Dissipative properties of the cosmological fluid

Stefan Flörchinger

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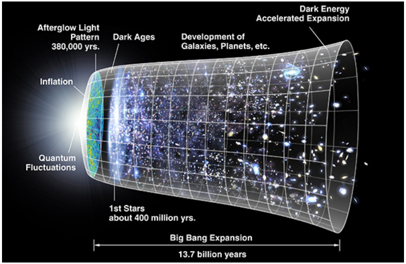
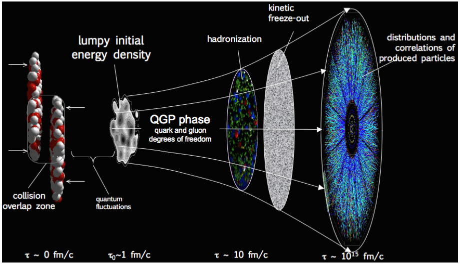
UNIVERSITÄT HEIDELBERG
ZUKUNFT SEIT 1386

Big bang – little bang: More than an analogy?





- cosmol. scale: MPc$= 3.1 \times 10^{22}$ m
- Gravity + QED + Dark sector
- one big event

- nuclear scale: fm$= 10^{-15}$ m
- QCD
- very many events

- initial conditions not directly accessible
- all information must be reconstructed from final state
- **dynamical description as a (viscous) fluid**

The dark matter fluid

- Heavy ion collisions

$$\mathscr{L}_{\text{QCD}} \quad \rightarrow \quad \text{fluid properties}$$

- Late time cosmology

$$\text{fluid properties} \quad \rightarrow \quad \mathscr{L}_{\text{dark matter}}$$

- Until direct detection of dark matter, it can only be observed via

$$T^{\mu\nu}_{\text{dark matter}}$$

Relativistic fluid dynamics

Energy-momentum tensor and conserved current

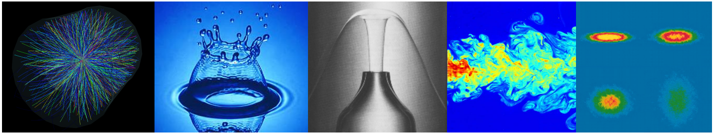
$$T^{\mu\nu} = (\epsilon + p + \pi_{\text{bulk}})u^\mu u^\nu + (p + \pi_{\text{bulk}})g^{\mu\nu} + \pi^{\mu\nu}$$
$$N^\mu = n\, u^\mu + \nu^\mu$$

- tensor decomposition w. r. t. fluid velocity u^μ
- pressure $p = p(\epsilon, n)$
- constitutive relations for viscous terms in derivative expansion
  - bulk viscous pressure $\pi_{\text{bulk}} = -\zeta\, \nabla_\mu u^\mu + \dots$
  - shear stress $\pi^{\mu\nu} = -\eta \left[\Delta^{\mu\alpha}\nabla_\alpha u^\nu + \Delta^{\nu\alpha}\nabla_\alpha u^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha\right] + \dots$
  - diffusion current $\nu^\alpha = -\kappa \left[\frac{nT}{\epsilon+p}\right]^2 \Delta^{\alpha\beta}\partial_\beta \left(\frac{\mu}{T}\right) + \dots$

Fluid dynamic equations from covariant **conservation laws**

$$\nabla_\mu T^{\mu\nu} = 0, \qquad \nabla_\mu N^\mu = 0.$$

Fluid dynamics



- Long distances, long times or strong enough interactions
- Needs **macroscopic** fluid properties
  - equation of state $p(\epsilon, n)$
  - shear viscosity $\eta(\epsilon, n)$
  - bulk viscosity $\zeta(\epsilon, n)$
  - heat conductivity $\kappa(\epsilon, n)$
  - relaxation times, ...
- For QCD no full *ab initio* calculation of transport properties possible yet but in principle fixed by **microscopic** properties encoded in $\mathscr{L}_{\mathrm{QCD}}$
- Ongoing experimental and theoretical effort to understand this in detail

Ideal fluid versus collision-less gas

- Many codes used in cosmology describe dark matter as **ideal, cold and pressure-less fluid**

$$T^{\mu\nu} = \epsilon\, u^{\mu} u^{\nu}$$

- Equation of state $p = 0$
- No shear stress and bulk viscous pressure $\pi^{\mu\nu} = \pi_{\text{bulk}} = 0$
- Dark matter is also modeled as **collision-less gas** of massive particles, interacting via gravity only
- Two pictures are **not consistent**

Dissipative properties

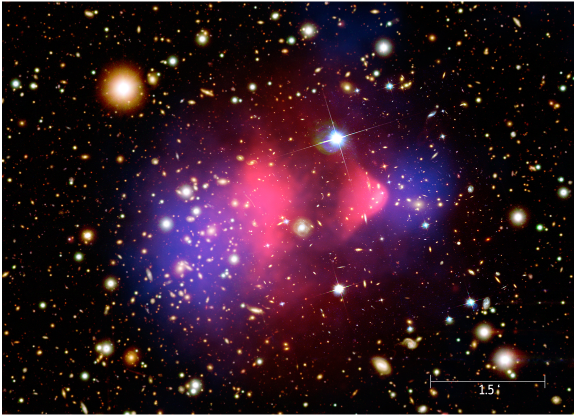
Viscosities

- Diffusive transport of momentum [Maxwell (1860)]
- Depend strongly on interaction properties
- Example: non-relativistic gas of particles with mass m, mean velocity $\bar{v}$, elastic $2 \to 2$ cross-section σ_{el}

