Electromagnetic Corrections to Hadronic Decays from LQCD

Guido Martinelli,
CERN & La Sapienza & INFN Roma & SISSA Trieste
September 14th 2016

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Kaon 2016 September 14^{th} 2016

Dipartimento di Fisica
Sapienza
Università di Roma

European Twisted Mass Collaboration meeting

INFN Istituto Nazionale di Fisica Nucleare

PLAN OF THE TALK

1) Physics Motivations

2) QED corrections to the hadron spectrum and to the hadronic amplitudes

3) $\pi^+ \rightarrow \mu^+ \nu_\mu \ (\gamma)$

4) Conclusion & Outlook

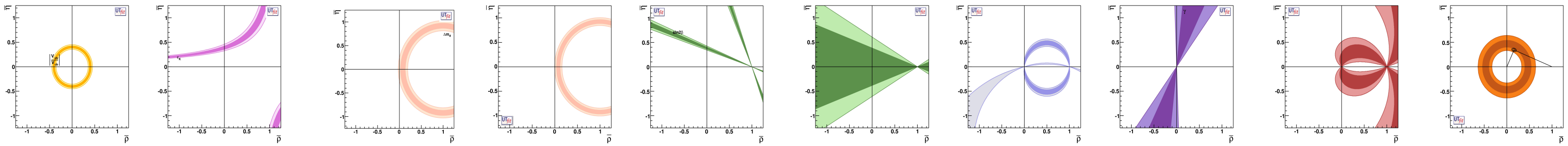
work done in collaboration with
N.Carrasco, V.Lubicz, C.T.Sachrajda,
F.Sanfillipo, S. Simula, N.Tantalo, C.Tarantino, M.Testa

Physics Motivations: Flavor and New Physics

Flavour phenomenology plays a fundamental role in indirect searches of New Physics:

- *looks for deviation from the SM whatever the origin*
- *needs good theoretical control of the SM contribution only*
- *in general cannot provide precise information on the NP scale, but a positive result would be a strong evidence that NP is not too far (i.e. in the multi-TeV region)*

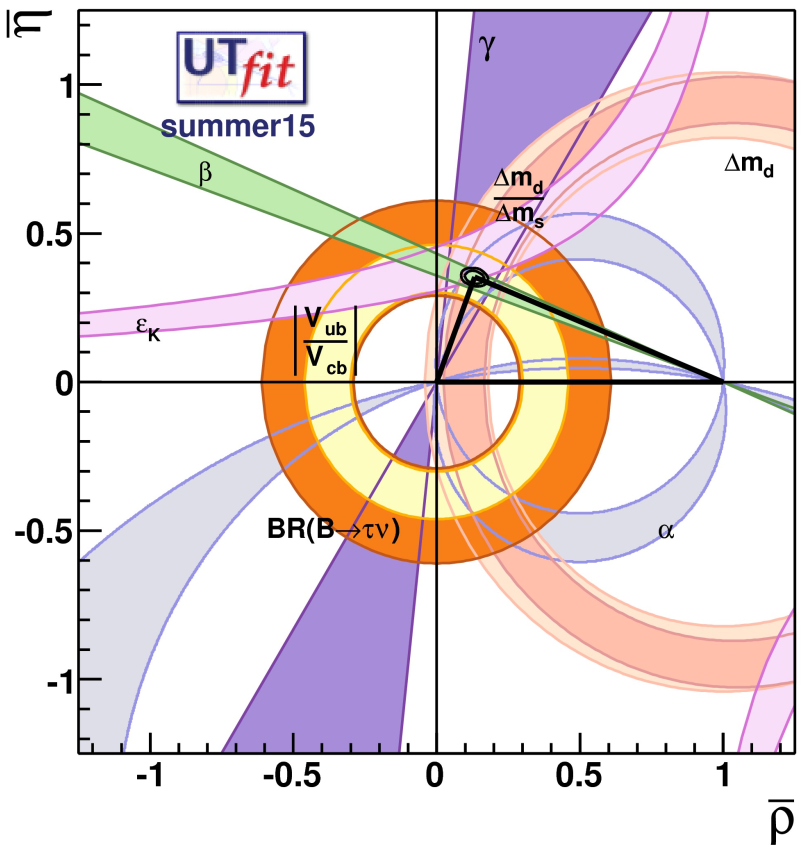
the path leading to TeV NP is narrower after the results of the LHC



2016 results

$\overline{\rho} = 0.153 \pm 0.013$ $\overline{\eta} = 0.343 \pm 0.011$

In the hadronic sector, the SM CKM pattern represents the principal part of the flavor structure and of CP violation

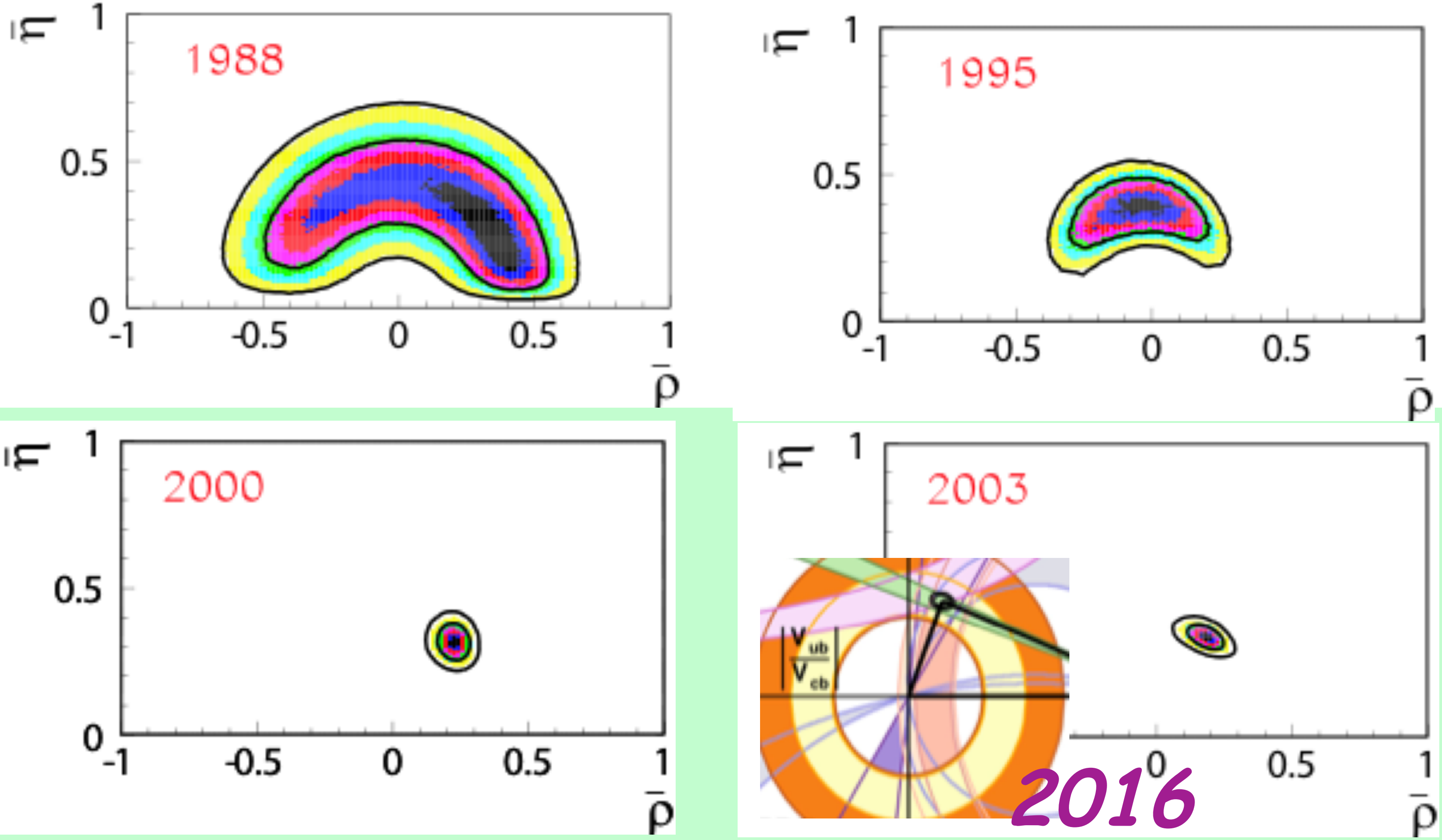


$\alpha = (92.0 \pm 2.0)^0$
$\sin 2\beta = 0.696 \pm 0.018$
$\beta = (21.82 \pm 0.72)^0$
$\gamma = (65.8 \pm 1.9)^0$
$A = 0.833 \pm 0.012$
$\lambda = 0.22497 \pm 0.00069$

Consistence on an over constrained fit of the CKM parameters

CKM matrix is the dominant source of flavour mixing and CP violation

PROGRESS SINCE 1988



The accuracy of lattice calculations of the hadron spectrum (and hence of the quark masses) and of the decay constants and form factors is such that isospin breaking effects cannot be neglected anymore:

FLAG Collaboration, arXiv:1607.00299

$N_f = 2+1$ $m_{ud} = 3.37(8)$ MeV $m_s = 92.0(2.1)$ MeV

$m_s/m_{ud} = 27.43(31)$ $\varepsilon = 3\%-6\%$

$N_f = 2+1+1$

$m_{ud} = 3.70(17)$MeV $m_s = 93.9(1.1)$ MeV

$m_s/m_{ud} = 27.30(34)$

$f_\pi{}^* = 130.2(1.4)$ MeV $f_K = 155.36(0.4)$ MeV $\varepsilon = 0.26\%$

$f_K/f_\pi = 1.1933(29)$ $\varepsilon = 0.24\%$ $F^{K\pi}(0) = 0.9704(32)$ $\varepsilon = 0.34\%$

Phenomenological relevance of precision physics in the Standard Model and beyond

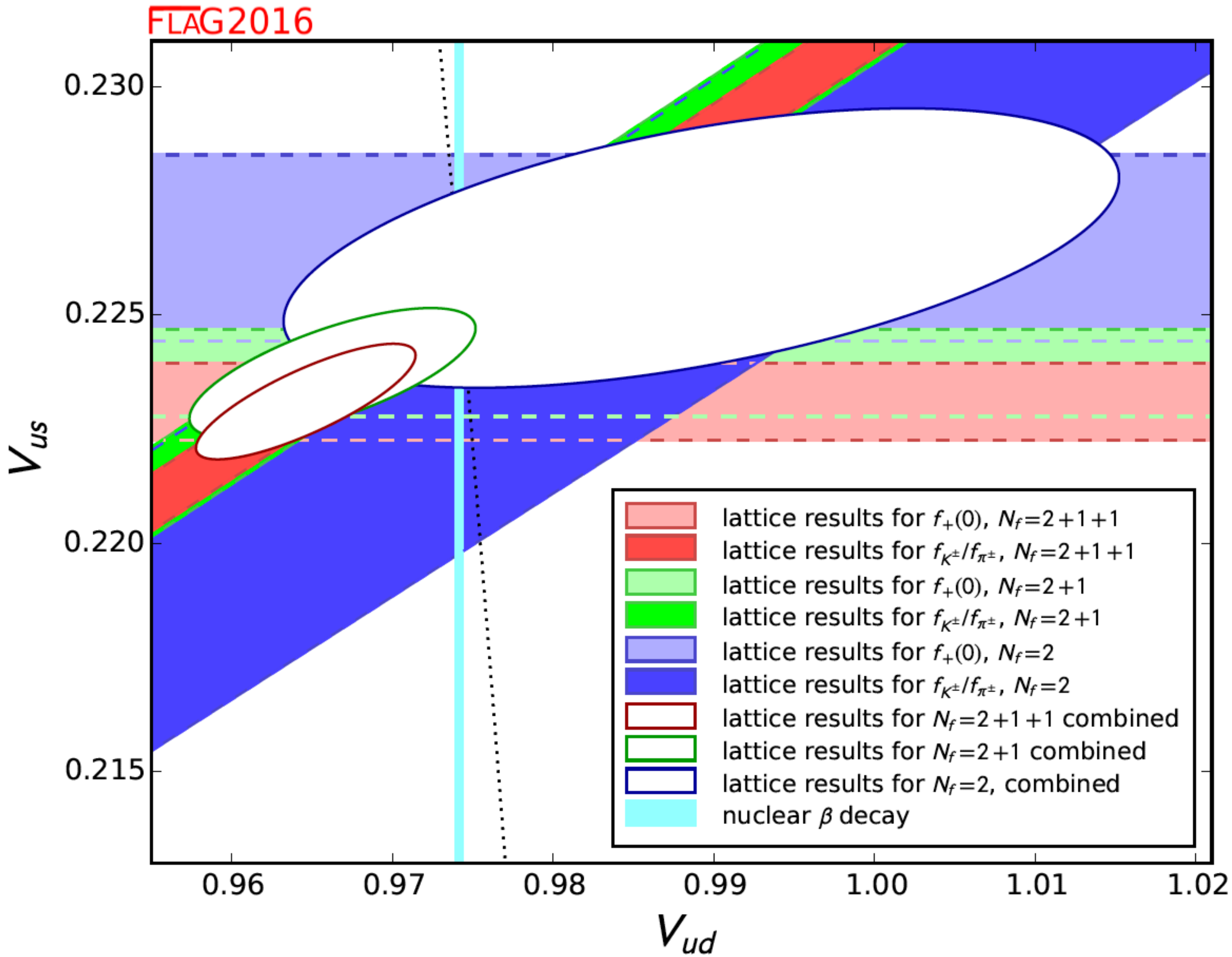
$|V_{us}| F^{K\pi}(0) = 0.2165(4)$ - exp ε =0.2%

