Lattice Kaon Physics

An Overview

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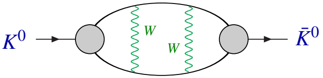
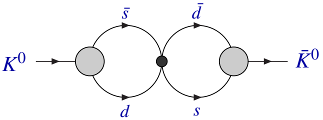
- The mission of lattice calculations is to evaluate hadronic effects.
- "Standard" lattice calculations in flavour physics are of matrix elements of local operators between single hadron states $\langle h_2(p_2)|O(0)|h_1(p_1)\rangle$ (or $\langle 0|O(0)|h(p)\rangle$).
- For example, in the evaluation of ϵ_K, we need to calculate (schematically)



(gluons and quark loops not shown.)

- The process is short-distance dominated and so we can approximate the above by a perturbatively calculable (Wilson) coefficient C times



where the black dot represents the insertion of the local operator $(\bar{s}\gamma_\mu(1-\gamma^5)d)\,(\bar{s}\gamma_\mu(1-\gamma^5)d)$.

  - In the standard model only this single operator contributes.
  - In generic BSM theories there are 5 possible $\Delta S = 2$ operators contributing.

Such standard calculations have been performed for a long time!

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94. **The Kaon B Parameter and K- pi and K- pi pi Transition Amplitudes on the Lattice**
M.B. Gavela (Madrid, Autonoma U.), L. Maiani, S. Petrarca, F. Rapuano (Rome U. & INFN, Rome), G. Martinelli (CERN), O. Pene (Orsay, LPT), Christopher T. Sachrajda (Southampton U.). Nov 1987.
Published in **Nucl.Phys. B306 (1988) 677**
CERN-TH-4905/87
DOI: 10.1016/0550-3213(88)90439-7
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95. **Pion Structure Functions From Lattice {QCD}**
G. Martinelli, Christopher T. Sachrajda (CERN). Jun 1987. 13 pp.
Published in **Phys.Lett. B196 (1987) 184**
CERN-TH-4766/87
DOI: 10.1016/0370-2693(87)90601-0
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96. **The Pion's structure function: A Lattice computation of the first two moments**
Christopher T. Sachrajda (CERN & Southampton U.). 1987.
Conference: C87-06-25
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Detailed record

97. **A Lattice Calculation of the Second Moment of the Pion's Distribution Amplitude**
G. Martinelli, Christopher T. Sachrajda (CERN). Jan 1987. 12 pp.
Published in **Phys.Lett. B190 (1987) 151**
CERN-TH-4637-87
DOI: 10.1016/0370-2693(87)90858-6
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98. **Nucleon Wave Functions From Lattice Gauge Theories: Renormalization of Baryonic Operators**
D.G. Richards, Christopher T. Sachrajda, C.J. Scott (Southampton U.). Sep 29, 1986. 32 pp.
Published in **Nucl.Phys. B286 (1987) 683**
SHEP-85-86-24
DOI: 10.1016/0550-3213(87)90459-7
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
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Introduction (cont.)

- In recent years the precision with which such standard quantities can be computed has improved immensely.
  - Computations can now be performed at with 2, 2+1, 2+1+1 dynamical quarks at physical masses.
- Some sample results from the FLAG collaboration: arXiv:1607.00299

	$N_f = 2$	$N_f = 2+1$	$N_f = 2+1+1$
f_K/f_π	1.205(6)(17)	1.192(5)	1.193(3)
$\hat{B}_K$	0.727(22)(12)	0.7625(97)	0.717(18)(16)
$f_+(0)$	0.9560(57)(62)	0.9677(27)	0.9704(24)(22)

  - Lattice studies of neutral-kaon mixing will be reviewed by Weonjong Lee.
- In this talk I want to introduce some new directions in lattice kaon physics $\Rightarrow$ prospectus for Kaon 2019.
  - The topics will then be discussed in more detail in the following talks.

Outline of talk:

1. Introduction
2. Isospin breaking contributions to decay amplitudes — **G.Martinelli**
3. Long-distance contributions to flavour changing processes — **X.Feng**

$$\int\!\!\int d^4x\, d^4y\, \langle f\,|\,T[Q_1(x)\,Q_2(y)]\,|\,i\rangle\,.$$

   (a) Δm_K and ϵ_K
   (b) Rare kaon decays
4. $K \to \pi\pi$ decays. — **N.Garron**
5. Summary and conclusions

Thank you to my collaborators from the RBC-UKQCD Collaboration and from Rome (em corrections) for such stimulating collaborations on the topics of this talk.