$$\eta = \frac{m\,\bar{v}}{3\,\sigma_{\text{el}}} \qquad\qquad \zeta = 0$$

- Interesting additional **information about dark matter**

Self-interaction of dark matter

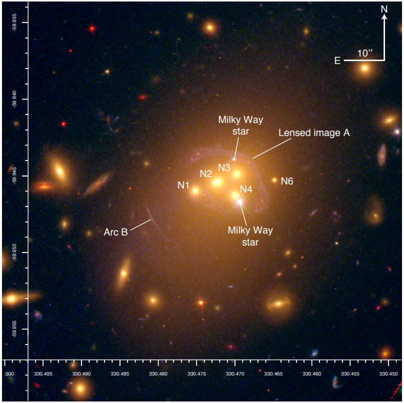


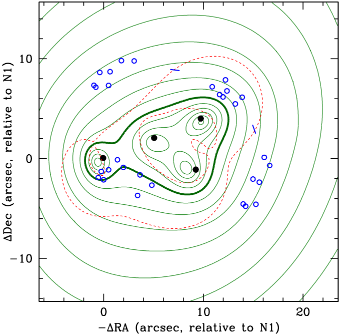
Gravitational lensing and x-ray image of "bullet cluster" 1E0657-56

- so far: dark matter is non-interacting → can collide without stopping

$$\frac{\sigma_{\text{el}}}{m} \lesssim 1.2 \, \frac{\text{cm}^2}{\text{g}}$$

Is dark matter self-interacting?





Galaxy cluster Abell 3827

[Massey *et al.*, MNRAS 449, 3393 (2015)]

- Offset between stars and dark matter falling into cluster
- Is this a first indication for a dark matter self-interaction?

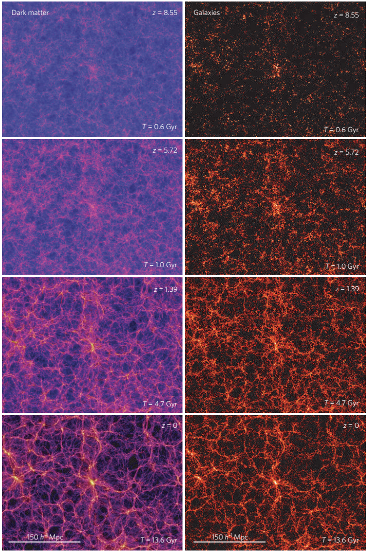
$$\frac{\sigma_{\text{el}}}{m} \approx 3\frac{\text{cm}^2}{\text{g}} \approx 0.5\frac{\text{b}}{\text{GeV}} \qquad \text{(under debate)}$$

[Kahlhoefer, Schmidt-Hoberg, Kummer & Sarkar, MNRAS 452, 1 (2015)]

Cosmological structure formation

- How do **viscosities influence structure formation**?
- Does viscous fluid dynamics help to **understand large scale structure (semi) analytically**?

Dark matter | Visible galaxies



[Springel, Frenk & White, Nature 440, 1137 (2006)]

How is structure formation modified?

Linear dynamics

- energy conservation ($\theta = \vec{\nabla} \cdot \vec{v}$)

$$\dot{\delta\epsilon} + 3\tfrac{\dot{a}}{a}\delta\epsilon + \bar{\epsilon}\,\theta = 0$$

- Navier-Stokes equation

$$\bar{\epsilon}\left[\dot{\theta} + \tfrac{\dot{a}}{a}\theta - k^2\psi\right] + \tfrac{1}{a}\left(\zeta + \tfrac{4}{3}\eta\right)k^2\theta = 0$$

- Poisson equation

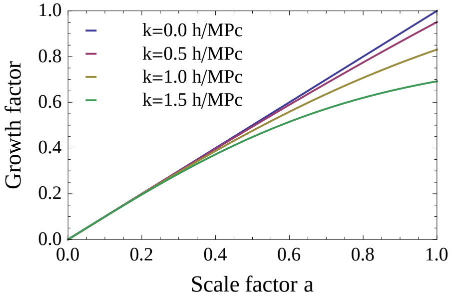
$$-k^2\psi = 4\pi G_{\mathsf{N}} a^2 \delta\epsilon$$

Scalar perturbations ($\delta = \frac{\delta\epsilon}{\bar{\epsilon}}$)

$$\ddot{\delta} + \left[\frac{\dot{a}}{a} + \frac{\zeta + \frac{4}{3}\eta}{a\bar{\epsilon}}k^2\right]\dot{\delta} - 4\pi G_{\mathsf{N}}\bar{\epsilon}\,\delta = 0$$

Viscosites slow down gravitational collapse but do not wash out structure

Structure formation with viscosities



[Blas, Floerchinger, Garny, Tetradis & Wiedemann, JCAP, in press (2015)]

- k-dependent growth factor for scalar modes
- Could be tested by observation of large scale structure
- Depends on $\zeta + \frac{4}{3}\eta$ as function of time (or density)

