

LHCb searches for the strong CP-violating decays $\eta \rightarrow \pi\pi$ and $\eta' \rightarrow \pi\pi$

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on behalf of the LHCb collaboration

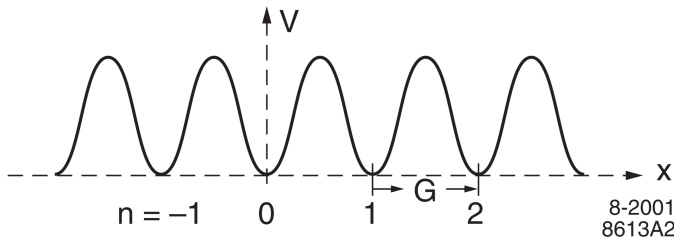


KAON
14/09/2016

The structure of the θ -vacuum

The strong CP problem arises from the structure of the vacuum in QCD

- Quantised non-Abelian Yang-Mills gauge theory
- Distinct states of zero field energy exist, connected by local gauge transformations G
- Discrete infinity of vacuum states $|n\rangle$, indexed by the winding number of the transformation $n \in \mathbb{Z}$
- Gauge invariant superposition of n -vacua: $|\theta\rangle = \sum_n e^{in\theta} |n\rangle$



[Phys. Lett. B63 (1976) 334-340]

[Phys. Rev. Lett. 37 (1976) 172]

The strong CP problem

This structure has as a consequence the presence of a natural CP-violating term in the QCD lagrangian (" θ term"):

$$\mathcal{L}_\theta = \theta \frac{g_s^2}{64\pi^2} \epsilon_{\mu\nu\alpha\beta} G_a^{\mu\nu} G_a^{\alpha\beta}$$

but no CP violation is observed in strong processes

- Observable phase is $\theta_{QCD} = \theta - \theta'$, where θ' is the chiral phase of the quarks mass matrix
- $\theta_{QCD} < 10^{-9}$ from n EDM measurements [Atom. Nuclei (2007) 70: 349]
- Fine tuning problem

Possible solutions to the strong CP problem include:

- Peccei-Quinn axions [Phys. Rev. Lett. 38 (1977) 1440]
- Extra space-time dimensions [Phys. Lett. B203 (1988) 121]
- Massless up quark [Phys. Rev. Lett. 90 (2003) 021601]
- String theory [arXiv:hep-ph/9307214]
- Quantum gravity [Phys. Rev. D47 (1993) 5565]

The decays $\eta^{(\prime)} \rightarrow \pi^+ \pi^-$

The decays $\eta \rightarrow \pi^+ \pi^-$ and $\eta' \rightarrow \pi^+ \pi^-$ would violate CP in the Standard Model via weak interaction (through mediation by a virtual K_s^0): ¹

- $\mathcal{B}(\eta \rightarrow \pi^+ \pi^-) < 2 \times 10^{-27}$
- $\mathcal{B}(\eta' \rightarrow \pi^+ \pi^-) < 4 \times 10^{-29}$

Via the strong interaction, based on θ_{QCD} from $nEDM$ measurements: ²

- $\mathcal{B}(\eta^{(\prime)} \rightarrow \pi^+ \pi^-) < 3 \times 10^{-17}$

Current world best limits:

- $\mathcal{B}(\eta \rightarrow \pi^+ \pi^-) < 1.3 \times 10^{-5}$ at 90% CL (KLOE, $\phi(1020) \rightarrow \eta\gamma$)³
- $\mathcal{B}(\eta' \rightarrow \pi^+ \pi^-) < 5.5 \times 10^{-5}$ at 90% CL (BESIII, $J/\psi \rightarrow \gamma\pi^+\pi^-$)⁴

¹[Phys. Rev. D52 (1995) 248]

²*Idem*

³[Phys. Lett. B606 (2005) 276]

⁴[Phys. Rev. D84 (2011) 032006]