Talk by G.Martinelli

- "Standard" QCD calculations have been performed in the isospin limit, i.e. with $m_u = m_d$, so to improve the precision still further isospin breaking effects (including electromagnetism) need to be included.
- These are

$$O(\frac{m_u - m_d}{\Lambda_{\rm QCD}}) \quad \text{and} \quad O(\alpha)\,,$$

i.e. $O(1\%)$ or so.

- Such calculations for the spectrum have been performed for a few years now, with perhaps the most noteworthy result being BMW Collaboration, arXiv:1406.4088

$$m_n - m_p = 1.51(16)(23)\,\text{MeV}$$

to be compared to the experimental value of 1.2933322(4) MeV.

- I stress that including electromagnetic effects, where the photon is massless of course, required considerable theoretical progress, e.g.

$$\int \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2}\cdots \Rightarrow \frac{1}{L^3T}\sum_k \frac{1}{k^2}\cdots$$

and we have to control the contribution of the zero mode in the sum.

Talk by G.Martinelli

- Calculating electromagnetic corrections to decay amplitudes has an added major complication, not present in computations of the spectrum,

the presence of infrared divergences

- This implies that when studying weak decays, such as e.g. $K^+ \to \ell^+\nu$ the physical observable must include soft photons in the final state

$$\Gamma(K^+ \to \ell^+\nu_\ell(\gamma)) = \Gamma(K^+ \to \ell^+\nu_\ell) + \Gamma(K^+ \to \ell^+\nu_\ell\gamma)\,.$$

F.Bloch and A.Nordsieck, PR 52 (1937) 54

- Last year we proposed a method for including electromagnetic corrections in decay amplitudes and are developing it further as well as testing it numerically.

N.Carrasco et al., arXiv:1502.00257

- I stress that in order to implement this method successfully, it will be be necessary to work with the experimental community to ensure that we are calculating quantities which correspond to the experimental measurements.

3(a) - Long-Distance Effects - The $K_L - K_S$ Mass Difference

N.H.Christ, T.Izubuchi, CTS, A.Soni & J.Yu (RBC-UKQCD), arXiv:1212.5931
Z.Bai, N.H.Christ, T.Izubuchi, CTS, A.Soni & J.Yu (RBC-UKQCD), arXiv:1406.0916
Z.Bai (RBC-UKQCD), arXiv:1411.3210

$$\boxed{\Delta m_K \equiv m_{K_L} - m_{K_S} = 3.483(6) \times 10^{-12}\,\text{MeV}.}$$

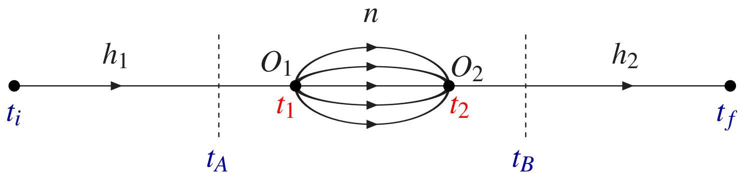
- Historically led to the prediction of the energy scale of the charm quark.

  Mohapatra, Rao & Marshak (1968); GIM (1970); Gaillard & Lee (1974)
- Tiny quantity $\Rightarrow$ places strong constraints on BSM Physics.
- Within the standard model, Δm_K arises from K^0–$\bar{K}^0$ mixing at second order in the weak interactions:

$$\boxed{\Delta m_K = 2\mathcal{P} \sum_\alpha \frac{\langle \bar{K}^0 \,|\, H_W \,|\alpha\rangle \, \langle\alpha \,| H_W \,|\, K^0\rangle}{m_K - E_\alpha}\,,}$$

  where the sum over $|\alpha\rangle$ includes an energy-momentum integral.
- I will use Δm_K to illustrate generic features present in the evaluation of long-distance effects.

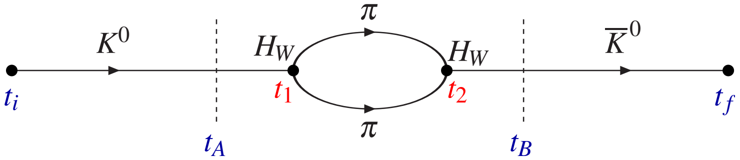
The fiducial volume



- How do you prepare the states $h_{1,2}$ in the generic integrated correlation function:

$$\int d^4x \int d^4y \, \langle h_2 \,|\, T\{O_1(x)\, O_2(y)\} \,|\, h_1 \rangle \,,$$

  when the time of the operators is integrated?
- The practical solution is to integrate over a large subinterval in time $t_A \leq t_{x,y} \leq t_B$, but to create h_1 and to annihilate h_2 well outside of this region.
- This is the natural modification of standard field theory for which the asymptotic states are prepared at $t \to \pm\infty$ and then the operators are integrated over all time.
- This approach has been successfully implemented in the projects which will be reviewed by Xu Feng.



