

*Observables, forgó megoldások  
X = Z = R, Y esetében*

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# *Our starting equations*

Consider the non-relativistic hydrodynamical problem, as given by the continuity, Euler and energy equations:

$$\begin{aligned}\partial_t n + \nabla \cdot (n\mathbf{v}) &= 0, \\ \partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} &= -(\nabla p)/(mn), \\ \partial_t \epsilon + \nabla \cdot (\epsilon\mathbf{v}) &= -p\nabla \cdot \mathbf{v},\end{aligned}$$

$\mathbf{v}$  flow velocity field,  $n$  particle number,  $p$  pressure,  $m$  particle mass,  $\epsilon$  energy density

## The closing Equation of State

$$p = nT, \quad \epsilon = \kappa(T)nT,$$

$T$  temperature,  $\kappa(T)$  compressibility  
for ideal gas 3/2

# *The Ansatz for the solution*

self-similar, ellipsoidally symmetric density and flow profiles

$$T(\mathbf{r}, t) = T(t),$$

$$n(\mathbf{r}, t) = n_0 \frac{V_0}{V} \exp \left\{ -\frac{r_x^2}{2X^2} - \frac{r_y^2}{2Y^2} - \frac{r_z^2}{2Z^2} \right\}$$

$$\mathbf{v}(\mathbf{r}, t) = \left( \frac{\dot{X}}{X} r_x + \omega r_z, \frac{\dot{Y}}{Y} r_y, \frac{\dot{Z}}{Z} r_z - \omega r_x \right)$$

*where*

$$(X, Y, Z) = (X(t), Y(t), Z(t))$$

$$V = XYZ,$$

*is known from a long time*

S.V. Akkelin, T. Csörgő, B. Lukács, Yu. M. Sinyukov and M. Weiner,  
Phys. Lett. **B505** (2001) 64.

# The final ordinary differentialequation for the time propagation

$$X = Z \equiv R, \quad \dot{X} = \dot{Z} \equiv \dot{R},$$

$$R\ddot{R} - R^2\omega^2 = \ddot{Y}Y = \frac{T}{m_0},$$

$$\dot{T} \frac{d}{dT} (T\kappa(T)) + T \frac{\dot{V}}{V} = 0,,$$

$$\omega(t) = \omega_0 \frac{R_0^2}{R^2(t)}.$$

The time dependence of the  
temperature is

$$\frac{V_0}{V} = \exp [\kappa(T) - \kappa(T_0)] \exp \int_{T_0}^T \frac{dT'}{T'} \kappa(T')$$

Initial conditions

$$V_0 = V(t_0)$$

$$T_0 = T(t_0)$$

$$T = T_0 \left( \frac{V_0}{V} \right)^{1/\kappa}$$

# The observables and the new solutions

*the emission function at a constant freeze-out temperature*

$$S(t, \mathbf{r}', \mathbf{k}') \propto e^{-\frac{(\mathbf{x}\mathbf{k}' - m\mathbf{v}')^2}{2mT_f} - \frac{r_x'^2}{2X_f^2} - \frac{r_y'^2}{2Y_f^2} - \frac{r_z'^2}{2Z_f^2}} \delta(t - t_f)$$

*single particle spectrum*

$$S \propto \exp \left\{ -\frac{T_x r_x^2}{2T_f X_f^2} + \frac{p_x \dot{X}_f - p_z \omega_f X_f}{X_f T_f} r_x - \frac{p_x^2}{2mT_f} \right\} \times$$

$$\times \exp \left\{ -\frac{T_y r_y^2}{2T_f Y_f^2} + \frac{p_y \dot{Y}_f}{Y_f T_f} r_y - \frac{p_y^2}{2mT_f} \right\} \times$$

$$\times \exp \left\{ -\frac{T_z r_z^2}{2T_f Z_f^2} + \frac{p_z \dot{Z}_f + p_x \omega_f Z_f}{Z_f T_f} r_z - \frac{p_z^2}{2mT_f} \right\}$$