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Search strategy at LHCb: exploit the large sample of prompt charm mesons to search for $\eta^{(\prime)} \rightarrow \pi^+ \pi^-$ in $D_{(s)}^+ \rightarrow \pi^+ \pi^+ \pi^-$ as narrow peaks in the inclusive $\pi^+ \pi^-$ mass spectra

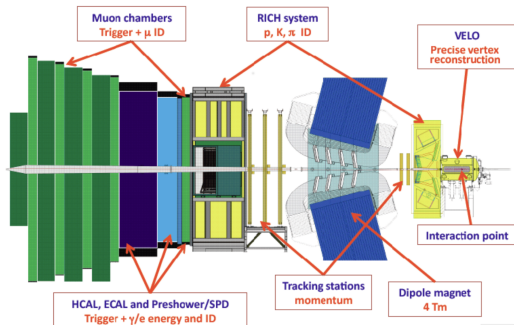
¹[Phys. Rev. D52 (1995) 248]

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- Single arm spectrometer designed for high precision flavour physics measurements
- Pseudorapidity range $\eta \in [2, 5]$
- Very good particle identification
- Good momentum and IP resolution
- Excellent primary and secondary vertices reconstruction



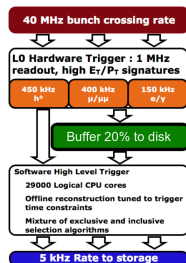
Datasets and trigger

Total integrated luminosity:

Run 1

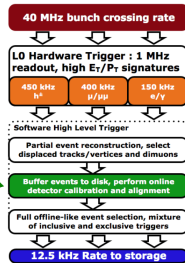
1 fb⁻¹ at $\sqrt{s} = 7$ TeV (2011)

2 fb⁻¹ at $\sqrt{s} = 8$ TeV (2012)



Run 2

0.3 fb⁻¹ at $\sqrt{s} = 13$ TeV (2015)



Run 2 trigger system:

- Real time alignment and calibration
- Turbo stream: online reconstruction of candidates with offline quality
- No need for additional offline reconstruction

$$\mathcal{B}(\eta^{(\prime)} \rightarrow \pi^+ \pi^-) = \frac{N(\eta^{(\prime)})}{N(D_{(s)}^+ \rightarrow \pi^+ \pi^+ \pi^-)} \times \frac{\mathcal{B}(D_{(s)}^+ \rightarrow \pi^+ \pi^+ \pi^-)}{\mathcal{B}(D_{(s)}^+ \rightarrow \pi^+ \eta^{(\prime)})} \times \frac{1}{\epsilon_{D_{(s)}^+}(\eta^{(\prime)})}$$

$$\mathcal{B}(\eta^{(\prime)} \rightarrow \pi^+ \pi^-) = \frac{N(\eta^{(\prime)})}{N(D_{(s)}^+ \rightarrow \pi^+ \pi^+ \pi^-)} \times \frac{\mathcal{B}(D_{(s)}^+ \rightarrow \pi^+ \pi^+ \pi^-)}{\mathcal{B}(D_{(s)}^+ \rightarrow \pi^+ \eta^{(\prime)})} \times \frac{1}{\epsilon_{D_{(s)}^+}(\eta^{(\prime)})}$$

- $N(\eta^{(\prime)})$ is the number of $\eta^{(\prime)} \rightarrow \pi^+ \pi^-$ candidates in the $\pi^+ \pi^-$ mass spectrum

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- $N(D_{(s)}^+ \rightarrow \pi^+ \pi^+ \pi^-)$ is the fitted number of $D_{(s)}^+ \rightarrow \pi^+ \pi^+ \pi^-$ candidates, after a multivariate selection, in an optimised mass window