- Δm_K is given by

$$\Delta m_K \equiv m_{K_L} - m_{K_S} = 2\mathcal{P}\sum_\alpha \frac{\langle \bar{K}^0\,|\,\mathcal{H}_W\,|\,\alpha\rangle\,\langle\alpha\,|\,\mathcal{H}_W\,|\,K^0\rangle}{m_K - E_\alpha} = 3.483(6)\times 10^{-12}\,\mathrm{MeV}.$$

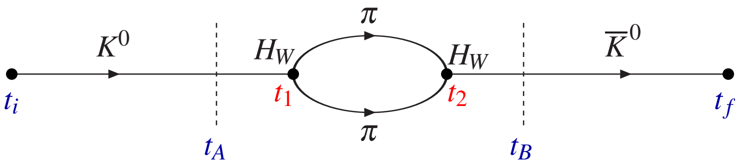
- The above correlation function gives ($T = t_B - t_A + 1$)

$$C_4(t_A,t_B;t_i,t_f) \;=\; |Z_K|^2 e^{-m_K(t_f-t_i)}\sum_n \frac{\langle \bar{K}^0\,|\,\mathcal{H}_W\,|\,n\rangle\,\langle n\,|\,\mathcal{H}_W\,|\,K^0\rangle}{(m_K-E_n)^2}\times \left\{e^{(M_K-E_n)T} - (m_K-E_n)T - 1\right\}.$$

- From the coefficient of T we can therefore obtain

$$\Delta m_K^{\rm FV} \equiv 2\sum_n \frac{\langle \bar{K}^0\,|\,\mathcal{H}_W\,|\,n\rangle\,\langle n\,|\,\mathcal{H}_W\,|\,K^0\rangle}{(m_K-E_n)}\,.$$

Exponentially growing exponentials



$$C_4(t_A,t_B;t_i,t_f) \;=\; |Z_K|^2 e^{-m_K(t_f-t_i)} \sum_n \frac{\langle \bar{K}^0\,|\,\mathcal{H}_W\,|\,n\rangle\,\langle n\,|\,\mathcal{H}_W\,|\,K^0\rangle}{(m_K-E_n)^2} \times \left\{ e^{(M_K-E_n)T} - (m_K-E_n)T - 1 \right\}.$$

- The presence of terms which (potentially) grow exponentially in T is a generic feature of calculations of matrix elements of bilocal operators.

Ultraviolet divergences in the calculation of Δm_K

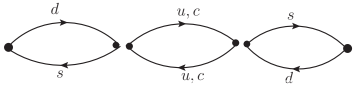
- The $\Delta S = 1$ effective Weak Hamiltonian takes the form:

$$H_W = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} V_{qd} V^*_{q's} (C_1 Q_1^{qq'} + C_2 Q_2^{qq'})$$

where the $\{Q_i^{qq'}\}_{i=1,2}$ are current-current operators, defined as:

$$\begin{aligned} Q_1^{qq'} &= (\bar{s}_i \gamma^\mu (1-\gamma^5) d_i)\,(\bar{q}_j \gamma^\mu (1-\gamma^5) q'_j) \\ Q_2^{qq'} &= (\bar{s}_i \gamma^\mu (1-\gamma^5) d_j)\,(\bar{q}_j \gamma^\mu (1-\gamma^5) q'_i)\,. \end{aligned}$$

- As the two H_W approach each other, we have the potential of new ultraviolet divergences.
  - Taking the u-quark component of the operators $\Rightarrow$ a quadratic divergence.



  - GIM mechanism & $V-A$ nature of the currents $\Rightarrow$ elimination of both quadratic and logarithmic divergences.
  - This is not the case for ϵ_K or for $K \to \pi\nu\bar{\nu}$ rare kaons decays.

Complete calculation of Δm_K

Z.Bai, N.H.Christ, T.Izubuchi, CTS, A.Soni & J.Yu, arXiv:1406.0916

- We reported on a full calculation of Δm_K on a $24^3 \times 64 \times 16$ lattice (with DWF and the Iwasaki gauge action), $m_\pi = 330$ MeV, $m_K = 575$ MeV, $m_c^{\overline{\rm MS}}(2\,\text{GeV}) = 949$ MeV, $(1/a = 1.729(28)$ GeV and $am_{\rm res} = 0.00308(4))$.
- At these unphysical parameters we find

$$\Delta m_K = 3.19(41)(96) \times 10^{-12}\,\text{MeV}\,,$$

to be compared to the physical value 3.483(6)$\times 10^{-12}$ MeV.