$|V_{us}| f_K / |V_{ud}| f_\pi$ =0.2760(4) ε =0.2%

$|V_{ud}|$= 0.97417(22) ε =0.02%

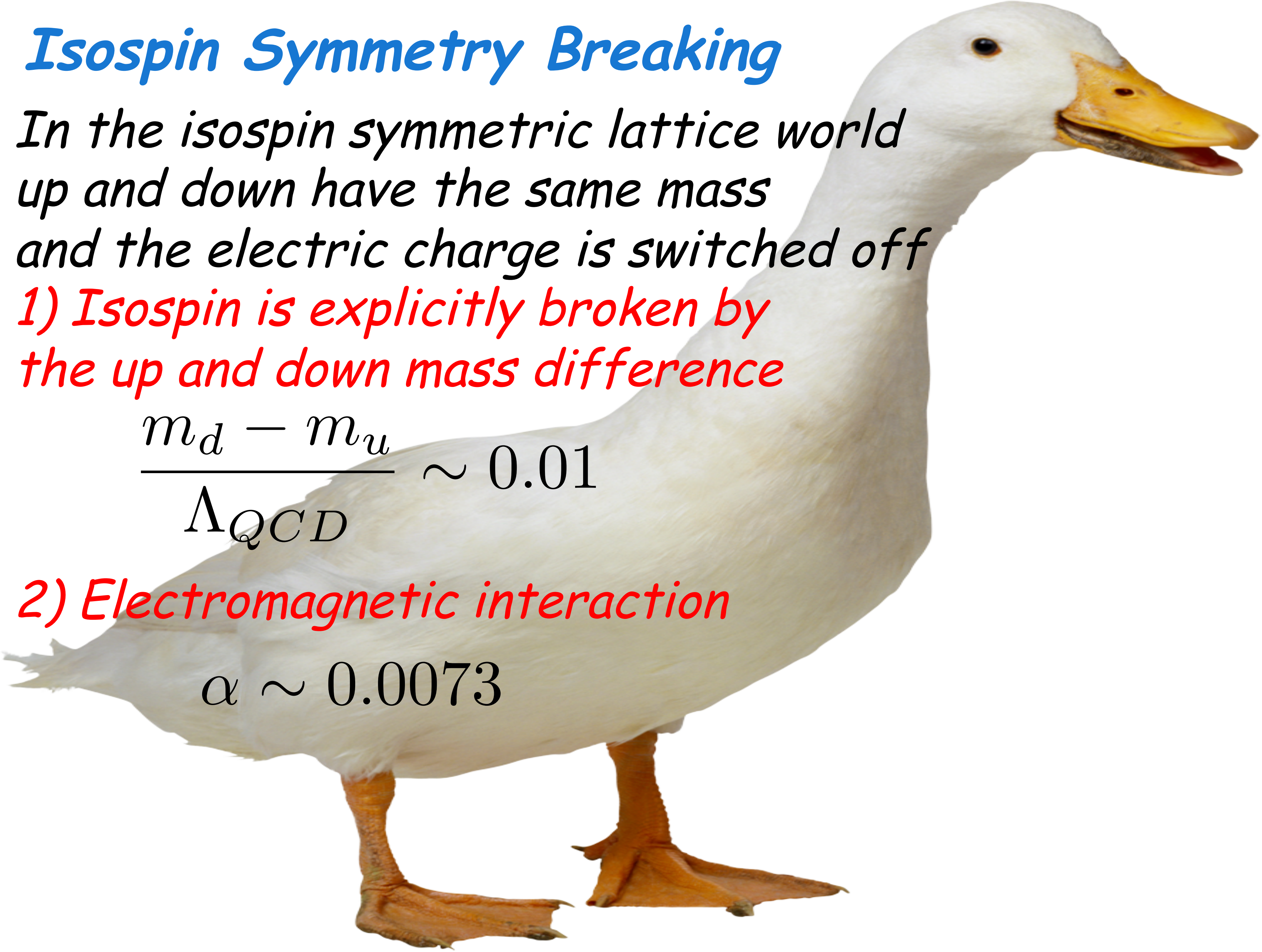
$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2 = 1$ in the SM ($|V_{ub}|^2 \approx 1.6\ 10^{-5}$)

STANDARD MODEL UNITARITY TRIANGLE ANALYSIS (FLAG)



- $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2 = 0.9998(5)$ or $0.9999(6)$ *from semileptonic and leptonic respectively*

Isospin Symmetry Breaking



In the isospin symmetric lattice world
up and down have the same mass
and the electric charge is switched off

1) Isospin is explicitly broken by
the up and down mass difference

$$\frac{m_d - m_u}{\Lambda_{QCD}} \sim 0.01$$

2) Electromagnetic interaction

$$\alpha \sim 0.0073$$

Non-compact lattice QED

- Naively discretised **Maxwell action**:

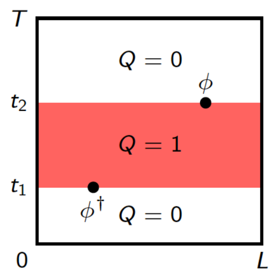
$$S[A_\mu] = \frac{1}{4}\sum_{\mu,\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2$$

- Pure gauge theory is **free**, it can be solved **exactly**
- Gauge invariance is preserved

Charge states in a finite box

In a finite box with periodic boundary conditions, Gauss law forbids states with nonzero charge

$$Q = \int d^3x \, j_0(t, \mathbf{x}) = \int d^3x \, \partial_k E_k(t, \mathbf{x}) = 0$$

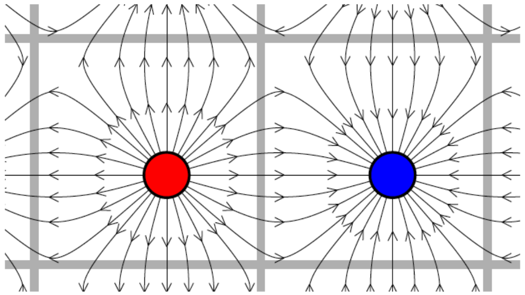


Some proposed methods

- Remove the global zero-mode of the gauge field ($\mathrm{QED_{TL}}$) **
- Restrict the global zero-mode of the gauge field
- Remove the spatial zero-mode of the gauge field in each timeslice ($\mathrm{QED_L}$) **
- Massive photon.
- $C^\star$ boundary conditions.

All these approaches are equivalent if the infinite-volume limit is take before any other limit (large-t limit in 2-point functions, continuum limit, massless photon limit). In general the infinite-volume limit does not commute with the other limits.

Courtesy by A. Patella at Lattice 2016



$$D_{\mu\nu}[k] = \frac{1}{L^4} \frac{a^2 G_{\mu\nu}[k]}{4\sum_{\mu=1,..4} \sin^2(k_\mu a/2)} \quad **$$

$$\frac{1}{4\sum_{\mu=1,..4} \sin^2(k_\mu a/2)} = 0 \quad \text{when} \quad k_\mu = 0 \quad \text{or} \quad \vec{k} = 0$$

QED_{TL} finite-volume effects

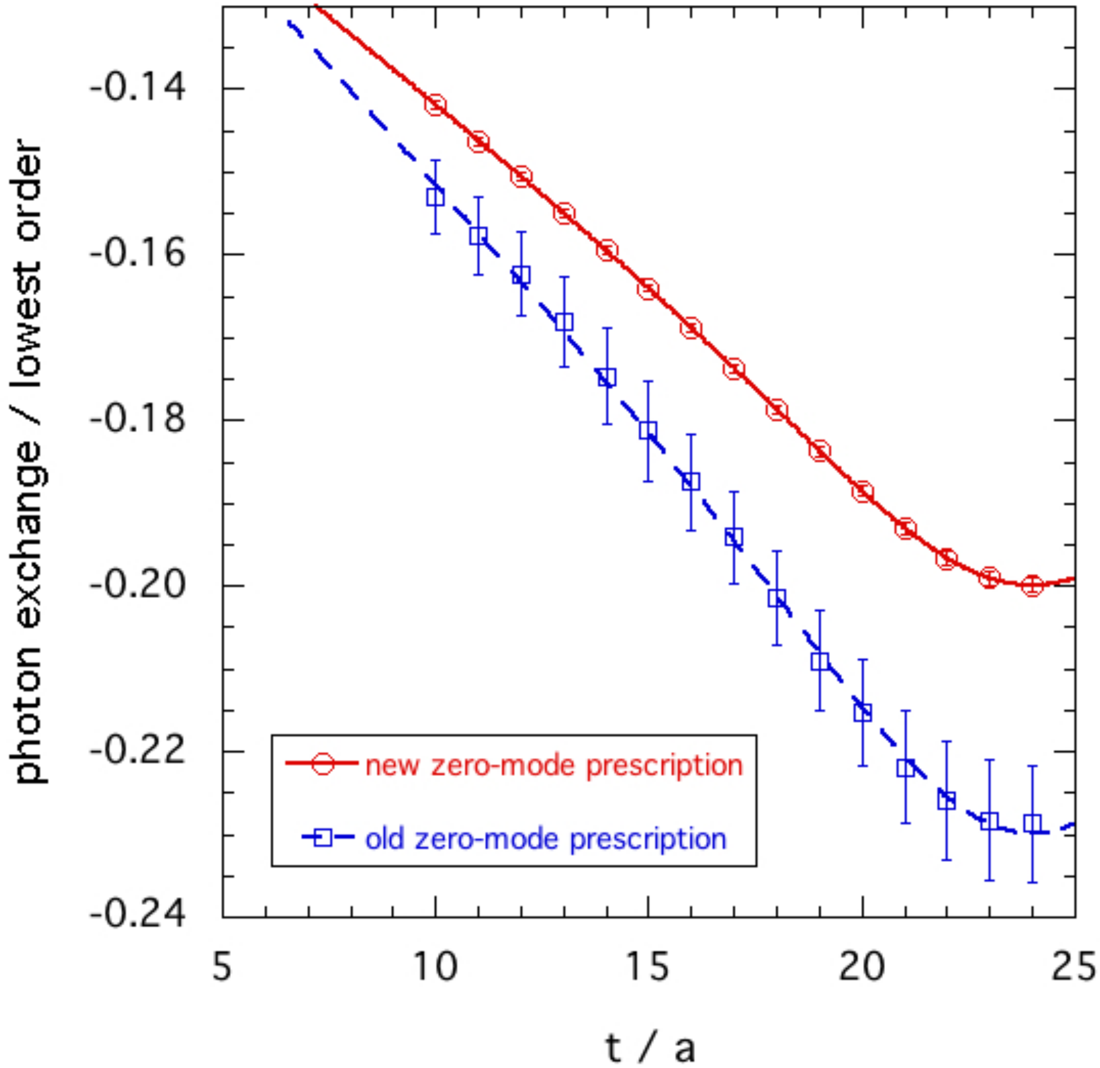
- Example — 1-loop QED_{TL} [BMWc, 2014]:

$$m(T,L) \underset{T,L\to+\infty}{\sim} m\left\{1 - q^2\alpha\left[\frac{\kappa}{2mL}\left(1+\frac{2}{mL}\left[1-\frac{\pi}{2\kappa}\frac{T}{L}\right]\right) - \frac{3\pi}{(mL)^3}\left[1-\frac{\coth(mT)}{2}\right] - \frac{3\pi}{2(mL)^4}\frac{L}{T}\right]\right\}$$

Finite volume effects depend on the regulator of the zero mode, but this is not relevant to the following discussion.