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- **BFs** are taken from the Particle Data Group

$$\mathcal{B}(\eta^{(\prime)} \rightarrow \pi^+\pi^-) = \frac{N(\eta^{(\prime)})}{N(D_{(s)}^+ \rightarrow \pi^+\pi^+\pi^-)} \times \frac{\mathcal{B}(D_{(s)}^+ \rightarrow \pi^+\pi^+\pi^-)}{\mathcal{B}(D_{(s)}^+ \rightarrow \pi^+\eta^{(\prime)})} \times \frac{1}{\epsilon_{D_{(s)}^+}(\eta^{(\prime)})}$$

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- BF's are taken from the Particle Data Group
- $\epsilon_{D_{(s)}^+}(\eta^{(\prime)})$ is a small correction to the relative efficiency as a function of the $\pi^+\pi^-$ mass

$$\mathcal{B}(\eta^{(\prime)} \rightarrow \pi^+\pi^-) = \frac{N(\eta^{(\prime)})}{N(D_{(s)}^+ \rightarrow \pi^+\pi^+\pi^-)} \times \frac{\mathcal{B}(D_{(s)}^+ \rightarrow \pi^+\pi^+\pi^-)}{\mathcal{B}(D_{(s)}^+ \rightarrow \pi^+\eta^{(\prime)})} \times \frac{1}{\epsilon_{D_{(s)}^+}(\eta^{(\prime)})}$$

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- BFs are taken from the Particle Data Group
- $\epsilon_{D_{(s)}^+}(\eta^{(\prime)})$ is a small correction to the relative efficiency as a function of the $\pi^+\pi^-$ mass
- CL_s method used to calculate limits on $\mathcal{B}(\eta^{(\prime)} \rightarrow \pi^+\pi^-)$

Candidates $D_{(s)}^+ \rightarrow \pi^+ \pi^+ \pi^-$ are required to satisfy loose selection criteria:

- Three good quality, high- p_T tracks coming from a displaced vertex
- Each track consistent with pion hypothesis
- Three pions invariant mass in the range 1820 – 2020 MeV/ c^2
- Opposite-sign pion pairs invariant mass in the range 300 – 1650 MeV/ c^2
- $\sim 20 \times 10^6$ candidates each for D^+ and D_s^+ from Run 1
- $\sim 7 \times 10^6$ candidates each for D^+ and D_s^+ from Run 2

A Boosted Decision Tree is used to further suppress background

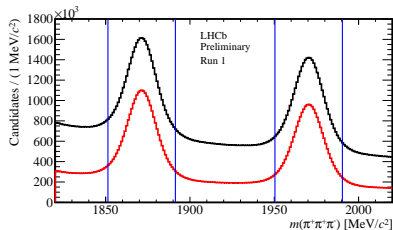
- 21 input variables: track fit quality, particle identification probabilities, decay vertex fit quality

Optimisation of BDT

[LHCb-PAPER-2016-046]

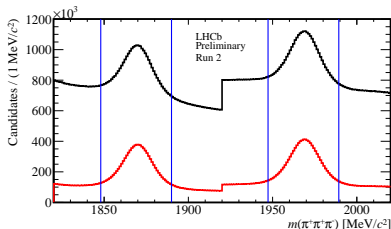
In order to maximise the sensitivity, the optimum cut values on the BDT response and on the mass window are chosen to maximise the statistical significance:

$$FoM = \frac{N_S}{\sqrt{N_S + N_B}}$$



Run 1

- For D^+ :
 - Mass window 20 MeV/c^2
- For D_s^+ :
 - Mass window 20 MeV/c^2



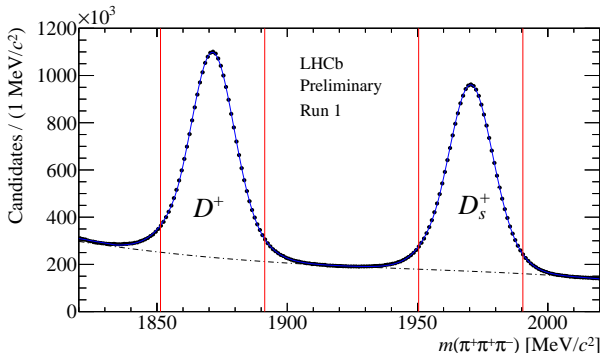
Run 2

- For D^+ :
 - Mass window 21 MeV/c^2
- For D_s^+ :
 - Mass window 21 MeV/c^2

Fits to mass spectra after BDT

[LHCb-PAPER-2016-046]

