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CP violation in $K \to \pi\pi$, Status and Prospects

Nicolas Garron

University of Birmingham, 14^{th} of September 2016

Kaon 2016, Edinburgh 8^{th} of September 2016

- CP violation in $K \to \pi\pi$ decays, short introduction
- Overview of the lattice computation
- Results and comparison
- Status and Prospect

This talk is based on a work done by the RBC-UKQCD Collaboration

The RBC & UKQCD collaborations

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Renwick Hudspith

An important Feature of our collaboration

We work with Domain-Wall fermions

⇒ At finite lattice spacing, Chiral-Flavour symmetry are preserved

- Numerically more expensive (harder to accumulate statistic)
- But we can compute quantities which are very hard for other (cheaper) formulations
- Computation of $K \to \pi\pi$ almost hopeless without chiral fermions

An important Feature of our collaboration

We work with Domain-Wall fermions

$\Rightarrow$ At finite lattice spacing, Chiral-Flavour symmetry are preserved "almost exactly"

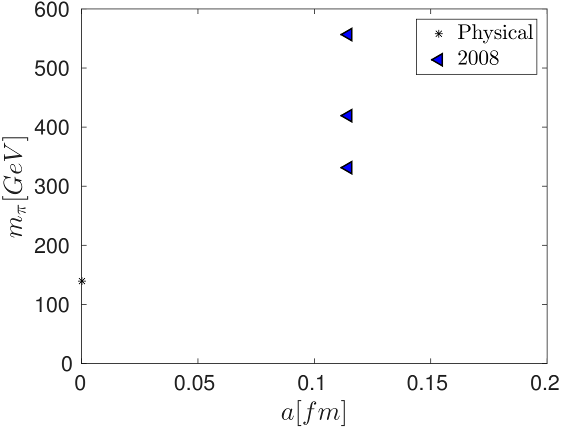
- Numerically more expensive (harder to accumulate statistic)
- But we can compute quantities which are very hard for other (cheaper) formulations
- Computation of $K \to \pi\pi$ almost hopeless without chiral fermions

Although there is a computation at threshold done with Wilson fermions Ishizuka, Ishikawa, Ukawa, Yoshié '15

The authors use a clever trick to avoid the dangerous mixing with lower dimension operators

Going light

RBC-UKQCD $N_f = 2 + 1$ DWF - Landscape (since 2008)



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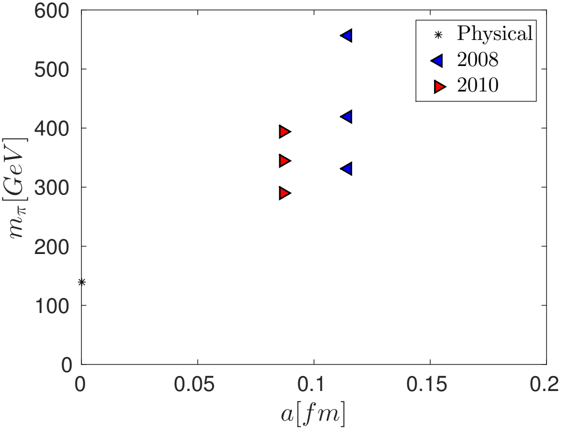

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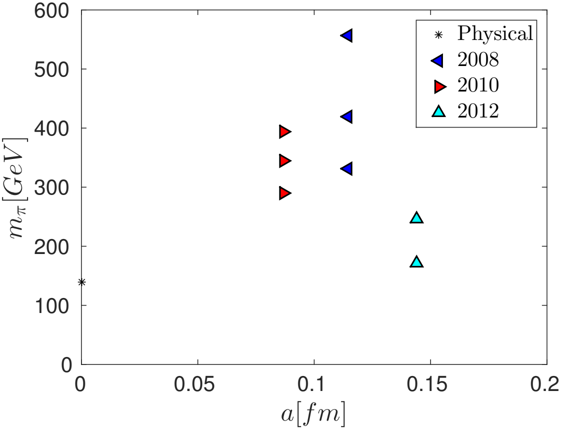

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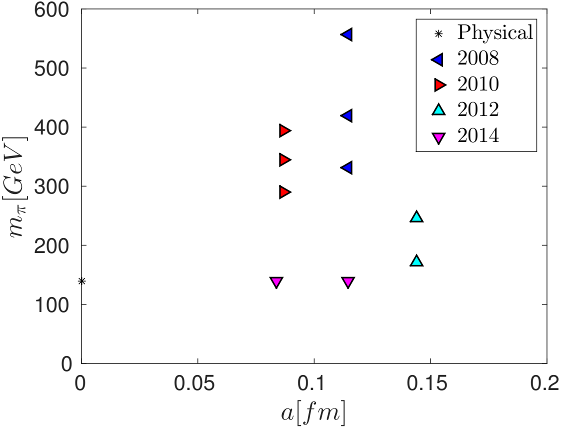

