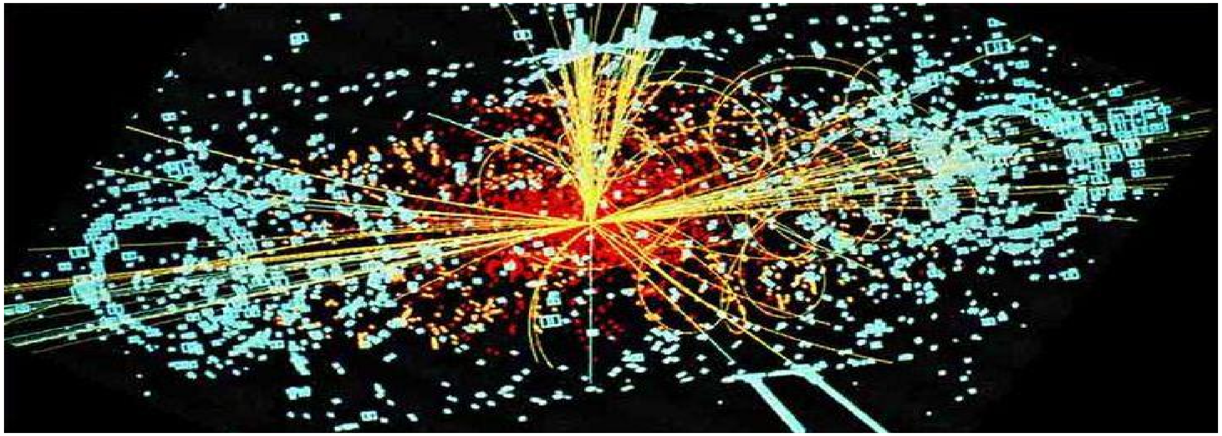
Hydrodynamical estimation of initial energy density for pp collision at CERN-LHC energies



Csörgő Tamás, Chun-bin Yang
Máté Csanád, Ze-fang Jiang

IOPP(CCNU) is in Wuhan, Hubei, China

Outline

- **A description of accelerating solutions of relativistic perfect fluid hydrodynamics;**
  (T. Csörgő, M. I. Nagy, M. Csanád. Phys. Lett. **B663** (2008) 306)
  (M. I. Nagy, T. Csörgő, M. Csanád. Phys. Rev. **C77** (2008), 024908)

- **Pseudorapidity distribution and Initial energy density of Charged particle at CERN-LHC Energies;**

- **Summary.**

Part 1. Recaptulation of an exact hydro solution

1. The equations of relativistic hydrodynamics

The energy-momentum tensor (prefect fluid):

$$T_{\mu\nu} = (\varepsilon + p)u_{\mu}u_{\nu} - pg_{uv}$$

The metric tensor:

$$g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$$

u^{μ} and ε, and p are, respectively, the 4-velocity, energy density, and pressure of fluid. ε and p are related by the equation of state(EoS):

$$\varepsilon = \kappa p$$

$1/\kappa = c_s^2$ is the speed of sound.

The energy-momentum conservation law

$$\partial_{\nu} T^{\mu\nu} = 0$$

The relativistic Euler equation and the energy conservation equation as below:

$$wu^{\mu\nu}\partial_{\nu}u^{\mu} = (g^{\mu\rho} - u^{\mu}u^{\rho})\partial_{\rho}p$$

$$w\partial_{\mu}u^{\mu} = -u^{\mu}\partial_{\mu}\varepsilon$$

The general form of the charge conservation equations is as follows :

$$\sum_{i} \mu_i \partial_{\mu}(n_i u^{\mu}) = 0$$

2. The exact solutions and the Rapidity distribution.

4 different sets of the parameters λ, κ, and d.

Possible cases are

Case	λ	d	κ	φ
a.)	2	**R**	d	0
b.)	1/2	**R**	1	(κ+1)/κ
c.)	3/2	**R**	(4d-1)/3	(κ+1)/κ
d.)	1	**R**	**R**	0
e.)	**R**	**1**	1	0

(λ = 1 is the Hwa-Bjorken solution in 1+1 dimensions.):

The velocity field and the pressure is expressed as :

$$v = \tanh \lambda\eta , \quad p = p_0 \left(\frac{\tau_0}{\tau}\right)^{-\lambda d \frac{(\kappa+1)}{\kappa}} \left(\cosh \frac{\eta}{2}\right)^{-(d-1)\phi}$$

(T. Csörgő, M. I. Nagy, M. Csanád. Phys. Lett. **B663** (2008) 306)
(M. I. Nagy, T. Csörgő, M. Csanád. Phys. Rev. **C77** (2008), 024908)

