1 - \boldsymbol{\pi} \cdot \boldsymbol{\pi}}$.

$$g = \begin{bmatrix} F(h)^2 g_{ab}(\pi) & 0 \\ 0 & 1 \end{bmatrix}$$

$g_{ab}(\boldsymbol{\pi})$ is the metric on a sphere of radius v.
No off-diagonal terms because of $O(3)$ symmetry, and can always pick $g_{hh} = 1$ by a field redefinition of h.

Curvature of S^3

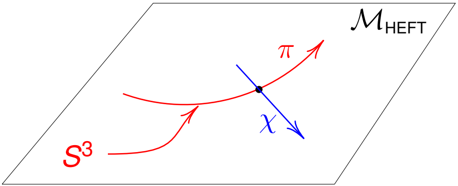
First look at curvature of S^3:

$$\widehat{R}_{abcd}(\varphi) = \frac{1}{v^2}\left(g_{ac}(\varphi)g_{bd}(\varphi) - g_{ad}(\varphi)g_{bc}(\varphi)\right),$$

$$\widehat{R}_{bd}(\varphi) = \frac{1}{v^2}(N_\varphi - 1)g_{bd}(\varphi) = \frac{2}{v^2}g_{bd}(\varphi),$$

$$\widehat{R} = \frac{1}{v^2}N_\varphi(N_\varphi - 1) = \frac{6}{v^2},$$

since S^3 is a maximally symmetric space.



Curvature of $\mathcal{M}$

Now look at the full manifold $\mathcal{M}$. Indices $\in \{a, h\}$

$$R_{abcd}(\phi) = \left[\frac{1}{v^2} - (F'(h))^2\right] F(h)^2 \left(g_{ac}g_{bd} - g_{ad}g_{bc}\right),$$

$$R_{ahbh}(\phi) = -F(h)F''(h)g_{ab},$$

$$R_{bd}(\phi) = \left\{\left[\frac{1}{v^2} - (F'(h))^2\right](N_\varphi - 1) - F''(h)F(h)\right\} g_{bd},$$

$$R_{hh}(\phi) = -\frac{N_\varphi F''(h)}{F(h)},$$

$$R(h) = \left[\frac{1}{v^2} - (F'(h))^2\right] \frac{N_\varphi(N_\varphi - 1)}{F(h)^2} - \frac{2N_\varphi F''(h)}{F(h)}.$$

Define dimensionless radial functions $\mathfrak{R}_4(h)$, $\mathfrak{R}_2(h)$ and $\mathfrak{R}_0(h)$

$$R_{abcd} = \mathfrak{R}_4(h)\widehat{R}_{abcd},$$
$$R_{bd} = \mathfrak{R}_2(h)\widehat{R}_{bd},$$
$$R = \mathfrak{R}_0(h)\widehat{R},$$

$$\mathfrak{R}_4(h) = \left[1 - v^2(F'(h))^2\right] F(h)^2,$$
$$\mathfrak{R}_2(h) = \left[1 - v^2(F'(h))^2\right] - \frac{v^2 F''(h)F(h)}{(N_\varphi - 1)},$$
$$\mathfrak{R}_0(h) = \left[1 - v^2(F'(h))^2\right] \frac{1}{F(h)^2} - \frac{2v^2 F''(h)}{(N_\varphi - 1)F(h)},$$

$$F(h)^4\,\mathfrak{R}_0(h) + \mathfrak{R}_4(h) = 2F(h)^2\,\mathfrak{R}_2(h)\,.$$

In SM

$$F(h) = 1 + \frac{h}{v}\,.$$

even though S^3 is curved, $\mathcal{M}$ is flat, and all the curvatures vanish.
In HEFT, one considers a general radial function

$$F(h) = 1 + c_1 \left(\frac{h}{v}\right) + \frac{1}{2} c_2 \left(\frac{h}{v}\right)^2 + \ldots,$$