<u>Hadron masses are infrared finite</u>

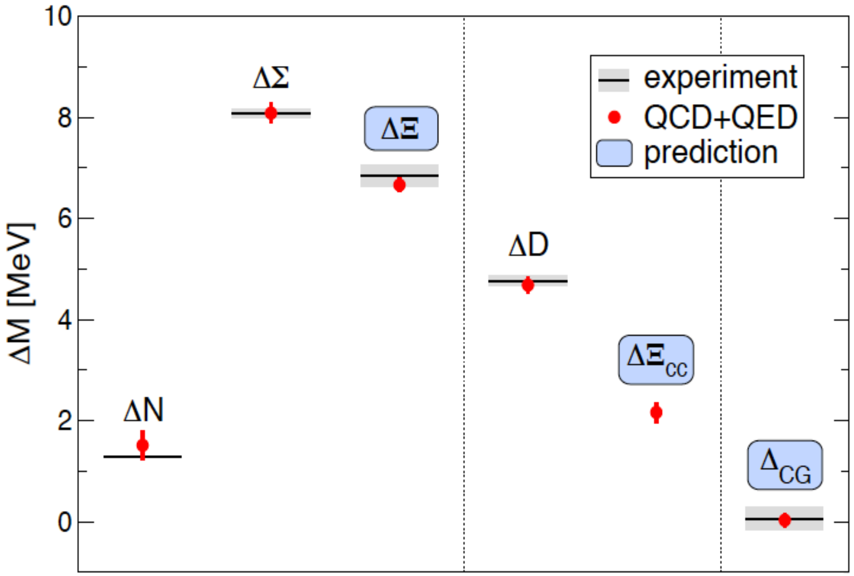
$$m_{\mathrm{QED_L}}(T,L) \underset{T,L\to+\infty}{=} m\left\{1 - q^2\alpha\left[\frac{\kappa}{2mL}\left(1+\frac{2}{mL}\right) - \frac{3\pi}{(mL)^3}\right]\right\}$$



BMW: Baryon masses

Borsanyi *et al.*, Science 347 (2015) 1452-1455

Liu @ Lattice 2016
see also talk by A. Patella



Analytic understanding of power-law finite-volume corrections to masses of stable states, in the fully-relativistic theory.

Fodor *et al.*, Phys. Lett. B **755**, 245 (2016)

Davoudi, Savage, Phys. Rev. D **90**, no. 5, 054503 (2014)

Large volume simulations: physical size up to $M_\pi L = 8.1$ with a 64×80^3 lattice.

QED Corrections to Hadron Masses, or $SU(3)_c \times U(1)$ on the Lattice

QED corrections to the hadron masses only require an ultraviolet cutoff

1) We need a physical condition for any renormalizable coupling to fix the scale i.e. to renormalize the strong (and the electromagnetic) coupling;
2) We must fix the masses of a certain number of hadrons, corresponding to the different flavors, to their physical value;
3) All the other hadron masses are finite and can be predicted
4) Quark masses are determined in your preferred renormalization scheme

QED (Isospin) Corrections in Hadronic Processes

After the renormalization of the $SU(3)_c \times U(1)$ Lagrangian you still need

1) The renormalization of the operators mediating the physical process of interest (e.g. the Weak effective Hamiltonian). But this is not a novelty;
2) A complex procedure to remove the infrared cutoff because in general the amplitudes, contrary to the masses, are infrared divergent.

A method to solve this problem is presented . This will be done by discussing an explicit example and will allow the discussion of some important theoretical subtelties

How to solve the problem of the infrared divergences discussed through an explicit example

$$\pi \to \ell + \nu_\ell + (\gamma)$$

N.Carrasco, V.Lubicz, G.M., C.T.Sachrajda, S. Simula, F.Sanfillippo, N.Tantalo, C.Tarantino, M.Testa

NOTE: Chiral Perturbation Theory is NOT Used

Leptonic decays at tree level

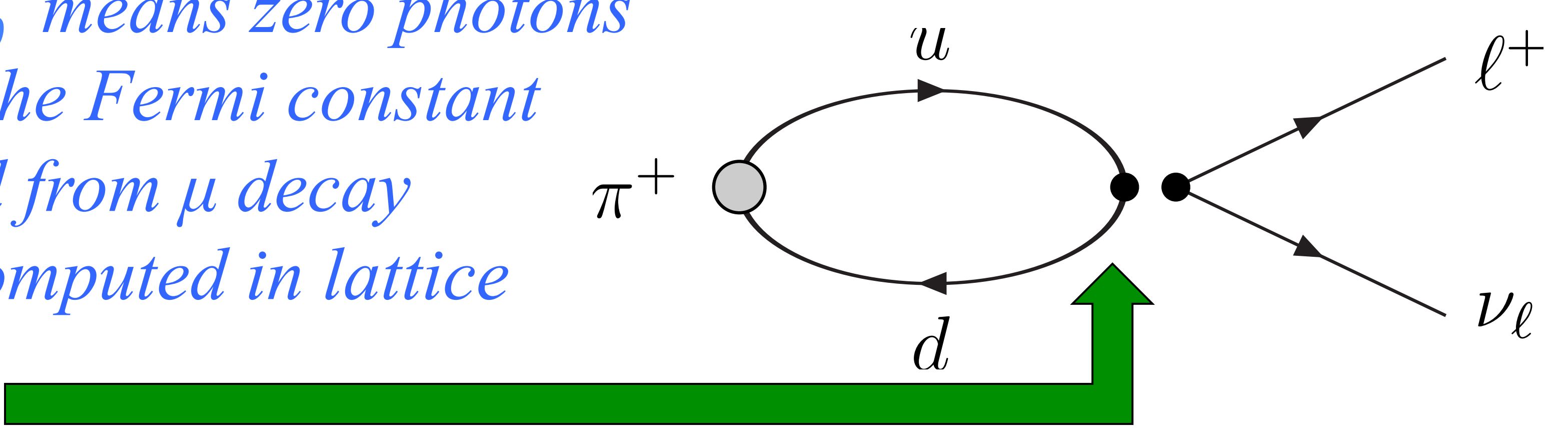
Since the mass of the pion is much lower than M_W we use the effective Hamiltonian

$$H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{ud}^{*} (\bar{d}\gamma^{\mu}(1-\gamma^5)u)\,(\bar{\nu}_{\ell}\gamma_{\mu}(1-\gamma^5)\ell)$$

from which we compute

$$\Gamma_0^{\text{tree}}(\pi^+ \to \ell^+\nu_\ell) = \frac{G_F^2\,|V_{ud}|^2 f_\pi^2}{8\pi}\, m_\pi\, m_\ell^2 \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2$$

- *0 in Γ_0 means zero photons*
- *G_F is the Fermi constant defined from μ decay*
- *f_π is computed in lattice QCD*



Leptonic decays at O(α) – The ultraviolet matching in the ``W Regularization”

If G_F is the Fermi constant defined at O(α) from μ decay in the standard (convention dependent) way

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left[1-\frac{8m_e^2}{m_\mu^2}\right]\left[1+\frac{\alpha}{2\pi}\left(\frac{25}{4}-\pi^2\right)\right]$$

S.M.Berman, PR 112 (1958) 267; T.Kinoshita and A.Sirlin, PR 113 (1959) 1652

then the effective Hamiltonian in the W-regularization is given by (Sirlin PRD 22 (80) 971)

$$H_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{ud}^* \left(1 + \frac{\alpha}{\pi}\log\frac{M_Z}{M_W}\right)(\bar d \gamma^\mu (1-\gamma^5) u)\,(\bar\nu_\ell \gamma_\mu (1-\gamma^5)\ell)$$

matching the (Wilson) lattice to the W-regularization.

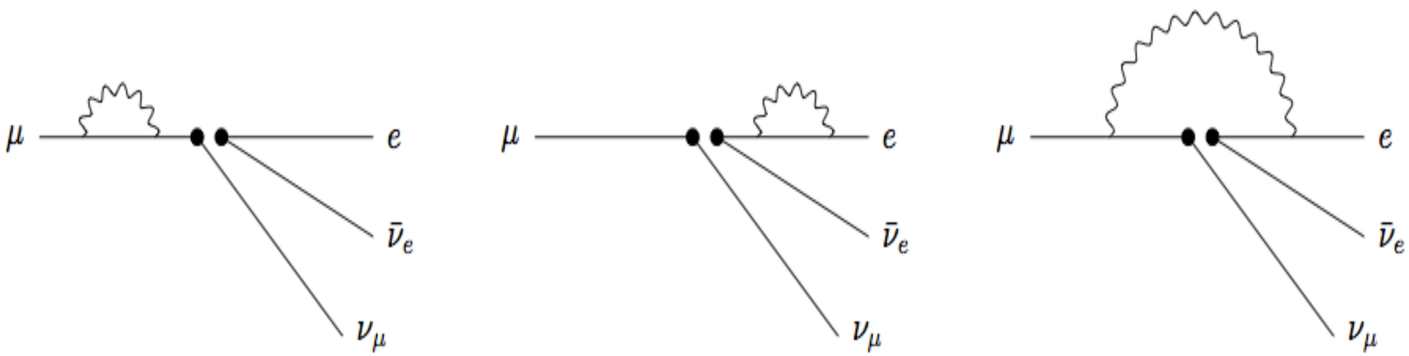
$$O_1^{\rm W-reg} = \left(1 + \frac{\alpha}{4\pi}\left(2\log a^2 M_W^2 - 15.539\right)\right) O_1^{\rm bare} + \frac{\alpha}{4\pi}\left(0.536\, O_2^{\rm bare} + 1.607\, O_3^{\rm bare} - 3.214\, O_4^{\rm bare} - 0.804\, O_5^{\rm bare}\right)$$

W Regularization

- The results for the widths are expressed in terms of G_F, the Fermi constant ($G_F = 1.16632(2) \times 10^{-5}\,\mathrm{GeV}^{-2}$). This is obtained from the muon lifetime:

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left[1 - \frac{8m_e^2}{m_\mu^2}\right]\left[1 + \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right]$$

  - This expression can be viewed as the definition of G_F. Many EW corrections are absorbed into the definition of G_F; the explicit $O(\alpha)$ corrections come from the following diagrams in the effective theory:



  together with the diagrams with a real photon.