- Background model: 4th order Chebyshev polynomial, plus six shapes accounting for $D_s^+ \rightarrow K^+\pi^+\pi^-$, $D_s^+ \rightarrow \pi^+\pi^+\pi^-\pi^0$ and $D_{(s)}^+ \rightarrow (\eta^{(\prime)} \rightarrow \pi^+\pi^-\gamma)\pi^+$
- Signal model: sum of Gaussian and double sided Crystal Ball

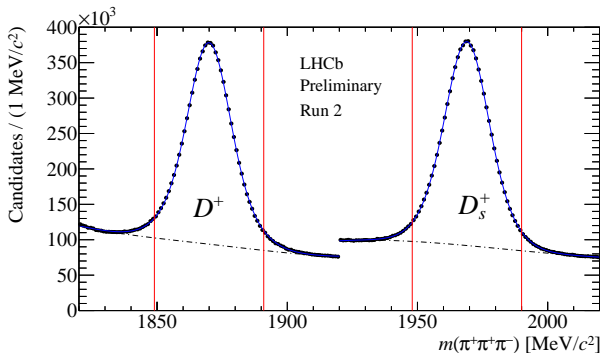


- For the D^+ window:
 - Signal: 1.88×10^7
 - Background: 9.77×10^6
- For the D_s^+ window:
 - Signal: 1.75×10^7
 - Background: 6.92×10^6

Fits to mass spectra after BDT

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- Signal model: sum of Gaussian and double sided Crystal Ball



- For the D^+ window:
 - Signal: 6.09×10^6
 - Background: 4.04×10^6
- For the D_s^+ window:
 - Signal: 6.26×10^6
 - Background: 3.90×10^6

The analysis starts from a known number of $D_{(s)}^+$ candidates

- No normalisation channel
- No absolute efficiency correction needed
- Relative efficiency correction: variations of the efficiency from a uniform distribution in the $m(\pi^+\pi^-)$ spectrum
- The ϵ_{rel} factors are included in the limit setting calculation

Run 1

- From Monte Carlo samples
- $\epsilon_{rel}^{D^+}(\eta) = 0.85 \pm 0.01$
- $\epsilon_{rel}^{D^+}(\eta') = 1.01 \pm 0.01$
- $\epsilon_{rel}^{D_s^+}(\eta) = 0.80 \pm 0.01$
- $\epsilon_{rel}^{D_s^+}(\eta') = 1.03 \pm 0.01$

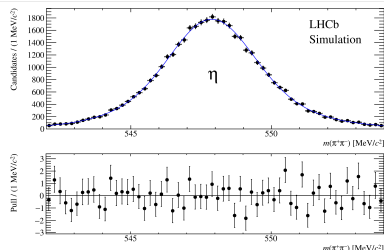
Run 2

- Run 1 data / Run 2 data
- $\epsilon_{rel}^{D^+}(\eta) = 0.87 \pm 0.01$
- $\epsilon_{rel}^{D^+}(\eta') = 1.02 \pm 0.01$
- $\epsilon_{rel}^{D_s^+}(\eta) = 0.84 \pm 0.01$
- $\epsilon_{rel}^{D_s^+}(\eta') = 1.00 \pm 0.01$

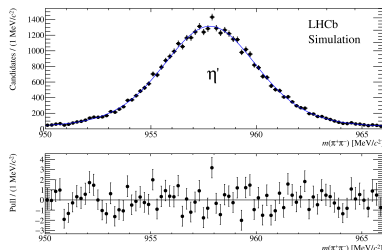
[LHCb-PAPER-2016-046]