In this case,

$$\begin{aligned}
\mathfrak{r}_4 &\equiv \mathfrak{R}_4(0) = 1 - c_1^2, \\
\mathfrak{r}_2 &\equiv \mathfrak{R}_2(0) = 1 - c_1^2 - \frac{1}{2} c_2, \\
\mathfrak{r}_0 &\equiv \mathfrak{R}_0(0) = 1 - c_1^2 - c_2,
\end{aligned}$$

with $\mathfrak{r}_0 + \mathfrak{r}_4 = 2\mathfrak{r}_2$,

Experimental Consequences

$$\begin{aligned}\mathscr{L} &= \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{v^2}{8}F(h)^2\left[2g^2 W_\mu^+ W^{-\mu} + (g^2+g'^2)Z_\mu Z^\mu\right] \\ &= \frac{1}{2}\partial_\mu h \partial^\mu h + \left[1 + 2c_1\frac{h}{v} + (c_1^2 + c_2)\frac{h^2}{v^2} + \ldots\right] \\ &\times \left[M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu\right].\end{aligned}$$

$v \sim 246\,\mathrm{GeV}$ is fixed by the gauge boson masses

S parameter:

$$\Delta S = \frac{1}{12\pi}\mathfrak{r}_4 \log\left(\frac{\Lambda^2}{M_Z^2}\right).$$

Experimental Consequences

Barbieri, Bellazzini, Rychkov, Varagnolo

The scattering amplitude of longitudinal W-bosons W_L depends on the curvature:

$$\mathcal{A}\left(W_L W_L \to W_L W_L\right) = \frac{s+t}{v^2}\mathfrak{r}_4\,,$$
$$\mathcal{A}\left(W_L W_L \to hh\right) = \frac{s}{v^2}\left(\mathfrak{r}_4 - \mathfrak{r}_0\right).$$

The scale of new physics governing the mass of these resonances is $\Lambda \sim 4\pi v/\sqrt{\mathfrak{r}}$

Note that this result is in accordance with the scenario of the Higgs boson as a Goldstone boson (Georgi, Kaplan) where resonances are expected at $\Lambda \sim 4\pi f$.

Radiative Corrections

$$Z[J] \equiv e^{iW[J]} = \int D\phi \exp\left[i\left(S[\phi] + \int J\,\phi\right)\right] .$$

$$\Gamma[\widetilde{\phi}] = W[J] - J\widetilde{\phi}, \qquad \widetilde{\phi} = \frac{\delta W}{\delta J}.$$

At one-loop Jackiw

$$\Gamma[\widetilde{\phi}] = S[\widetilde{\phi}] + \frac{i}{2} \ln\det\left(\frac{\delta^2 S}{\delta\phi^{\mathrm{i}}\delta\phi^{\mathrm{j}}}\right)_{\phi=\widetilde{\phi}} .$$

The variation of the action is computed using

$$\phi = \widetilde{\phi} + \eta$$

and expanding in η.

Gauged action:

$$\mathscr{L} = \frac{1}{2} g_{ab}(\varphi) D_\mu \varphi^a D^\mu \varphi^b + J_a \varphi^a,$$

$$\delta S = -\int \mathrm{d}x \; g_{ab}(\widetilde{\varphi}) \eta^a [\mathscr{D}^\mu (D_\mu \widetilde{\varphi})]^b \,,$$

where

$$\mathscr{D}_\mu \equiv \partial_\mu \delta^a_b + \mathcal{D}_b \left(D^\mu \varphi^a\right), \qquad \mathcal{D}_a V^b = \frac{\partial V^b}{\partial \varphi^a} + \Gamma^b_{ac}(\widetilde{\varphi}) V^c,$$

is the covariant derivative on the scalar manifold $\mathcal{G}/\mathcal{H} = S^3$,

It is important to remember that the metric and tensors are in *scalar field space*, which is curved. Spacetime is flat in our analysis.

The second variation of the action is

$$\delta^2 S = \frac{1}{2}\int \mathrm{d}x \Big[g_{ab}(\mathscr{D}_\mu\eta)^a(\mathscr{D}^\mu\eta)^b - R_{abcd} D_\mu\widetilde{\varphi}^a D^\mu \widetilde{\varphi}^c \eta^b\eta^d$$
$$- g_{af}\Gamma^f_{bc}\eta^b\eta^c(\mathscr{D}^\mu D_\mu\widetilde{\varphi})^a \Big].$$

The last term results in the non-covariant terms found in one-loop calculations in the literature

Appelquist and Bernard; Gavela, Kanshin, Machado, Saa

The origin of the non-covariant terms can be traced back to the expansion of $S[\phi+\eta]$ in η to compute the second variation. Under a coordinate transformation

