Higher order effects in εpsilon'/εpsilon

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Contents

- **Motivation**
- **Effective Field Theories**
- **Perturbative Calculations**
- **ε'/ε in the SM**
- **Future Improvements**
- **Conclusions**

Direct CP Violation Exists

A non-zero value of Re(ε'/ε) signals that direct CP Violation exists

$$Re\left(\frac{\varepsilon'}{\varepsilon}\right) = \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2\right)$$

The measured quantity is the double ratio of the decay widths

$$R = \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = \frac{\Gamma(K_L \to \pi^0\pi^0)\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_L \to \pi^+\pi^-)\Gamma(K_S \to \pi^0\pi^0)}$$

(a long series of precision counting experiments)

From NA48 and KTeV collaborations,

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}$$

ε'/ε Current Situation

Matrix elements can now be determined on the Lattice [Blum et. al., Bai et. al. `15]

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\rm SM} = (1.9 \pm 4.5) \times 10^{-4}$$

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_{\rm exp} = (16.6 \pm 2.3) \times 10^{-4}$$

2.9σ

[Buras, Gorbahn, Jäger, Jamin '15] **(using input from Lattice results)**

Tension between the theoretical prediction and the experimental data

NEW ANOMALY???

Disentangle the Flavor Puzzle

Is flavor violation completely governed by the CKM matrix?

Yes → **Minimal Flavor Violation (MFV)**

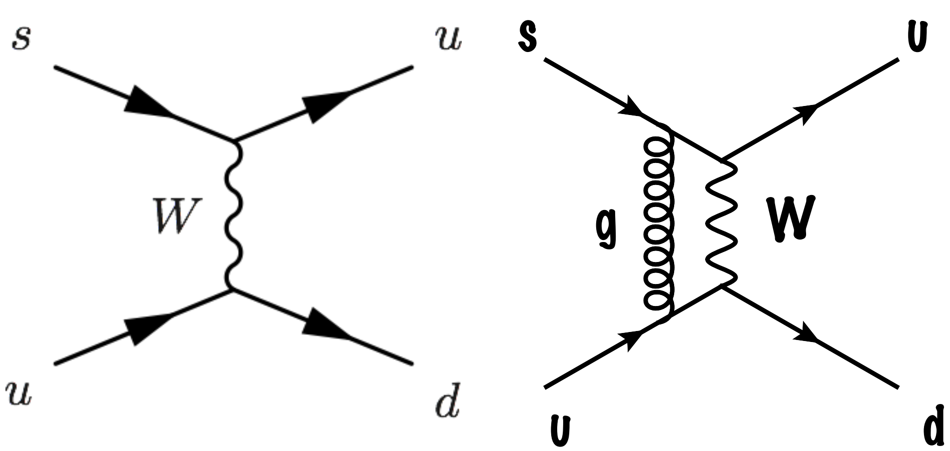
NO → **New sources of Flavor Symmetry Breaking at 10 TeV scale**

Deeper Understanding of the SM is crucial

CPV in Kaon Decays

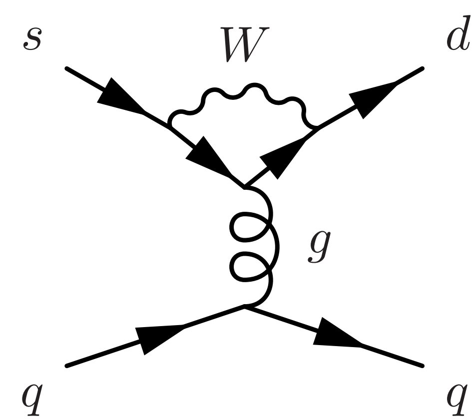
[Buras et.al., Ciuchini et. al. `92 `93]

