The Bialas-Bzdak model of elastic pp scattering

Results with proton-antiproton elastic scattering data

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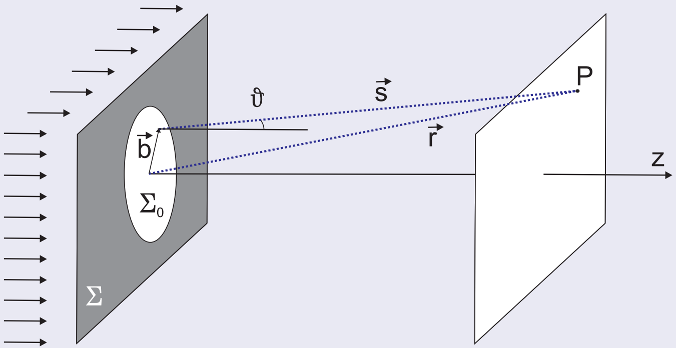
Light scattered by a hole Σ_0



The Kirchhoff integral (U_0, k are the amplitude and momentum of the incoming wave):

$$U(x,y,z) = -\frac{\mathrm{i}\,k}{4\pi} U_0 \int_{\Sigma_0} \mathrm{d}^2\vec{b}\,(1+\cos\vartheta)\frac{e^{iks}}{s} \qquad (1)$$

From the Kirchhof integral to Fraunhofer diffraction

Far from the obstacle (Fraunhofer limit):

$$U(x,y,z) = -\frac{\mathrm{i}k}{2\pi}U_0\frac{e^{\mathrm{i}kr}}{r}\int \mathrm{d}^2\vec{b}\; S(\vec{b})\, e^{-\mathrm{i}\vec{q}\cdot\vec{b}} \quad (\text{disk } \Sigma_0) \tag{2}$$

where $S(\vec{b}) = 1 - \Gamma(\vec{b})$ and the profile function:

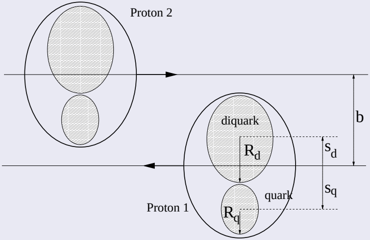
$$\Gamma(\vec{b}) = \begin{cases} 1 & \text{if } \vec{b} \in \Sigma_0\,, \\ 0 & \text{otherwise}\,. \end{cases} \tag{3}$$

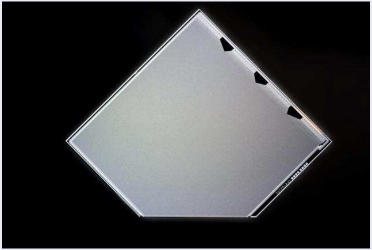
The scattering amplitude:

$$f(q) = \mathrm{i}k\int_0^\infty \mathrm{d}b\, b\, J_0(qb)\,\Gamma(b) \tag{4}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = |f(\vec{q})|^2 \tag{5}$$

High-energy particle diffraction: light waves $\longrightarrow$ wave functions, screen $\longrightarrow$ silicon detector





The scattering amplitude in the eikonal approximation:

$$T(s,\Delta) = 2\pi \int_0^{\infty} J_0\left(\Delta \cdot b\right) t_{\rm el}(s,b) b \, {\rm d}b, \tag{6}$$

where $t_{\rm el}(s,b) = {\rm i}\left[1 - e^{-\Omega(s,b)}\right]$ and $t \simeq -\Delta^2$. See Eq. (4)!

Cross-sections and others

Differential cross-section:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{1}{4\pi} \left| T(s,\Delta) \right|^2 \qquad (7)$$

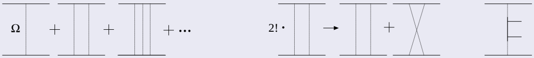
Total cross-section:

$$\sigma_{\mathrm{tot}} = 2 \, \mathrm{Im} \, T(s,\Delta)|_{t=0} \qquad (8)$$

The real to imaginary part ratio:

$$\rho = \frac{\mathrm{Re}\, T(s,0)}{\mathrm{Im}\, T(s,0)} \qquad (9)$$

The (seemingly!) simple equations in QFT: multi meson exchange



(a) (b) (c)

The profile function in the BB model:

Partial wave equation → impact parameter b ! Unitarity:

$$2\,\mathrm{Im}\,t_{\rm el}(s,b) = \left|t_{\rm el}(s,b)\right|^2 + \tilde{\sigma}_{\rm inel}(s,b) \quad (10)$$

The profile function becomes the function of the inelastic scattering probability:

$$t_{\rm el}(s,b) = \mathrm{i}\left[1-\sqrt{1-\tilde{\sigma}_{\rm inel}(s,b)}\right] \quad (11)$$