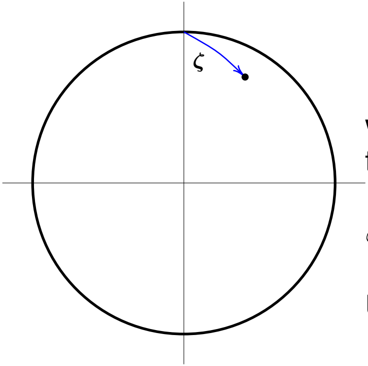
$$\varphi'^{\,a} = f^a(\varphi) \qquad\qquad (\varphi' + \eta')^a = f^a(\varphi+\eta)$$

$$\eta'^{\,a} = \frac{\partial\varphi'^{\,a}}{\partial\varphi^b}\eta^b + \frac{1}{2}\frac{\partial^2\varphi'^{\,a}}{\partial\varphi^b\partial\varphi^c}\eta^b\eta^c + \ldots$$

Geodesic Coordinates

Relate $\widetilde{\varphi} + \delta\varphi$ and $\widetilde{\varphi}$ in a covariant way using geodesics

Honerkamp, Gasser and Honerkamp (1971)



The geodesic equation is

$$\frac{\mathrm{d}^2\varphi^a}{\mathrm{d}\lambda^2} + \Gamma^a_{bc}\frac{\mathrm{d}\varphi^b}{\mathrm{d}\lambda}\frac{\mathrm{d}\varphi^c}{\mathrm{d}\lambda} = 0,$$

with the power series solution

$$\varphi^a(\lambda) = \widetilde{\varphi}^a + \zeta^a\lambda - \frac{1}{2}\Gamma^a_{bc}\zeta^b\zeta^c\lambda^2 + \ldots$$

Use

$$\varphi^a = \widetilde{\varphi}^a + \zeta^a - \frac{1}{2}\Gamma^a_{bc}(\widetilde{\varphi})\zeta^b\zeta^c + \ldots$$

$$\tilde{\mathcal{D}}_a S = \frac{\delta S}{\delta \varphi^a}\,, \qquad \tilde{\mathcal{D}}_b \tilde{\mathcal{D}}_a S = \frac{\delta^2 S}{\delta \varphi^b \delta \varphi^a} - \Gamma^c_{ab} \frac{\delta S}{\delta \varphi^c}\,.$$

The first variation is the same as before, but the second variation is modified to

$$\delta^2 S = \frac{1}{2} \int \mathrm{d}x \left[g_{ab} (\mathscr{D}_\mu \zeta)^a (\mathscr{D}^\mu \zeta)^b - R_{abcd} D_\mu \widetilde{\varphi}^a D^\mu \widetilde{\varphi}^c \zeta^b \zeta^d \right],$$

which is covariant.

The difference is a field redefinition, so that the second variation of the action $\delta^2 S$ changes by an equation of motion term $\Gamma^c_{ab} \delta S / \delta \varphi^c$.

Non-covariant terms found vanish on-shell; they can be eliminated by a field redefinition which does not change the S-matrix.

$$\mathscr{L} = \frac{1}{2} D_\mu \phi^T D^\mu \phi - \frac{1}{2} \phi^T X \phi$$

The infinite part of the one-loop correction to a general Lagrangian has been computed 't Hooft

$$\Delta\Gamma = \frac{1}{16\pi^2\epsilon} \int \mathrm{d}x \, \mathrm{Tr} \left\{ \frac{\Gamma_{\mu\nu}^2}{24} + \frac{X^2}{4} \right\},$$

where the antisymmetric tensor $\Gamma_{\mu\nu}$ and scalar X are:

$$\begin{aligned}
[\Gamma_{\mu\nu}]^{\mathfrak{i}}{}_{\mathfrak{j}} &= [\mathscr{D}_\mu , \mathscr{D}_\nu] = R^{\mathfrak{i}}_{\mathfrak{jkl}} D^\mu \phi^{\mathfrak{k}} D^\nu \phi^{\mathfrak{l}} + (\mathscr{A}^{\mu\nu})^{\mathfrak{i}}_{\mathfrak{j}} \,, \\
X^{\mathfrak{i}}{}_{\mathfrak{j}} &= \mathcal{D}^{\mathfrak{i}} \mathcal{D}_{\mathfrak{j}} \mathcal{I} - R^{\mathfrak{i}}_{\mathfrak{kjl}} D^\mu \phi^{\mathfrak{k}} D_\mu \phi^{\mathfrak{l}}, \\
(\mathscr{A}^{\mu\nu})^{\mathfrak{i}}_{\mathfrak{j}} &= \left(\partial_{[\mu} A^B_{\nu]} + f^B_{CD} A^C_\mu A^D_\nu \right) \mathcal{D}_{\mathfrak{j}} t^{\mathfrak{i}}_B,
\end{aligned}$$

and $R^{\mathfrak{i}}_{\mathfrak{jkl}}(\phi)$ is the Riemann tensor of $\mathcal{M}$.