Current-Current



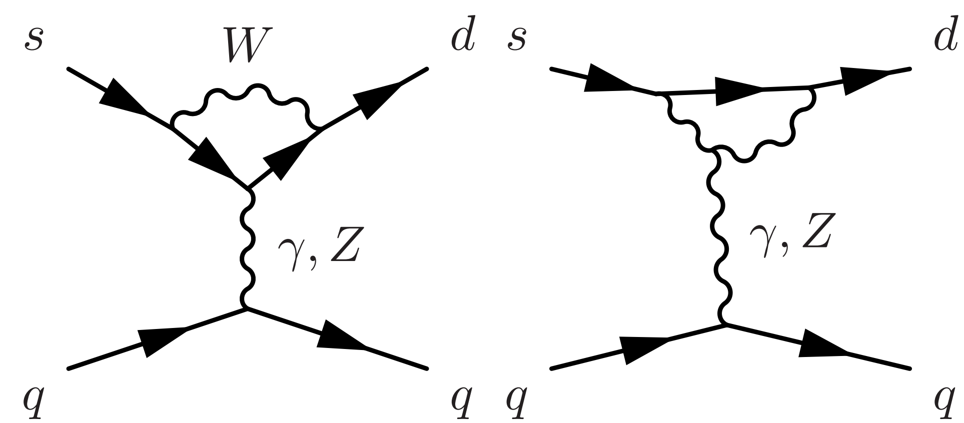
$\mathcal{O}_1, \mathcal{O}_2$

QCD-Penguins



$\mathcal{O}_3 - \mathcal{O}_6$

EW-Penguins



$\mathcal{O}_7 - \mathcal{O}_{10}$

$$V_{ij}^{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \mathcal{O}\begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$\lambda = \mathcal{O}(0.2)$

s → d $\lambda^5 \sim 10^{-4}$

The CP violation is small because of flavor suppression

Effective Field Theories

$$H_{eff} = V_{CKM} \sum \underbrace{C_i(\mu)}_{\text{short-distance}} \underbrace{O_i}_{\text{Long-distance}}$$

short-distance Long-distance

Weak Effective Theory

Effective Hamiltonian at μ < m_c

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{10} \left(z_i(\mu) + \tau y_i(\mu) \right) \mathcal{O}_i$$

$$\tau \equiv -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}}$$

perturbative Wilson coeffs.

Only the Imaginary part of τ is responsible for CPV
(everything else is pure-real)

Long Distance: [Blum et. al., Bai et. al. `15]

Lattice QCD calculation of the Matrix Elements

$$\langle (\pi\pi)_I | \mathcal{O}_i | K \rangle = \langle \mathcal{O}_i \rangle_I$$

by RBC-UKQCD.

Short Distance

Traditional Basis | Modern Basis

i) [Misiak, Bobeth, Urban]
ii) [Gambino,Buras, U.H]
iii) [Gorbahn, Haisch]
iv) [Gorbahn, Brod]
v) [Buras, M.Jamin, M.E.L]
vi) [Blum et. al., Bai et. al. '15]

	Energy scale	Fields	Effective Theory	
Matching at μW	Mw	γ,g, H τ, μ, e, νi b, c, s, d, u	A5	i) NNLO QCD + Current-Current ii) NNLO EW penguins iii) NNLO in QCD ADM/RGE
Matching at μb	mb	γ,g τ, μ, e, νi c, s, d, u	A4	iv) NNLO QCD + Current-Current iii) NNLO in QCD ADM/RGE
Matching at μc	mc	γ,g μ, e, νi s, d, u	A3	V) NLO All operators iii) NNLO in QCD ADM/RGE
Lattice QCD	MLattice	u, s, d	Lattice QCD	V) NLO for all operators except Q8g

NNLO Operator Basis

The traditional basis requires the calculation of traces with γ5

$$\mathcal{O}_{5,6} = (\bar{s}_i d_j)_{\text{V-A}} \sum_{u,d,s} (\bar{q}_k q_l)_{\text{V+A}}$$