  - These diagrams are evaluated in the W-regularisation in which the photon propagator is modified by:

A.Sirlin, PRD 22 (1980) 971

$$\frac{1}{k^2} \to \frac{M_W^2}{M_W^2 - k^2}\,\frac{1}{k^2}\,. \qquad \left(\frac{1}{k^2} = \frac{1}{k^2 - M_W^2} + \frac{M_W^2}{M_W^2 - k^2}\,\frac{1}{k^2}\right)$$

W Regularization

$$H_{\rm eff} = -\frac{G_F}{\sqrt{2}}\, V_{ud}^* \left(1 + \frac{\alpha}{\pi}\log\frac{M_Z}{M_W}\right) (\bar{d}\gamma^\mu(1-\gamma^5)u)\,(\bar{\nu}_\ell\gamma_\mu(1-\gamma^5)\ell)$$

matching the (Wilson) lattice to the W-regularization.

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where

$$O_1 = (\bar{d}\gamma^\mu(1-\gamma^5)u)\,(\bar{\nu}_\ell\gamma_\mu(1-\gamma^5)\ell) \qquad O_2 = (\bar{d}\gamma^\mu(1+\gamma^5)u)\,(\bar{\nu}_\ell\gamma_\mu(1-\gamma^5)\ell)$$

$$O_3 = (\bar{d}(1-\gamma^5)u)\,(\bar{\nu}_\ell(1+\gamma^5)\ell) \qquad O_4 = (\bar{d}(1+\gamma^5)u)\,(\bar{\nu}_\ell(1+\gamma^5)\ell)$$

$$O_5 = (\bar{d}\sigma^{\mu\nu}(1+\gamma^5)u)\,(\bar{\nu}_\ell\sigma_{\mu\nu}(1+\gamma^5)\ell)\,.$$

Rate at O(α)

$$\Gamma(\Delta E) = \Gamma_0 + \Gamma_1(\Delta E)$$

$|V_{ud}|$

where
$$\Gamma(\Delta E) = \int_0^{\Delta E} dE_\gamma \frac{d\Gamma}{dE_\gamma}$$

contrary to the hadron masses

at O(α) both Γ_0 and $\Gamma_1(\Delta E)$ are

INFRARED DIVERGENT

although the divergence cancel in the sum

F. Bloch, A. Nordsieck Phys.Rev. 52 (1937) T.D. Lee, M. Nauenberg Phys.Rev. 133 (1964)

and the infinite volume limit cannot be separately taken

MASTER FORMULA for the rate at O(α)

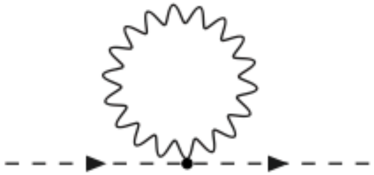
$$\Gamma(\Delta E) = \lim_{V\to\infty}(\Gamma_0 - \Gamma_0^{\rm pt}) + \lim_{V\to\infty}(\Gamma_0^{\rm pt} + \Gamma_1(\Delta E))$$

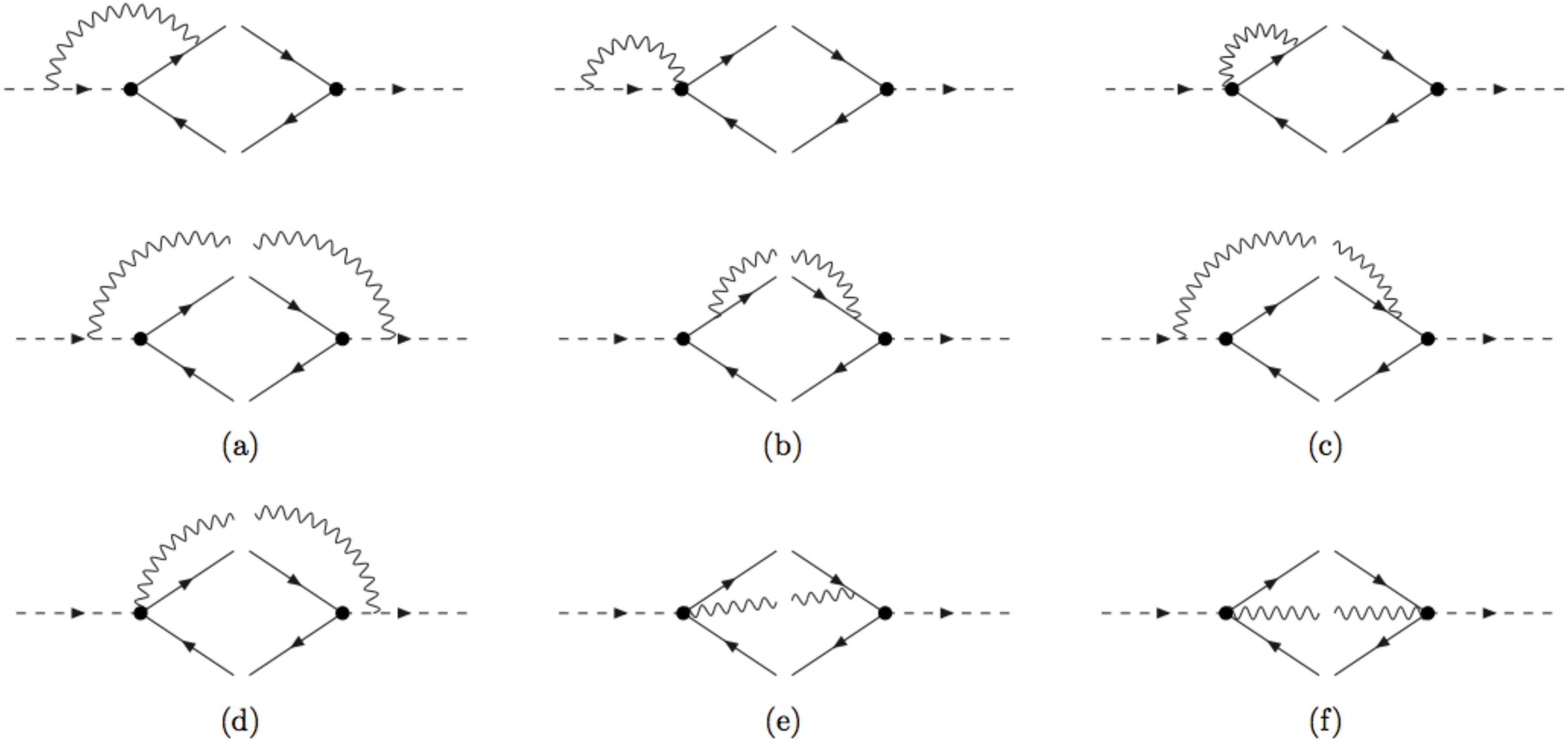
pt = point-like & perturbative

- the infrared divergences in Γ_0 and Γ_0^{pt} are exactly the same and cancel in the difference
- $\Gamma(\Delta E) = \Gamma_0^{pt} + \Gamma_1(\Delta E)$ is infrared finite since is a physical, well defined quantity *F. Bloch, A. Nordsieck Phys.Rev. 52 (1937) T.D. Lee, M. Nauenberg Phys.Rev. 133 (1964)*
- the infrared divergences in $\Delta\Gamma_0(L) = \Gamma_0 - \Gamma_0^{pt}$ and $\Gamma(\Delta E) = \Gamma_0^{pt} + \Gamma_1(\Delta E)$ cancel separately hence they can be regulated with different infrared cutoff
- Γ_0 and Γ_0^{pt} are also ultraviolet finite

We now discuss the two terms, $\Delta\Gamma_0(L)$ and $\Gamma(\Delta E)$

and





Leptonic decays at O(α) – Perturbative Calculation of $\Gamma(\Delta E) = \Gamma_0^{pt} + \Gamma_1(\Delta E)$

U.V. & Infrared finite but contains $\log(M_W)$ & $\log(\Delta E)$

$$\Gamma(\Delta E) = \Gamma_0^{\rm tree} \times \left(1 + \frac{\alpha}{4\pi}\left\{3\log\left(\frac{m_\pi^2}{M_W^2}\right) + \log\left(r_\ell^2\right) - 4\log(r_E^2) + \frac{2-10r_\ell^2}{1-r_\ell^2}\log(r_\ell^2)\right.\right.$$
$$-2\frac{1+r_\ell^2}{1-r_\ell^2}\log(r_E^2)\log(r_\ell^2) - 4\frac{1+r_\ell^2}{1-r_\ell^2}{\rm Li}_2(1-r_\ell^2) - 3$$
$$+\left[\frac{3+r_E^2-6r_\ell^2+4r_E(-1+r_\ell^2)}{(1-r_\ell^2)^2}\log(1-r_E) + \frac{r_E(4-r_E-4r_\ell^2)}{(1-r_\ell^2)^2}\log(r_\ell^2)\right.$$
$$\left.\left.\left.-\frac{r_E(-22+3r_E+28r_\ell^2)}{2(1-r_\ell^2)^2} - 4\frac{1+r_\ell^2}{1-r_\ell^2}{\rm Li}_2(r_E)\right]\right\}\right)$$

We think that this is a new result;
$\Gamma(\Delta E_1)$ *T.Kinoshita, PRL 2 (1959) 477*

$$r_E = \frac{2\Delta E}{m_\pi} \qquad r_\ell = \frac{m_\ell}{m_\pi}$$

Leptonic decays at O(α) – Perturbative Calculation of $\Gamma(\Delta E) = \Gamma_0^{pt} + \Gamma_1(\Delta E)$

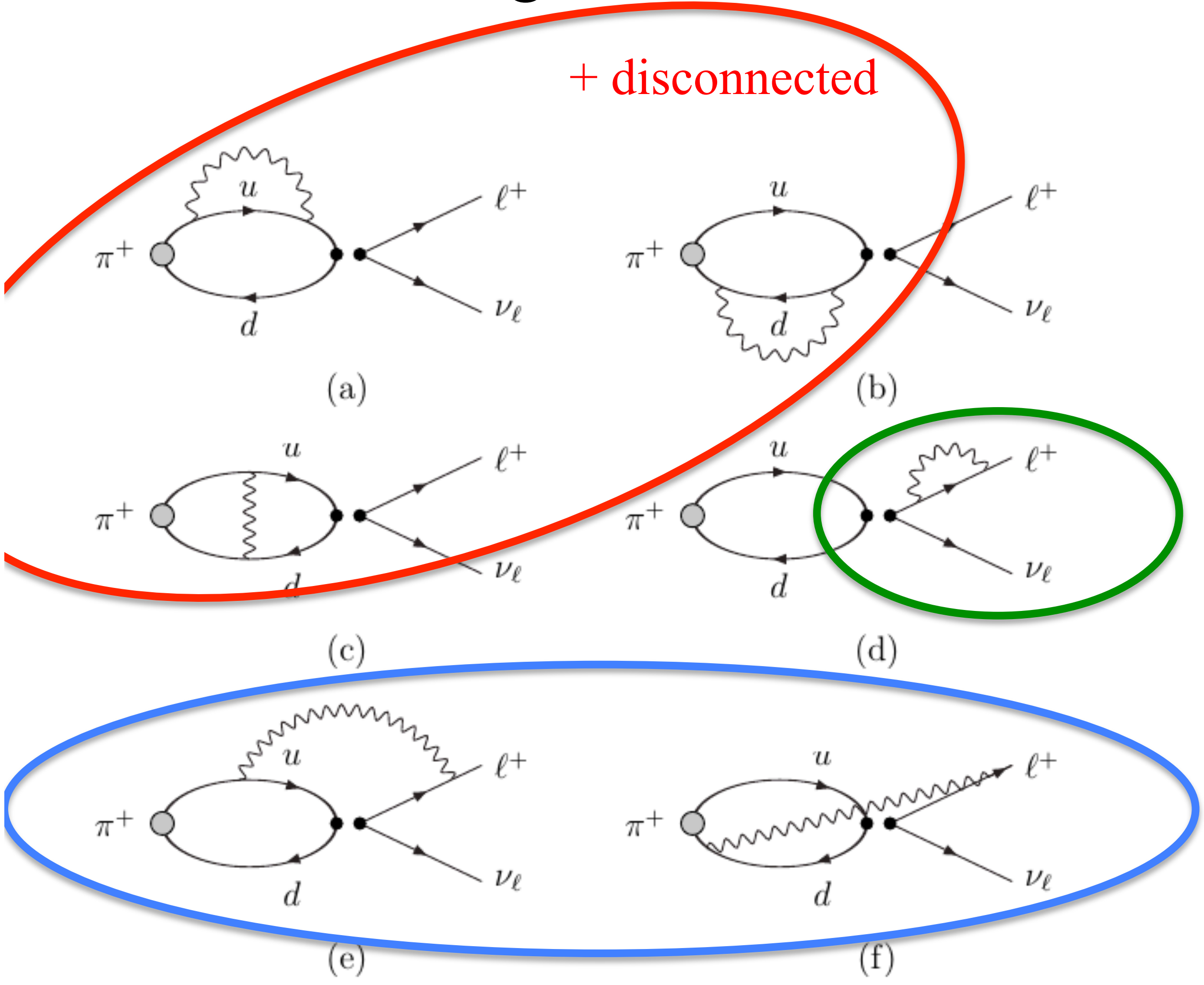
- The total rate is readily computed by setting r_E to its maximum value, namely $r_E = 1 - r_\ell^2$, giving