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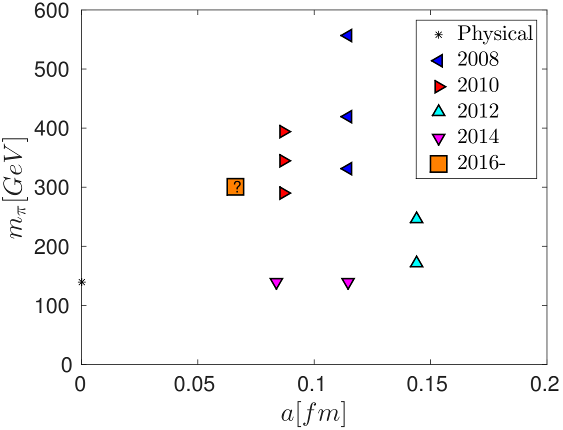


$K \to \pi\pi$ and CP violation

Background: Kaon decays and CP violation

- First discovery of CP violation was made in kaon system in 1964 (Christenson, Cronin, Fitch and Turlay)
- Noble prize in 1980 (Cronin and Fitch)
- Direct CP violation discovered in kaon decays [NA31, KTeV, NA48, '90-99]
- Very nice measurements of both direct and indirect CP violation (numbers from [PDG 2011])

$$\begin{cases} \textit{Indirect} \quad |\varepsilon| = (2.228 \pm 0.011) \times 10^{-3} \\ \textit{Direct} \quad Re\left(\frac{\varepsilon'}{\varepsilon}\right) = (1.65 \pm 0.26) \times 10^{-3} \end{cases}$$

- Theoretically:

  Relate indirect CP violation parameter (ε) to neutral kaon mixing (B_K)

  B_K is now computed on the lattice with a few-percent precision

  But the first realistic theoretical computstion of ε' has only been achieved last year

- Sensitivity to new physics expected

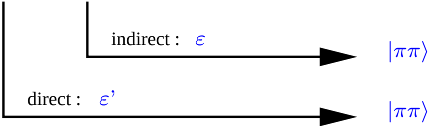
Background: Kaon decays and CP violation

Flavour eigenstates $\begin{pmatrix} K^0 = \bar{s}\gamma_5 d \\ \overline{K}^0 = \bar{d}\gamma_5 s \end{pmatrix} \neq$ CP eigenstates $|K^0_{\pm}\rangle = \frac{1}{\sqrt{2}}\{|K^0\rangle \mp |\overline{K}^0\rangle\}$

They are mixed in the physical eigenstates $\begin{cases} |K_L\rangle \sim |K^0_-\rangle + \bar{\varepsilon}|K^0_+\rangle \\ |K_S\rangle \sim |K^0_+\rangle + \bar{\varepsilon}|K^0_-\rangle \end{cases}$

Direct and indirect CP violation in $K \to \pi\pi$

$$|K_L\rangle \propto |K_-\rangle + \varepsilon|K_+\rangle$$



$$\varepsilon = \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})} = |\varepsilon| e^{i\phi_\varepsilon} \sim \bar{\varepsilon}$$

$K \to \pi\pi$ amplitudes

Two isospin channels: $\Delta I = 1/2$ and $\Delta I = 3/2$

$$K \to (\pi\pi)_{\mathrm{I}=0,2}$$

Corresponding amplitudes defined as

$$A[K \to (\pi\pi)_{\mathrm{I}}] = A_{\mathrm{I}} \exp(i\delta_{\mathrm{I}}) \qquad \text{/w } \mathrm{I} = 0, 2 \qquad \delta = \text{strong phases}$$

$\Delta I = 1/2$ rule

$$\omega = \frac{\mathrm{Re} A_2}{\mathrm{Re} A_o} \sim 1/22 \qquad \text{(experimental number)}$$

Amplitudes are related to the parameters of CP violation $\varepsilon, \varepsilon'$ via (in the isospin limit)

$$\varepsilon' = \frac{i\omega \exp(i\delta_2 - \delta_0)}{\sqrt{2}} \left[\frac{\mathrm{Im}(A_2)}{\mathrm{Re} A_2} - \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0} \right]$$