# *The observables and the new solutions*

*Using the new notation:*

$$T_x = T_f + m \left( \dot{X}_f^2 + \omega_f^2 Z_f^2 \right),$$

$$T_y = T_f + m \dot{Y}_f^2,$$

$$T_z = T_f + m \left( \dot{Z}_f^2 + \omega_f^2 X_f^2 \right).$$

*new terms from  
the rotation*



# The observables II

$$\begin{aligned}
 C(\mathbf{K}', \mathbf{q}') &= 1 + \lambda \exp \left( -q_x'^2 R_x'^2 - q_y'^2 R_y'^2 - q_z'^2 R_z'^2 \right), \\
 \mathbf{K}' &= \mathbf{K}'_{12} = 0.5(\mathbf{k}'_1 + \mathbf{k}'_2), \\
 \mathbf{q}' &= \mathbf{q}'_{12} = \mathbf{k}'_1 - \mathbf{k}'_2 = (q'_x, q'_y, q'_z), \\
 R_x'^{-2} &= X_f^{-2} \left( 1 + \frac{m}{T_f} (\dot{X}_f^2 + Y_f^2 \omega^2) \right), \\
 R_y'^{-2} &= Y_f^{-2} \left( 1 + \frac{m}{T_f} \dot{Y}_f^2 \right), \\
 R_z'^{-2} &= Z_f^{-2} \left( 1 + \frac{m}{T_f} (\dot{Z}_f^2 + X_f^2 \omega^2) \right).
 \end{aligned}$$

*the two-particle BECF  
new terms from the  
rotation*

$$C_2(\mathbf{K}, \mathbf{q}) = 1 + \lambda \exp \left( - \sum_{i,j=s,o,l} q_i q_j R_{ij}^2 \right),$$

$$\begin{aligned}
 R_s^2 &= R_y'^2 \cos^2 \phi + R_x'^2 \sin^2 \phi, \\
 R_o^2 &= R_x'^2 \cos^2 \phi + R_y'^2 \sin^2 \phi + \beta_t^2 \Delta t^2, \\
 R_l^2 &= R_z'^2 \cos^2 \theta + R_x'^2 \sin^2 \theta + \beta_1^2 \Delta t^2, \\
 R_{ol}^2 &= 0 + \beta_t \beta_1 \Delta t^2, \\
 R_{os}^2 &= (R_x'^2 - R_y'^2) \cos \phi \sin \phi, \\
 R_{sl}^2 &= 0.
 \end{aligned}$$

*the transverse momentum  
of the measured pair*

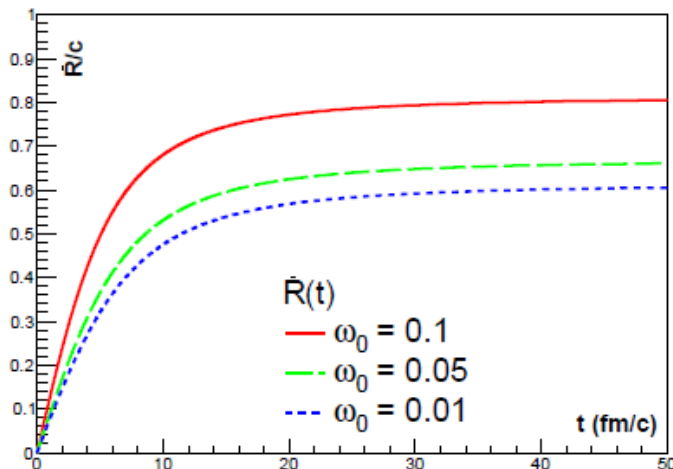
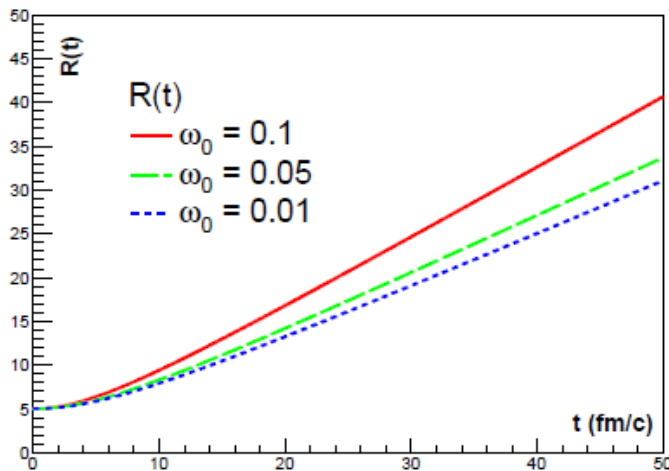
# Results I