3.1 The rapidity distribution

Freeze-out condition: the freeze-out hypersurface is pseudo-orthogonal to the four velocity field u^u , and the temperature at η = 0 reachs a given T_f valve.

$$(\frac{\tau_f}{\tau})^{\lambda\text{-}1}\cosh((\lambda-1)\eta)=1$$

The expression of rapidity distribution as below:

$$\frac{\mathrm{d}N}{dy}\approx\left.\frac{\mathrm{d}N}{dy}\right|_{y=0}\cosh^{\pm\frac{\alpha}{2}-1}(\frac{y}{\alpha})e^{-\frac{m}{T_f}[\cosh^{\alpha}(\frac{y}{\alpha})-1]}$$

with $\alpha=\dfrac{2\lambda\text{ -}1}{\lambda\text{ -}1}$.

3.2 The energy density estimation

Follow Bjorken's method, the initial energy density for acclerationless, boost-invariant Hwa-Bjorken flows

$$\varepsilon_0 = \frac{< m_t >}{(R^2\pi)\tau_0}\frac{dN}{d\eta_0}, \qquad (\eta_0 = \eta = y)$$

Here τ_0 is the proper-time of thermalization.

J. D. Bjroken, Phys. Rev. D 27, 140 (1983)

For an accelerating flow, the initial energy density

$$\frac{\varepsilon_c}{\varepsilon_{Bj}} = (2\lambda - 1)(\frac{\tau_f}{\tau_0})^{\lambda-1}, \qquad \lambda > 1$$

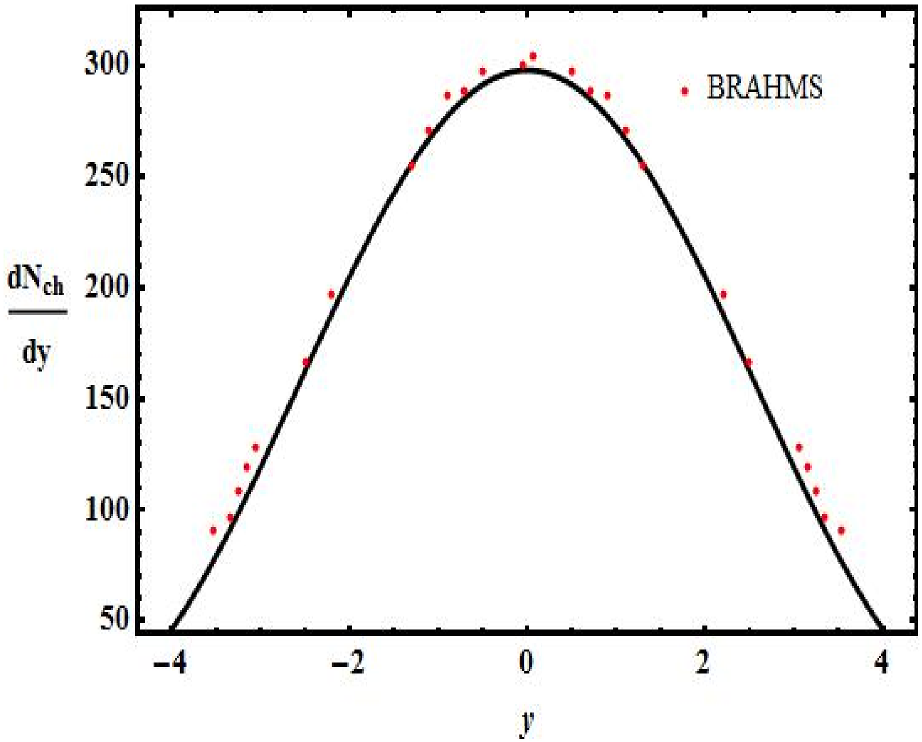
Effect of the pressure (→ $(2\lambda - 1)$)

