Current Status of ε_K with lattice QCD inputs

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Contents

LANL–SWME Collaboration
1998 — Present

LANL–SWME Collaboration I

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  Dr. Jon Bailey (R.A. Prof.),
  9 graduate students.

- University of Washington (SWME):
  Prof. Stephen Sharpe

- Brookhaven National Laboratory (SWME):
  Dr. Chulwoo Jung (Staff Scientist)

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- Los Alamos National Laboratory:
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  Dr. Tanmoy Bhattacharya (Staff)
  Dr. Boram Yoon (Staff)
  Dr. Yong-Chull Jang (Postdoc)
- University of Bielefeld (SWME):
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Lattice Gauge Theory Research Center (SNU)

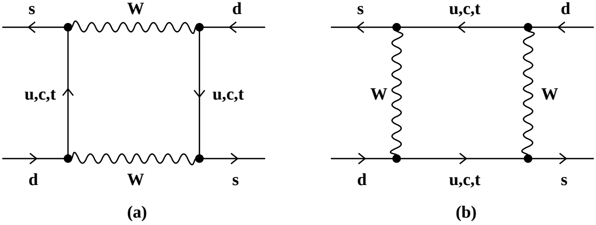
- Center Leader: Prof. Weonjong Lee.
- Research Assistant Professor: Dr. Jon Bailey
- 9 graduate students
- Secretary: Mrs. Sora Park.
- more details on `http://lgt.snu.ac.kr/`.

Group Photo (2014)

CP Violation in Neutral Kaons

Kaon Eigenstates and ε

- Flavor eigenstates, $K^0 = (\bar{s}d)$ and $\bar{K}^0 = (s\bar{d})$ mix via box diagrams.



Kaon Eigenstates and ε

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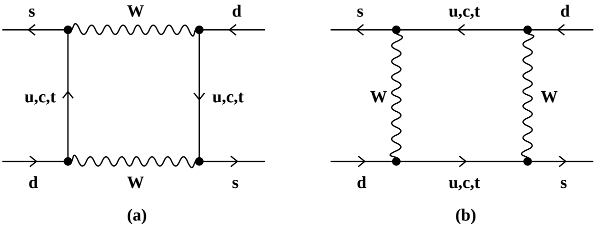

- CP eigenstates K_1(even) and K_2(odd).

$$K_1 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0) \qquad K_2 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0)$$

Kaon Eigenstates and ε

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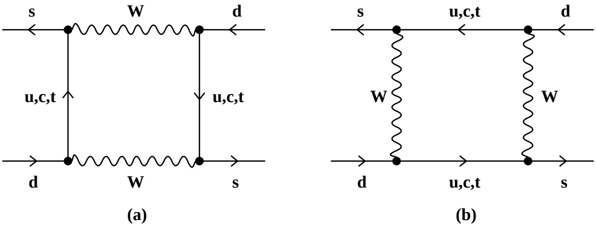


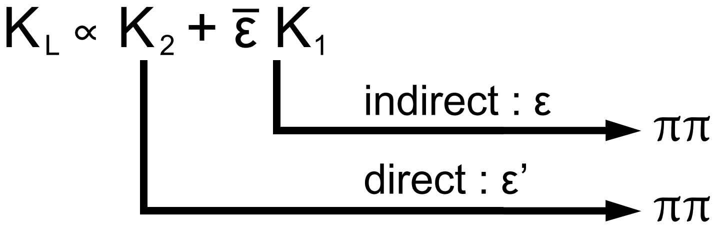
- CP eigenstates K_1(even) and K_2(odd).

$$K_1 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0) \qquad K_2 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0)$$

- Neutral Kaon eigenstates K_S and K_L.

$$K_S = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}(K_1 + \bar{\varepsilon}K_2) \qquad K_L = \frac{1}{\sqrt{1+|\bar{\varepsilon}|^2}}(K_2 + \bar{\varepsilon}K_1)$$

Indirect CP violation and direct CP violation



ε_K and $\hat{B}_K$, V_{cb} I

- Definition of ε_K

$$\varepsilon_K = \frac{A[K_L \to (\pi\pi)_{I=0}]}{A[K_S \to (\pi\pi)_{I=0}]}$$

- Master formula for ε_K in the Standard Model.