$$\Gamma^{pt} = \Gamma_0^{tree} \times \left\{ 1 + \frac{\alpha}{4\pi} \left(3\log\left(\frac{m_\pi^2}{M_W^2}\right) - 8\log(1-r_\ell^2) - \frac{3r_\ell^4}{(1-r_\ell^2)^2}\log(r_\ell^2) - 8\frac{1+r_\ell^2}{1-r_\ell^2}\mathrm{Li}_2(1-r_\ell^2) + \frac{13-19r_\ell^2}{2(1-r_\ell^2)} + \frac{6-14r_\ell^2-4(1+r_\ell^2)\log(1-r_\ell^2)}{1-r_\ell^2}\log(r_\ell^2) \right) \right\}.$$

- This result agrees with the well known results in literature providing an important check of our calculation.

Leptonic decays at O(α) – The first term of the Master Formula $\Delta\Gamma(L) = \Gamma_0 - \Gamma_0^{pt}$

- Each of the two terms is U.V. finite but contains $\log(M_W)$
- Infrared divergences cancel in the difference

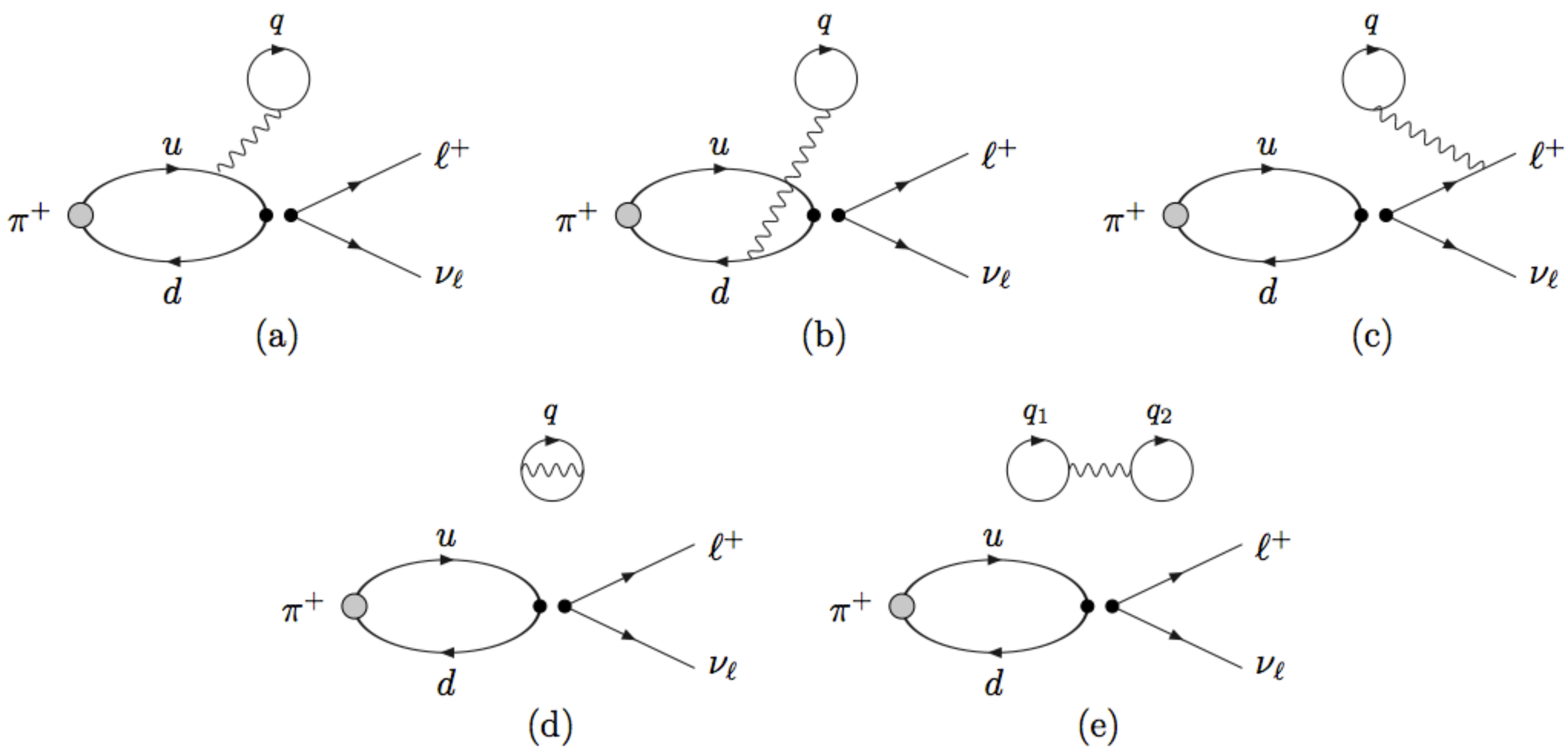


at this order we can take the difference of the amplitudes

Can be computed as discussed in arXiv: 1303.4896, Phys.Rev. D87(2013)

NOT by including the electromagnetic field in the action

DISCONNECTED DIAGRAMS



$$\Gamma_0^{pt}$$

Universality of the logarithmically divergent term and of the 1/L correction

(Tantalo at Lattice 2016, to appear)

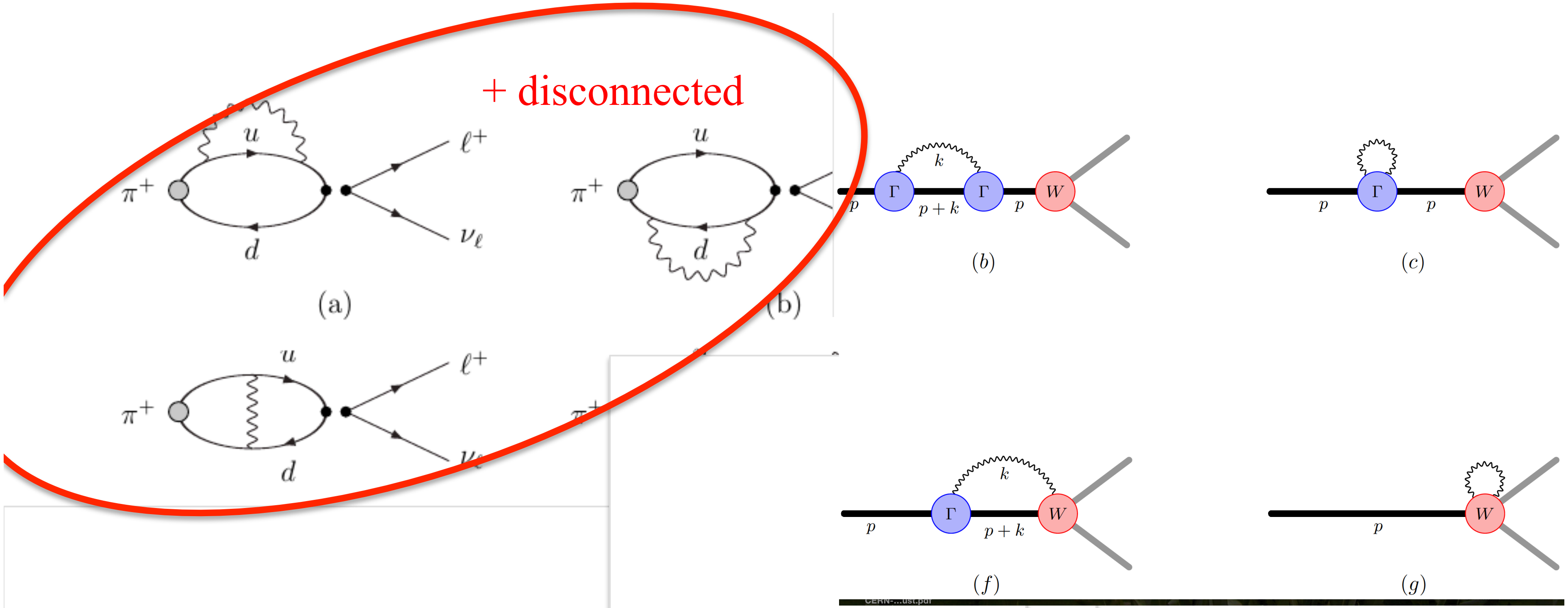
$$\Gamma_0^{\mathrm{pt}}(L) = C_0(r_\ell) + \tilde{C}_0(r_\ell)\mathrm{Log}\left[m_P L\right] + \frac{C_1(r_\ell)}{m_P L} + \dots$$

Depends on the ir regularization. The regularization dependent part does not depend on the internal structure of the hadron

Does NOT depend on the ir regularization or on the internal structure of the hadron

Thus $\Delta\Gamma(L) = \Gamma_0 - \Gamma_0^{pt}$ = Infrared finite, independent of the regularization up to $O(1/L^2)$

$$\Delta\Gamma(L) = \Gamma_0 - \Gamma_0^{pt}$$



Universality demonstrated via skeleton expansion or effective theory

- Wilson twisted-mass action for sea and valence up/down quarks, Osterwalder-Seiler action for valence strange (and charm) quark
- Iwasaki action for the gluons
- maximal twist guarantees an automatic O(a)-improvement for the above non-unitary setup

Courtesy of Silvano Simula Lattice 2016

gauge ensembles from the European Twisted Mass Collaboration (ETMC)

N_f = 2+1+1 dynamical sea quarks

three values of the lattice spacing: a ~ 0.0885 (36), 0.0815 (30), 0.0619 (18) fm

lattice sizes from 1.8 to 3 fm

$3 < M_\pi L < 6$

pion masses from 225 to 500 MeV

the strange quark mass at each β is calculated using the physical m_s mass and Z_m obtained by ETMC in NPB 887 (2014)