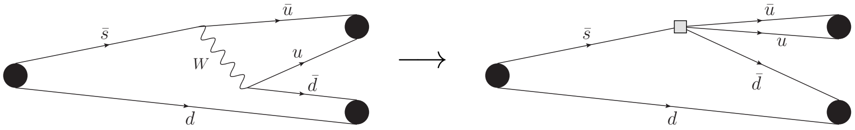
$$\varepsilon = e^{i\phi_\varepsilon} \left[\frac{\mathrm{Im}\langle \bar{K}^0 | H_{\mathrm{eff}}^{\Delta S=2} | K^0 \rangle}{\Delta m_K} + \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0} \right]$$

$\Rightarrow$ Related to $K^0 - \bar{K}^0$ mixing

Overview of the computation

Overview of the computation

- Operator Product expansion



- In the $N_f = 3$ theory, describe $K \to (\pi\pi)_{\mathrm{I}=0,2}$ with an effective Hamiltonian [Buchalla, Buras, Lautenbacher '96]

$$H^{\Delta s=1} = \frac{G_F}{\sqrt{2}} \Big\{ \sum_{i=1}^{10} \big(V_{ud} V_{us}^* z_i(\mu) - V_{td} V_{ts}^* y_i(\mu) \big) Q_i(\mu) \Big\}$$

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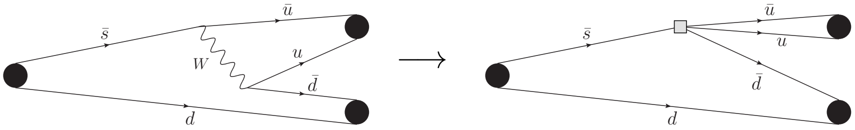

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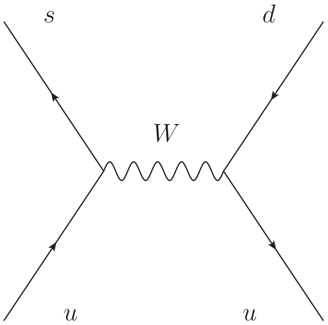
- Amplitude given by $A \propto \langle \pi\pi | H^{\Delta s=1} | K \rangle$
- Short distance effects factorized in the Wilson coefficients y_i, z_i , computed at NLO in [BBL '96]
- Long distance effects factorized in the matrix elements

$$\langle \pi\pi | Q_i(\mu) | K \rangle \longrightarrow \text{task for the Lattice}$$

See reviews by [Buras, Christ @ Kaon'09, Lellouch @ Les Houches'09, Sachrajda @ Lattice '10], ...

4-quark operators

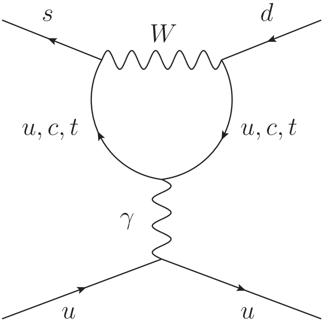
Current diagrams



$Q_1 = (\bar{s}d)_{\mathrm{V-A}}(\bar{u}u)_{\mathrm{V-A}}$ $\qquad$ $Q_2 =$ color mixed

4-quark operators

Electroweak penguins

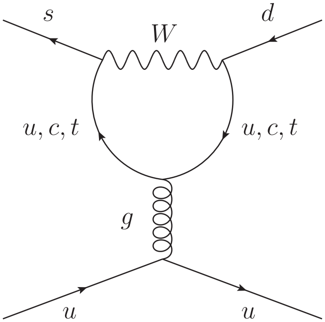


$$Q_7 = \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} e_q (\bar{q}q)_{\mathrm{V+A}} \qquad Q_8 = \text{color mixed}$$

$$Q_9 = \frac{3}{2}(\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} e_q (\bar{q}q)_{\mathrm{V-A}} \qquad Q_{10} = \text{color mixed}$$

4-quark operators

QCD penguins



$$Q_3 = (\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} (\bar{q}q)_{\mathrm{V-A}} \qquad Q_4 = \text{color mixed}$$

$$Q_5 = (\bar{s}d)_{\mathrm{V-A}} \sum_{q=u,d,s} (\bar{q}q)_{\mathrm{V+A}} \qquad Q_6 = \text{color mixed}$$

$SU(3)_L \otimes SU(3)_R$ and isospin decomposition

Irrep of $SU(3)_L \otimes SU(3)_R$

$$
\begin{aligned}
\overline{3} \otimes 3 &= 8 + 1 \\
\overline{8} \otimes 8 &= 27 + \overline{10} + 10 + 8 + 8 + 1
\end{aligned}
$$