With real part:

$$t_{\rm el}(s,b) = \mathrm{i}\left[1-e^{-\mathrm{i}\,\mathrm{Im}\,\Omega(s,b)}\sqrt{1-\tilde{\sigma}_{\rm inel}(s,b)}\right] \quad (12)$$

The inelastic scattering probability function in the BB model:

The average (over configurations) of inelastic probabilities:

$$\tilde{\sigma}_{\rm inel}(b) = \left\langle \sigma(h;\vec{b}) \right\rangle_H = \int\limits_{-\infty}^{+\infty} \ldots \int\limits_{-\infty}^{+\infty} {\rm d}h\, p(h) \cdot \sigma(h;\vec{b}) \qquad (13)$$

Inelastic scatt. prob. of a composite particle (Glauber):

$$\sigma(h;\vec{b}) = 1 - \prod_a \prod_b \left[1 - \sigma_{ab}(\vec{b} + \vec{s}_a^{\,\prime} - \vec{s}_b)\right] \qquad (14)$$

and the elementary inelastic scattering probabilities:

$$\sigma_{ab}\left(\vec{s}\right) = A_{ab} e^{-s^2/S_{ab}^2},\ S_{ab}^2 = R_a^2 + R_b^2 \qquad (15)$$

Distributions

The probability density of collisions:

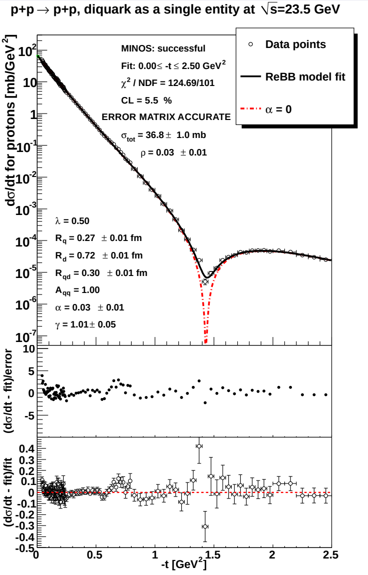
$$p(h) = D(\vec{s}_q, \vec{s}_d) \cdot D(\vec{s}_q^{\,\prime}, \vec{s}_d^{\,\prime})\,. \tag{16}$$

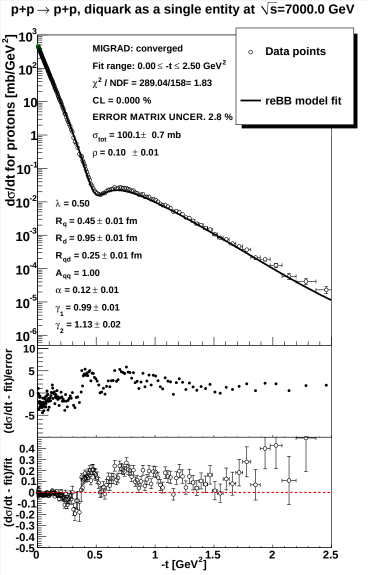
Quark-diquark probability distribution:

$$D\left(\vec{s}_q, \vec{s}_d\right) = \frac{1+\lambda^2}{R_{qd}^2\,\pi} e^{-(s_q^2+s_d^2)/R_{qd}^2} \delta^2(\vec{s}_d + \lambda\vec{s}_q)\,, \tag{17}$$

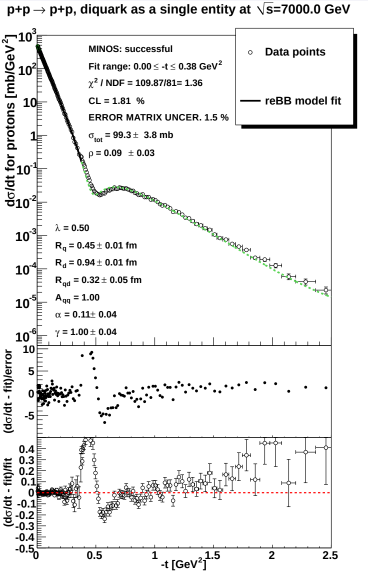
$$\lambda = \frac{m_q}{m_d} \tag{18}$$

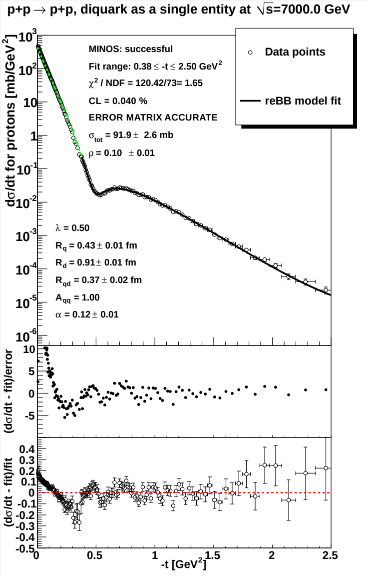
ISR and TOTEM data fits (ReBB paper with Tamás and Máté)