  - Agreement with physical value may well be fortuitous, but it is nevertheless reassuring to obtain results of the correct order.
  - Systematic error dominated by discretization effects related to the charm quark mass, which we estimate at 30%.
  - Here $m_K < 2m_\pi$ and so we do not have exponentially growing two-pion terms.
- Ziyuan Bai later reported on an exploratory calculation with $m_\pi = 171$ MeV, $m_K = 492$ MeV and with two unphysical c-quark masses aimed at studying the contribution from two-pion intermediate states.
  - Two-pion contribution to Δm_K is very small and the corresponding finite-volume corrections were negligible. Z.Bai, arXiv:1411.3210

Δm_K and ϵ_K

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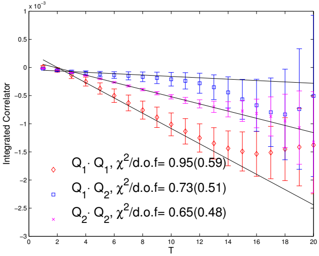
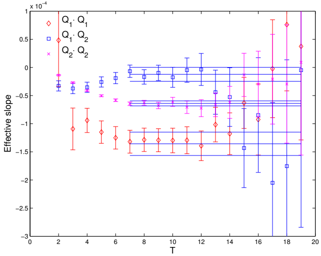
- Examples of the slopes for Δm_K Z.Bai et al., arXiv:1406.0916





- At lattice 2016 this July, Z.Bai presented his latest exploratory studies of the long-distance contributions to ϵ_K, albeit at unphysical masses. He showed that he was able to perform the renormalization with the bilinear operators.
- Conclusion - We now understand how to perform these calculations, which will be performed on the next generation of machines - timescale O(2-3 years).

3(b) Rare Kaon Decays

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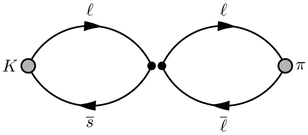
Talk by X.Feng

- The FCNC rare-kaon decays $K \to \pi\ell^+\ell^-$ or $K \to \pi\nu\bar\nu$ are, as we all know, particularly important in tests of the Standard Model and hence in signatures of New Physics.
- The evaluation of the long-distance contributions to the amplitudes for rare kaon decays $K \to \pi\ell^+\ell^-$ or $K \to \pi\nu\bar\nu$ also requires the evaluation of matrix elements of the form
$$\int\!\!\int d^4x\, d^4y\, \langle f\,|\,T[Q_1(x)\,Q_2(y)]\,|\,i\rangle\,.$$
- The general features we have seen for Δm_K and ϵ_K are also present here:
  - The requirement to define a fiducial volume.
  - The presence of exponentially growing terms when there are intermediate states lighter than the kaon.
  - The need, in general, to control the additional UV divergences as Q_1 and Q_2 approach each other.

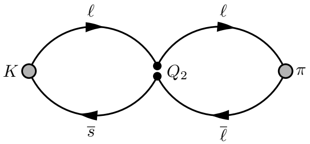
$$T_i^\mu = \int d^4x\, e^{-iq\cdot x}\,\langle \pi(p)\,|\,\mathrm{T}\{J^\mu_{\mathrm{em}}(x)\,Q_i(0)\,\}\,|\,K(k)\rangle\,,$$
$$= \frac{\omega_i(q^2)}{(4\pi)^2}\,\left\{q^2(p+k)^\mu - (m_K^2 - m_\pi^2)\,q^\mu\right\}\,.$$

- The CP-conserving decays $K^+ \to \pi^+\ell^+\ell^-$ and $K_S \to \pi^0\ell^+\ell^-$ are dominated by long-distance hadronic effects induced by single photon exchange.
- Lattice QCD results can be compared with experimental data and with ChPT-based phenomenological results.
- Results for K_S decays $\Rightarrow$ evaluation of the significant interference between direct and indirect CP-violation in $K_L \to \pi^-\ell^+\ell^-$ decays.
- The theoretical framework for computing $K^+ \to \pi^+\ell^+\ell^-$ and $K_S \to \pi^0\ell^+\ell^-$ decay amplitudes was presented in N.H.Christ, X.Feng, A.Portelli and CTS, arXiv:1507.03094
- If the conserved electromagnetic current is used for J_{em} and we work in the four-flavour theory then GIM $\Rightarrow$ that there are no additional divergences as J_{em} and Q_i approach each other. G.Isidori, G.Martinelli and P.Turchetti, hep-lat/0506026
  - This is not possible for $K \to \pi\nu\bar{\nu}$ decays. **Talk by X.Feng**
- This framework was very recently used to perform the first exploratory numerical calculations for $K^+ \to \pi^+\ell^+\ell^-$ decays. N.H.Christ et al., arXiv:1608.07585

- For example for K^+ decays we need to evaluate the diagrams obtained by inserting the current at all possible locations in the three point function (and adding the disconnected diagrams):



W



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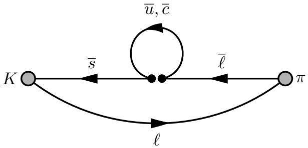

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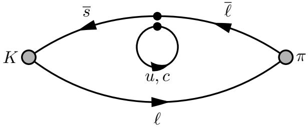


E

- W=Wing, C=Connected, S=Saucer, E=Eye.
- For K_S decays there is an additional topology with a gluonic intermediate state.