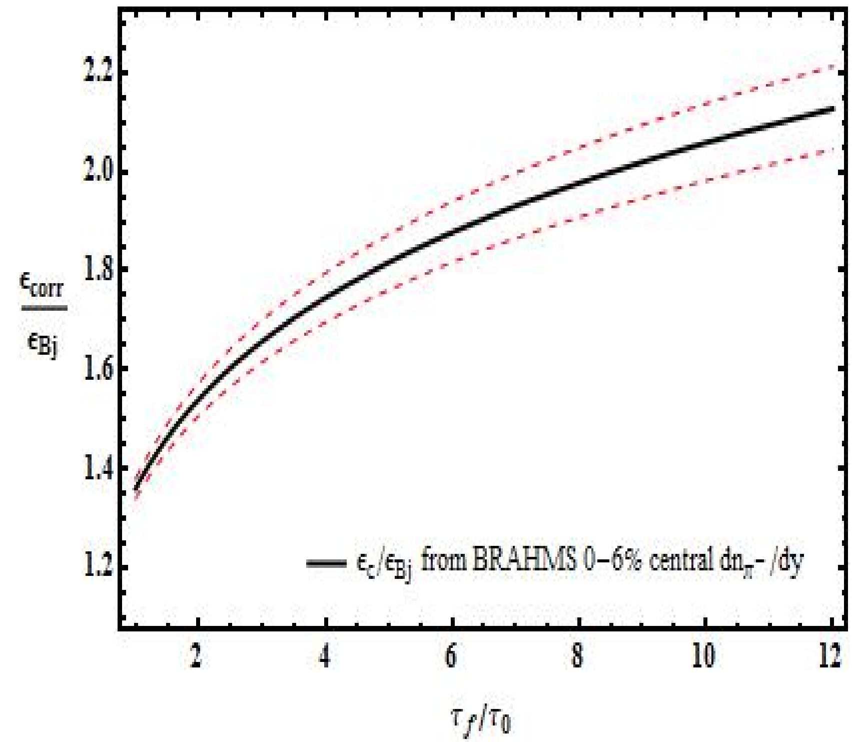
The expansion of intial volume element (→ $(\frac{\tau_f}{\tau_0})^{\lambda-1}$)

Here τ_f is the proper-time of freeze-out.

T. Csörgő, M. I. Nagy, M. Csanád. Phys. Lett. **B663** (2008) 306

For 200 GeV AuAu collision at RHIC:





$$\lambda = 1.18 \pm 0.01, \quad \tau_f / \tau_0 = 8 \pm 2 \mathrm{fm}/c,$$

$$\varepsilon_{corr} = (2.0 \pm 0.1)\varepsilon_{Bj} = 10.0 \pm 0.5 \mathrm{GeV/fm}^3.$$

M. I. Nagy, T. Csörgő, M. Csanád. Phys. Rev. **C77** (2008), 024908

I. G. Bearden el al [BRAHMS], Phys. Rev. Lett 94, 162301 (2005)

Part 2. New initial energy density estimation of pp collision at LHC

1. The pseudo-rapidity distribution

The relation between rapidity and pseudo-rapidity can be writen as follows:

$$\eta = \frac{1}{2}\ln\frac{\sqrt{(m^2+p_T^2)\cosh^2 y - m^2} + \sqrt{m^2+p_T^2}\sinh y}{\sqrt{(m^2+p_T^2)\cosh^2 y - m^2} - \sqrt{m^2+p_T^2}\sinh y}$$

The relation between rapidity distribution and pseudo-rapidity distribution can be writen as follows:

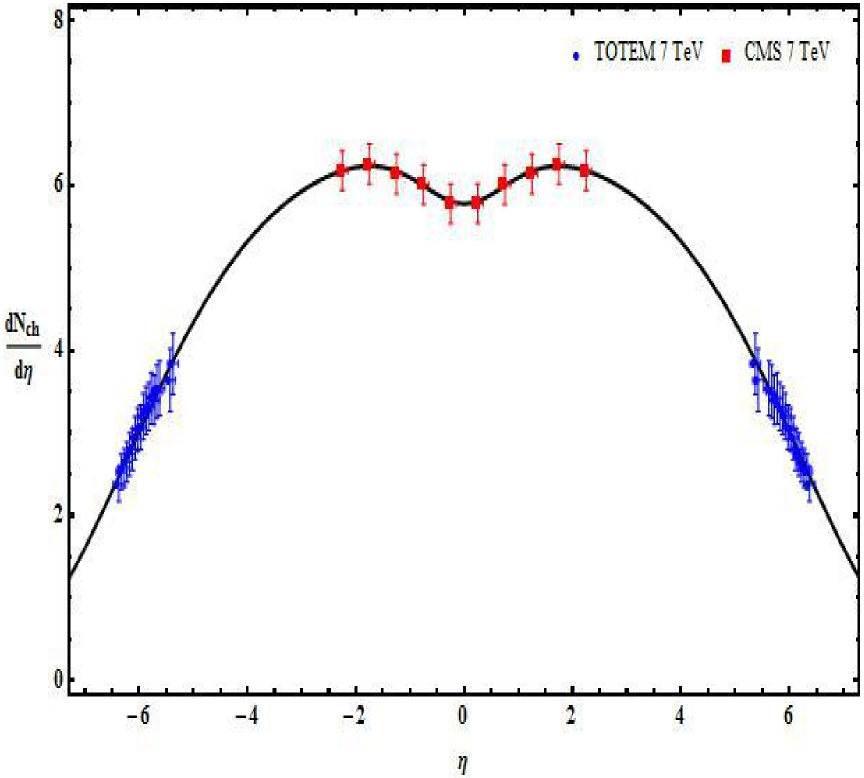
$$\frac{dN}{d\eta} \approx \frac{\overline{p}}{\overline{E}}\frac{dn}{dy} = \frac{\overline{p_T}\cosh\eta}{\sqrt{m^2+\overline{p_T^2}}}\frac{dN}{dy}$$