Relevant operators transform under $(27, 1), (8, 8)$ and $(8, 1)$ of $SU(3)_L \otimes SU(3)_R$

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Decomposition of the 4-quark operators gives

$$\begin{aligned} Q_{1,2} &= Q_{1,2}^{(27,1),\Delta I=3/2} + Q_{1,2}^{(27,1),\Delta I=1/2} + Q_{1,2}^{(8,8),\Delta I=1/2} \\ Q_{3,4} &= Q_{3,4}^{(8,1),\Delta I=1/2} \\ Q_{5,6} &= Q_{5,6}^{(8,1),\Delta I=1/2} \\ Q_{7,8} &= Q_{7,8}^{(8,8),\Delta I=3/2} + Q_{7,8}^{(8,8),\Delta I=1/2} \\ Q_{9,10} &= Q_{9,10}^{(27,1),\Delta I=3/2} + Q_{9,10}^{(27,1),\Delta I=1/2} + Q_{9,10}^{(8,8),\Delta I=1/2} \end{aligned}$$

see eg [Claude Bernard @ TASI'89] and [RBC'01]

$SU(3)_L \otimes SU(3)_R$ and isospin decomposition

In four dimension, using Fierz transformation, one observes that

$$\begin{aligned} Q_1 + Q_4 &= Q_2 + Q_3 \\ 3Q_1 - Q_3 &= 2Q_9 \\ Q_1 - Q_3 &= 2(Q_{10} - Q_2) \end{aligned}$$

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Our 7-operator basis

$$\begin{aligned} Q'_1 &= 3Q_1 + 2Q_2 - Q_3 && (27, 1) \\ Q'_2 &= \frac{1}{5}(2Q_1 - 2Q_2 + Q_3) && (8, 1) \\ Q'_3 &= \frac{1}{5}(-3Q_1 + 3Q_2 + Q_3) && (8, 1) \\ Q'_{5,6,7,8}i &= Q_{5,6,7,8} && (8, 8) \end{aligned}$$

$SU(3)_L \otimes SU(3)_R$ and isospin decomposition

$$
\begin{array}{rcccl}
(27,1) & Q'_1 & = & Q'^{(27,1),\Delta I=3/2}_1 + Q'^{(27,1),\Delta I=1/2}_1 \\
(8,1) & Q'_2 & = & Q'^{(8,1),\Delta I=1/2}_2 \\
 & Q'_3 & = & Q'^{(8,1),\Delta I=1/2}_3 \\
 & Q'_5 & = & Q'^{(8,1),\Delta I=1/2}_5 \\
 & Q'_6 & = & Q'^{(8,1),\Delta I=1/2}_6 \\
(8,8) & Q'_7 & = & Q'^{(8,8),\Delta I=3/2}_7 + Q'^{(8,8),\Delta I=1/2}_7 \\
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 & Q_6' & = & Q_6'^{(8,1),\Delta I=1/2} \\
(8,8) & Q_7' & = & Q_7'^{(8,8),\Delta I=3/2} + Q_7'^{(8,8),\Delta I=1/2} \\
 & Q_8' & = & Q_8'^{(8,8),\Delta I=3/2} + Q_8'^{(8,8),\Delta I=1/2}
\end{array}
$$

Only 3 operators contribute to the $\Delta I = 3/2$ channel

Lattice computation $\langle \pi\pi | Q_i | K \rangle$

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Three main ingredients

- The finite volume bare matrix elements $\langle \pi\pi | Q_i | K \rangle_{FV}^{bare}$
- The renormalization Matrix Z_{ij}
- The phase shift (Lellouch-Lüscher factor)

- We want to extract the physical 2-pion state with momenta $p = |\vec{p}_\pi|$

$$2\sqrt{p^2 + m_\pi^2} = E_{\pi\pi} = m_K$$

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- In a naive simulation, the ground state is unphysical $|\pi(\vec{0})\pi(\vec{0})\rangle$

  $|\pi(\vec{p})\pi(\vec{p})\rangle$ would be an excited state

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  $\Rightarrow$ We don't want to use multiple-exponential fits.