Time evolutions of the axes  $X(t) = Z(t) = R(t)$

Initial conditions:

$$R_0 = 5 \text{ fm},$$

$$Y_0 = 5 \text{ fm}$$





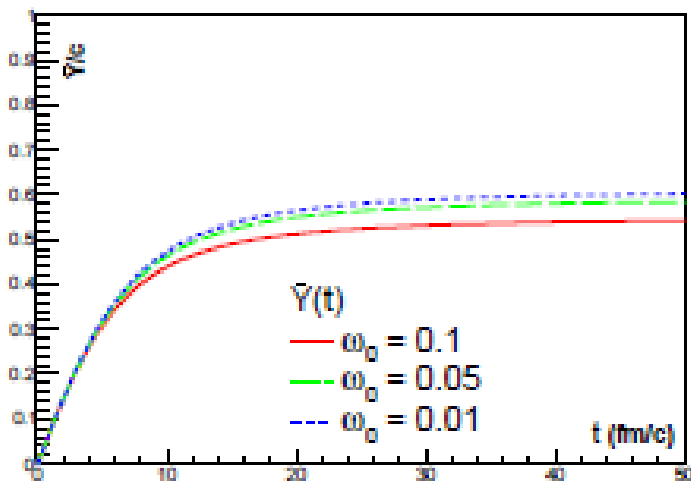
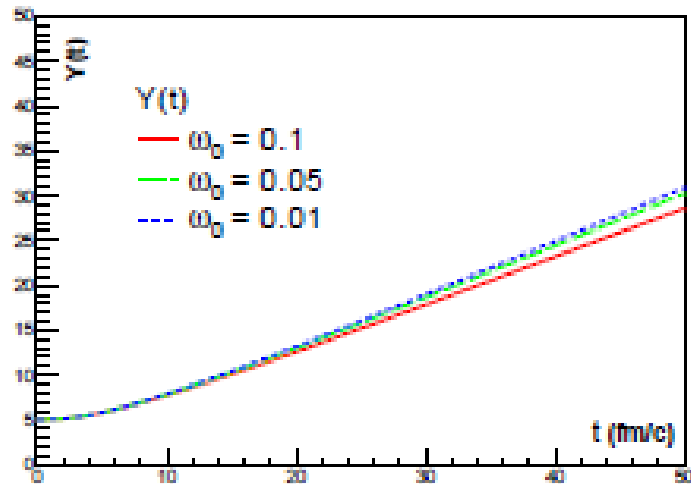
# Results II

Time evolutions of the axes  $Y(t)$

Initial conditions:

$$R_0 = 5 \text{ fm},$$

$$Y_0 = 5 \text{ fm}$$



# Results III

The time evolution of the temperature and the angular velocity

Initial conditions:

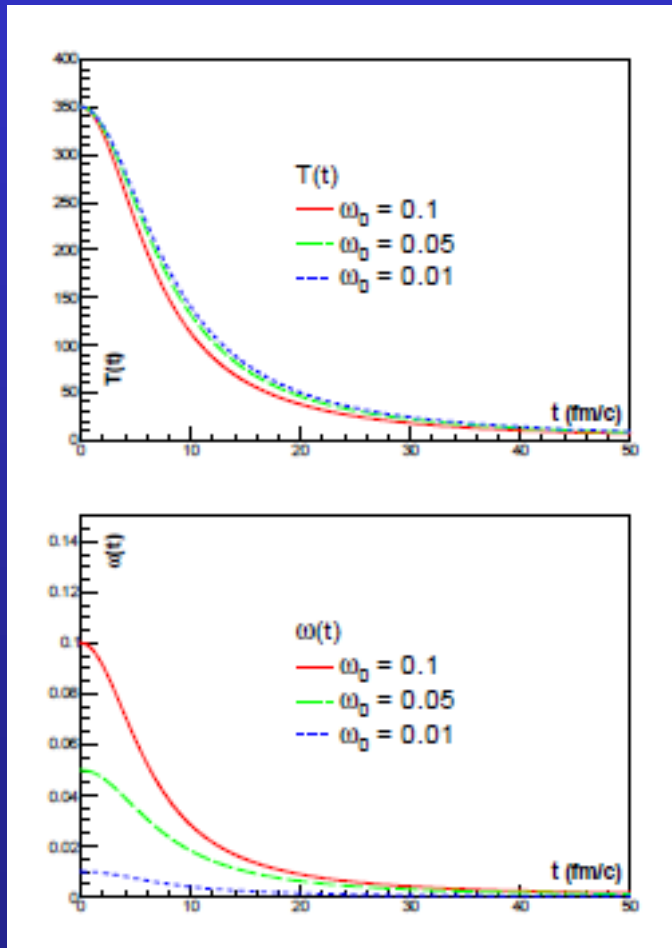
$$R_0 = 5 \text{ fm},$$

$$Y_0 = 5 \text{ fm}$$

$$\text{Kappa} = 3/2$$

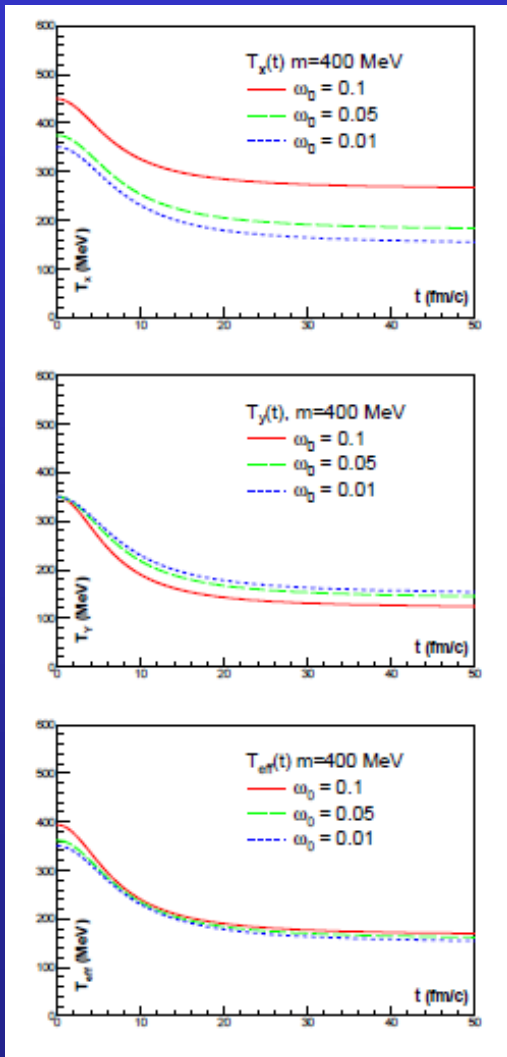
$$T_f = 140 \text{ MeV}$$

$$\text{at } 8 - 10 \text{ fm/c}$$



# Results IV

Freeze-out time dependence of the slope parameters  $T_x = T_z$

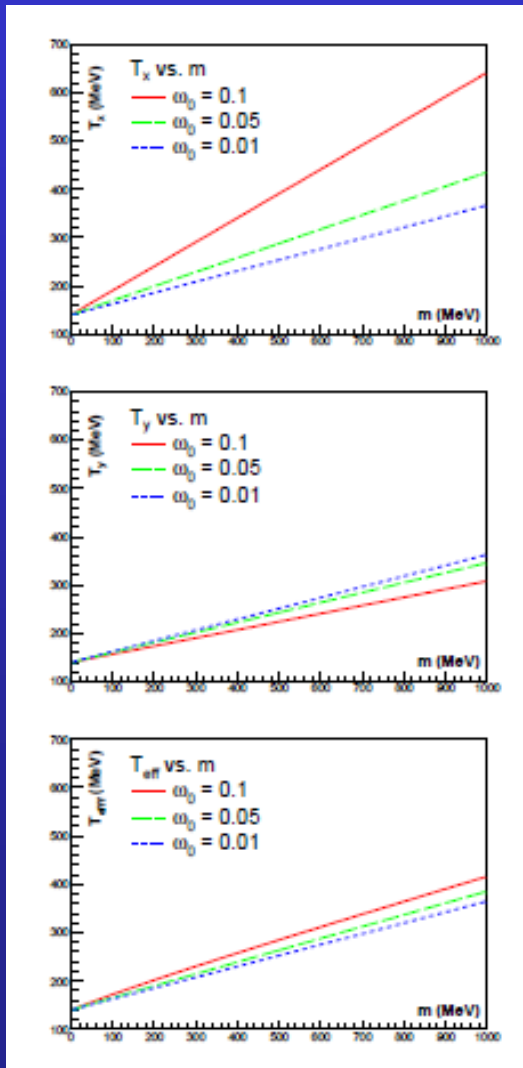


Initial conditions:  
 $R_0 = 5$  fm,  
 $Y_0 = 5$  fm  
 $Kappa = 3/2$   
 $T_f = 140$  MeV