“Fundamental” and “effective” viscosity

Two types of viscosities for cosmological fluid

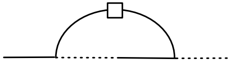
1. Momentum transport by particles or radiation
   - governed by interactions
   - from linear response theory [Green (1954), Kubo (1957)]
   - close to equilibrium

2. Momentum transport in the inhomogeneous, coarse-grained fluid
   - governed by non-linear fluid mode couplings
   - determined perturbatively [Blas, Floerchinger, Garny, Tetradis & Wiedemann]
   - non-equilibrium
   - heavy ions: anomalous plasma viscosity [Asakawa, Bass & Müller (2006)]
     eddy viscosity [Romatschke (2008)]

Effective viscosity and sound velocity from perturbative matching

[Blas, Floerchinger, Garny, Tetradis & Wiedemann, JCAP, in press (2015)]

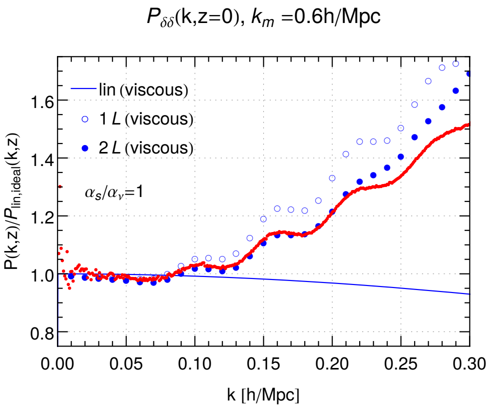
$$\left(\zeta + \tfrac{4}{3}\eta\right)_{\text{eff}}, \ \left(c_s^2\right)_{\text{eff}} \quad \sim$$



- Consider theory with a coarse-graining scale k_m
- Statistical fluctuation with $|\vec{k}| > k_m$ modify effective propagator
- Leading correction for growing mode can be matched to good approximation to **effective viscosity and sound velocity** terms
- similar to 1-PI scheme
- No free parameter except k_m

Large scale structure and effective viscosities

Dark matter density power spectrum in the BAO range

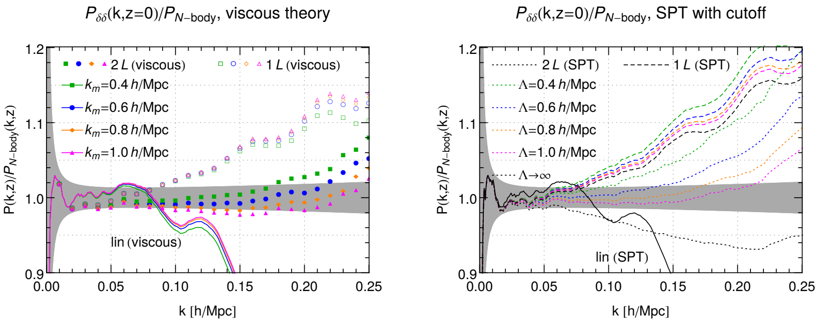


[Blas, Floerchinger, Garny, Tetradis & Wiedemann, JCAP, in press (2015)]

- 2-Loop calculation with **effective viscosity and sound velocity**
- agrees with N-body simulations up to $k = 0.2\, h/\text{MPc}$
  [related: Effective field theory of LSS, Baumann, Nicolis, Senatore & Zaldarriaga (2012), Carrasco, Hertzberg & Senatore (2012), ...]

Comparison with standard perturbation theory

[Blas, Floerchinger, Garny, Tetradis & Wiedemann, JCAP, in press (2015)]



- Convergence properties of theory with effective viscosities are better

Precision cosmology can measure shear stress

- Scalar excitations in gravity

$$ds^2 = a^2 \left[-(1+2\psi)d\eta^2 + (1-2\phi)dx_i dx_i \right]$$

  with two Newtonian potentials ψ and ϕ.

- Einsteins equations imply

$$\left(\partial_i \partial_j - \tfrac{1}{3}\delta_{ij}\partial_k^2\right)(\phi - \psi) = 8\pi G_{\mathsf{N}} a^2 \; \pi_{ij}\big|_{\mathsf{scalar}}$$

  with scalar part of shear stress

$$\pi_{ij}\big|_{\mathsf{scalar}} = \left(\partial_i \partial_j - \tfrac{1}{3}\delta_{ij}\partial_k^2\right)\tilde{\pi}$$

- Detailed data at small redshift e.g. from Euclid satellite (esa, 2020) [Amendola *et al.* (2012)]
  - ψ can be measured via acceleration of matter
  - $\psi + \phi$ can be meaured by weak lensing and Sachs-Wolfe effect
  - fluid velocity can be accessed by redshift space distortions
- New quantitative precise insights into fluid properties of dark matter

Bulk viscosity

- Bulk viscous pressure is negative for expanding universe

$$\pi_{\text{bulk}} = -\zeta\,\nabla_\mu u^\mu = -\zeta\,3H < 0$$

- Negative effective pressure

$$p_{\text{eff}} = p + \pi_{\text{bulk}} < 0$$

  would act similar to dark energy in Friedmann's equations
  [Murphy (1973), Padmanabhan & Chitre (1987), Fabris, Goncalves & de Sa Ribeiro (2006), Li & Barrow (2009), Velten & Schwarz (2011), Gagnon & Lesgourgues (2011), ...]