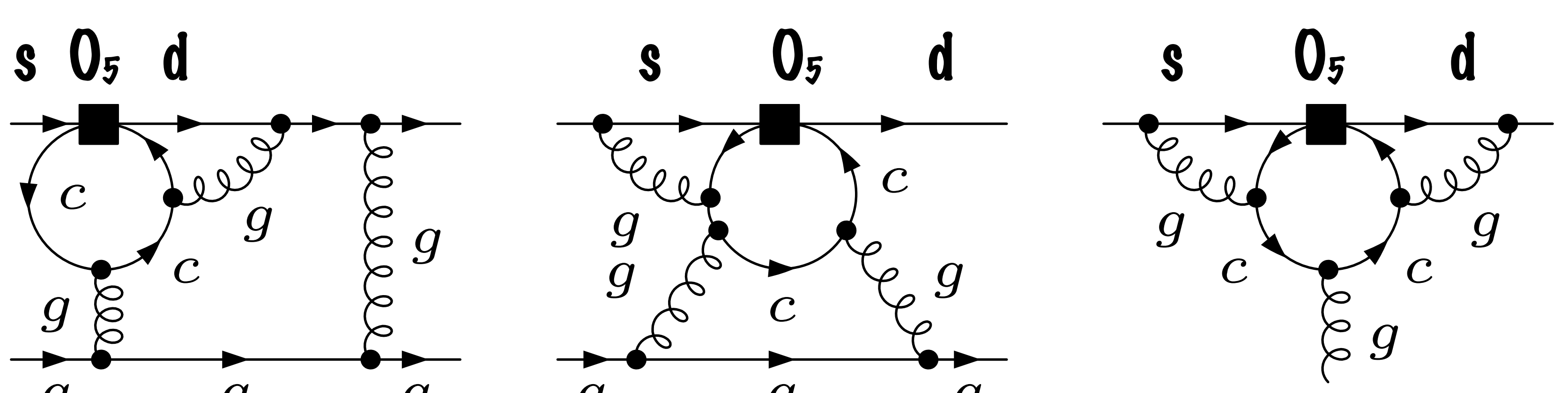
→ **Issues with the treatment of the γ5 in D dimensions**

Higher order calculations can be significantly simplified if we use a different operator basis: Modern basis

[Misiak et al.]

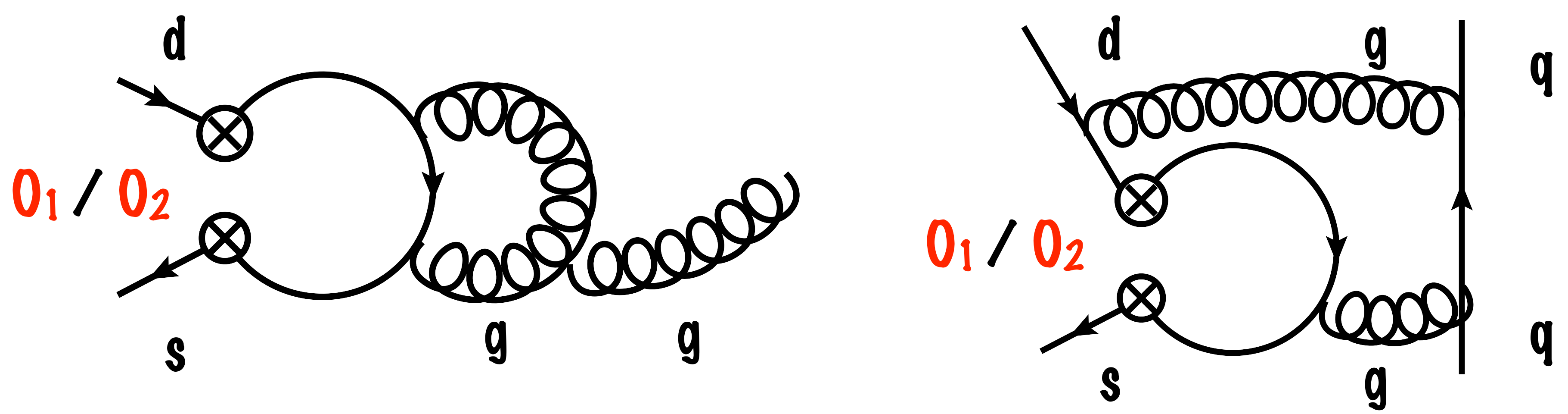
$$\mathcal{O}^{m}_{5,6} = (\bar{s}_i \gamma_\mu \gamma_\nu \gamma_\rho P_L d_j)_{\text{V-A}} \sum_{u,d,s} (\bar{q}_k \gamma^\mu \gamma^\nu \gamma^\rho q_l)$$