Signal PDF for the CL_s method: double Gaussian PDF extracted from $D^+ \rightarrow (\eta^{(\prime)} \rightarrow \pi^+\pi^-)\pi^+$ Monte Carlo

- Resolution for $\eta^{(\prime)}$ cross-checked using $K_s^0 \rightarrow \pi^+\pi^-$ from data
- Results are consistent, with the difference assigned as systematic error



$$\sigma = 2.2 \pm 0.1 \text{ MeV}/c^2$$



$$\sigma = 2.7 \pm 0.1 \text{ MeV}/c^2$$

[LHCb-PAPER-2016-046]

BF limits are dominated by the statistical errors of the $\pi^+\pi^-$ mass spectra.
Sources of systematic uncertainty considered:

- Errors on the branching fractions, taken from the PDG
- Errors on the fitted numbers of D^+ and D_s^+ , dominated by fit residuals
- Uncertainty in variation of efficiency with $\pi^+\pi^-$ mass
- Uncertainties in the background PDF parameters
- Uncertainty on the $\pi^+\pi^-$ mass resolution

Source	Systematic uncertainty			
	$D^+ \rightarrow \eta\pi^+$	$D_s^+ \rightarrow \eta\pi^+$	$D^+ \rightarrow \eta'\pi^+$	$D_s^+ \rightarrow \eta'\pi^+$
BRs	8.5%	7.0%	8.8%	7.8%
Number of $D_{(s)}^+$ (Run 1)	2%	2%	2%	2%
Number of $D_{(s)}^+$ (Run 2)	1%	1%	1%	1%
Efficiency variation (Run 1)	1%	1%	1%	1%
Efficiency variation (Run 2)	1%	1%	1%	1%
Background fit (Run 1)	0.2%	0.2%	0.4%	0.3%
Background fit (Run 2)	0.3%	0.5%	0.4%	0.4%
Mass resolution (Run 1)	5%	5%	5%	5%
Mass resolution (Run 2)	10%	10%	10%	10%

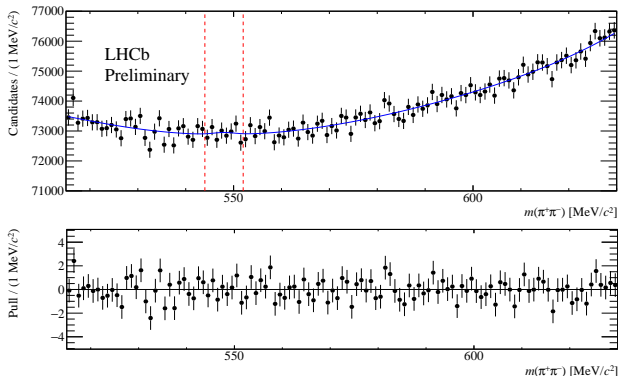
Fits to the $\pi^+\pi^-$ mass spectra

[LHCb-PAPER-2016-046]

The $\pi^+\pi^-$ mass spectra are fitted with a sum of:

- Background: 4th order Chebyshev polynomial
- Signal: double Gaussian from fit to MC

All fitted $\eta^{(\prime)}$ yields are compatible with zero



Sum of the four individual contributions, η region

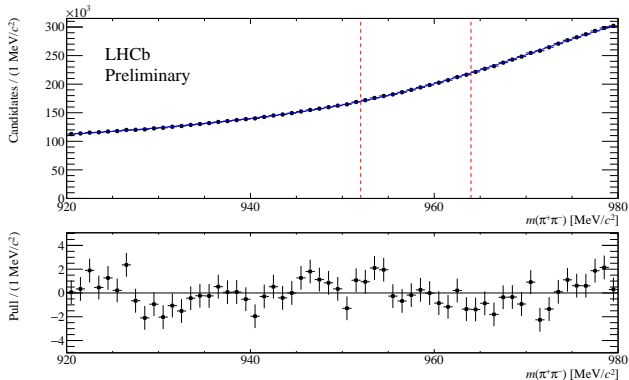
Fits to the $\pi^+\pi^-$ mass spectra

[LHCb-PAPER-2016-046]