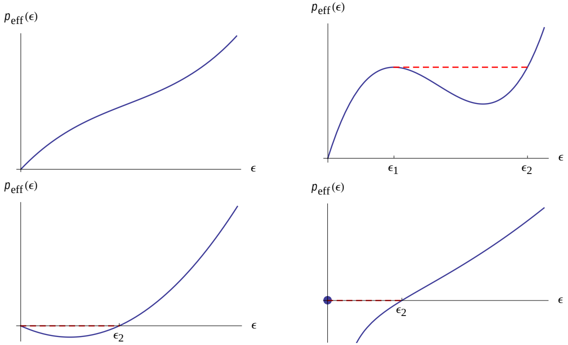
- **Is negative effective pressure physical?**
- In context of heavy ion physics: instability for $p_{\text{eff}} < 0$ ("cavitation")
  [Torrieri & Mishustin (2008), Rajagopal & Tripuraneni (2010), Buchel, Camanho & Edelstein (2014), Habich & Romatschke (2015), Denicol, Gale & Jeon (2015)]
- What precisely happens at the instability?

Is negative effective pressure physical?

- Kinetic theory

$$p_{\text{eff}}(x) = \int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}^2}{3E_{\vec{p}}} f(x, \vec{p}) \geq 0$$

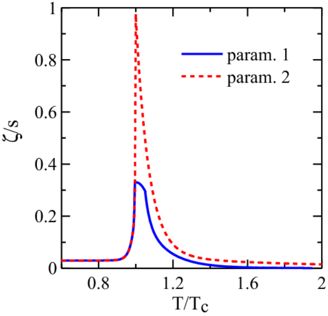
- Stability argument



If there is a vacuum with $\epsilon = p_{\text{eff}} = 0$, phases with $p_{\text{eff}} < 0$ cannot be mechanically stable. (But could be metastable.)

Bulk viscosity in heavy ion physics

- In heavy ion physics people start now to consider bulk viscosity.
- Becomes relevant close to chiral crossover



[Denicol, Gale & Jeon (2015)]

- Is there a first-order phase transition triggered by the expansion?
- What is the relation to chemical and kinetic freeze-out?
- More detailed understanding needed, both for heavy ion physics and cosmology

Backreaction: General idea

- for $0+1$ dimensional, non-linear dynamics

$$\dot{\varphi} = f(\varphi) = f_0 + f_1\,\varphi + \tfrac{1}{2} f_2\,\varphi^2 + \ldots$$

- one has for expectation values $\bar{\varphi} = \langle \varphi \rangle$

$$\dot{\bar{\varphi}} = f_0 + f_1\,\bar{\varphi} + \tfrac{1}{2} f_2\,\bar{\varphi}^2 + \tfrac{1}{2} f_2\,\langle (\varphi - \bar{\varphi})^2 \rangle + \ldots$$

- evolution equation for expectation value $\bar{\varphi}$ depends on two-point correlation function or spectrum $P_2 = \langle (\varphi - \bar{\varphi})^2 \rangle$
- evolution equation for spectrum depends on bispectrum and so on
- more complicated for higher dimensional theories
- more complicated for gauge theories such as gravity

Backreaction in gravity

- Einstein's equations are non-linear.
- Important question [G. F. R. Ellis (1984)]: If Einstein's field equations describe small scales, including inhomogeneities, do they also hold on large scales?
- Is there a sizable backreaction from inhomogeneities to the cosmological expansion?
- Difficult question, has been studied by many people [Ellis & Stoeger (1987); Mukhanov, Abramo & Brandenberger (1997); Unruh (1998); Buchert (2000); Geshnzjani & Brandenberger (2002); Schwarz (2002); Wetterich (2003); Räsänen (2004); Kolb, Matarrese & Riotto (2006); Brown, Behrend, Malik (2009); Gasperini, Marozzi & Veneziano (2009); Clarkson & Umeh (2011); Green & Wald (2011); ...]
- Recent reviews: [Buchert & Räsänen, Ann. Rev. Nucl. Part. Sci. 62, 57 (2012); Green & Wald, Class. Quant. Grav. 31, 234003 (2014)]
- No general consensus but most people believe now that **gravitational backreaction is rather small**.
- In the following we look at a new **backreaction on the matter side** of Einstein's equations.

Fluid equation for energy density

First order viscous fluid dynamics

$$u^\mu \partial_\mu \epsilon + (\epsilon + p)\nabla_\mu u^\mu - \zeta\Theta^2 - 2\eta\sigma^{\mu\nu}\sigma_{\mu\nu} = 0$$

For $\vec{v}^2 \ll c^2$ and Newtonian potentials $\Phi, \Psi \ll 1$

$$\begin{aligned}&\dot{\epsilon} + \vec{v}\cdot\vec{\nabla}\epsilon + (\epsilon + p)\left(3\tfrac{\dot{a}}{a} + \vec{\nabla}\cdot\vec{v}\right)\\ &= \tfrac{\zeta}{a}\left[3\tfrac{\dot{a}}{a} + \vec{\nabla}\cdot\vec{v}\right]^2 + \tfrac{\eta}{a}\left[\partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \tfrac{2}{3}(\vec{\nabla}\cdot\vec{v})^2\right]\end{aligned}$$