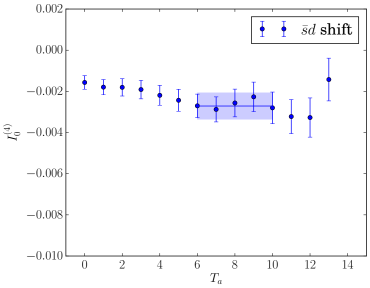
Exploratory numerical study

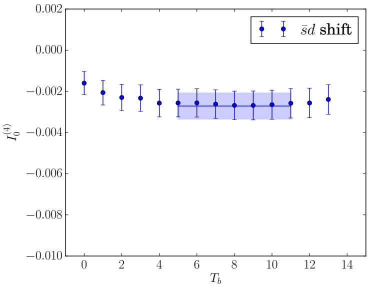
N.Christ, X.Feng, A.Jüttner, A.Lawson, A.Portelli and CTS, arXiv:1608.07585

- The numerical study is performed on the $24^3 \times 64$ DWF+Iwasaki RBC-UKQCD ensembles with $m_\pi \simeq 430$ MeV, $m_K \simeq 625$ MeV, $m_c^{\overline{\text{MS}}}(2\,\text{GeV}) \simeq 530$ MeV, $a^{-1} \simeq 1.73$ fm.
- 128 configurations were used with $\vec{k} = \vec{0}$ and $\vec{p}$ =(1,0,0), (1,1,0) and (1,1,1) in units of $2\pi/L$. (The (1,1,1) case is still being completed.)
- With this kinematics we are in the unphysical region, $q^2 < 0$.
- The charm quark is also lighter than physical .
- The calculation is performed using the conserved vector current (5-dimensional).

Method 2 for $\vec{p} = (1,0,0)$

N.Christ, X.Feng, A.Jüttner, A.Lawson, A.Portelli and CTS, arXiv:1608.07585





$$A_0(q^2) = -0.0027(6).$$

$K \to \pi\nu\bar{\nu}$ Decays

N.H.Christ, X.Feng, A.Portelli and CTS, arXiv:1605.04442

- I don't need to mention at this meeting that these FCNC processes provide ideal probes for the observation of new physics effects.
- The dominant contributions from the top quark $\Rightarrow$ they are also very sensitive to V_{ts} and V_{td}.
- Experimental results and bounds:

$$\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{exp}} = 1.73^{+1.15}_{-1.05} \times 10^{-10}$$

A.Artamonov et al. (E949), arXiv:0808.2459

$$\mathrm{Br}(K_L \to \pi^0\nu\bar{\nu}) \leq 2.6 \times 10^{-8} \ \text{at 90\% confidence level},$$

J.Ahn et al. (E291a), arXiv:0911.4789

- Sample recent theoretical predictions:

$$\begin{aligned}\mathrm{Br}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{SM}} &= (9.11 \pm 0.72) \times 10^{-11} \\ \mathrm{Br}(K_L \to \pi^0\nu\bar{\nu})_{\mathrm{SM}} &= (3.00 \pm 0.30) \times 10^{-11},\end{aligned}$$

A.Buras, D.Buttazzo, J.Girrbach-Noe, R.Knejgens, arXiv:1503.02693

- To what extent can lattice calculations reduce the theoretical uncertainty?

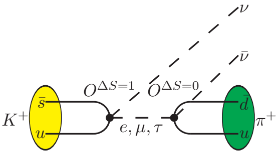
To what extent can lattice calculations reduce the uncertainty?

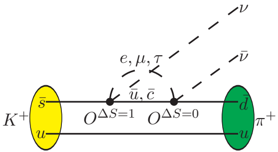
- $K \to \pi\nu\bar{\nu}$ decays are SD dominated and the hadronic effects can be determined from CC semileptonic decays such as $K^+ \to \pi^0 e^+ \nu$.
- LD contributions, i.e. contributions from distances greater than $1/m_c$ are negligible for K_L decays and are expected to be $\leq 5\%$ for for K^+ decays.
  - K_L decays are therefore one of the cleanest places to search for the effects of new physics.
  - The aim of our study is to compute the LD effects in K^+ decays.
    These provide a significant, if probably still subdominant, contribution to the theoretical uncertainty (which is dominated by the uncertainties in CKM matrix elements).
  - A phenomenological estimate of the long distance effects, estimated these to enhance the branching fraction by 6% with an uncertainty of 3%.