$$\varepsilon_K = \exp(i\theta)\,\sqrt{2}\,\sin(\theta)\left(C_\varepsilon\, X_{\mathsf{SD}}\, \hat{B}_K + \frac{\xi_0}{\sqrt{2}} + \xi_{\mathsf{LD}} \right) + \mathcal{O}(\omega\varepsilon') + \mathcal{O}(\xi_0 \Gamma_2/\Gamma_1)$$

$$X_{\mathsf{SD}} = \mathrm{Im}\lambda_t \Big[\mathrm{Re}\,\lambda_c\, \eta_{cc}\, S_0(x_c) - \mathrm{Re}\,\lambda_t\, \eta_{tt}\, S_0(x_t) - (\mathrm{Re}\,\lambda_c - \mathrm{Re}\,\lambda_t)\, \eta_{ct}\, S_0(x_c, x_t) \Big]$$

ε_K and $\hat{B}_K$, V_{cb} II

$$\lambda_i = V_{is}^* V_{id}, \qquad x_i = m_i^2 / M_W^2, \qquad C_\varepsilon = \frac{G_F^2 F_K^2 m_K M_W^2}{6\sqrt{2}\,\pi^2 \Delta M_K}$$

$$\frac{\xi_0}{\sqrt{2}} = \frac{1}{\sqrt{2}} \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0} \approx -5\%$$

ξ_{LD} = Long Distance Effect $\approx 2\%$ $\longrightarrow$ systematic error

- Inami-Lim functions:

$$S_0(x_i) = x_i \left[\frac{1}{4} + \frac{9}{4(1-x_i)} - \frac{3}{2(1-x_i)^2} - \frac{3x_i^2 \ln x_i}{(1-x_i)^3} \right],$$

$$S_0(x_i, x_j) = \left\{ \frac{x_i x_j}{x_i - x_j} \left[\frac{1}{4} + \frac{3}{2(1-x_i)} - \frac{3}{4(1-x_i)^2} \right] \ln x_i - (i \leftrightarrow j) \right\} - \frac{3 x_i x_j}{4(1-x_i)(1-x_j)}$$

ε_K and $\hat{B}_K$, V_{cb} III

$$S_0(x_t) \longrightarrow +70\%$$
$$S_0(x_c, x_t) \longrightarrow +44\%$$
$$S_0(x_c) \longrightarrow -14\%$$

- Dominant contribution (≈70%) comes with $|V_{cb}|^4$.

$$\mathrm{Im}\lambda_t \cdot \mathrm{Re}\lambda_t = \bar{\eta}\lambda^2 |V_{cb}|^4 (1-\bar{\rho})$$
$$\mathrm{Re}\lambda_c = -\lambda(1-\frac{\lambda^2}{2}) + \mathcal{O}(\lambda^5)$$
$$\mathrm{Re}\lambda_t = -(1-\frac{\lambda^2}{2})A^2\lambda^5(1-\bar{\rho}) + \mathcal{O}(\lambda^7)$$
$$\mathrm{Im}\lambda_t = \eta A^2 \lambda^5 + \mathcal{O}(\lambda^7)$$

ε_K and $\hat{B}_K$, V_{cb} IV

- Definition of $\hat{B}_K$ in standard model.

$$B_K = \frac{\langle \bar{K}_0 | [\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{s}\gamma_\mu(1-\gamma_5)d] | K_0 \rangle}{\frac{8}{3}\langle \bar{K}_0 | \bar{s}\gamma_\mu\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu\gamma_5 d | K_0 \rangle}$$

$$\hat{B}_K = C(\mu) B_K(\mu), \qquad C(\mu) = \alpha_s(\mu)^{-\frac{\gamma_0}{2b_0}} [1 + \alpha_s(\mu) J_3]$$

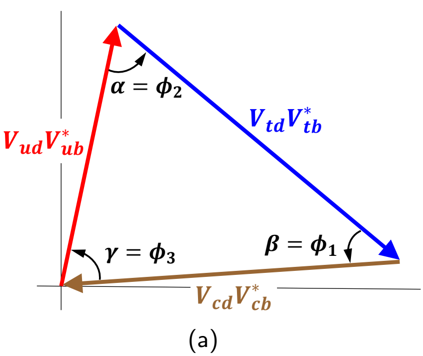
- Experiment:

$$\varepsilon_K = (2.228 \pm 0.011) \times 10^{-3} \times e^{i\phi_\varepsilon}$$

$$\phi_\varepsilon = 43.52(5)^\circ$$

ε_K on the lattice

Unitarity Triangle → $(\bar{\rho},\ \bar{\eta})$



Global UT Fit and Angle-Only-Fit (AOF)

Global UT Fit

- Input: $|V_{ub}|/|V_{cb}|$, Δm_d, $\Delta m_s/\Delta m_d$, ε_K, and $\sin(2\beta)$.
- Determine the UT apex $(\bar{\rho}, \bar{\eta})$.
- Take λ from

$$|V_{us}| = \lambda + \mathcal{O}(\lambda^7)\,,$$

which comes from K_{l3} and $K_{\mu 2}$.