The R terms multiplied by terms with two derivatives of ϕ, so generate $O(p^4)$ terms at one-loop if $\mathcal{M}$ is not flat.

HEFT: One Loop Correction

$$\mathscr{L} = \frac{1}{2}\partial_\mu h \, \partial^\mu h + \frac{1}{2} F(h)^2 g_{ab}(\varphi) D_\mu \varphi^a D_\mu \varphi^b - V(h) + K(h)\, w^i u^i(\varphi),$$

$$w^i = \bar{q}_L \sigma^i \, Y_q \, q_R + \bar{\ell}_L \sigma^i \, Y_\ell \, \ell_R + \text{h.c.},$$

The one-loop divergent contribution from scalar loops can be computed to be $\Delta\Gamma = 1/(32\pi^2\epsilon) Z$,

$$
\begin{aligned}
Z &= \frac{1}{2}\left(V'' - K'' w\cdot u\right)^2 + \left((K/(vF))'\right)^2\left[w\cdot w - (w\cdot u)^2\right] + \frac{1}{2}N_\varphi\left[\left(\frac{F''}{F}\right)(\partial_\mu h\partial^\mu h) - \frac{V'F'}{F} + (w\cdot u)\left(\frac{F'K'}{F} - \frac{K}{v^2F^2}\right)\right]^2 \\
&- \left[\left(v^2FF''\right)\left(V'' - K''u\cdot w\right) + (N_\varphi - 1)\left[1-(vF')^2\right]\left\{-\frac{V'F'}{F} + (w\cdot u)\left(\frac{F'K'}{F} - \frac{K}{v^2F^2}\right)\right\}\right](D_\mu u\cdot D^\mu u) \\
&- \left[\frac{1}{3}(vF'')^2 + (N_\varphi - 1)\left[1-(vF')^2\right]\frac{F''}{F}\right](\partial_\nu h\partial^\nu h)(D_\mu u\cdot D^\mu u) + \frac{2}{3}\left[1-(vF')^2\right]^2(D_\mu u\cdot D_\nu u)^2 \\
&+ \left[\frac{1}{2}(v^2FF'')^2 + \frac{3N_\varphi - 7}{6}\left[1-(vF')^2\right]^2\right](D_\mu u\cdot D^\mu u)^2 + \frac{4}{3}(vF'')^2(\partial^\mu h\partial^\nu h)(D_\mu u\cdot D_\nu u) - 2F''(\partial^\mu h)\left(K/F\right)'(w\cdot D_\mu u) \\
&- \frac{1}{3}\left[1-(vF')^2\right](D^\mu u)^T A_{\mu\nu}(D^\nu u) - \frac{2}{3}(vF')(vF'')(\partial_\mu h)(D_\nu u)^T A^{\mu\nu}u + \frac{1}{12}\mathrm{tr}(A_{\mu\nu}A^{\mu\nu}) + \frac{1}{6}\left[(vF')^2 - 1\right]u^T(A_{\mu\nu}A^{\mu\nu})u,
\end{aligned}
$$

agrees with F.-K. Guo, P. Ruiz-Femenía, and J. J. Sanz-Cillero

Many applications

1. Chiral Perturbation Theory

$$\mathscr{L} = \frac{1}{2}\partial_\mu h\,\partial^\mu h + \frac{1}{2}F(h)^2 g_{ab}(\varphi)D_\mu\varphi^a D_\mu\varphi^b - V(h) + K(h)\,w^i u^i(\varphi),$$

If $F(h), K(h)$ are constants, get the non-linear sigma model.

Reproduce the known results, including for the $\mathcal{O}(p^4)$ RGE in chiral perturbation theory

2. SM

$$\mathscr{L} = \frac{1}{2}\partial_\mu h\,\partial^\mu h + \frac{1}{2}F(h)^2 g_{ab}(\varphi) D_\mu\varphi^a D_\mu\varphi^b - V(h) + K(h)\, w^i u^i(\varphi),$$

$$F(h) = K(h) = v + h \qquad\qquad V(h) = \frac{\lambda}{4}(h^2 + 2hv)^2$$

the HEFT Lagrangian reduces to the SM Higgs Lagrangian.