G.Isidori, F.Mescia and C.Smith, hep-ph/0503107

- Lattice QCD can provide a first-principles determination of the LD contribution with controlled errors.
  - Given the NA62 experiment, it is timely to perform a lattice QCD calculation of these effects.

- For this doubly weak decay there are a number of novel diagrams to evaluate:

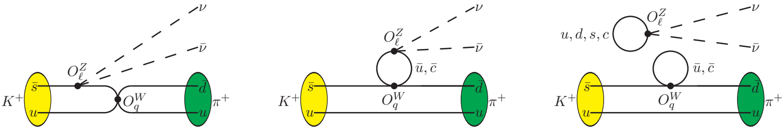




WW-diagrams

$$\mathcal{H}_{\rm eff}^{\rm LO} = \frac{G_F}{\sqrt{2}} \sum_{q,\ell} \left(V_{qs}^* O_{q\ell}^{\Delta S=1} + V_{qd} O_{q\ell}^{\Delta S=0} \right) + \frac{G_F}{\sqrt{2}} \sum_q \lambda_q O_q^W + \frac{G_F}{\sqrt{2}} \sum_\ell O_\ell^Z \,,$$

$$\begin{aligned} O_{q\ell}^{\Delta S=1} &= C_{\Delta S=1}^{\overline{\rm MS}}(\mu)\,\left[(\bar{s}q)_{V-A}\,(\bar{\nu}_\ell \ell)_{V-A}\right]^{\overline{\rm MS}}(\mu), \\ O_{q\ell}^{\Delta S=0} &= C_{\Delta S=0}^{\overline{\rm MS}}(\mu)\,\left[(\bar{\ell}\nu_\ell)_{V-A}\,(\bar{q}d)_{V-A}\right]^{\overline{\rm MS}}(\mu). \end{aligned}$$



Z-exchange diagrams

$$\mathcal{H}_{\rm eff}^{\rm LO} = \frac{G_F}{\sqrt{2}} \sum_{q,\ell} \left(V_{qs}^* O_{q\ell}^{\Delta S=1} + V_{qd} O_{q\ell}^{\Delta S=0} \right) + \frac{G_F}{\sqrt{2}} \sum_q \lambda_q O_q^W + \frac{G_F}{\sqrt{2}} \sum_\ell O_\ell^Z \,,$$

$$\begin{aligned} O_q^W &= C_1^{\overline{\rm MS}}(\mu)\, Q_{1,q}^{\overline{\rm MS}}(\mu) + C_2^{\overline{\rm MS}}(\mu)\, Q_{2,q}^{\overline{\rm MS}}(\mu), \\ O_\ell^Z &= C_Z^{\overline{\rm MS}}(\mu) \left[J_\mu^Z \, \bar{\nu}_\ell \gamma^\mu (1-\gamma_5) \nu_\ell \right]^{\overline{\rm MS}} (\mu) \end{aligned}$$

N.H.Christ, X.Feng, A.Portelli and CTS, arXiv:1605.04442

- The general issues encountered in computing long-distance effects (additional ultra-violet divergences, subtraction or suppression of growing unphysical exponential terms and FV effects which fall as powers of the volume) must also be dealt with here.
- An important element of this paper is a detailed explanation of how to handle the additional ultra-violet divergences, eliminating the need to perform perturbation theory at scales of $O(m_c)$.
- An exploratory study of $K^+ \to \pi^+\nu\bar\nu$ decays is also underway and the parameters and preliminary results will be presented by Xu Feng in his talk.

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Talk by N.Garron

- In 2015 RBC-UKQCD published our first result for ϵ'/ϵ computed at physical quark masses and kinematics, albeit still with large errors:

$$\left.\frac{\epsilon'}{\epsilon}\right|_{\text{RBC-UKQCD}} = (1.38 \pm 5.15 \pm 4.59) \times 10^{-4}$$

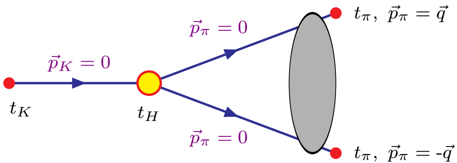
to be compared with

$$\left.\frac{\epsilon'}{\epsilon}\right|_{\text{Exp}} = (16.6 \pm 2.3) \times 10^{-4}\,.$$

RBC-UKQCD, arXiv:1505.07863

- This is by far the most complicated project that I have ever been involved with.
- This single result hides much important (and much more precise) information which we have determined along the way.
- In this section I will review the main obstacles to computing $K \to \pi\pi$ decay amplitudes, the techniques used to overcome them and our main results.