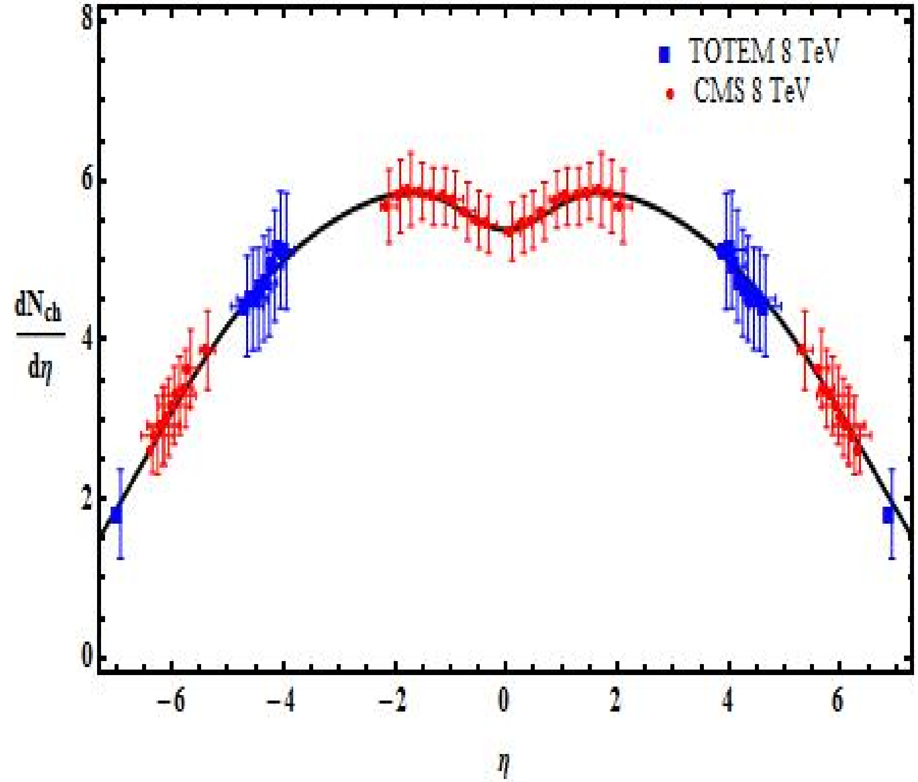
From model

$$\overline{p_T} = \frac{T_{eff}}{1+\frac{\sigma^2}{2}y^2}, \quad \sigma^2 = \frac{T_0 T_{eff}}{m_0^2(\Delta y^2 + \frac{T_0}{m_0})}, \quad T_{eff} = T_0 + \frac{m_0 < u_T >^2}{1+\frac{m_0}{T_0}}$$

T. Csörgő, B. Lörstad Phys. Rev. **C54 (1996)**, 1390

The pseudo-rapidity distribution at CMS+TOTEM **7 TeV** and **8 TeV** pp collision data.





1. V. Khachatryan, el al [CMS], Phys. Rev. Lett 105, 022002 (2010);
2. The TOTEM Collaboration, Eur. Phys. Lett, 98 (2012) 31002;
3. G. Antchev, el al[TOTEM], arXiv: 1411.4963 (2014);
4. The CMS and TOTEM Collaborations, Eur. Phys. J. C (2014) 74:3053.

The energy density estimation at CMS+TOTEM 7 TeV and 8 TeV pp collision.

$$\varepsilon_{Bj} = \frac{<E>}{(R^2\pi)\tau_0}\frac{dN}{d\eta_0}\bigg|_{\eta=\eta_0}$$

Estimation made by Bjorken

The initial energy density are under-estimatied by Bjorken formula, the new corrected are:

$$\frac{\varepsilon_{corr}}{\varepsilon_{Bj}} = (2\lambda-1)\left(\frac{\tau_f}{\tau_0}\right)^{\lambda-1}\left(\frac{\tau_f}{\tau_0}\right)^{(\lambda-1)(1-c_s^2)}, \qquad \lambda > 1$$