The $\pi^+\pi^-$ mass spectra are fitted with a sum of:

- Background: 4th order Chebyshev polynomial
- Signal: double Gaussian from fit to MC

All fitted $\eta^{(\prime)}$ yields are compatible with zero



Sum of the four individual contributions, η' region

No evidence of signal is observed from fits, limits set with the CL_s method

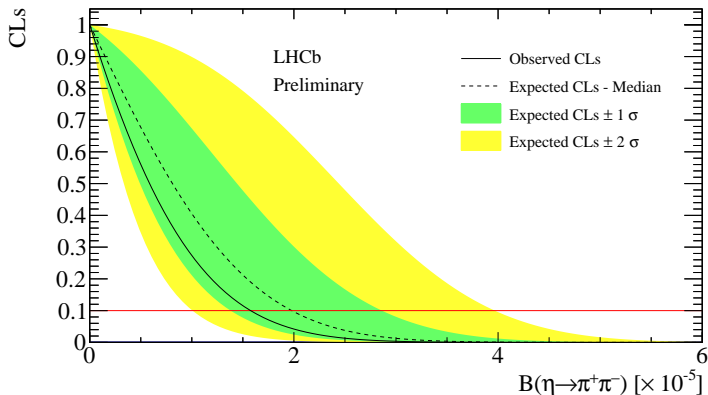
- PDF for background hypothesis: 4th order Chebyshev polynomial, extracted from data
- PDF for signal hypothesis: sum of two Gaussians, extracted from MC

CL_s method algorithm:

- Scan of $\mathcal{B}(\eta^{(\prime)} \rightarrow \pi^+\pi^-)$ in the range $[0, 6] \times 10^{-5}$
- $N(\eta^{(\prime)} \rightarrow \pi^+\pi^-)$ is calculated for each value of $\mathcal{B}(\eta^{(\prime)} \rightarrow \pi^+\pi^-)$ and used as signal yield
- Profile likelihood ratio as test statistic: $t_N = -2 \log \frac{L(N, \theta)}{L(\hat{N}, \hat{\theta})}$

Limit setting for $\mathcal{B}(\eta \rightarrow \pi^+\pi^-)$

[LHCb-PAPER-2016-046]



Expected limit (preliminary) at 90% CL: 2.0×10^{-5}

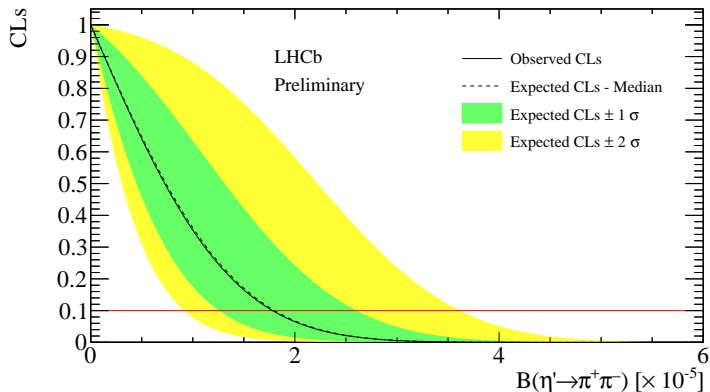
Observed limit (preliminary) at 90% CL: 1.6×10^{-5}

World best limit⁵ at 90% CL: 1.3×10^{-5}

⁵[Phys. Lett. B606 (2005) 276]

Limit setting for $\mathcal{B}(\eta' \rightarrow \pi^+\pi^-)$

[LHCb-PAPER-2016-046]