- Disadvantage: unwanted correlation between $(\bar{\rho}, \bar{\eta})$ and ε_K.

AOF

- Input: $\sin(2\beta)$, $\cos(2\beta)$, $\sin(\gamma)$, $\cos(\gamma)$, $\sin(2\beta+\gamma)$, $\cos(2\beta+\gamma)$, and $\sin(2\alpha)$.
- Determine the UT apex $(\bar{\rho}, \bar{\eta})$.
- Take λ from $|V_{us}| = \lambda + \mathcal{O}(\lambda^7)\,,$ which comes from K_{l3} and $K_{\mu 2}$.
- Use $|V_{cb}|$ to determine A.

$$|V_{cb}| = A\lambda^2 + \mathcal{O}(\lambda^7)$$

- Advantage: NO correlation between $(\bar{\rho}, \bar{\eta})$ and ε_K.

Inputs of Angle-Only-Fit (AOF)

- $A_{\mathsf{CP}}(J/\psi\ K_s) \to S_{\psi K_s} = \sin(2\beta)$ with assumption of $S_{\psi K_s} \ggg C_{\psi K_s}$.
- $(B \to DK) + (B \to [K\pi]_D K) +$ (Dalitz method) give $\sin(\gamma)$ and $\cos(\gamma)$.
- $S(D^-\pi^+)$ and $S(D^+\pi^-)$ give $\sin(2\beta + \gamma)$ and $\cos(2\beta + \gamma)$.
- $(B^0 \to \pi^+\pi^-) + (B^0 \to \rho^+\rho^-) + (B^0 \to (\rho\pi)^0)$ give $\sin(2\alpha)$.
- Combining all of these gives β, γ, and α, which leads to the UT apex $(\bar\rho,\ \bar\eta)$.

Wolfenstein Parameters

Input Parameters for Angle-Only-Fit (AOF)

- ϵ_K, $\hat{B}_K$, and $|V_{cb}|$ are used as inputs to determine the UT angles in the global fit of `UTfit` and `CKMfitter`.
- Instead, we can use angle-only-fit result for the UT apex $(\bar{\rho}, \bar{\eta})$.
- Then, we can take λ independently from

$$|V_{us}| = \lambda + \mathcal{O}(\lambda^7)\,,$$

which comes from K_{l3} and $K_{\mu 2}$.

- Use $|V_{cb}|$ instead of A.

$$|V_{cb}| = A\lambda^2 + \mathcal{O}(\lambda^7)$$

λ	0.22537(61)	[1] CKMfitter
	0.2255(6)	[1] UTfit
	0.2253(8)	[1] $\lvert V_{us}\rvert$ (AOF)
$\bar{\rho}$	0.117(21)	[1] CKMfitter
	0.124(24)	[1] UTfit
	0.139(29)	[2] UTfit (AOF)
$\bar{\eta}$	0.353(13)	[1] CKMfitter
	0.354(15)	[1] UTfit
	0.337(16)	[2] UTfit (AOF)

Input Parameters of B_K, V_{cb} and others

B_K

$\hat{B}_K$	$0.7625(97)$	[3] FLAG
	$0.7379(47)(365)$	[4] SWME
	$0.7499(24)(150)$	[5] RBC-UK

$|V_{cb}| \times 10^3$

$B \to X_c \ell \bar{\nu}$	$42.00(64)$	[6]
$B \to D^* \ell \bar{\nu}$	$39.04(49)(53)(19)$	[7]
$B \to D \ell \bar{\nu}$	$40.70(100)(20)$	[8]
ex-combined	$39.62(60)$	wlee