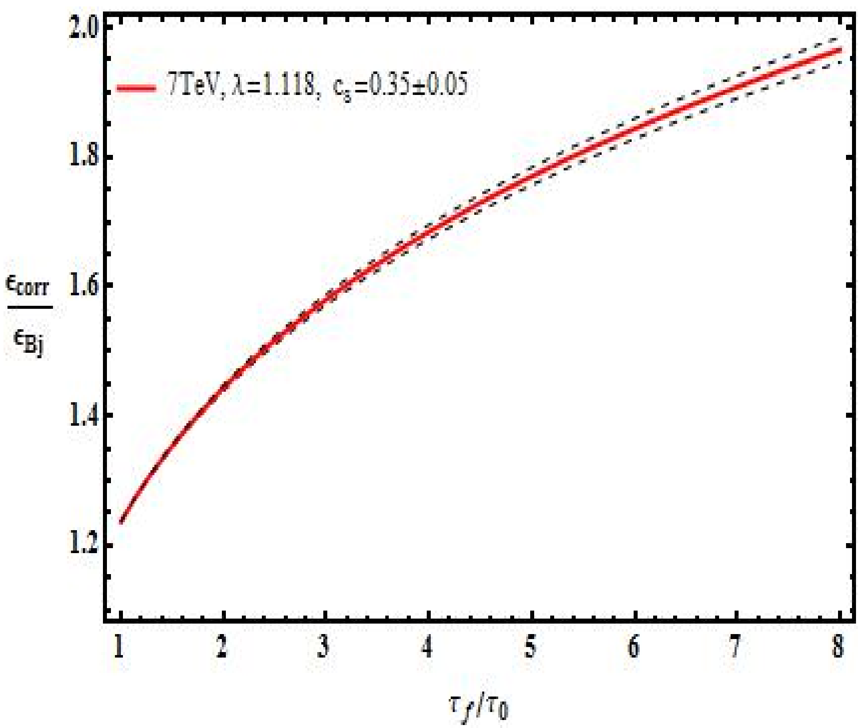
Effect of the pressure

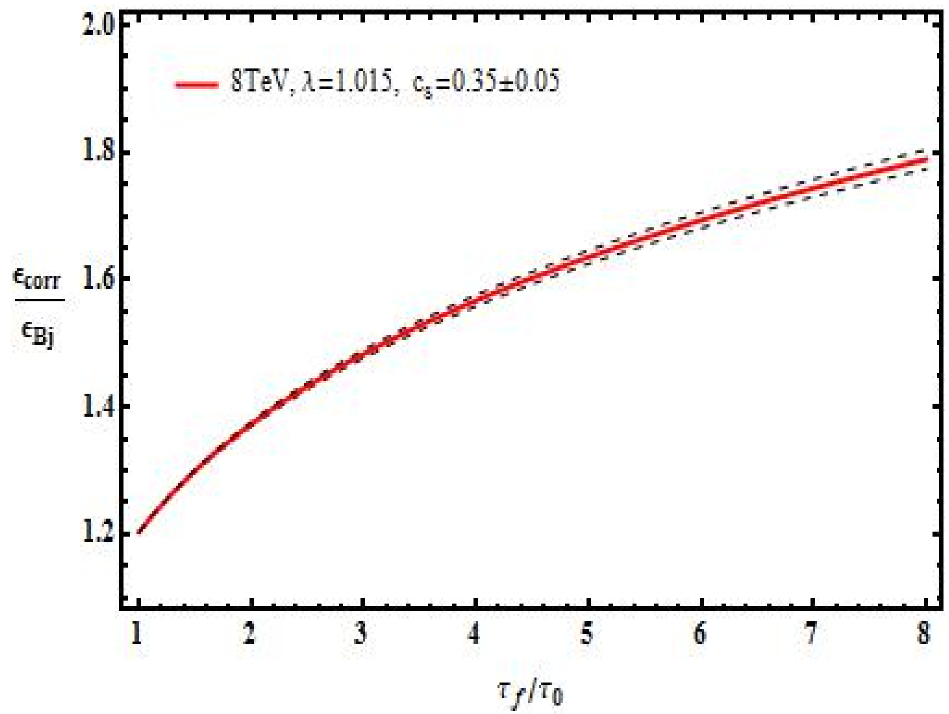
The expansion of intial volume element

Pressure/energy/non-ideal EoS

T. Csörgő, M. I. Nagy, M. Csanád. Phys. Lett. **B663** (2008) 306

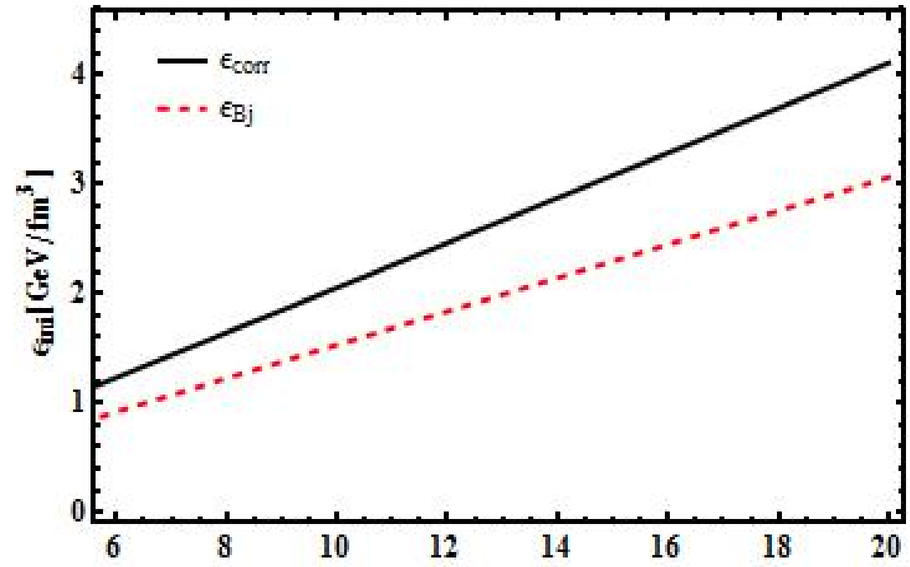
CMS+TOTEM 7 TeV and 8 TeV pp collision.