$\Rightarrow$ We use some tricks to eliminate the unphysical state

See talk by Chris Sachrajda

Without going through the details, to simulate the physical kinematics:

- For the $\Delta I = 3/2$ channel, we can combine the Wigner-Eckart theorem with peculiar boundary conditions (in the valence sectors)

  $\Rightarrow$ We can use already exisiting ensembles

- For the $\Delta I = 1/2$ channel, we have to use something else, we choose G-parity boundary conditions [Wiese '92, Kim, Christ '02 '03 '09] and [C. Kelly @lat'15]

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Consequences

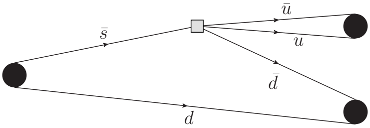
We have several lattice spacings for $\Delta I = 3/2$ but only one for $\Delta I = 1/2$

Extraction of the bare matrix elements

Compute a correlator

$$C^i_{K\pi\pi} = \langle 0|J_{\pi\pi}(t_{\pi\pi})Q_i(t_Q)J^\dagger_K(t_K)|0\rangle$$

$$\longrightarrow \mathrm{e}^{-m_K(t_Q-t_K)}\,\mathrm{e}^{-E_{\pi\pi}(t_{\pi\pi}-t_Q)}\,\langle 0|J_{\pi\pi}(0)|\pi\pi\rangle\,\langle\pi\pi|Q_i(0)|K\rangle\,\langle K|J^\dagger_K(0)|0\rangle$$



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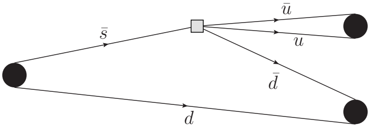

Needs also

$$C_K(t) = \langle\, 0\,|\,J_K(t)\,J_K^\dagger(0)\,|\,0\,\rangle \longrightarrow |\langle\, K\,|\,J_K^\dagger(0)\,|\,0\,\rangle|^2 e^{-m_K t}$$

$$C_{\pi\pi}(t) = \langle\, 0\,|\,J_{\pi\pi}(t)\,J_{\pi\pi}^\dagger(0)\,|\,0\,\rangle \longrightarrow |\langle\, 0|\,J_{\pi\pi}(0)\,|\,\pi\pi\,\rangle|^2 e^{-E_{\pi\pi} t}$$

And compute the ratios

$$R(t_Q) \equiv \frac{C_{K\pi\pi}(t_K, t_Q, t_{\pi\pi})}{C_K(t_Q - t_K)\,C_{\pi\pi}(t_{\pi\pi} - t_Q)} \longrightarrow \frac{\langle\pi\pi|Q_i|K\rangle}{\langle\, 0|\,J_{\pi\pi}(0)\,|\,\pi\pi\,\rangle\,\langle\, K\,|\,J_K^\dagger(0)\,|\,0\,\rangle}$$

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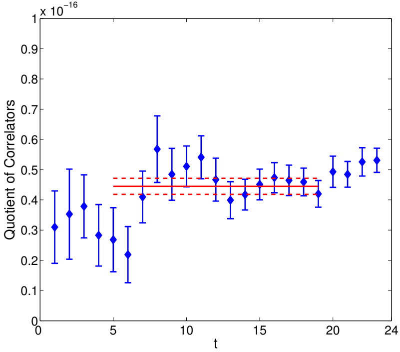

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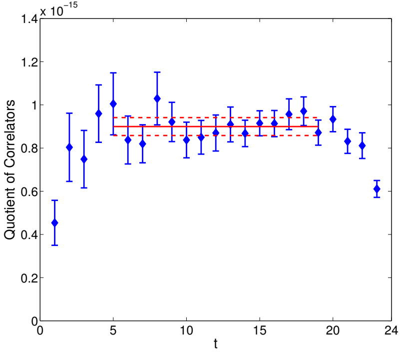

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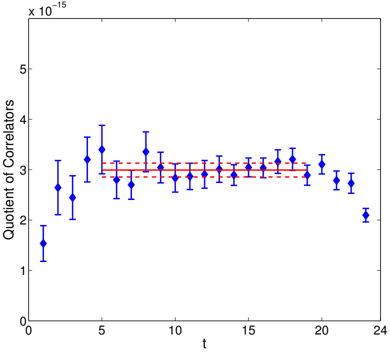

With our boundary conditions we “give” momenta to the pions $|\mathbf{p}| = \pm\pi/L$