# Results V

The slope parameters  $T_x = T_z$  and the effective temperature

$$\frac{1}{T_{\text{eff}}} = \frac{1}{2} \left( \frac{1}{T_x} + \frac{1}{T_y} \right)$$



Initial conditions:

$$R_0 = 5 \text{ fm},$$

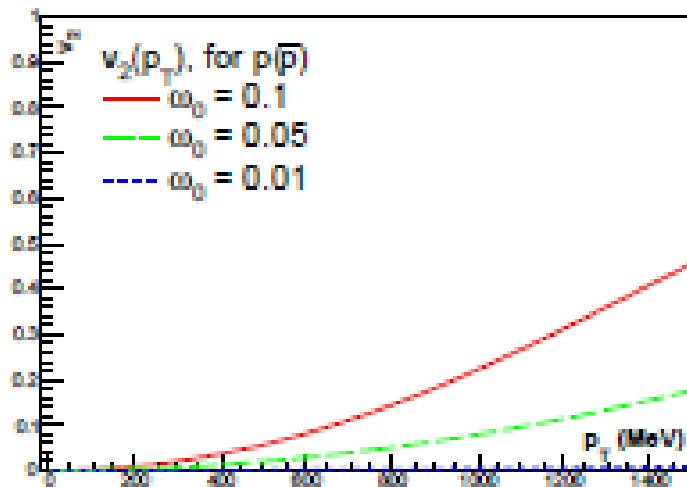
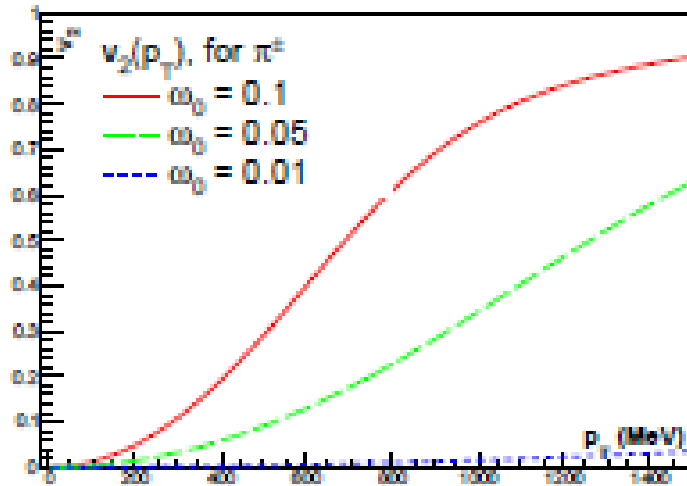
$$Y_0 = 5 \text{ fm}$$

$$\text{Kappa} = 3/2$$

$$T_f = 140 \text{ MeV}$$

# Results VI

$V_2$  for pions (upper) and protons (lower)



Initial conditions:

$$R_0 = 5 \text{ fm},$$

$$Y_0 = 5 \text{ fm}$$

$$\text{Kappa} = 3/2$$

$$T_f = 140 \text{ MeV}$$

$$\frac{dn}{2\pi p_t dp_t dp_z} \propto \exp\left(-\frac{p_t^2}{2mT_{\text{eff}}} - \frac{p_z^2}{2mT_z}\right) I_0(w)$$

$$\frac{dn}{dp_z p_t dp_t d\phi} = \frac{dn}{2\pi dp_z p_t dp_t} \left[ 1 + 2 \sum_{n=1}^{\infty} v_n \cos(n\phi) \right]$$