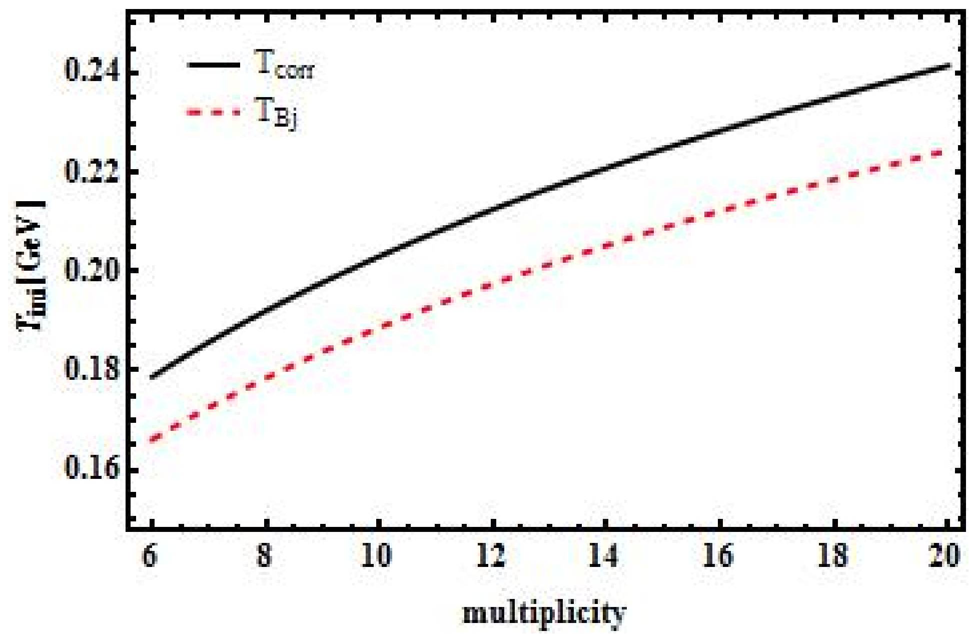


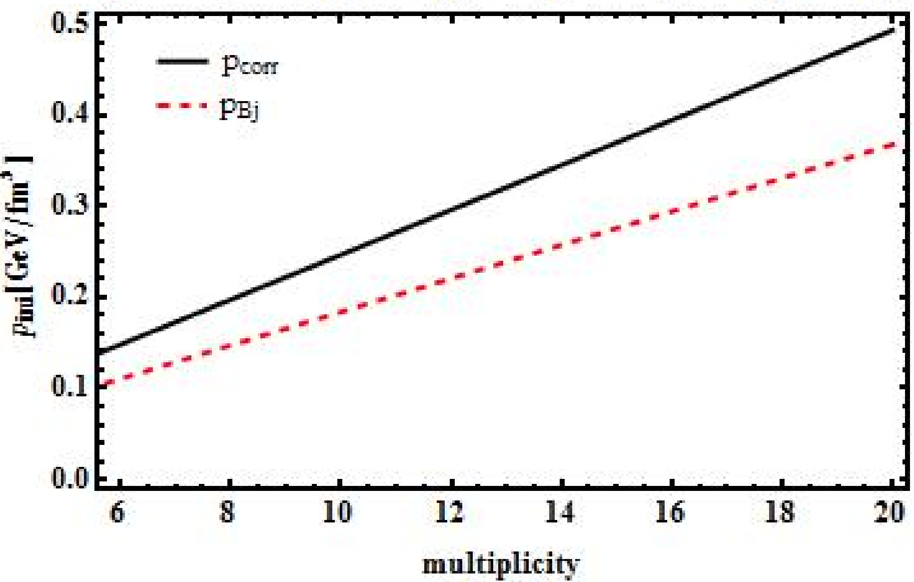


$\sqrt{S}$	ε_{Bj}	ε_{c}	ε_{corr}	λ	c_s^2	$dN/d\eta\big\|_{\eta=0}$
7 TeV	0.902	1.20989	1.3001	1.118	0.12	5.895(NSD)
8 TeV	0.902	1.16283	1.2367	1.101	0.12	5.38(Inelastic)