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The infinite volume matrix element is given by

$$\langle\pi\pi|H_W|K\rangle_\infty = F\langle\pi\pi|H_W|K\rangle_{FV}$$

where F is the Lellouch-Lüscher factor [Lellouch Lüscher '00, Lin et al '01]

$$F^2 = 8\pi q \left(\frac{\partial\phi}{\partial q} + \frac{\partial\delta}{\partial q}\right) \frac{m_K E_{\pi\pi}^2}{p^3}$$

and p is the magnitude of the momentum of each pion in the center-of-mass frame

$$\begin{aligned} 2\sqrt{p^2 + m_\pi^2} &= E_{\pi\pi} \\ q &= \frac{pL}{2\pi} \end{aligned}$$

δ is the s-wave phase shift

ϕ is a kinematic function defined in [Lellouch Lüscher '00]

Once $E_{\pi\pi}$ has been measured and q_π determined, δ can be calculated using the Lüscher quantization condition [Lüscher 1990]

$$n\pi = \delta(q) + \phi(q)$$

$\Rightarrow$ have to compute $\partial\delta/\partial \boldsymbol{q}_\pi$

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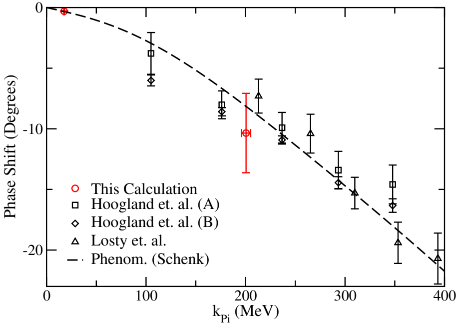

Phase shift for $(\pi\pi)_{I=2}$

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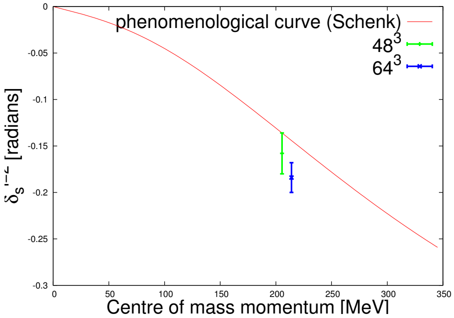

Phase shift for $(\pi\pi)_{I=2}$

- For $(\pi\pi)_{I=2}$ we find $\delta_2 = -11.6(2.5)(1.2)^o$
- For $(\pi\pi)_{I=0}$ we find $\delta_0 = 23.8(4.9)(1.2)^o$

δ_0 differs from phenomenology [Colangelo, Gasser, Leutwyler '01, Colangelo, Passemar, Stoffer '15]

$$\delta_2 = -8.3(0.15) \text{ and } \delta_0 = 38.0(1.3)$$

Values from Gilberto Colangelo @ NA62 Physics Handbook MITP Workshop

$\Rightarrow$ Is there a issue there ? Discretisation effect ?

Thanks to Emilie Passemar

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Note that this value has very little effect on the amplitudes

Numerical Results

- First computation (2012): Physical kinematic, Near physical vaule of the pion mass

  But only one coarse lattice spacing

  IDSDR $32^3 \times 64$, with $a^{-1} \sim 1.37\ \mathrm{GeV} \Rightarrow a \sim 0.14\ \mathrm{fm}$, $L \sim 4.6\ \mathrm{fm}$

$K \to (\pi\pi)_{I=2}$ Results

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- New computation:

  two lattice spacing, $n_f = 2 + 1$, large volume at the physical point

  New discretisation of the Domain-Wall fermion forumlation: Möbius Brower, Neff, Orginos '12

  - $48^3 \times 96$, with $a^{-1} \sim 1.729\ \mathrm{GeV} \Rightarrow a \sim 0.11\ \mathrm{fm}$, $L \sim 5.5\ \mathrm{fm}$
  - $64^3 \times 128$ with $a^{-1} \sim 2.358\ \mathrm{GeV} \Rightarrow a \sim 0.084\ \mathrm{fm}$, $L \sim 5.4\ \mathrm{fm}$
  - $am_{res} \sim 10^{-4}$

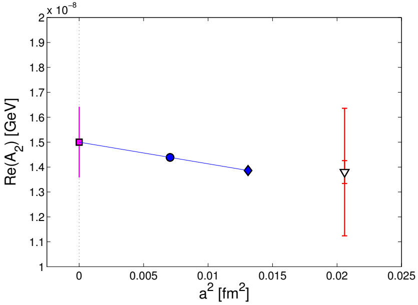
$K \to (\pi\pi)_{I=2}$ 2015 Results

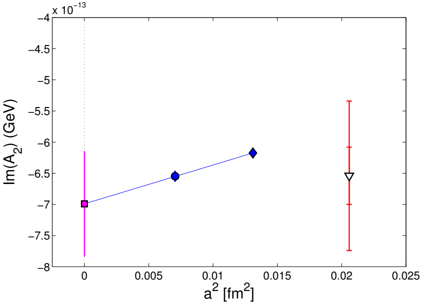
2012 Blum, Boyle, Christ, N.G.,Goode, Izubuchi, Jung, Kelly, Lehner, Lightman, Liu, Lytle, Mawh