The Maiani-Testa Theorem



- $K \to \pi\pi$ correlation function is dominated by lightest state, e.g. for $I = 2$ with $\vec{p}_K = 0$ this is the state with two-pions at rest. Maiani and Testa, PL B245 (1990) 585

$$C(t_\pi) = A_1 e^{-2m_\pi t_\pi} + A_2 e^{-2E_\pi t_\pi} + \cdots$$

(For $I = 0$ there is also a constant term A_0 on the right-hand side.)

  - Solution 1: Study an excited state. Lellouch and Lüscher, hep-lat/0003023
  - Solution 2: Introduce suitable boundary conditions such that the $\pi\pi$ ground state is $|\pi(\vec{q})\pi(-\vec{q})\rangle$. RBC-UKQCD, C.h.Kim hep-lat/0311003

(For B-decays, with so many intermediate states below threshold, this is the main obstacle to producing reliable calculations.)

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- Requiring that the $E^0_{\pi\pi} = m_K \Rightarrow$ the volume must be tuned accordingly.
  non-trivial

- Moreover - since the two-pion potential is attractive in the $I = 0$ channel and repulsive in the $I = 2$ channel, on a given volume

$$E^{0,\,I=0}_{\pi\pi} < E^{0,\,I=2}_{\pi\pi}$$

  and the tuning has to be done separately in each channel.

- For the evaluation of A_2, it is sufficient to impose antiperiodic boundary conditions for the d-quark.
  - Isospin breaking by the boundary conditions is harmless here.

CTS & G.Villadoro, hep-lat/0411033

- For A_0 this is not possible and we have had to develop the implementation of G-parity boundary conditions in which $(u, d) \to (\bar{d}, -\bar{u})$ at the boundary .

U. Wiese, Nucl.Phys. B375 (1992) 45 , RBC-UKQCD, C.h.Kim hep-lat/0311003

  - This has been the key development making the calculation of A_0 possible.

Results for A_2

- Our first results for A_2 at physical kinematics were obtained at a single, rather coarse, value of the lattice spacing ($a \simeq 0.14$ fm). Estimated discretization errors at 15%. arXiv:1111.1699, arXiv:1206.5142
- Our recent results were obtained on two new ensembles, 48^3 with $a \simeq 0.11$ fm and 64^3 with $a \simeq 0.084$ fm so that we can make a continuum extrapolation:

$$\begin{aligned} \mathrm{Re}(A_2) &= 1.50(4)_{\mathrm{stat}}(14)_{\mathrm{syst}} \times 10^{-8}\ \mathrm{GeV}. \\ \mathrm{Im}(A_2) &= -6.99(20)_{\mathrm{stat}}(84)_{\mathrm{syst}} \times 10^{-13}\ \mathrm{GeV}\,. \end{aligned}$$

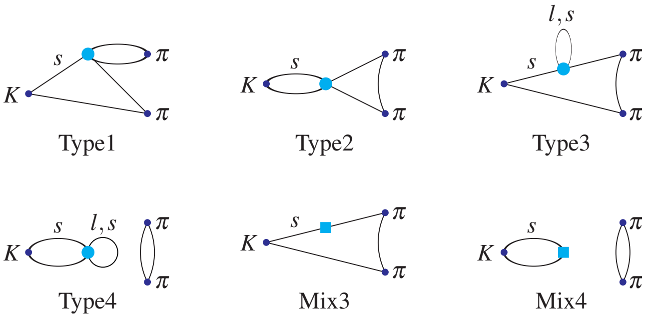
arXiv:1502.00263

- (The experimental result is $\mathrm{Re}(A_2) = 1.4787(31) \times 10^{-8}$. $\mathrm{Im}(A_2)$ is unknown.)
- Although the precision can still be significantly improved (partly by perturbative calculations), the calculation of A_2 at physical kinematics can now be considered as standard.

Calculation of A_0

RBC-UKQCD Collaboration, arXiv:1505.07863

- The calculation is much more difficult for the $K \to (\pi\pi)_{I=0}$ amplitude A_0:
  - G-parity boundary conditions, disconnected diagrams, vacuum subtraction, ultra-violet power divergences, $\cdots$



  - $|\pi^+(\pi/L)\pi^-(\text{-}\pi/L)\rangle$ has a different energy from $|\pi^0(\vec{0})\pi^0(\vec{0})\rangle$.
  - We have developed the implementation of G-parity boundary conditions in which $(u,d) \to (\bar{d}, -\bar{u})$ at the boundary .