Others

G_F	$1.1663787(6) \times 10^{-5}$ GeV^{-2}	[1]
M_W	$80.385(15)$ GeV	[1]
$m_c(m_c)$	$1.2733(76)$ GeV	[9]
$m_t(m_t)$	$163.3(2.7)$ GeV	[10]
η_{cc}	$1.72(27)$	[11]
η_{tt}	$0.5765(65)$	[12]
η_{ct}	$0.496(47)$	[13]
θ	$43.52(5)^\circ$	[1]
m_{K^0}	$497.614(24)$ MeV	[1]
ΔM_K	$3.484(6) \times 10^{-12}$ MeV	[1]
F_K	$156.2(7)$ MeV	[1]

Current Status of exclusive $|V_{cb}|$ in 2016

- $B \to D^* \ell \bar{\nu}$ at zero recoil: (in units of 10^{-3})

$$V_{cb} = 39.04 \pm 0.49(\text{exp}) \pm 0.53(\text{QCD}) \pm 0.19(\text{QED})$$

from PRD89.114504(2014) FNAL-MILC

- $B \to D \ell \bar{\nu}$ at non-zero recoil: (in units of 10^{-3})

$$V_{cb} = 40.7 \pm 1.0(\text{QCD+exp}) \pm 0.2(\text{QED})$$

from arxiv:1511.06884 by Carleton Detar

1. FNAL-MILC: PRD92, 034506 (2015)
2. HPQCD: PRD92, 054510 (2015)
3. Babar: PRD79, 012002 (2009)
4. Belle: EPSC of HEP 306 (2015), EPSC of HEP 824 (2015)

Current Status of inclusive $|V_{cb}|$ in 2016

- $B \to X_c \ell \bar{\nu}$: (in units of 10^{-3})

$$V_{cb} = 42.00 \pm 0.64$$ from arxiv:1606.06174

- $B \to X_u \ell \bar{\nu}$ (in units of 10^{-3})

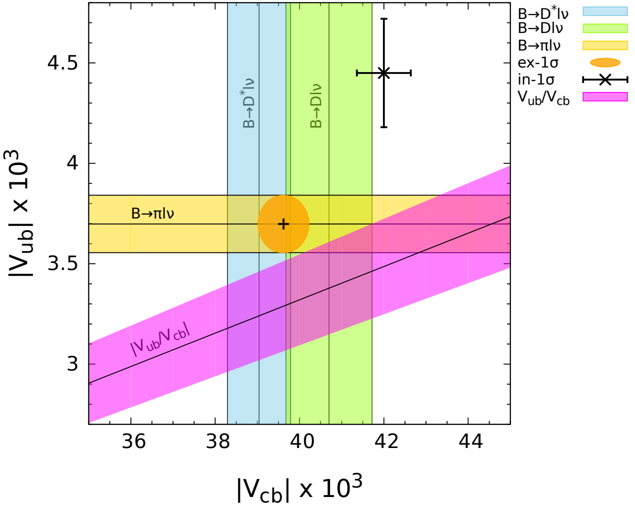
$$V_{ub} = 4.45 \pm 0.16(\text{exp}) \pm 0.22(\text{th})$$ from arxiv:1412.7515 HFAG

- $|V_{ub}|/|V_{cb}| = 0.1060 \pm 0.0067.$
- By the way, LHCb data combined with lattice form factors:

$$|V_{ub}|/|V_{cb}| = 0.083 \pm 0.004(\text{exp}) \pm 0.004(\text{lat})$$

- There is a 2.6σ tension in $|V_{ub}|/|V_{cb}|$.

Current Status of $|V_{cb}|$ in 2016



ξ_0
Indirect Method

$$\xi_0 = \frac{\mathrm{Im} A_0}{\mathrm{Re}\, A_0}, \quad \xi_2 = \frac{\mathrm{Im} A_2}{\mathrm{Re}\, A_2}.$$

ξ_0	$-1.63(19) \times 10^{-4}$	RBC-UK-2015 [14]

- RBC-UKQCD calculated $\mathrm{Im} A_2$. $\mathrm{Im} A_2 \to \xi_2 \to \varepsilon'_K/\varepsilon_K \to \xi_0$

$$\mathrm{Re}\left(\frac{\varepsilon'_K}{\varepsilon_K}\right) = \frac{1}{\sqrt{2}|\varepsilon_K|}\omega\left(\xi_2 - \xi_0\right).$$

  Other inputs ω, ε_K and $\varepsilon'_K/\varepsilon_K$ are taken from the experimental values.
- Here, we choose an approximation of $\cos(\phi_{\epsilon'} - \phi_\epsilon) \approx 1$.
- $\phi_\epsilon = 43.52(5)$, $\phi_{\epsilon'} = 42.3(1.5)$
- Isopspin breaking effect: (at most 20% of ξ_0) $\to$ (1% in ε_K) $\to$ neglected!