→ No trace of γ5



NNLO Matching

O_1 & O_2 have the largest Wilson Coefficients y_i for $\mu_c > m_c$.



The calculation produces several types of structures,

$$(\bar{s}_i \gamma^\mu P_L T^a_{ij} d_j) G^a_\mu k_1^2 \quad (\bar{s}_i \gamma_\nu T^a_{ij} P_L d_j) G^a_\mu k_1^\mu k_2^\nu \quad \ldots$$

– more than operators.

Renormalisation O_1/O_2

Divergencies in 4 flavor theory canceled by 3 flavor theory:

One-loop matching coefficient × one-loop operator mixing

$A_{full} = A_{eff}$ results then in finite threshold corrections for O_3 - O_6

Additional Check:

All results can be projected onto the Physical and EOM vanishing Operator Basis.

The log(μ) dependence cancels analytically.

Note:

Evanescent Operators only contribute in f=4 theory at NNLO

Change of Basis

Lattice results are presented in the Traditional Basis.

A change of basis in Dimensional Regularization is equivalent to

a rotation (R) plus a change of scheme (δZ) [Gorbahn, Haisch]

Physical quantities do not depend on the renormalisation scheme.

The scheme dependence of the
Wilson coefficients and the ME
cancels out in the product

QCD penguins at μ_{lat}=1.3 GeV scale

We compute

RGE

$$y_i^{\rm CMM}(\mu_{\rm lat}) = U(\mu_{\rm Lat},\mu_c).M(\mu_c).U(\mu_c,\mu_b).M(\mu_b).U(\mu_b,\mu_W).y_i(\mu_W)$$

Threshold corrections

We transform the Wilson coefficients to the the traditional basis

$$y_i^{\bf tra}(\mu_{\rm lat}) = (R^{-1})^T.(1-\delta Z^T).y_i^{\rm CMM}(\mu_{\rm lat})$$

Alternatively, we can use the formula

$$y_i^{\bf tra}(\mu_{\rm lat}) = U_{\bf tra}^{(1/2)}(\mu_{\rm Lat}).(R^{-1})^T.U^{(-1/2)}(\mu_c).M(\mu_c).U(\mu_c,\mu_b).M(\mu_b).U(\mu_b,\mu_W).y_i(\mu_W)$$