Expected limit (preliminary) at 90% CL: 1.8×10^{-5}

Observed limit (preliminary) at 90% CL: 1.8×10^{-5}

World best limit⁶ at 90% CL: 5.5×10^{-5}

⁶[Phys. Rev. D84 (2011) 032006]

- New method to search for strong CP violation in $\eta^{(\prime)} \rightarrow \pi^+ \pi^-$ decays using LHCb's large sample of $D_{(s)}^+ \rightarrow \pi^+ \pi^+ \pi^-$ decays
- No signal is observed for both decays and we set (preliminary) limits at 90% CL:
 - $\mathcal{B}(\eta \rightarrow \pi^+ \pi^-) < 1.6 \times 10^{-5}$
 - $\mathcal{B}(\eta' \rightarrow \pi^+ \pi^-) < 1.8 \times 10^{-5}$
- Limit on η competitive with KLOE
- Limit on η' is a factor $\times 3$ better than BESIII
- Statistics is the limiting factor
- New data taking and Run 2 trigger strategy will allow to get more stringent limits
- Paper in preparation: LHCb-PAPER-2016-046

BACKUP

$\epsilon_{D^+}(\eta)$ (Run 1)	0.85 ± 0.01
$\epsilon_{D^+}(\eta)$ (Run 2)	0.87 ± 0.01
$\epsilon_{D_s^+}(\eta)$ (Run 1)	0.80 ± 0.01
$\epsilon_{D_s^+}(\eta)$ (Run 2)	0.84 ± 0.01
$\epsilon_{D^+}(\eta')$ (Run 1)	1.01 ± 0.01
$\epsilon_{D^+}(\eta')$ (Run 2)	1.02 ± 0.01
$\epsilon_{D_s^+}(\eta')$ (Run 1)	1.03 ± 0.01
$\epsilon_{D_s^+}(\eta')$ (Run 2)	1.00 ± 0.01
$N(D^+ \rightarrow \pi^+\pi^+\pi^-)$ (Run 1)	$(1.88 \pm 0.04) \times 10^7$
$N(D_s^+ \rightarrow \pi^+\pi^+\pi^-)$ (Run 1)	$(1.75 \pm 0.03) \times 10^7$
$N(D^+ \rightarrow \pi^+\pi^+\pi^-)$ (Run 2)	$(6.09 \pm 0.06) \times 10^6$
$N(D_s^+ \rightarrow \pi^+\pi^+\pi^-)$ (Run 2)	$(6.26 \pm 0.06) \times 10^6$
$\mathcal{B}(D^+ \rightarrow \pi^+\pi^+\pi^-)$	$(3.29 \pm 0.20) \times 10^{-3}$
$\mathcal{B}(D_s^+ \rightarrow \pi^+\pi^+\pi^-)$	$(1.09 \pm 0.05) \times 10^{-2}$
$\mathcal{B}(D^+ \rightarrow \eta\pi^+)$	$(3.66 \pm 0.22) \times 10^{-3}$
$\mathcal{B}(D_s^+ \rightarrow \eta\pi^+)$	$(1.70 \pm 0.09) \times 10^{-2}$
$\mathcal{B}(D^+ \rightarrow \eta'\pi^+)$	$(4.84 \pm 0.31) \times 10^{-3}$
$\mathcal{B}(D_s^+ \rightarrow \eta'\pi^+)$	$(3.94 \pm 0.25) \times 10^{-2}$

Sensitivity to other channels (I)