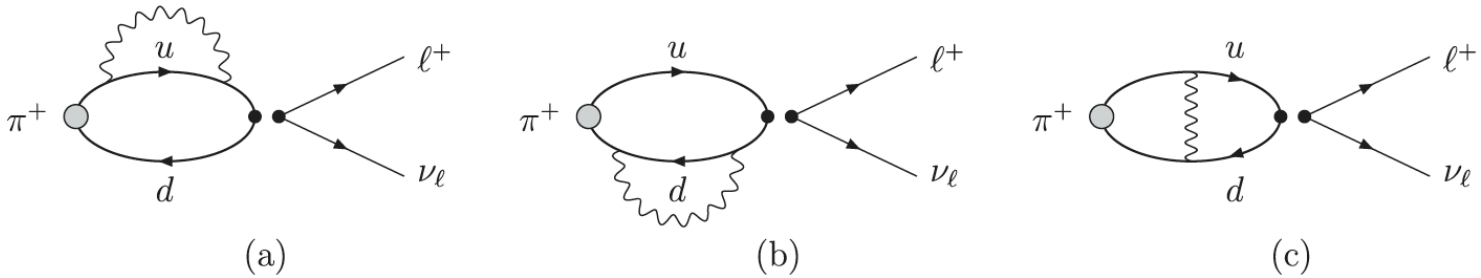
ensemble	β	V/a^4	$a\mu_{sea} = a\mu_\ell$	$a\mu_\sigma$	$a\mu_\delta$	N_{cfg}	$a\mu_s$	M_{π^+} (MeV)	M_{K^+} (MeV)	L (fm)	$M_\pi L$
A30.32	1.90	$32^3 \times 64$	0.0030	0.15	0.19	150	0.0236	278	564	2.9	4.0
A40.32			0.0040			100		318	573		4.6
A50.32			0.0050			150		351	581		5.1
A40.24		$24^3 \times 48$	0.0040			150		325	579	2.1	3.5
A60.24			0.0060			150		387	594		4.2
A80.24			0.0080			150		444	615		4.8
A100.24			0.0100			150		496	636		5.4
A40.20		$20^3 \times 48$	0.0040			150		331	583	1.8	3.0
B25.32	1.95	$32^3 \times 64$	0.0025	0.135	0.170	150	0.0209	261	542	2.6	3.5
B35.32			0.0035			150		304	551		4.1
B55.32			0.0055			150		377	574		5.0
B75.32			0.0075			80		438	596		5.8
B85.24		$24^3 \times 48$	0.0085			150		468	609	2.0	4.7
D15.48	2.10	$48^3 \times 96$	0.0015	0.12	0.1385	100	0.0161	226	526	3.0	3.4
D20.48			0.0020			100		257	529		3.9
D30.48			0.0030			100		313	546		4.8

all the relevant correlation functions calculated thanks to the

PRACE project **Pra10_2693**: "QED corrections to meson decay rates in LQCD"

18 Mcore-hours on the BG/Q system Fermi at Cineca (Italy), April 2015 - March 2016

* virtual photons between quarks: lattice calculation



$$\delta C^{(qq)}(t) = -\frac{1}{2} \sum_{\vec{x}, x_1, x_2} \langle 0 | T \left\{ J^{\rho}_{ew}(0)\, j^{em}_{\mu}(x_1)\, j^{em}_{\mu}(x_2)\, \phi^{\dagger}_{PS}(\vec{x}, -t) \right\} | 0 \rangle \Delta_{em}(x_1, x_2) \frac{p^{\rho}_{PS}}{M_{PS}}$$

(V-A) quark current; em quark current; PS interpolating field; photon propagator

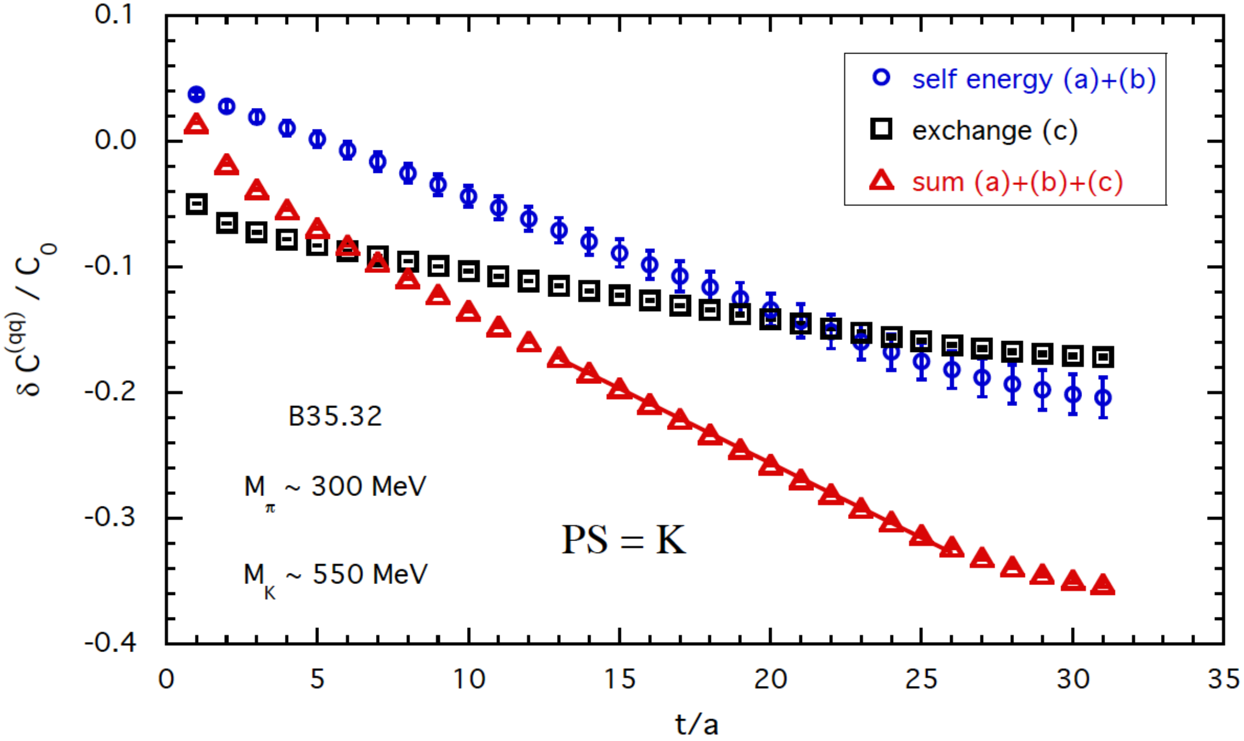
tree level: $C_0(t) = \sum_{\vec{x}} \langle 0 | T \left\{ J^{\rho}_{ew}(0)\, \phi^{\dagger}_{PS}(\vec{x}, -t) \right\} | 0 \rangle \frac{p^{\rho}_{PS}}{M_{PS}}$

large time distances: $C_0(t) + \alpha_{em} \delta C^{(qq)}(t) \xrightarrow[t>>a]{} \frac{Z_{PS} A^{(qq)}_{PS}}{2 M_{PS}} \left[e^{-M_{PS} t} - e^{-M_{PS}(T-t)} \right]$

$$M_{PS} = M^{(0)}_{PS} + \alpha_{em} \delta M_{PS}, \quad A^{(qq)}_{PS} = A^{(0)}_{PS} + \alpha_{em} \delta A^{(qq)}_{PS}, \quad Z_{PS} = Z^{(0)}_{PS} + \alpha_{em} \delta Z_{PS}$$

$$\frac{\delta C^{(qq)}(t)}{C_0(t)} \xrightarrow[t>>a]{} \frac{\delta\left[Z_{PS} A^{(qq)}_{PS} \right]}{Z^{(0)}_{PS} A^{(0)}_{PS}} + \frac{\delta M_{PS}}{M^{(0)}_{PS}} f(t) \qquad f(t) \equiv M^{(0)}_{PS} \left(\frac{T}{2} - t \right) \frac{e^{-M^{(0)}_{PS} t} + e^{-M^{(0)}_{PS}(T-t)}}{e^{-M^{(0)}_{PS} t} - e^{-M^{(0)}_{PS}(T-t)}} - 1 \approx -M^{(0)}_{PS} t$$

***** δM_{PS} from the slope and $\delta\left[Z_{PS} A^{(qq)}_{PS} \right]$ from the intercept *****



$\delta \mathrm{M}_{PS}$ from the slope

$\delta\left[Z_{PS} A_{PS}^{(qq)}\right]$ from the intercept

need to subtract δZ_{PS}

$$\delta C^{PS}(t) = -\frac{1}{2}\int d^3\vec{x}\, d^4x_1 d^4x_2 \langle 0|T\left\{\phi_{PS}(0) j_\mu^{em}(x_1) j_\mu^{em}(x_2)\phi_{PS}^\dagger(\vec{x},-t)\right\}|0\rangle \Delta_{em}(x_1,x_2)$$

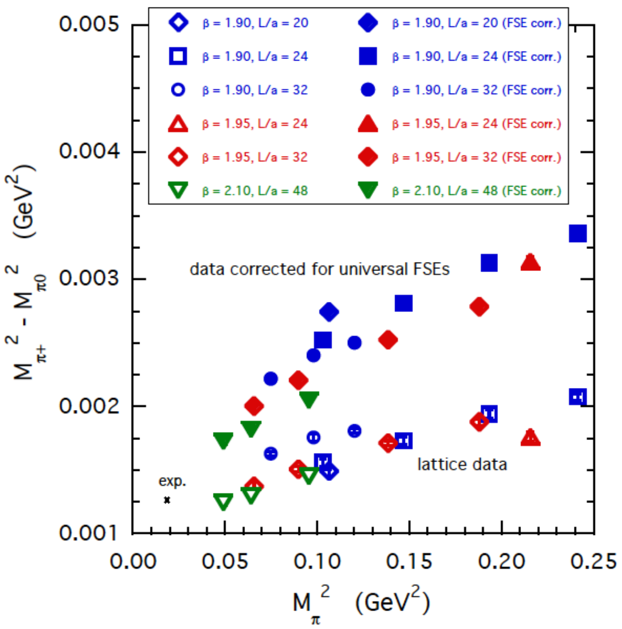
tree level: $C_0^{PS}(t) = \int d^3\vec{x} \langle 0|T\left\{\phi_{PS}(0)\,\phi_{PS}^\dagger(\vec{x},-t)\right\}|0\rangle$

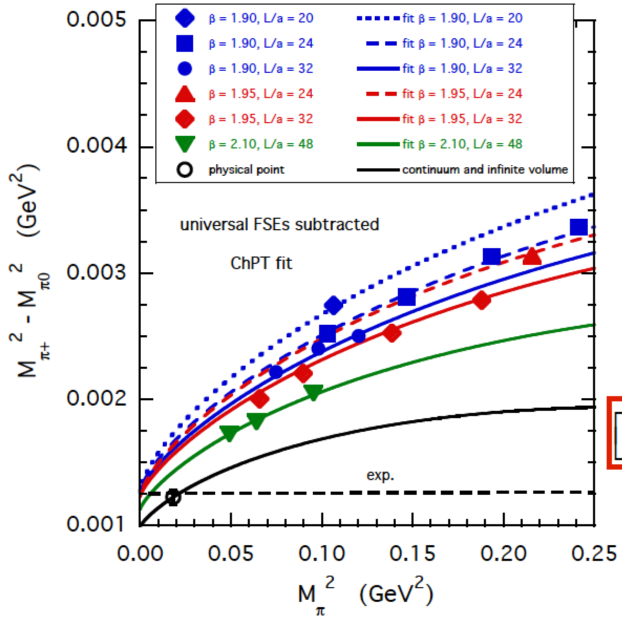
$$\frac{\delta C^{PS}(t)}{C_0^{PS}(t)} \xrightarrow[t>>a]{} 2\frac{\delta[Z_{PS}]}{Z_{PS}^{(0)}} - \frac{\delta M_{PS}}{M_{PS}^{(0)}} + \frac{\delta M_{PS}}{M_{PS}^{(0)}} f^{PS}(t) \qquad f^{PS}(t) \equiv M_{PS}^{(0)}\left(\frac{T}{2}-t\right)\frac{e^{-M_{PS}^{(0)}t} - e^{-M_{PS}^{(0)}(T-t)}}{e^{-M_{PS}^{(0)}t} + e^{-M_{PS}^{(0)}(T-t)}} \approx -M_{PS}^{(0)}t$$