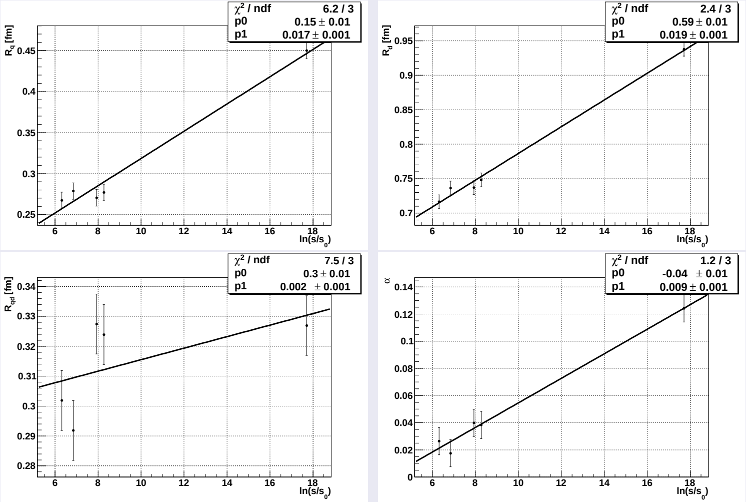


TOTEM data fits with two ranges

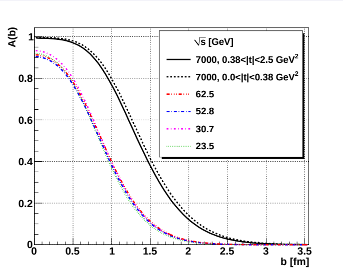


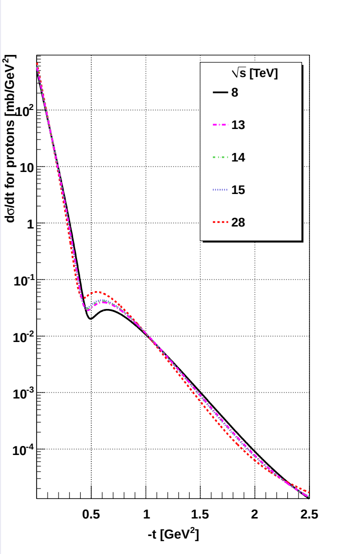


The excitation functions of the parameters



Shadow pr. + extrapolations to future LHC energies and beyond

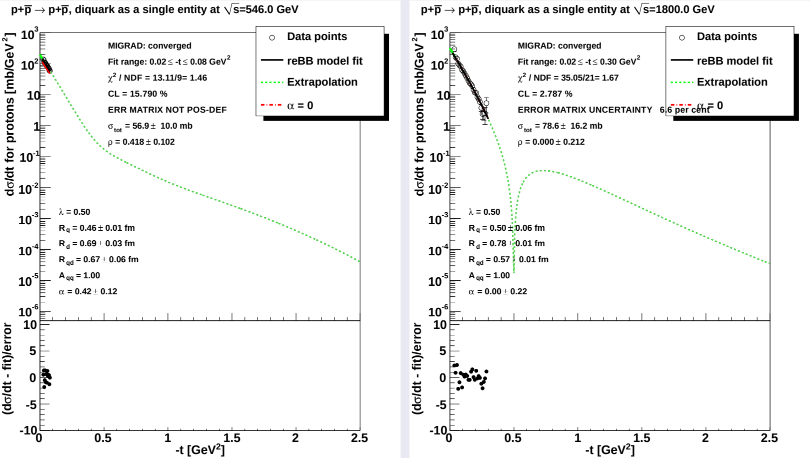




The increasing total cross-section with increasing $\sqrt{s}$

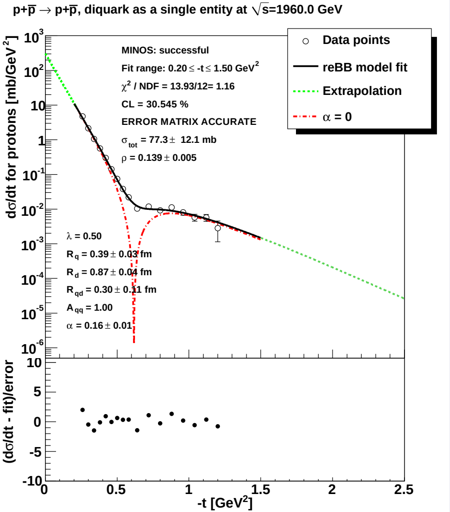
- Pomeron trajectory !? BFKL dynamics: emission and absorption of parton cascades: leads to diffusion like expansion of the proton, see the Lund model (next talk)
- QED with massive photon in 2 space dimensions (b space): expanding fermion (proton)