$$v_{2n+1} = 0, \quad v_{2n} = \frac{I_n(w)}{I_0(w)}.$$

# Results VII

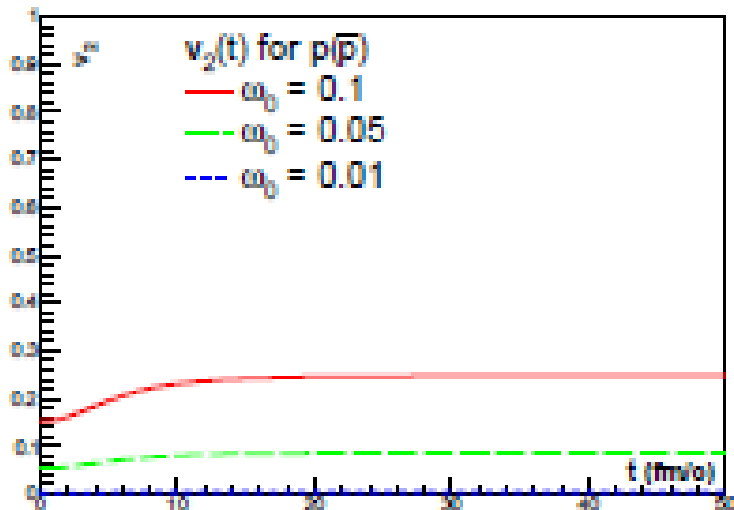
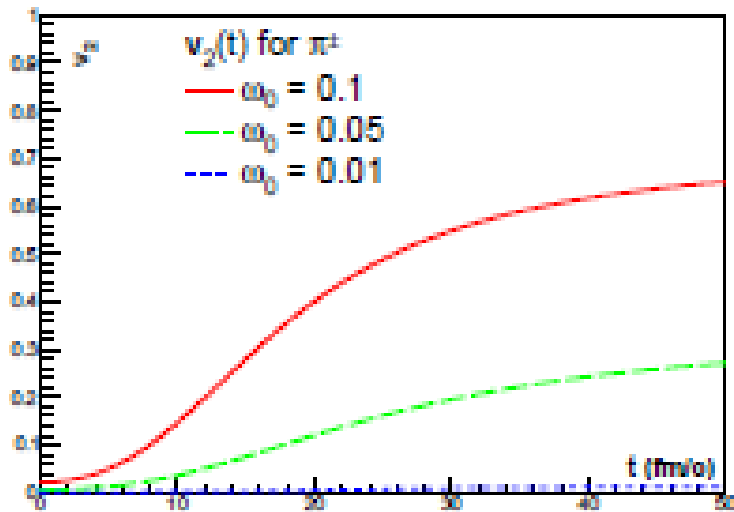
The freeze-out time dependence of  $v_2$  for pions (upper) and for protons (lower) at fixed  $p_T$  (300 MeV/c) and (1000 MeV/c)

Initial conditions:

$R_0 = 5$  fm,

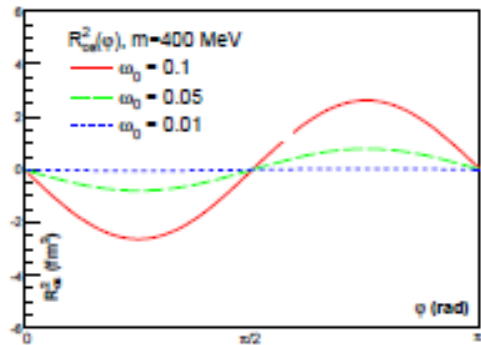
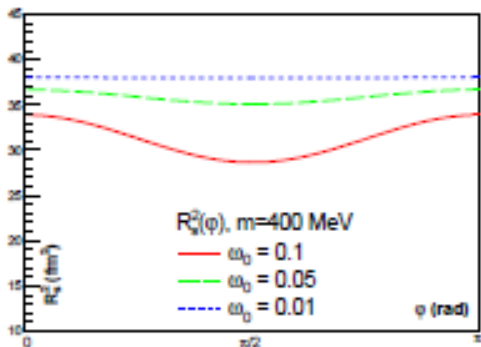
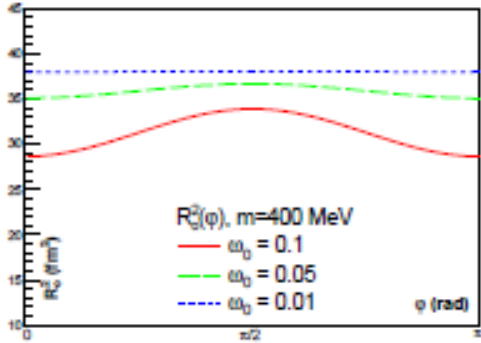
$Y_0 = 5$  fm