$\delta \mathrm{M}_{PS}$ from the slope and $\delta[Z_{PS}]$ from the intercept

***** nice consistency between δM_{PS} extracted from δC and δC^{PS} *****

preliminary results for PS meson masses





pion

universal FSE: $-\alpha_{em}\kappa\left(2+M_\pi L\right)/L^2$

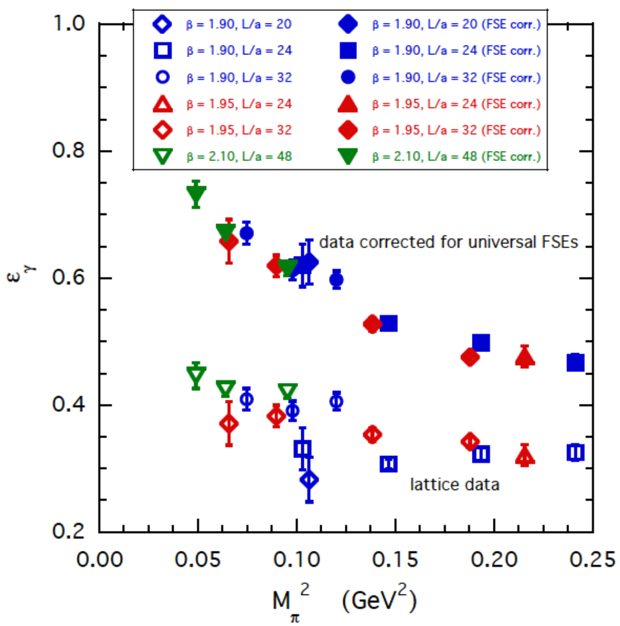
Hayakawa&Uno [PTP '08]

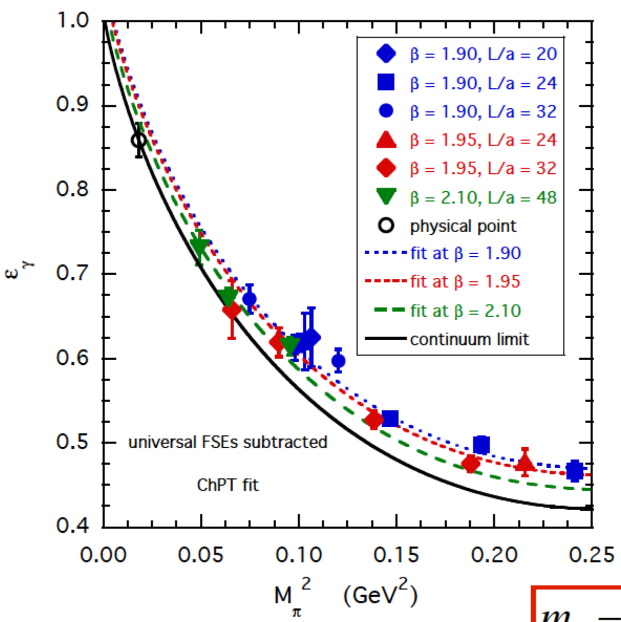
BMW: QED$_L$ on T^4 [Science '15]

residual FSE still visible

$$\left[M_{\pi^+}^2 - M_{\pi^0}^2\right]^{phys} = 1.226\,(58)_{stat}\,(96)_{syst}\,10^{-3}\ \mathrm{GeV}^2$$

$$\left[M_{\pi^+}^2 - M_{\pi^0}^2\right]^{exp} = 1.2612\,(1)\,10^{-3}\ \mathrm{GeV}^2$$





kaon

$$\varepsilon_\gamma = \frac{\left[M_{K^+}^2 - M_{K^0}^2 + M_{\pi^0}^2 - M_{\pi+}^2\right]^{QED}}{M_{\pi^+}^2 - M_{\pi^0}^2}$$

$$\varepsilon_\gamma^{phys} = 0.833\,(22)_{stat}\,(28)_{syst}\,(\ldots)_{qQED}$$

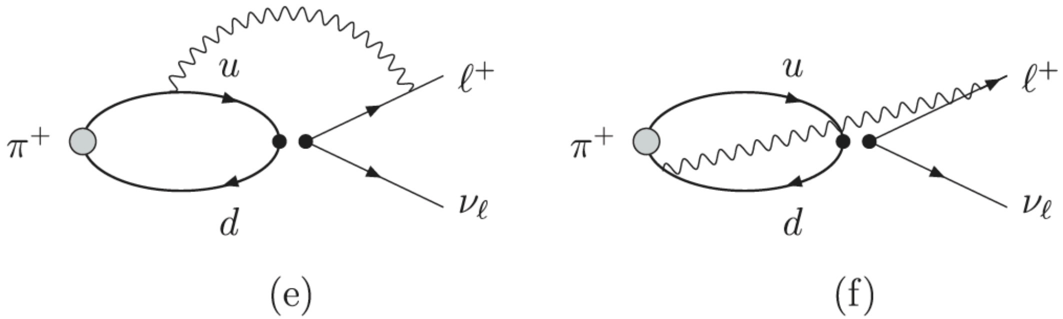
$$\varepsilon_\gamma^{FLAG} = 0.7\,(3)$$ [arXiv:1607.00299]

$$\delta_{IB} M_K = -4.66\,(6)_{stat}\,(22)_{syst}$$

$\overline{MS}(2\ \mathrm{GeV})$

$$m_d - m_u = 2.69\,(5)_{stat}\,(13)_{syst}\,(\ldots)_{qQED}\ \mathrm{MeV}$$

* virtual photons between quarks and final lepton: lattice calculation



[times the tree-level leptonic part]

$$\delta C^{(q\ell)}(t) = -\sum_{\vec{x},x_1,x_2} \langle 0 | T\left\{ J^{\rho}_{ew}(0) j^{em}_{\mu}(x_1) \phi^{\dagger}_{PS}(\vec{x},-t) \right\} | 0 \rangle \Delta_{em}(x_1,x_2) e^{E_\ell t_2 - i\vec{p}_\ell \cdot \vec{x}_2}$$

$$\cdot\, \overline{u}(p_\nu)\gamma_\rho(1-\gamma_5) S^\ell(0,x_2)\gamma_\mu v(p_\ell)\left[\overline{v}(p_\ell)\gamma_\sigma(1-\gamma_5)u(p_\nu)\frac{p^\sigma_{PS}}{M_{PS}}\right]$$

$S^\ell(0,x) =$ free twisted-mass lepton propagator $\quad E_\ell = \sqrt{m_\ell^2 + \vec{p}_\ell^{\,2}}, \quad E_\ell + E_\nu = M^{(0)}_{PS} \quad \vec{p}_\ell$ injected via non-periodic b.c.

tree-level: $C^{(q\ell)}_0(t) = C_0(t) Tr(p_\ell, p_{PS})$

leptonic trace: $Tr(p_\ell, p_{PS}) = \overline{u}(p_\nu)\gamma_\rho(1-\gamma_5)v(p_\ell)\overline{v}(p_\ell)\gamma_\sigma(1-\gamma_5)u(p_\nu)\frac{p^\rho_{PS}}{M_{PS}}\frac{p^\sigma_{PS}}{M_{PS}}$

* expanding the (V-A) structure of the quark e.w. current:

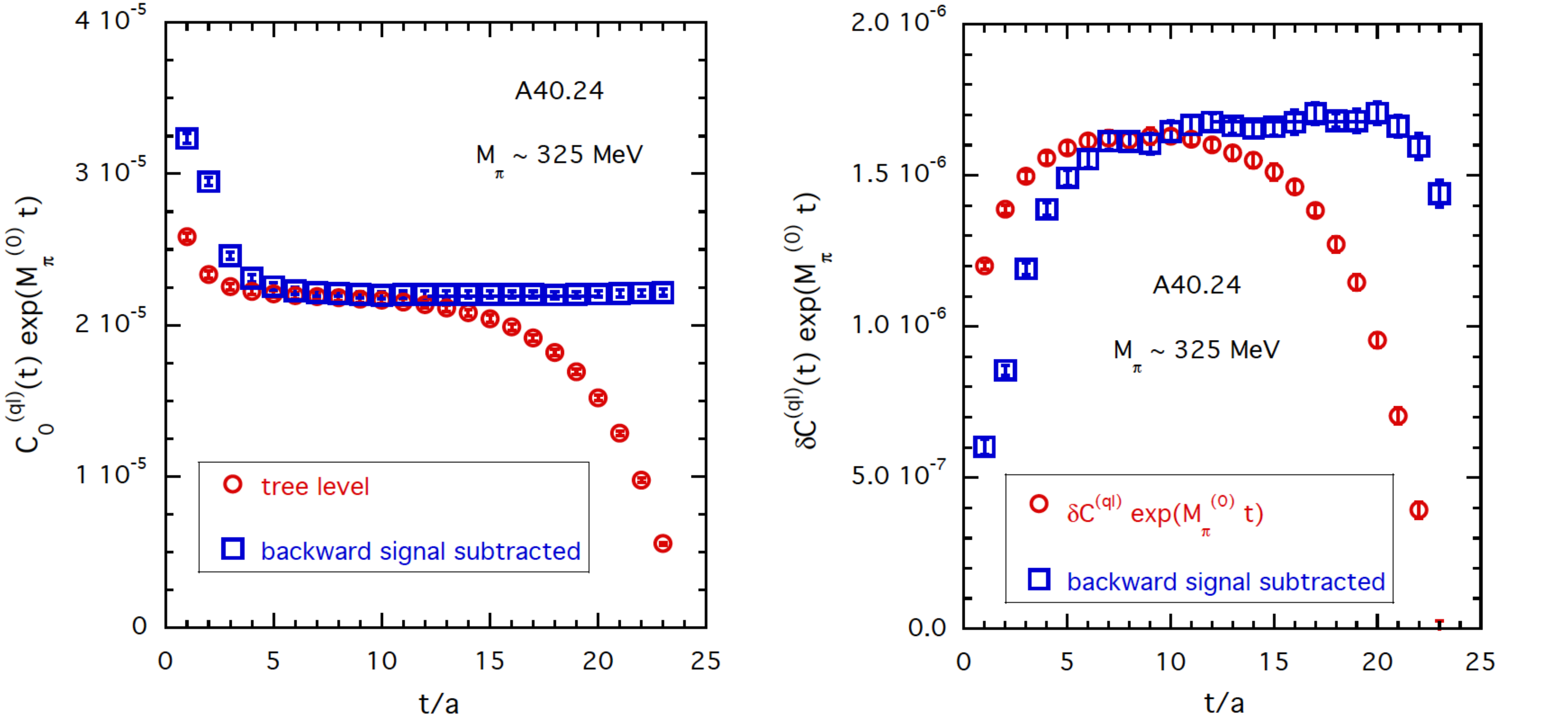
$$\delta C^{(q\ell)}(t) = Z_A\left[\delta C^{(V_0)}(t) + \delta C^{(V_k)}(t)\right] + Z_V\left[\delta C^{(A_0)}(t) + \delta C^{(A_k)}(t)\right]$$

(twisted-mass renormalization)

$$\delta C^{(q\ell)}(t) \xrightarrow[t>>a]{} \frac{Z^{(0)}_{PS}}{2M^{(0)}_{PS}} \delta A^{(q\ell)}_{PS}\, Tr(p_\ell, p_{PS})\left[e^{-M^{(0)}_{PS} t} \pm \text{backward signals}\right]$$

depending on the time/spatial components

* subtraction of backward signals: $$\bar{C}(t)e^{M_{PS}^{(0)}t} \equiv \frac{1}{2}\left[C(t)+\frac{C(t-1)-C(t+1)}{e^{M_{PS}^{(0)}}-e^{-M_{PS}^{(0)}}}\right]e^{M_{PS}^{(0)}t} \xrightarrow[t>>a]{} const.$$