R_{abcd} vanishes.

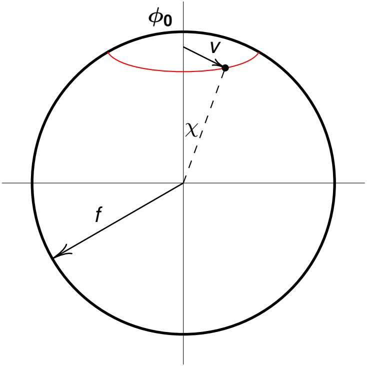
all order p^4 terms disappear, and the theory is renormalizable, even though in the field parametrization chosen here, renormalizability is not obvious.

3. Higgs as a Goldstone Boson

Assume the Higgs boson h and “eaten" scalars φ^a are Goldstone bosons resulting from dynamical breaking of a global symmetry at a high energy scale Composite Higgs: Kaplan and Georgi (1984)

Reviews: Contino, arXiv:1005.4269, Panico and Wulzer arXiv:1506.01961

$\mathcal{G}/\mathcal{H} = O(5)/O(4)$ Agashe, Contino, Pomarol



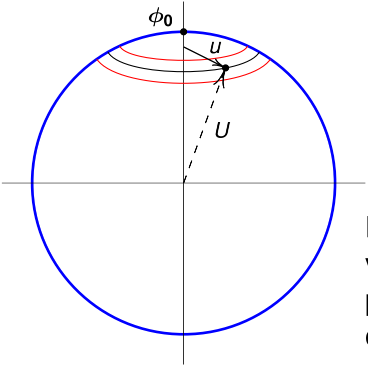
χ is the vacuum misalignment angle

Curvature of HEFT manifold $\mathcal{M}$ is $1/f^2$

Curvature of SM vacuum is $1/v^2$

3. Higgs as a Goldstone Boson

$\mathcal{G}/\mathcal{H} = O(5)/O(4)$



u: 4-dim unit vector
U: 5-dim unit vectors

$$\mathbf{U} = \begin{bmatrix} \sin\frac{h}{f}\,\mathbf{u} \\ \cos\frac{h}{f} \end{bmatrix}$$

$$F(h) = \sin(h/f)$$

Higher-dimensional operators: invariants constructed with u, multiplied by calculable singlet functions of h.

Drastic reduction in parameters

4. Higgs as a Dilaton

Goldberger, Grinstein, Skiba

Higgs boson from spontaneously broken scale invariance.

$$v \to v\, e^{\tau/v}, \qquad \tau \text{ is the dilaton field}$$

$$h/v = e^{\tau/v} - 1$$

$$F(h) = K(h) = v + h \implies R_{abcd} = 0$$

The p^4 dilaton-dilaton scattering term vanishes, which is related to the *a*-theorem Komargodski, Schwimmer

5. CCWZ

Standard CCWZ construction:

Expand $\xi = \exp(i\pi \cdot \boldsymbol{X})$ in a power series

$$1\xi^{-1}\partial_\mu \xi = \underbrace{\xi^{-1}\partial_\mu \xi\Big|_X}_{p_\mu} + \xi^{-1}\partial_\mu \xi\Big|_T$$

$$L \propto p_\mu^A p_\mu^A = \frac{1}{2} g_{ij}(\pi)\, \partial_\mu \pi^A \partial_\mu \pi^A$$

and multiple commutators written in terms of f_{abc} are the curvature of $\mathcal{G}/\mathcal{H}$.

$\mathcal{G}/\mathcal{H}$ not as simple as S^n for the general case.

Conclusions

- The S matrix depends on the geometric properties of $\mathcal{M}$.
- Can measure this experimentally
- Studying whether $\mathcal{L}$ is linear or nonlinear is meaningless
  - Is $\mathcal{M}$ flat or curved?
  - Is there an $O(4)$ invariant fixed point?
- Experiments probe the local properties of $\mathcal{M}$ near φ_0
- Geometrical method provides an efficient way to compute

  Gilkey — heat kernel method
- Understand some puzzles about the radiative corrections
- Study $\mathcal{M}$ and then see if you can construct it dynamically