Fluid dynamic backreaction in Cosmology

[Floerchinger, Tetradis & Wiedemann, PRL 114, 091301 (2015)]

Expectation value of energy density $\bar{\epsilon} = \langle \epsilon \rangle$

$$\frac{1}{a}\dot{\bar{\epsilon}} + 3H\left(\bar{\epsilon} + \bar{p} - 3\bar{\zeta}H\right) = D$$

with dissipative backreaction term

$$D = \frac{1}{a^2}\langle \eta \left[\partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \tfrac{2}{3}\partial_i v_i \partial_j v_j\right]\rangle + \frac{1}{a^2}\langle \zeta [\vec{\nabla} \cdot \vec{v}]^2 \rangle + \frac{1}{a}\langle \vec{v} \cdot \vec{\nabla}\left(p - 6\zeta H\right)\rangle$$

- D vanishes for unperturbed homogeneous and isotropic universe
- D has contribution from shear & bulk viscous dissipation and thermodynamic work done by contraction against pressure gradients
- dissipative terms in D are positive semi-definite
- for spatially constant viscosities and scalar perturbations only

$$D = \frac{\bar{\zeta} + \frac{4}{3}\bar{\eta}}{a^2}\int d^3q\; P_{\theta\theta}(q)$$

Dissipation of perturbations

[Floerchinger, Tetradis & Wiedemann, PRL 114, 091301 (2015)]

- Dissipative backreaction does not need negative effective pressure

$$\frac{1}{a}\dot{\bar{\epsilon}} + 3H\left(\bar{\epsilon} + \bar{p}_{\text{eff}}\right) = D$$

- D is an integral over perturbations, could become large at late times.
- Can it potentially accelerate the universe?
- Need additional equation for scale parameter a
- Use trace of Einstein's equations $R = 8\pi G_{\text{N}} T^{\mu}{}_{\mu}$

$$\frac{1}{a}\dot{H} + 2H^2 = \frac{4\pi G_{\text{N}}}{3}\left(\bar{\epsilon} - 3\bar{p}_{\text{eff}}\right)$$

  does not depend on unknown quantities like $\langle(\epsilon + p_{\text{eff}})u^{\mu}u^{\nu}\rangle$
- To close the equations one needs equation of state $\bar{p}_{\text{eff}} = \bar{p}_{\text{eff}}(\bar{\epsilon})$ and dissipation parameter D

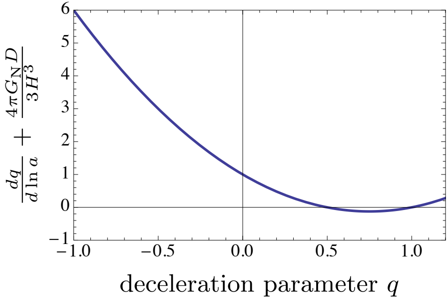
Deceleration parameter

[Floerchinger, Tetradis & Wiedemann, PRL 114, 091301 (2015)]

- assume now vanishing effective pressure $\bar{p}_{\text{eff}} = 0$
- obtain for deceleration parameter $q = -1 - \frac{\dot{H}}{aH^2}$

$$-\frac{dq}{d\ln a} + 2(q-1)\left(q - \frac{1}{2}\right) = \frac{4\pi G_\text{N} D}{3H^3}$$

- for $D = 0$ attractive fixed point at $q_* = \frac{1}{2}$ (deceleration)
- for $D > 0$ fixed point shifted towards $q_* < 0$ (acceleration)



Estimating viscous backreaction D

- For $\frac{4\pi G_\mathsf{N} D}{3H^3} \approx 4$ one could explain the current accelerated expansion ($q \approx -0.6$) by dissipative backreaction.
- Is this possible?
- In principle one can determine D for given equation of state and viscous properties from dynamics of structure formation.
- So far only rough estimates. If shear viscosity dominates:

$$D = \frac{1}{a^2}\langle \eta \left[\partial_i v_j \partial_i v_j + \partial_i v_j \partial_j v_i - \frac{2}{3}\partial_i v_i \partial_j v_j\right]\rangle \approx c_D \bar{\eta} H^2$$

  with $c_D = \mathcal{O}(1)$. Corresponds to $\Delta v \approx 100\,\text{km/s}$ for $\Delta x \approx 1\,\text{MPc}$

- Leads to

$$\frac{4\pi G_\mathsf{N} D}{3H^3} \approx \frac{c_D \bar{\eta} H}{2\rho_c}$$

  with $\rho_c = \frac{3H^2}{8\pi G_\mathsf{N}}$

Viscosities

- Relativistic particles / radiation contribute to shear viscosity

$$\eta = c_\eta \, \epsilon_R \, \tau_R$$

  - prefactor $c_\eta = \mathcal{O}(1)$
  - energy density of radiation ϵ_R
  - mean free time τ_R
- Bulk viscosity vanishes in situations with conformal symmetry but can be large when conformal symmetry is broken.
- For massive scalar particles with $\lambda\varphi^4$ interaction [Jeon & Yaffe (1996)]

$$\zeta \sim \frac{m^6}{\lambda^4 T^3} e^{2m/T}, \qquad \eta \sim \frac{m^{5/2} T^{1/2}}{\lambda^2} \qquad \text{for} \qquad \frac{T}{m} \ll 1$$

Estimating viscous backreaction D

Consider shear viscosity from radiation

$$\eta = c_\eta \, \epsilon_R \, \tau_R$$

Backreaction term

$$\frac{4\pi G_{\mathsf{N}} D}{3H^3} \approx \frac{c_D c_\eta}{2} \frac{\epsilon_R}{\rho_c} \tau_R H$$

- fluid approximation needs $\tau_R H < 1$
- for sizeable effect one would need $\epsilon_R/\rho_c = \mathcal{O}(1)$
- unlikely that D becomes large enough in this scenario

Needed refinements:

- full dynamics of perturbations
- second order fluid dynamics
- complete model(s)

Gravity and thermalization

Consider ensemble of massive particles interacting via gravity only. Start with some velocity distribution. Is there **equilibration/thermalization**...