ξ_0
Direct Method

- RBC-UKQCD calculated $\mathrm{Im} A_0$. $\mathrm{Im} A_0 \to \xi_0$.

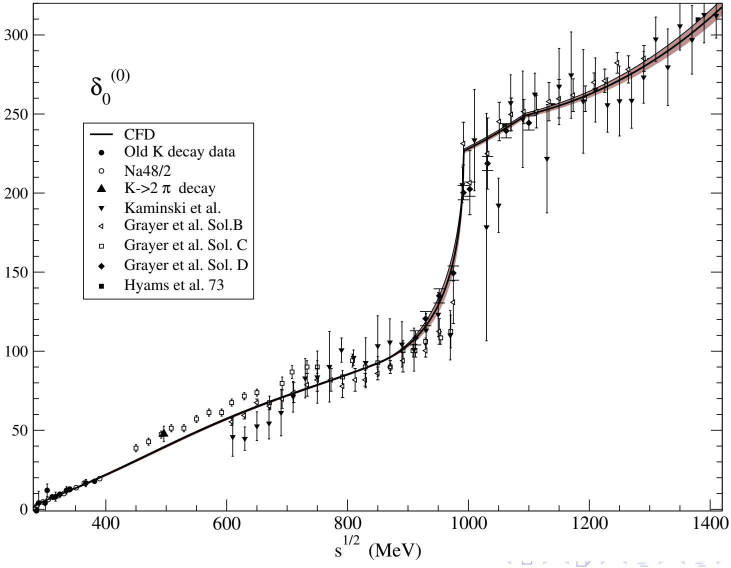
$$\xi_0 = \frac{\mathrm{Im} A_0}{\mathrm{Re}\, A_0} = -0.57(49) \times 10^{-4}$$

  Other input $\mathrm{Re}\, A_0$ is taken from the experimental value.
- RBC-UKQCD also calculated δ_0

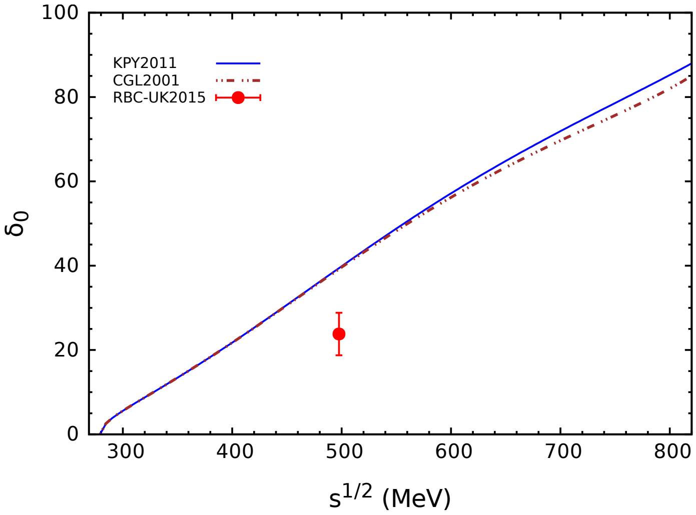
$$\delta_0 = 23.8(49)(12)^\circ$$

  This value is 3.0σ away from the experimental value: $\delta_0 = 39.1(6)^\circ$.
- This indicates that this method belongs to the category of exploratory study rather than precision measurement.
- Hence, we use the indirect method to determine ξ_0.

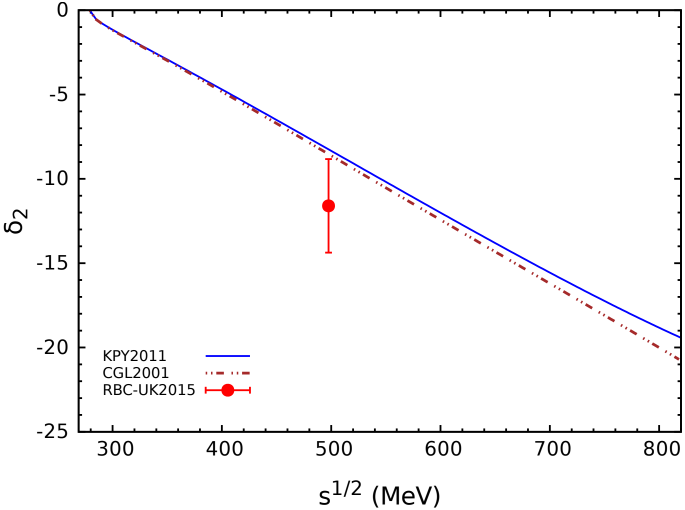
CFD analysis for δ_0: PRD83,074004 (2011)