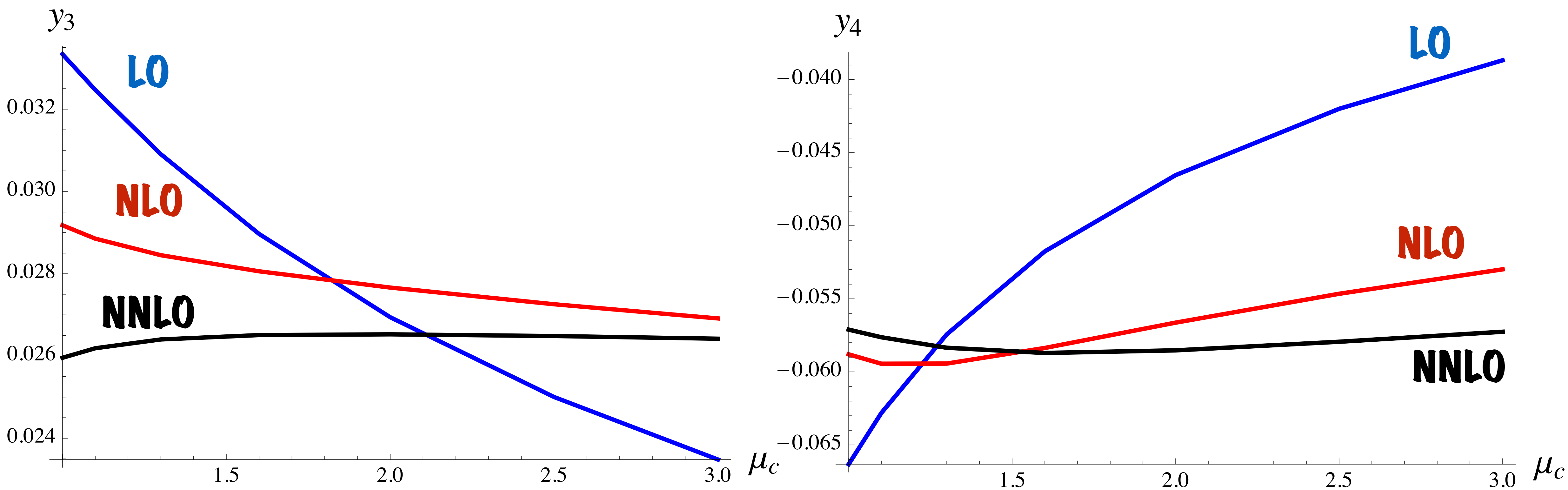
[Jäger's talk]