- ... in Newtonian gravity?
- ... in classical General relativity?
- ... in quantized gravity?

Analogy to other gauge theories suggests that **quantum properties are important for thermalization**

Dissipation by gravity

- Gravitational waves in viscous fluid have life time [Hawking (1966)]

$$\tau_G = \frac{1}{16\pi G_\mathsf{N} \eta}$$

- Diffusive momentum transport by graviton radiation induces viscosity

$$\eta \approx \epsilon_G \, \tau_G$$

  with energy density of gravitational field ϵ_G

- Can be solved for η and τ_G [Weinberg (1972)]

$$\eta = \sqrt{\frac{\epsilon_G}{16\pi G_\mathsf{N}}}, \qquad \tau_G = \sqrt{\frac{1}{16\pi G_\mathsf{N} \epsilon_G}}$$

- Can this really be independent of dark matter mass and density?
- Thermalization time $\sim m_\mathsf{P}/T^2$ is very large
- What determines dissipation on shorter time scales, when classical fields dominate?

Inhomogeneities in heavy ion collisions

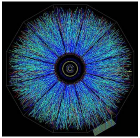
Inhomogeneities are main source of information in cosmology.

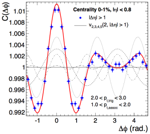
Similarly, in heavy ion collisions:

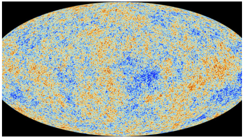
- **Initial fluid perturbations**: Event-by-event fluctuations around averaged fluid fields at time τ_0 and their evolution:
  - energy density ϵ
  - fluid velocity u^{μ}
  - shear stress $\pi^{\mu\nu}$
  - more general also: baryon number density n, electric charge density, electromagnetic fields, ...
- governed by universal evolution equations
- determine particle distributions after freeze-out, e.g. $v_n(p_T)$
- usefull to constrain **thermodynamic and transport properties of QCD**
- contain interesting information from early times

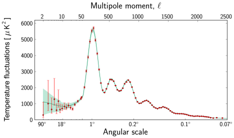
Fluid dynamic perturbation theory for heavy ions

[Floerchinger & Wiedemann, PLB 728, 407 (2014); JHEP 08 (2014) 005]









- goal: understand dynamics of heavy ion collisions better, constrain fluid properties of QCD from experimental results
- before: fully numerical fluid simulations e.g. [Heinz & Snellings (2013)]
- new: solve fluid equations for smooth and symmetric background and order-by-order in perturbations
- similar to cosmological perturbation theory
- good convergence properties [Floerchinger *et al.*, PLB 735, 305 (2014), Brouzakis *et al.* PRD 91, 065007 (2015)]
- some insights from heavy ion physics might be useful for cosmology

Conclusions

Dissipative properties of the cosmological fluid...

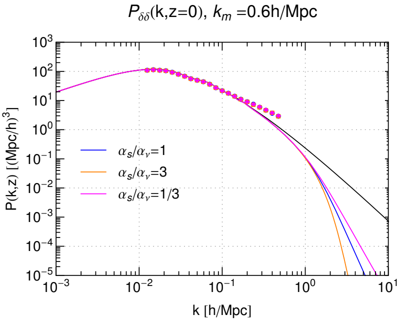
- ... are quite interesting
- ... could provide more detailed understanding of dark matter
- ... can be tested by precision cosmology
- ... can help to better understand large scale structure
- ... could even affect the cosmological expansion

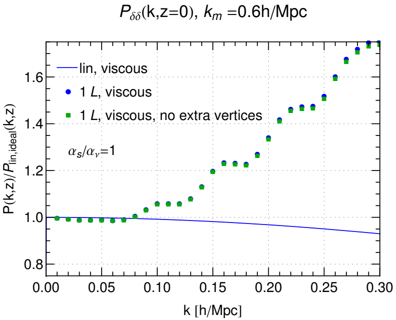
Interesting parallels between cosmology and heavy ion physics...