U. Wiese, Nucl.Phys. B375 (1992) 45 , RBC-UKQCD, C.h.Kim hep-lat/0311003

RBC-UKQCD Collaboration, arXiv:1505.07863

- Computations were performed on a $32^3 \times 64$ lattice with the Iwasaki and DSDR gauge action and $N_f = 2+1$ flavours of Möbius Domain Wall Fermions

$$a^{-1} = 1.379(7)\,\mathrm{GeV},\, m_\pi = 143.2(2.0)\,\mathrm{MeV},\, (E_\pi = 274.8(1.4)\,\mathrm{MeV})$$

- The $\pi\pi$ energies are

$$E_{\pi\pi}^{I=0} = (498 \pm 11)\,\mathrm{MeV} \quad E_{\pi\pi}^{I=2} = (565.7 \pm 1.0)\,\mathrm{MeV}$$

to be compared with $m_K = (490.6 \pm 2.4)\,\mathrm{MeV}$.

- Results:

$$\begin{aligned} \mathrm{Re}(A_0) &= 4.66(1.00)(1.26) \times 10^{-7}\,\mathrm{GeV} \\ \mathrm{Im}(A_0) &= -1.90(1.23)(1.08) \times 10^{-11}\,\mathrm{GeV} \\ \mathrm{Re}\frac{\epsilon'}{\epsilon} &= 1.38(5.15)(4.59) \times 10^{-4}, \end{aligned}$$

to be compared to the experimental results Re$(A_0) = 3.3201(18) \times 10^{-7}$ GeV and Re$(\epsilon'/\epsilon) = 16.6(2.3) \times 10^{-4}$.

Two features from the calculations

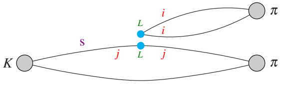
- Lüscher's quantisation condition $\Rightarrow E_{\pi\pi}^{I=0}$ corresponds to $\delta_0(m_K) = (23.8 \pm 4.9 \pm 1.2)^\circ$, which is somewhat smaller than phenomenological expectations.
  - For $I = 2$ are results are in line with expectations. arXiv:1502.00263

Two features from the calculations (cont)

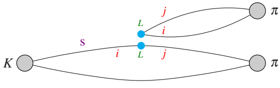
- Re A_2 is dominated by a simple operator:

$$O^{3/2}_{(27,1)} = (\bar{s}^i d^i)_L \left\{ (\bar{u}^j u^j)_L - (\bar{d}^j d^j)_L \right\} + (\bar{s}^i u^i)_L \, (\bar{u}^j d^j)_L$$

and two diagrams:

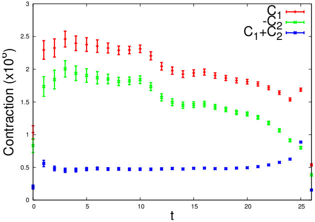


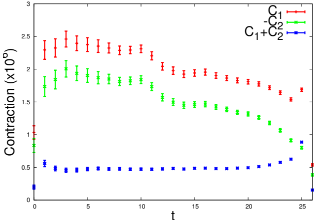
C_1



C_2

- Re A_2 is proportional to $C_1 + C_2$.
- The two dominant contributions to A_2 have opposite signs $\Rightarrow$ significant cancellation $\Rightarrow$ major contribution to the $\Delta I = 1/2$ rule. arXiv:1212.1474
  This is confirmed on our latest computation of A_2 arXiv:1502.00263





Conclusions

- For "standard" quantities such as f_K/f_π, B_K or $f^+(0)$, the precision of lattice calculations is now $O(1\%)$ or better. FLAG collaboration, arXiv:1607.00299
- To push *precision flavour physics* still further, therefore requires control of:
  - IB effects, including electromagnetic corrections; **Talk by Guido Martinelli**
  - long-distance contributions. **Talk by Xu Feng**
- In $K \to \pi\pi$ decays **Talk by Nicolas Garron**
  - as a result of our work, the computation of A_2 is now fully controlled (and becoming "standard");
  - the $\Delta I = 1/2$ rule has a number of components, of which the significant cancelation between the two dominant contributions to ReA_2 is a major one.
  - We have completed the first calculation of ϵ'/ϵ with controlled errors $\Rightarrow$ motivation for further refinement (systematic improvement by collecting more statistics, working on larger volumes, $\geq$2 lattice spacings etc.)
  - ϵ'/ϵ is now a quantity which is amenable to lattice computations.