Comparison of δ_0 between CFD and RBC-UKQCD



Comparison of δ_2 CFD and RBC-UKQCD



ξ_0
Comparison

Input Parameters: ξ_0

Method	Value	Reference
Indirect	$-1.63(19) \times 10^{-4}$	RBC-UK-2015 [14]
Direct	$-0.57(49) \times 10^{-4}$	RBC-UK-2015 [15]

ξ_{LD}

- Definition:

$$\xi_{\text{LD}} = \frac{m'_{\text{LD}}}{\sqrt{2}\,\Delta M_K}$$

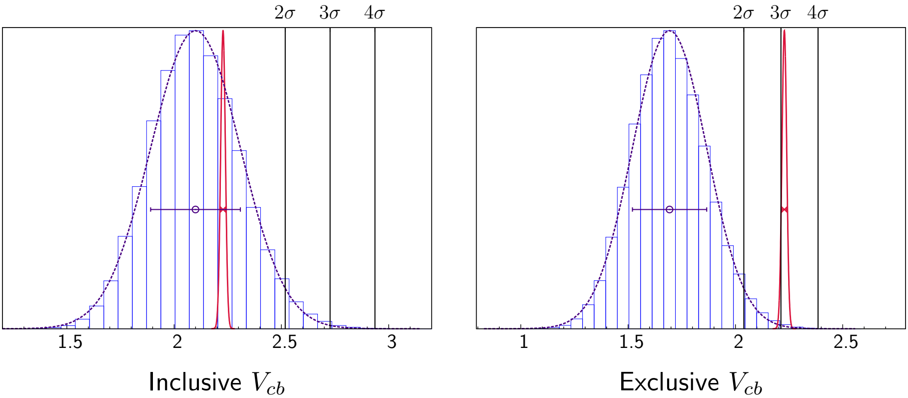
$$m'_{\text{LD}} = -\text{Im}\left[\mathcal{P}\sum_C \frac{\langle\overline{K}^0|H_{\text{w}}|C\rangle\langle C|H_{\text{w}}|K^0\rangle}{m_{K^0} - E_C}\right]$$

- Rough estimate in [PRD 88, 014508] gives

$$\xi_{\text{LD}} = (0 \pm 1.6)\%$$

- Precise lattice QCD calculation is not available yet.

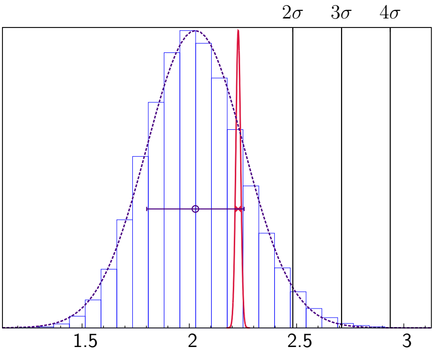
ϵ_K: FLAG $\hat{B}_K$, AOF of $(\bar{\rho}, \bar{\eta})$, V_{us}



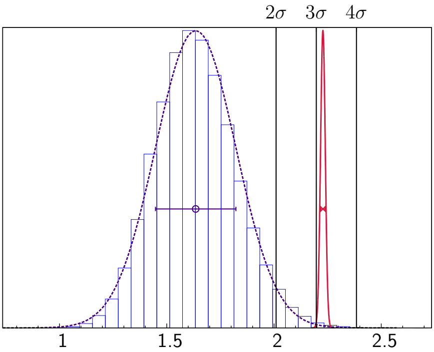
- With exclusive V_{cb}, it shows 3.2σ tension.

$$\epsilon_K^{Exp} = 2.228(11) \times 10^{-3}$$
$$\epsilon_K^{SM} = 1.69(17) \times 10^{-3}$$