- ... could help heavy ion physics to become more quantitative
- ... could help cosmology to better understand fluid properties

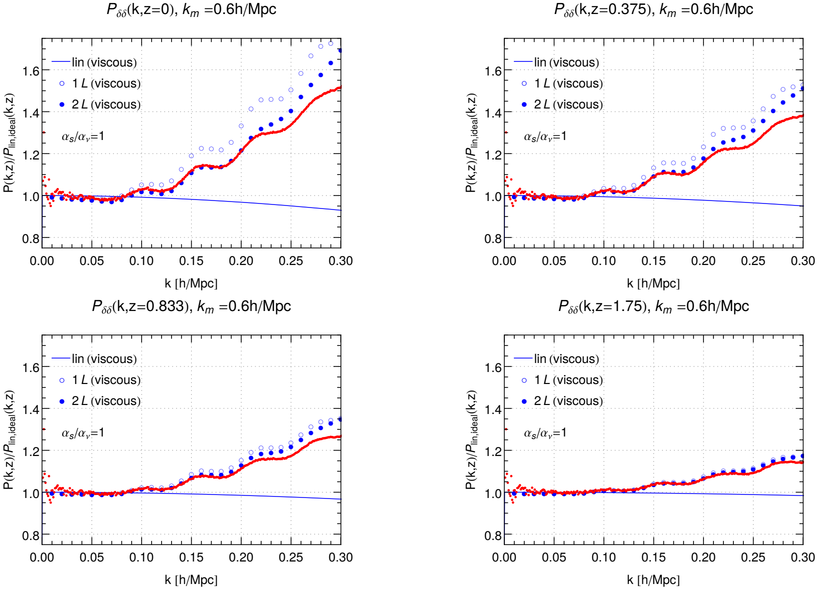
Backup slides

Perturbation theory with effective viscosity and sound velocity

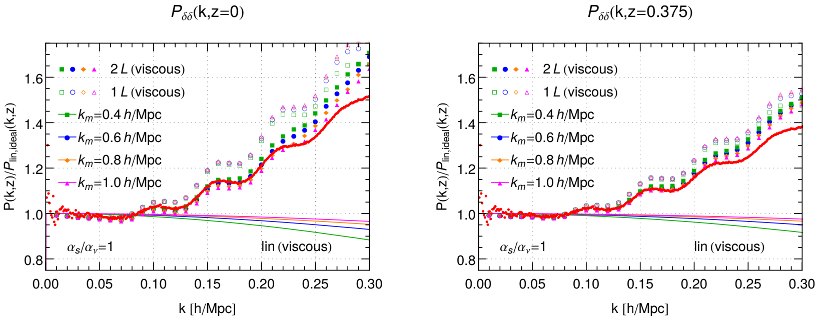




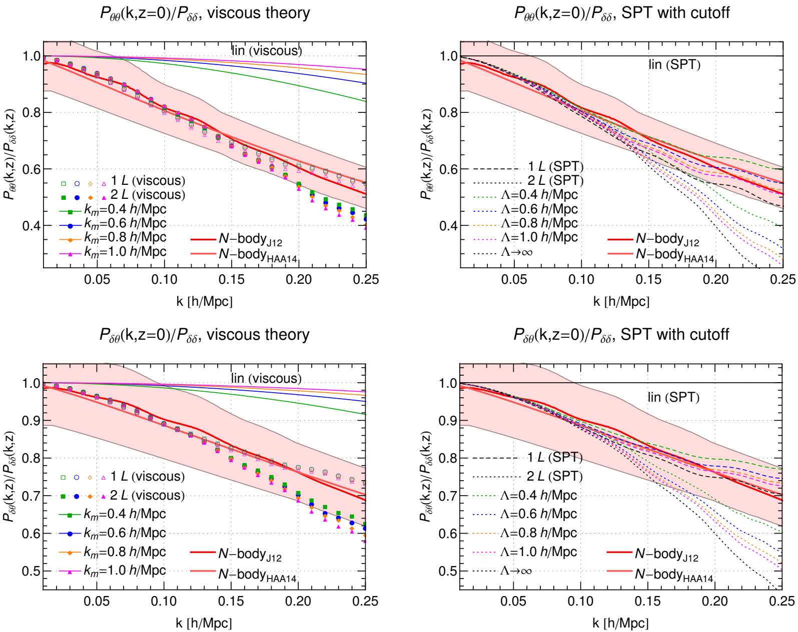
Power spectrum at different redshifts



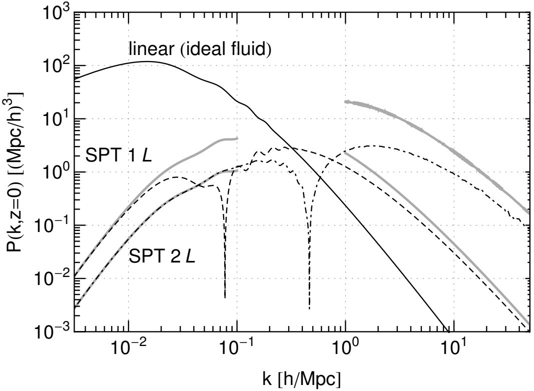
Dependence on matching scale



Velocity spectra



Power spectrum, standard perturbation theory



[D. Blas, M. Garny and T. Konstandin, JCAP 1309 (2013) 024]

Could viscous backreaction lead to ΛCDM-type expansion?