The initial energy density, Temperature and pressure estimate at CERN-LHC

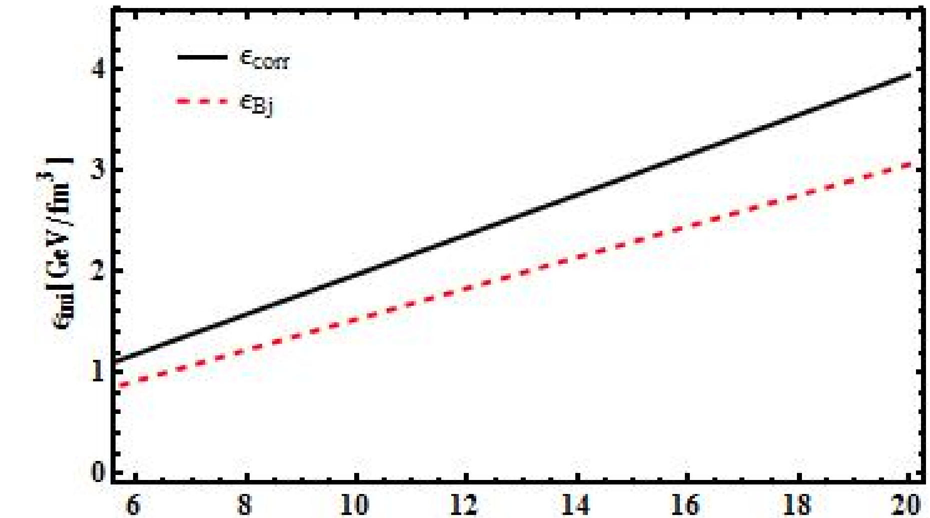


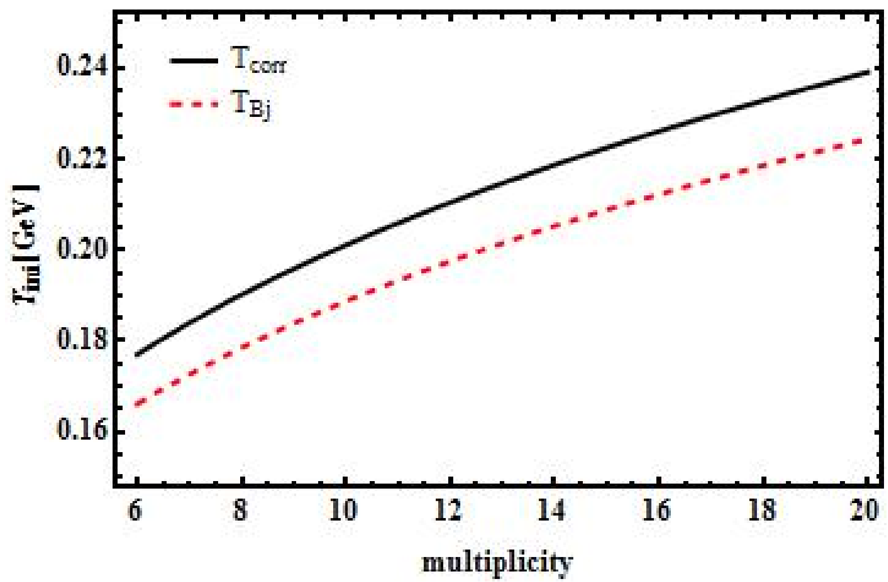


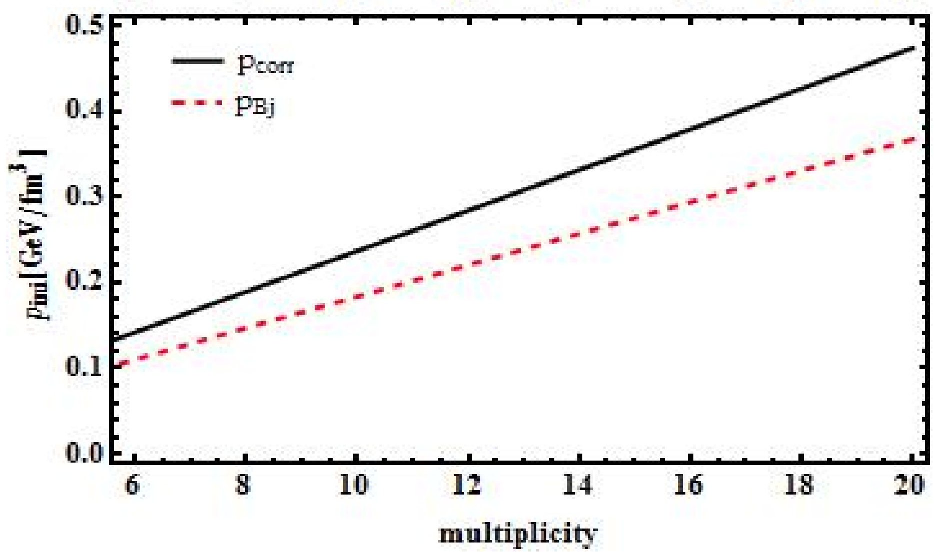


CMS+TOTEM 7 TeV

The initial energy density, Temperature and pressure estimate at CERN-LHC







CMS+TOTEM 8 TeV

Part 3. Summary

1. Exact hydro solution is recapitulated and applied to dn/dy in Au+Au at RHIC;
2. Initial energy density estimate in pp at LHC;
3. The initial energy density in pp at LHC is **bigger than 1 GeV/fm^3**

Thank you all very much

Write down these equations also in a three-dimensional notation.

$$\text{Euler}: \qquad \frac{w}{1-v^2}\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -(\nabla p + \mathbf{v}\frac{\partial p}{\partial p})$$

$$\text{Energy conservation}: \qquad \frac{1}{w}\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = -(\nabla \mathbf{v}) - \frac{1}{1-v^2}\frac{\mathrm{d}}{\mathrm{d}t}\frac{v^2}{2}$$

$$\text{Continuity equation}: \qquad \frac{\mathrm{d}}{\mathrm{d}t}\ln\frac{n}{\sqrt{1-v^2}} = -(\nabla \mathbf{v})$$

The thermodynamical quantities obey genreal rules.

2. Hydrodynamical equations in Rindler coordinates :

The definiTion of the Rindler coordinates (inside the lightcone):

$$t = \tau \cosh \eta, \qquad r = \tau \sinh \eta;$$

$$\tau = \sqrt{t^2 - r^2}, \qquad \eta = \operatorname{arctan} \frac{r}{t};$$