$Kappa = 3/2$



# Results VIII

The HBT radius parameters  
 $R_o^2$  (upper panel)  
 $R_s^2$  (middle panel)  
 $R_{os}^2$  (lower panel)



$$C(\mathbf{K}, \mathbf{q}) = 1 + \lambda \exp \left\{ -q_x^2 R_x^2 - q_y^2 R_y^2 - q_z^2 R_z^2 \right\}$$

$$\mathbf{K} = \mathbf{K}_{12} = \frac{1}{2} (\mathbf{p}_1 + \mathbf{p}_2),$$

$$\mathbf{q} = \mathbf{q}_{12} = (\mathbf{k}_1 - \mathbf{k}_2),$$

$$C_2(\mathbf{K}, \mathbf{q}) = 1 + \lambda \exp \left( - \sum_{i,j=s,o,l} q_i q_j R_{ij}^2 \right)$$

$$R_s^2 = R_y^2 \cos^2 \phi + R_x^2 \sin^2 \phi,$$

$$R_o^2 = R_x^2 \cos^2 \phi + R_y^2 \sin^2 \phi + \beta_t^2 \Delta t^2$$

$$R_l^2 = R_x^2 + \beta_l^2 \Delta t^2,$$

$$R_{ol}^2 = \beta_t \beta_l \Delta t^2,$$

$$R_{os}^2 = (R_x^2 - R_y^2) \cos \phi \sin \phi,$$

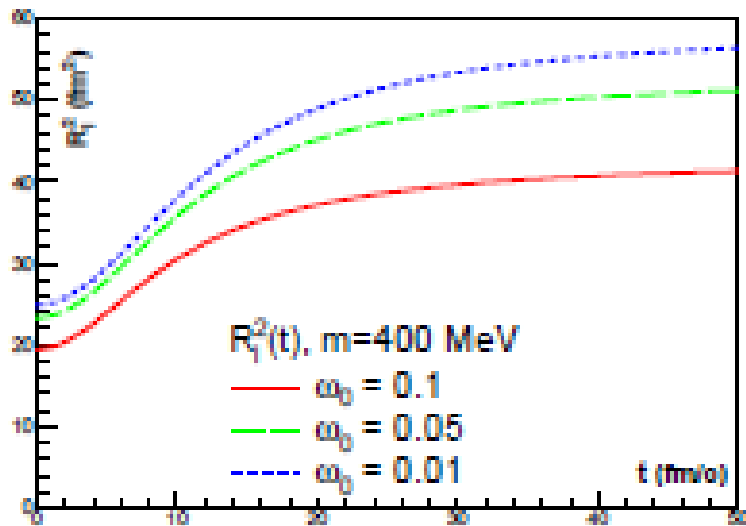
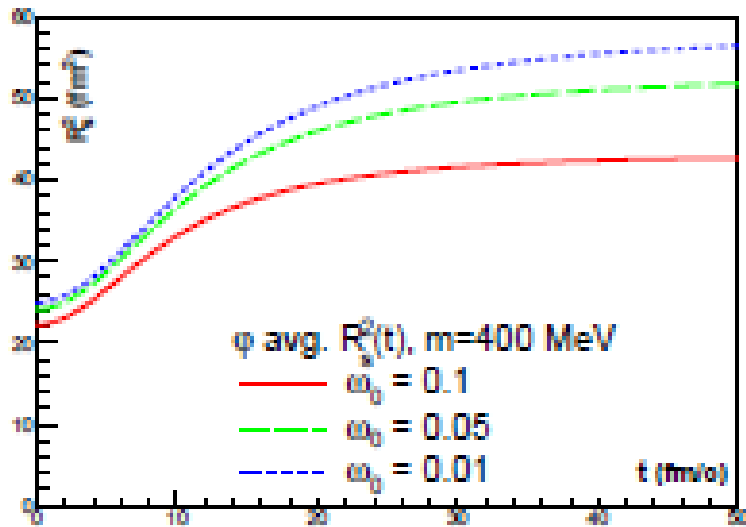
$$R_{sl}^2 = 0.$$

for  $m = 400 \text{ MeV}$

# Results IX

Freeze-out time dependence  
of the azimuthal average of  
 $R_s^2$  (upper) and  $R_l^2$  (lower)

$m = 400 \text{ MeV}$





*Thank  
You  
for  
Your  
Attention!*

*Questions, Remarks, Comments?...*