2-point plateau region

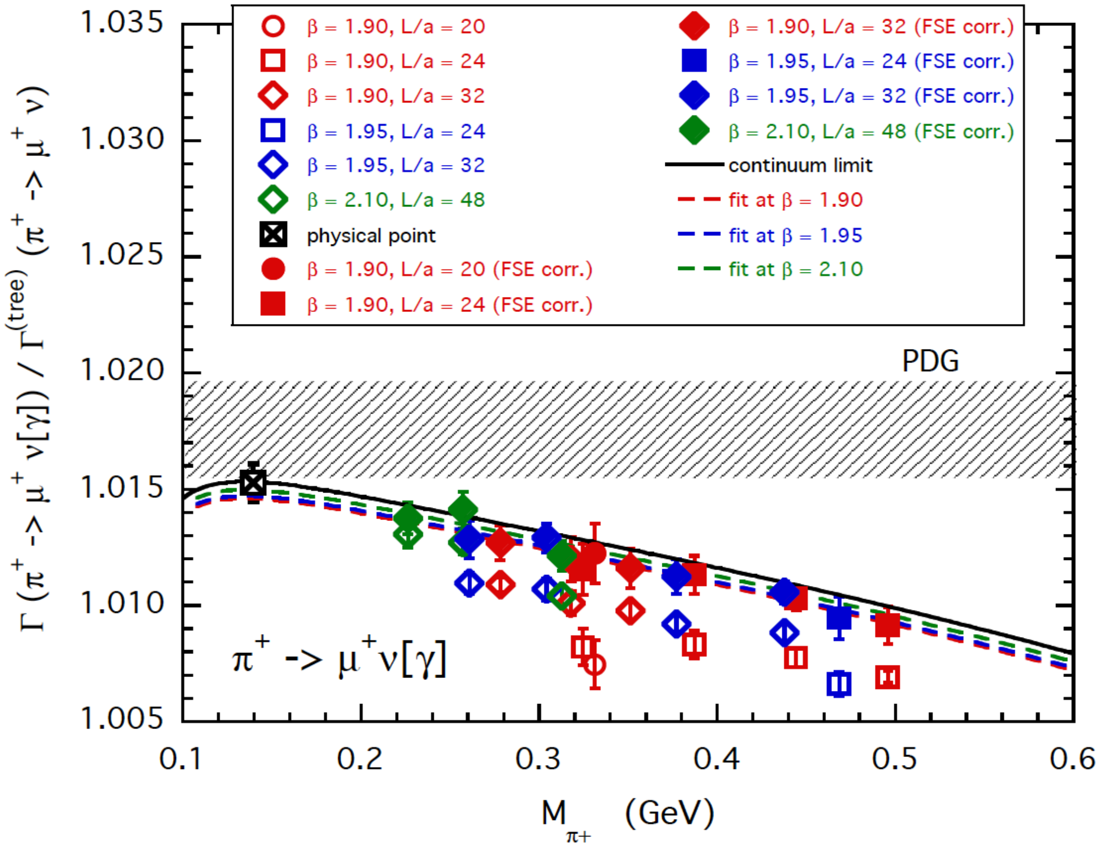
* after subtraction of backward signals: $$\frac{\delta\bar{C}^{(q\ell)}(t)}{\bar{C}_0^{(q\ell)}(t)} \xrightarrow[t>>a]{} \frac{\delta A_{PS}^{(q\ell)}}{A_{PS}^{(0)}}$$

* **chiral extrapolation** [Knecht et al., EPJC 12 (2000) 469]

updated with respect to Lattice 2016

$$R_\pi\left(\Delta E_\gamma^{\max}\right) = 1 + \alpha_{em}\left\{4\pi E(\mu) + \frac{3}{4\pi}\log\left(\frac{\xi}{\mu^2}\right) + A_1\xi + Da^2 + \delta\Gamma^{pt}\left(\Delta E_\gamma^{\max}\right) + K_\pi^{FSE}(L)\right\} \qquad \xi \equiv \frac{M_\pi^2}{(4\pi f_0)^2}$$

residual (structure-dependent) FSEs: $K_\pi^{FSE}(L) = \frac{K_2}{(M_\pi L)^2} + \frac{K_2^\ell}{(E_\ell L)^2}$ $\qquad E, A_1, D, K_2, K_2^\ell$: 5 free parameters



$$\pi^+ \to \mu^+\nu[\gamma]$$

$$\Delta E_\gamma^{\max} \cong 29.6 \text{ MeV}$$

open markers: lattice data with subtraction of universal FSEs up to 1/L

full markers: lattice data with subtraction of both universal and structure-dependent FSEs

$\chi_{d.o.f.} \simeq 0.7$

$$R_\pi^{phys}\left(\Delta E_\gamma^{\max}\right) = 1.0169\,(8)_{stat+fit}\,(11)_{chiral}\,(7)_{FSE}\,(2)_{a^2}\,(\ldots)_{qQED}$$
$$= 1.0169\,(8)_{stat+fit}\,(13)_{syst}\,(\ldots)_{qQED} = 1.0169\,(15)(\ldots)_{qQED}$$

$$\frac{R_\pi^{phys}\left(\Delta E_\gamma^{\max}\right)}{R_\pi^{PDG}\left(\Delta E_\gamma^{\max}\right)} = 0.9993\,(26)(\ldots)_{qQED}$$

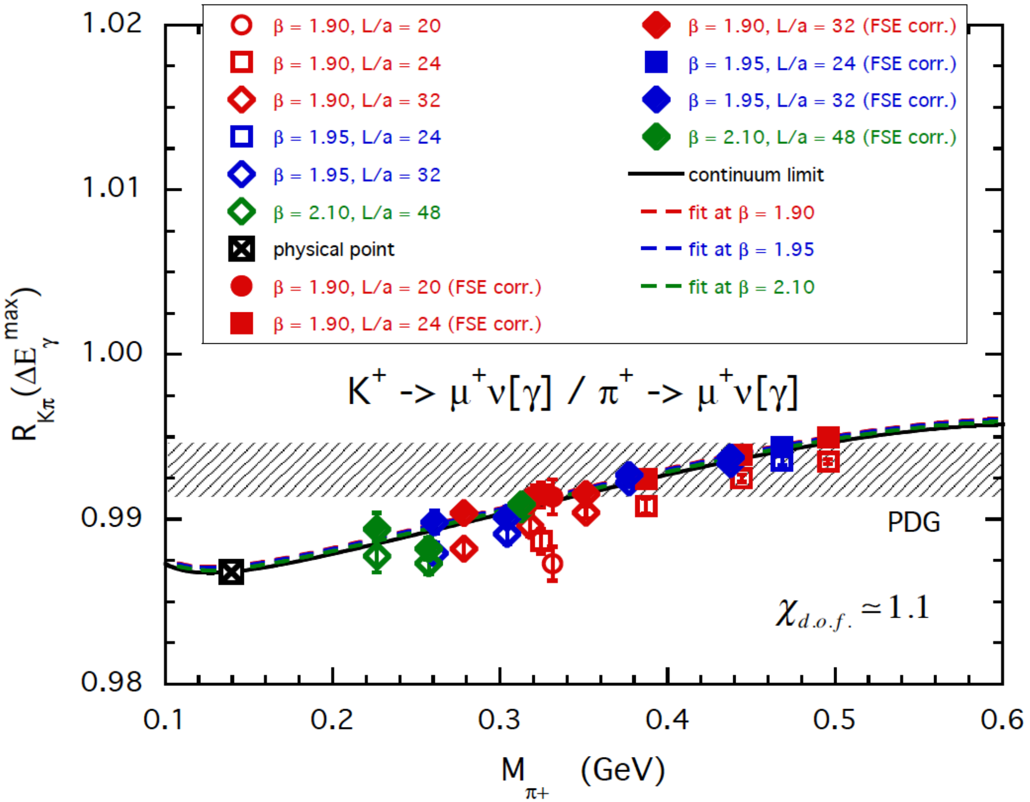
preliminary

* **K/π ratio:** $R_{K\pi}(\Delta E_\gamma) = 1 + R_K(\Delta E_\gamma) - R_\pi(\Delta E_\gamma)$

$$R_{K\pi}(\Delta E_\gamma^{max}) = 1 + \alpha_{em}\left\{\tilde{A}_0 - \frac{3}{4\pi}\log\left(\frac{M_\pi^2}{M_K^2}\right) + \tilde{A}_1\xi + \tilde{A}_2\xi^2 + \tilde{D}a^2 + \delta\Gamma_K^{pt}(\Delta E_\gamma^{max}) - \delta\Gamma_\pi^{pt}(\Delta E_\gamma^{max}) + K_{K\pi}^{FSE}(L)\right\}$$

$$K_{K\pi}^{FSE}(L) = \frac{\tilde{K}_2}{(M_K L)^2} + \frac{\tilde{K}_2^\ell}{(E_\ell^{(K)} L)^2} - \frac{K_2}{(M_\pi L)^2} - \frac{K_2^\ell}{(E_\ell^{(\pi)} L)^2}$$

$\tilde{A}_0, \tilde{A}_1, \tilde{A}_2, \tilde{D}, \tilde{K}_2, \tilde{K}_2^\ell$: 6 free parameters



$$\frac{K^+ \to \mu^+\nu[\gamma]}{\pi^+ \to \mu^+\nu[\gamma]} \qquad \Delta E_\gamma^{max} \cong 235.5 \text{ MeV}$$

PDG '16

$R_{K\pi}(\Delta E_\gamma^{max}) = 0.9931\,(17)$

used to get

$|V_{us}|\, f_{K^-} / |V_{ud}|\, f_{\pi^-} = 0.27599\,(29)_{exp}\,(24)_{R_{K\pi}}$

open markers: lattice data with subtraction of universal FSEs up to 1/L

full markers: lattice data with subtraction of both universal and structure-dependent FSEs

$$R_{K\pi}^{phys}(\Delta E_\gamma^{max}) = 0.9863\,(11)_{stat+fit}\,(6)_{chiral}\,(1)_{FSE}\,(1)_{a^2}\,(\ldots)_{qQED}$$
$$= 0.9863\,(11)_{stat+fit}\,(6)_{syst}\,(\ldots)_{qQED} = 0.9863\,(13)(\ldots)_{qQED}$$

preliminary

$$\frac{R_{K\pi}^{phys}(\Delta E_\gamma^{max})}{R_{K\pi}^{PDG}(\Delta E_\gamma^{max})} = 0.9931\,(21)(\ldots)_{qQED}$$

To conclude

- We have presented **a method to compute QED corrections** to hadronic processes;
- For these quantities the presence of infrared divergences in the intermediate stages of the calculation make the procedure much more complicated than in the case of the hadronic spectrum;
- In order to obtain the physical answer virtual corrections and real photon emissions must be combined together;
- It is not sufficient to add the electromagnetic interaction to the quark action, because separate explicit real and virtual emission diagrams must be evaluated for any given process;
- We have discussed a specific case, namely the radiative corrections to the leptonic decay of charged pseudoscalar mesons. **The method can e however be extended to many other cases like for example to semileptonic decays.**

To conclude

- The condition for the applicability of our strategy is that **there is a mass gap** between the decaying particle and the intermediate states generated by the emission of the photon, and that none of these states is lighter than the initial hadron.
- In the calculation of electromagnetic corrections a general issue is finite size effects. In this respect our method reduces to compute infrared finite, gauge invariant quantities for which we do expect **finite size corrections which are comparable to those encountered for the spectrum**. This expectation will be checked in forthcoming numerical studies.
- The implementation of our method, although challenging, is **within reach of the present lattice technology.** The accuracy necessary to make the results phenomenologically interesting is not exceedingly high since the effect that we want to predict is, in general, of the order of a few percent.

THANKS FOR YOUR ATTENTION