$$\frac{\partial}{\partial t} = \cosh \eta \frac{\partial}{\partial \tau} - \frac{\sinh \eta}{\tau} \frac{\partial}{\partial \eta};$$

$$\frac{\partial}{\partial r} = - \sinh \eta \frac{\partial}{\partial \tau} + \frac{\cosh \eta}{\tau} \frac{\partial}{\partial \eta};$$

$$\frac{d}{dt} = \frac{\cosh(\Omega - \eta)}{\cosh \Omega} \frac{\partial}{\partial \tau} + \frac{\sinh(\Omega - \eta)}{\tau \cosh \Omega} \frac{\partial}{\partial \eta};$$

$$\frac{d}{dr} = \frac{\sinh(\Omega - \eta)}{\cosh \Omega} \frac{\partial}{\partial \tau} + \frac{\cosh(\Omega - \eta)}{\tau \cosh \Omega} \frac{\partial}{\partial \eta};$$

The domain of the variables is $-\infty < \eta < +\infty, \; 0 \le \tau < \infty.$

Ω stands for the rapidity of the flow.

$$v = \tanh \Omega$$

$$Q = \frac{1}{\kappa + 1} \ln p$$

Using the assumption :

$$\Omega = \lambda \eta$$

Rewetring and rearranging the Euler and energy conservation equations :

$$\tanh((\lambda - 1)\eta)(\tau \frac{\partial Q}{\partial \tau} + \lambda) + \frac{\partial Q}{\partial \eta} = 0,$$

$$\kappa\tau \frac{\partial Q}{\partial \tau} + (d-1)\frac{\sinh \lambda\eta}{\sinh \eta}\cosh((\lambda - 1)\eta) + \lambda(\cosh^2(\lambda-1)\eta) - \kappa \sinh^2((\lambda-1)\eta)) = 0$$

The solution is easily obtained as (Inside the forward lightcone)

$$p = p_0 (\frac{\tau_0}{\tau})^{-K(\kappa+1)} \sinh^{-\frac{K+\lambda}{\lambda-1}}((\lambda - 1)\eta)$$