All the usual BB fit parameters are free ($\sqrt{s} = 546$ GeV, 1.8 TeV)



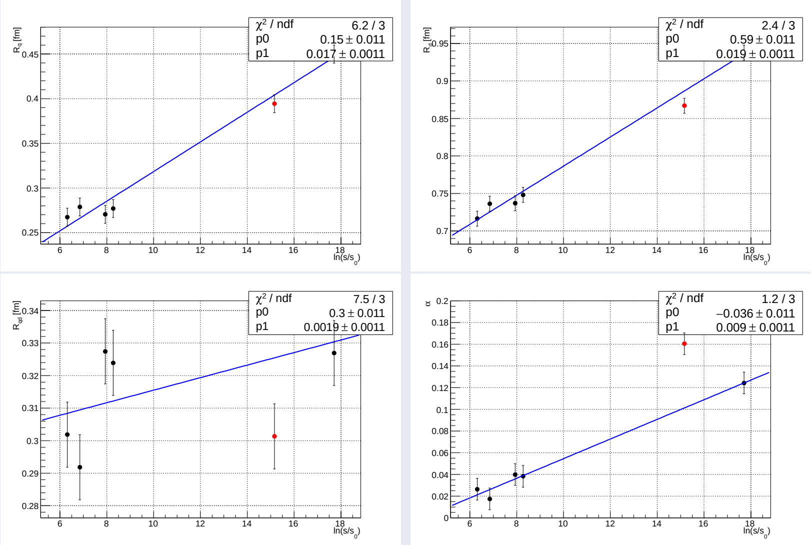
There are not enough points to pin down the shape.

All the usual BB fit parameters are free (1.96 TeV)

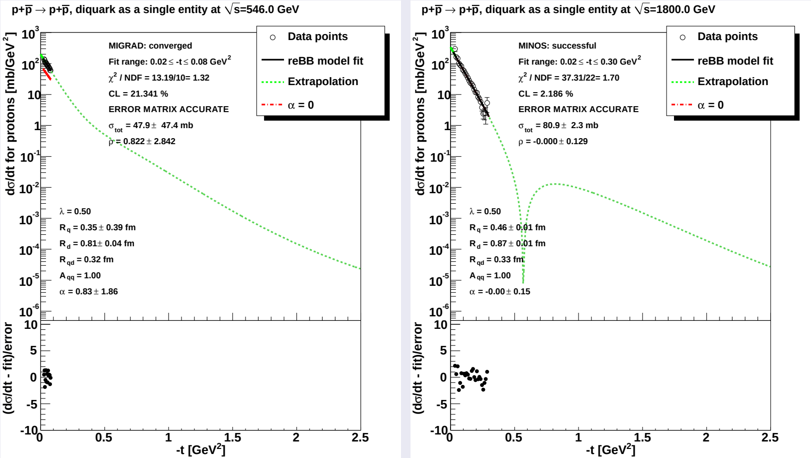


Ok.

The good fit at $\sqrt{s} = 1.96$ TeV compared with the extrapolations based only on *pp* fits of our ReBB paper

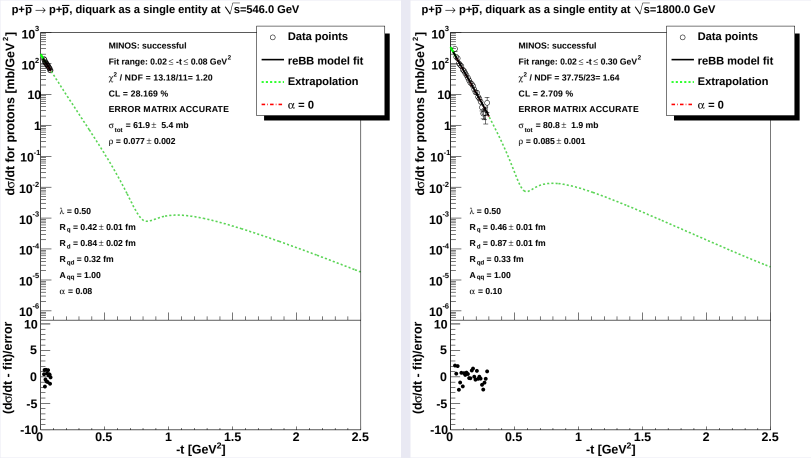


R_{qd} is fixed to the extrapolated value



Better fits.

Both R_{qd} and α is fixed to the extrapolated value



Ok.

Conclusions

- BB model is able to interpret pp, $p\bar{p}$ (and πp) data
- Trends in energy are consistent (more or less)
- Provides solid baseline for future physics studies