Channel	BF	η BF	η' BF
$D^+ \rightarrow \pi^+ \pi^+ \pi^-$	$(3.18 \pm 0.18) \times 10^{-3}$	$(3.53 \pm 0.21) \times 10^{-3}$	$(4.67 \pm 0.29) \times 10^{-3}$
$D_s^+ \rightarrow \pi^+ \pi^+ \pi^-$	$(1.09 \pm 0.05) \times 10^{-2}$	$(1.69 \pm 0.10) \times 10^{-2}$	$(3.94 \pm 0.25) \times 10^{-2}$
$D^+ \rightarrow K^+ \pi^+ \pi^-$	$(5.27 \pm 0.23) \times 10^{-4}$	$(1.08 \pm 0.17) \times 10^{-4}$	$(1.76 \pm 0.22) \times 10^{-4}$
$D_s^+ \rightarrow K^+ \pi^+ \pi^-$	$(6.5 \pm 0.4) \times 10^{-3}$	$(1.76 \pm 0.35) \times 10^{-3}$	$(1.8 \pm 0.6) \times 10^{-3}$
$D^0 \rightarrow K_S^0 \pi^+ \pi^-$	$(2.83 \pm 0.20) \times 10^{-2}$	$(4.79 \pm 0.30) \times 10^{-3}$	$(9.4 \pm 0.5) \times 10^{-3}$
$D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$	$(7.42 \pm 0.21) \times 10^{-3}$	$(1.09 \pm 0.16) \times 10^{-3}$	$(4.5 \pm 1.7) \times 10^{-4}$
$D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-$	$(8.08 \pm 0.21) \times 10^{-2}$	$X \times 10^{-Y}$	$(7.5 \pm 1.9) \times 10^{-3}$
$D^0 \rightarrow \phi(K^+ K^-) \pi^+ \pi^-$	$(5.11 \pm 0.68) \times 10^{-4}$	$(1.4 \pm 0.5) \times 10^{-4}$	$X \times 10^{-Y}$
$J/\psi \rightarrow \phi(K^+ K^-) \pi^+ \pi^-$	$(9.4 \pm 0.9) \times 10^{-4}$	$(7.5 \pm 0.8) \times 10^{-4}$	$(4.0 \pm 0.7) \times 10^{-4}$

Sensitivity to other channels (II)

Channel	R	R'	N_{sig}	N_{bkg}	f_{Comb}	α	α'
$D^+ \rightarrow \pi^+\pi^+\pi^-$	0.90 ± 0.07	0.68 ± 0.06	2.1×10^7	2.7×10^7	$\sqrt{2}$	4.2×10^{-4}	3.2×10^{-4}
$D_s^+ \rightarrow \pi^+\pi^+\pi^-$	0.64 ± 0.05	0.28 ± 0.02	2.0×10^7	2.1×10^7	$\sqrt{2}$	2.9×10^{-4}	1.3×10^{-4}
$D^+ \rightarrow K^+\pi^+\pi^-$	4.9 ± 0.8	3.0 ± 0.4	3.8×10^5	2.4×10^6	1	2.2×10^{-2}	1.3×10^{-2}
$D_s^+ \rightarrow K^+\pi^+\pi^-$	3.7 ± 0.8	4 ± 1	8.5×10^5	1.6×10^6	1	6.9×10^{-2}	7.4×10^{-3}
$D^0 \rightarrow K_S^0\pi^+\pi^-$	5.9 ± 0.6	3.0 ± 0.3	6.6×10^6	8.3×10^6	1	3.5×10^{-3}	1.8×10^{-3}
$D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$	7 ± 1	16 ± 6	5.1×10^6	6.2×10^6	$\sqrt{4}$	9.2×10^{-2}	2.1×10^{-2}
$D^0 \rightarrow K^-\pi^+\pi^+\pi^-$	10^\dagger	11 ± 3	5.0×10^7	1.0×10^7	$\sqrt{2}$	2.2×10^{-3}	2.4×10^{-3}
$D^0 \rightarrow \phi(K^+K^-)\pi^+\pi^-$	4 ± 1	5^\dagger	4.1×10^5	5.8×10^5	$\sqrt{2}$	1.4×10^{-2}	1.7×10^{-2}
$J/\psi \rightarrow \phi(K^+K^-)\pi^+\pi^-$	1.3 ± 0.2	2.4 ± 0.5	$X \times 10^Y$	$X \times 10^Y$	1	$X \times 10^{-Y}$	$X \times 10^{-Y}$