Re $A_2 = 1.381(46)_{\rm stat}(258)_{\rm syst}\ 10^{-8}\ {\rm GeV}$ Im $A_2 = -6.54(46)_{\rm stat}(12$

2015 Blum, Boyle, Christ, Frison, N.G., Janowski, Jung, Kelly, Lehner, Lytle, Mawhinney, Sachraj

Re $A_2 = 1.50(4)_{\rm stat}(14)_{\rm syst}\ 10^{-8}\ {\rm GeV}$ Im $A_2 = -6.99(20)_{\rm stat}(84)_{\rm syst}$





see also talk by T.Janowski @ lat'13

$$K \to (\pi\pi)_{I=0}$$

Physical kinematics for the $\Delta I = 1/2$ channel

For the $I = 0$ state (ie $\Delta I = 1/2$), we impose isospin-symetric BC to avoid mixing the $I = 0$ and $I = 2$ state

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We employ G-parity Boundary Conditions $G = Ce^{i\pi I_y}$ [Wiese '92, Kim, Christ '02 '03] and [C. Kelly @lat'15]

- Product of Charge conjugation C and π-isospin rotation
- Transforms (u, d) into $(\bar{d}, -\bar{u})$

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- Transforms (u, d) into $(\bar{d}, -\bar{u})$
- Ground state: π have momenta $\pm\pi/L$
- Use Lellouch-Lüscher to compute the phase shift

But requires the generation of dedicated ensembles

- First complete computation of $K \to \pi\pi$ (both isospin channel) with physical kinematics

  Bai, Blum, Boyle, Christ, Frison, N.G., Izubuchi, Jung, Kelly, Lehner, Mawhinney, Sachrajda, Soni, Zhang PRL'15

- Pion mass $m_\pi = 143.1(2.0)\ \mathrm{MeV}$, single lattice spacing $a \sim 0.14\ \mathrm{fm}$

  Kaon mass $m_K = 490.6(2.4)\ \mathrm{MeV}$

- Physical kinematics achieved with G-Parity boundary conditions

  [Wiese '92, Kim, Christ, '03 and '09

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- See talk by C.Kelly and proceeding from Lattice'14

A_0, 2015

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Another computation, [Ishizuka, Ishikawa, Ukawa, Yoshié '15] with Wilson fermions at threshold (unphysical kinematics)

After renormalisation at $\mu \sim 1.5$ GeV, we combine with the Wilson coefficients and find

i	Re(A_0)(GeV)	Im(A_0)(GeV)
1	$1.02(0.20)(0.07) \times 10^{-7}$	0
2	$3.63(0.91)(0.28) \times 10^{-7}$	0
3	$-1.19(1.58)(1.12) \times 10^{-10}$	$1.54(2.04)(1.45) \times 10^{-12}$
4	$-1.86(0.63)(0.33) \times 10^{-9}$	$1.82(0.62)(0.32) \times 10^{-11}$
5	$-8.72(2.17)(1.80) \times 10^{-10}$	$1.57(0.39)(0.32) \times 10^{-12}$
6	$3.33(0.85)(0.22) \times 10^{-9}$	$-3.57(0.91)(0.24) \times 10^{-11}$
7	$2.40(0.41)(0.00) \times 10^{-11}$	$8.55(1.45)(0.00) \times 10^{-14}$
8	$-1.33(0.04)(0.00) \times 10^{-10}$	$-1.71(0.05)(0.00) \times 10^{-12}$
9	$-7.12(1.90)(0.46) \times 10^{-12}$	$-2.43(0.65)(0.16) \times 10^{-12}$
10	$7.57(2.72)(0.71) \times 10^{-12}$	$-4.74(1.70)(0.44) \times 10^{-13}$
Tot	$4.66(0.96)(0.27) \times 10^{-7}$	$-1.90(1.19)(0.32) \times 10^{-11}$
Exp	$3.3201(18) \times 10^{-7}$	-

ε'/ε can be computed from

$$Re(\varepsilon'/\varepsilon) = Re\left\{\frac{i\omega\exp(i\delta_2-\delta_0)}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}(A_2)}{\mathrm{Re}A_2}-\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