[Floerchinger, Tetradis & Wiedemann, 1506.00407]

- Backreaction term $D(z)$ will be *some* function of redshift.
- For given dissipative properties $D(z)$ can be determined, but calculation is involved.
- One may ask simpler question: For what form of $D(z)$ would the expansion be as in the ΛCDM model?
- The *ad hoc* ansatz $D(z) = \text{const} \cdot H(z)$ leads to modified Friedmann equations

$$\bar{\epsilon} - \frac{D}{4H} = \frac{3}{8\pi G_\mathsf{N}} H^2, \qquad \bar{p}_\mathsf{eff} - \frac{D}{12H} = -\frac{1}{8\pi G_\mathsf{N}} \left(2\frac{1}{a}\dot{H} + 3H^2 \right)$$

- In terms of $\hat{\epsilon} = \bar{\epsilon} - \frac{D}{3H}$ one can write

$$\frac{1}{a}\dot{\hat{\epsilon}} + 3H(\hat{\epsilon} + \bar{p}_\mathsf{eff}) = 0, \qquad R + \frac{8\pi G_\mathsf{N} D}{3H} = -8\pi G_\mathsf{N}(\hat{\epsilon} - 3\bar{p}_\mathsf{eff})$$

- For $\bar{p}_\mathsf{eff} = 0$ these are standard equations for ΛCDM model with

$$\Lambda = \frac{2\pi G_\mathsf{N} D}{3H}$$

Modification of Friedmann's equations by backreaction 1

- For universe with fluid velocity inhomogeneities one cannot easily take direct average of Einstein's equations.
- However, fluid equation for energy density and trace of Einstein's equations can be used.
- By integration one finds modified Friedmann equation

$$H(\tau)^2 = \frac{8\pi G_{\mathsf{N}}}{3}\left[\bar{\epsilon}(\tau) - \int_{\tau_{\mathsf{I}}}^{\tau} d\tau' \left(\frac{a(\tau')}{a(\tau)}\right)^4 a(\tau')D(\tau')\right]$$

- Additive deviation from Friedmann's law for $D(\tau') > 0$
- Part of the total energy density is due to dissipative production

$$\bar{\epsilon} = \bar{\epsilon}_{\mathsf{nd}} + \bar{\epsilon}_{\mathsf{d}}$$

- Assume for dissipatively produced part

$$\dot{\bar{\epsilon}}_{\mathsf{d}} + 3\frac{\dot{a}}{a}(1+\hat{w}_{\mathsf{d}})\bar{\epsilon}_{\mathsf{d}} = aD$$

Modification of Friedmann's equations by backreaction 2

Leads to another variant of Friedmann's equation

$$H(\tau)^2 = \frac{8\pi G_{\mathsf{N}}}{3}\left[\bar{\epsilon}_{\mathsf{nd}}(\tau) + \int_{\tau_{\mathsf{I}}}^{\tau} d\tau' \left[\left(\frac{a(\tau')}{a(\tau)}\right)^{3+3\hat{w}_{\mathsf{d}}} - \left(\frac{a(\tau')}{a(\tau)}\right)^{4}\right] a(\tau') D(\tau')\right]$$

- If the dissipative backreaction D produces pure radiation, $\hat{w}_{\mathsf{d}} = 1/3$, it does not show up in effective Friedmann equation at all!
- For $\hat{w}_{\mathsf{d}} < 1/3$ there is a new component with positive contribution on the right hand side of the effective Friedmann equation.
- To understand expansion, parametrize for late times

$$D(\tau) = H(\tau)\left(\frac{a(\tau)}{a(\tau_0)}\right)^{-\kappa}\tilde{D}$$

with constants $\tilde{D}$ and κ.

- Hubble parameter as function of $(a_0/a) = 1 + z$

$$H(a) = H_0\sqrt{\Omega_\Lambda + \Omega_M\left(\frac{a_0}{a}\right)^3 + \Omega_R\left(\frac{a_0}{a}\right)^4 + \Omega_D\left(\frac{a_0}{a}\right)^\kappa}$$

- For $\kappa \approx 0$ the role of Ω_Λ and Ω_D would be similar.

Fluid dynamics in cosmology and heavy ion collisions

Cosmology

- Large part of cosmological evolution is governed by equilibrium thermodynamics or ideal fluid dynamics.
- Free-streaming of photons and neutrinos at late times.
- Matter in equilibrium at early times, drops out of equilibrium later.
- Gravitational interaction is long range and treated explicitely.

Heavy ion collisions

- Expansion governed by viscous fluid dynamics.
- Free streaming of hadrons at late times.
- Strong interactions are confined at low T and screened at high T, treated implicitly.

First steps towards fluid dynamic perturbation or response theory

- Linear perturbations around Bjorken flow [Floerchinger & Wiedemann (2011)]
- Linear perturbations around Gubser solution for conformal fluids
  [Gubser & Yarom (2010), Staig & Shuryak (2011), Springer & Stephanov (2013)]
- More detailed investigation of linear perturbations and first steps towards non-linear perturbations around Gubser solution
  [Hatta, Noronha, Torrieri, Xiao (2014)]
- Linear perturbations around general azimuthally symmetric initial state, realistic equation of state
  [Floerchinger & Wiedemann (2013)]
- Characterization of initial conditions by Bessel-Fourier expansion
  [Coleman-Smith, Petersen & Wolpert (2012), Floerchinger & Wiedemann (2013)]
- Comparison to full numerical solution shows good convergence properties of perturbative expansion
  [Floerchinger, Wiedemann, Beraudo, Del Zanna, Inghirami, Rolando (2013)]
- Related response formalism for expansion in eccentricities
  [Teaney & Yan (2012), Yan & Ollitrault (2015]