ϵ_K: SWME $\hat{B}_K$, AOF of $(\bar{\rho}, \bar{\eta})$, V_{us}



Inclusive V_{cb}



Exclusive V_{cb}

- With exclusive V_{cb}, it shows 3.1σ tension.

$$\epsilon_K^{Exp} = 2.228(11) \times 10^{-3}$$
$$\epsilon_K^{SM} = 1.63(19) \times 10^{-3}$$

Current Status of ε_K

- FLAG 2016: (in units of 1.0×10^{-3}, AOF)

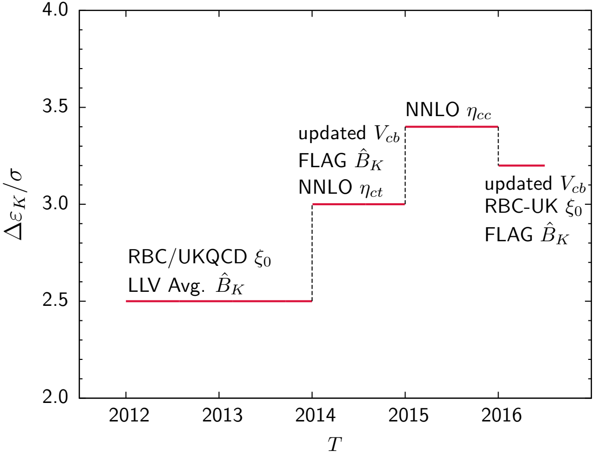
$$\varepsilon_K = 1.69 \pm 0.17 \qquad \text{for Exclusive } V_{cb} \text{ (Lattice QCD)}$$
$$\varepsilon_K = 2.10 \pm 0.21 \qquad \text{for Inclusive } V_{cb} \text{ (QCD Sum Rule)}$$

- Experiments:

$$\varepsilon_K = 2.228 \pm 0.011$$

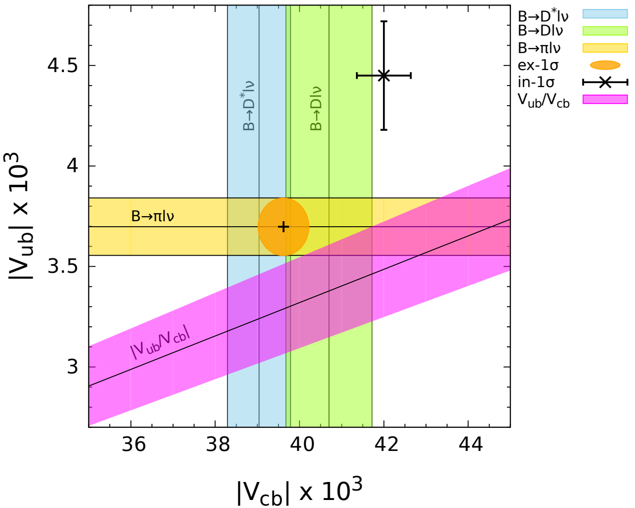
- Hence, we observe 3.2 σ difference between the SM theory (Lattice QCD) and experiments.
- What does this mean? $\longrightarrow$ Breakdown of SM ?

Time Evolution of $\Delta\varepsilon_K$ on the Lattice



- $\Delta\varepsilon_K \equiv \varepsilon_K^{\text{exp}} - \varepsilon_K^{\text{SM}}$

Current Status of $|V_{cb}|$ in 2016



Error Budget of Exclusive ε_K

source	error (%)	memo
V_{cb}	30.1	Exclusive Combined
$\bar{\eta}$	24.7	AOF
η_{ct}	19.5	$c-t$ Box
η_{cc}	8.2	$c-c$ Box
$\bar{\rho}$	6.6	AOF
m_t	3.0	top quark mass
ξ_{LD}	2.5	Long-distance
$\hat{B}_K$	1.8	FLAG
ξ_0	1.2	$\text{Im}(A_0)/\text{Re}(A_0)$
$\vdots$	$\vdots$	

To Do List in Lattice QCD

- We need to reduce overall errors on V_{cb}: $1.9\% \to 1.1\%$.
- We need to understand 3.0σ tension in δ_0.
- We need to reduce overall errors on ξ_0 and ξ_2.
- We need to reduce overall errors on $\bar{\eta}$.
- We need to update top quark mass $m_t^{\overline{\mathrm{MS}}}(m_t)$ with new sets of data on CMS and ATLAS.

Thank God for your help !!!

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