Combining our new value of $\mathrm{Im}A_0$ and δ_0 with

- our continuum value for $\mathrm{Im}A_2$
- the experimental value for ReA_0, ReA_2 and their ratio ω

we find

$$Re(\varepsilon'/\varepsilon) = 1.38(5.15)(4.43)\times 10^{-4}$$

whereas the experimental value is

$$Re(\varepsilon'/\varepsilon) = 16.6(2.3)\times 10^{-4} \qquad (\sim 2.1\sigma)$$

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Our errors are large, but are expected to dectrease rapidly

Status and Prospects

- $A \to (\pi\pi)_{I=2}$ becoming a "mature" quantity (continuum limit)
- First realistic computation of $A \to (\pi\pi)_{I=0}$ and ε'/ε (single lattice spacing)

- Room for improvement

  Renormalisation performed at $\sim 1.5\,\mathrm{GeV}$: running to higher scale

  Use finer lattice spacing and extrapolate to the continuum

  Control the mixing with lower dimension operators

  $\Rightarrow$ Error on ε'/ε should decrease rapidly

- Future improvements

  Isospin corrections

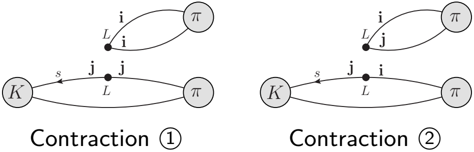
  Inclusion of the charm

BACKUP SLIDES

Enter at your own risks

On the $\Delta I = 1/2$ rule

Two kinds of contraction for each $\Delta I = 3/2$ operator



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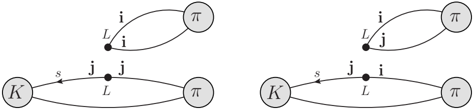

Contraction ①　　Contraction ②

- Re A_2 is dominated by the tree level operator (EWP $\sim 1\%$)

$$ReA_2 \sim ① + ②$$

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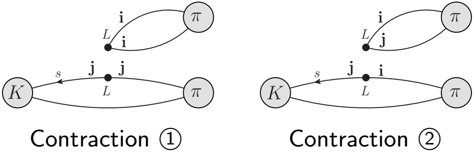

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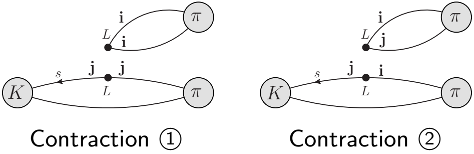

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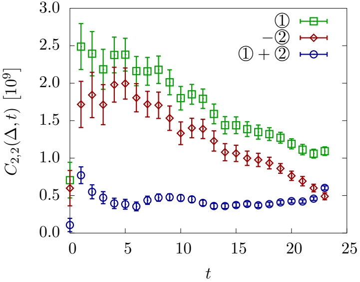
- Our computation: $② \sim -0.7①$



$\Rightarrow$ large cancellation in ReA_2

Toward an quantitative understanding of the $\Delta I = 1/2$ rule

ReA_0 is also dominated by the tree level operators

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Dominant contribution to Q_2^{lat} is $\propto$ (2② − ①) ⇒ Enhancement in ReA_0

Toward an quantitative understanding of the $\Delta I = 1/2$ rule

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$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} \sim \frac{2② - ①}{① + ②}$$

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With this unphysical computation (kinematics, masses) we find

$$\begin{aligned}\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} &= 9.1(2.1) \text{ for } m_K = 878 \text{ MeV } m_\pi = 422 \text{ MeV} \\ &= 12.0(1.7) \text{ for } m_K = 662 \text{ MeV } m_\pi = 329 \text{ MeV}\end{aligned}$$

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New Results, Physical Mass and kinematics

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = \frac{1.66(0.96)(0.27) \times 10^{-7}}{0.150(4)(14) \times 10^{-7}} \sim 31.0(11.1)$$

- Relative sign between ① and ② implies both a cancellation in $\mathrm{Re}A_2$ and an enhancement in $\mathrm{Re}A_0$
- See also analytic work in that direction, e.g. Pich, de Rafael '96, Bardeen, Buras, Gerard '87
- See also discussion in Lellouch @ Les Houches '09

Emerging understanding of the $\Delta I = 1/2$ rule

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- See also discussion in Lellouch @ Les Houches '09
- Similar observation done by the other lattice computation Ishizuka, Ishikawa, Ukawa, Yoshié '15

  $K \to \pi\pi$ amplitudes with unphysical kinematics (and Wilson fermions)