y_3 & y_4

μ_{lat} =1.3GeV

Traditional Basis

α_s at 3 Loops



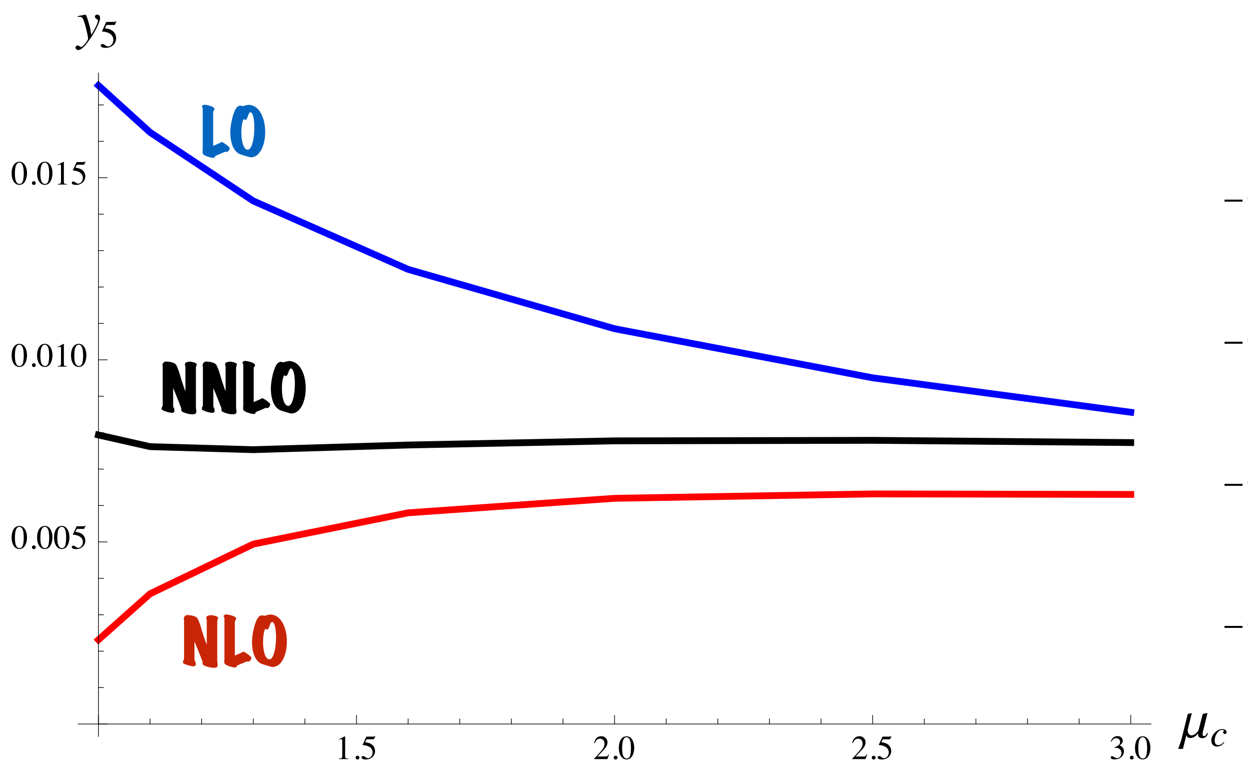
m_c at 2 Loops

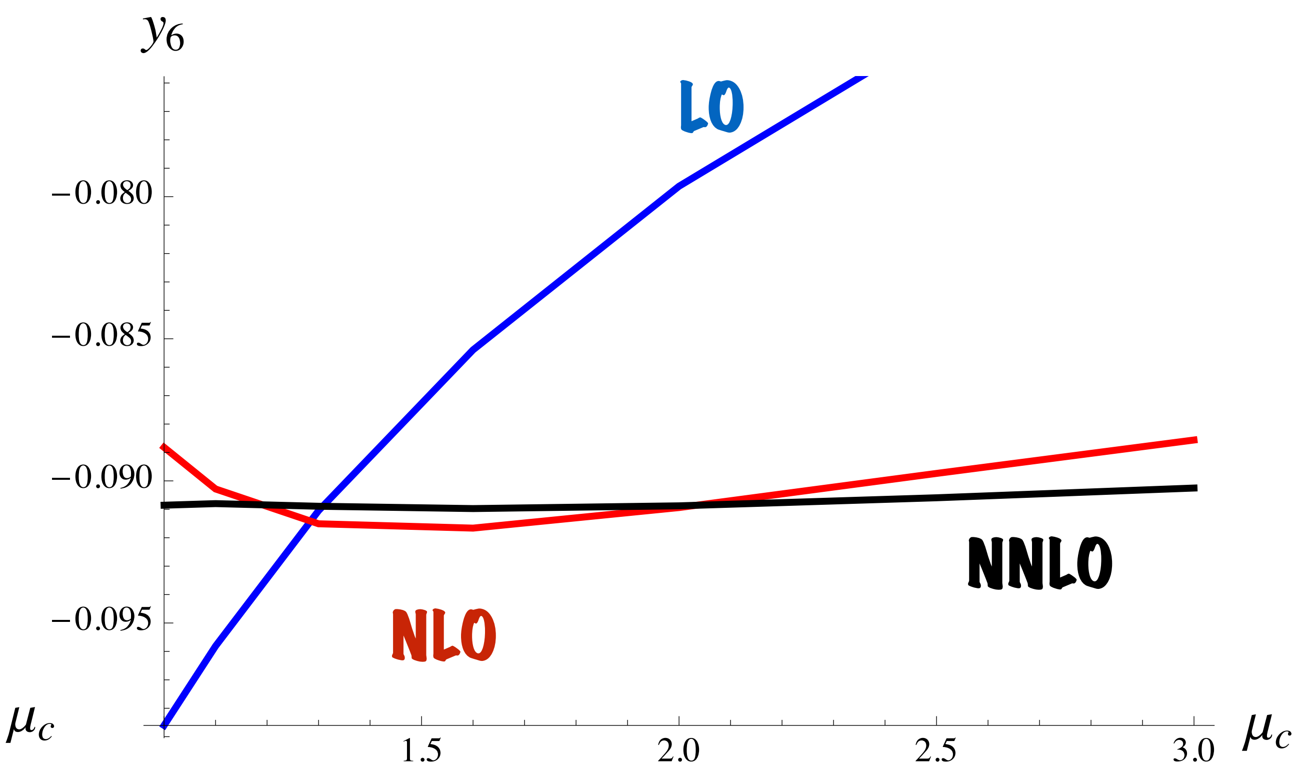
y_5 & y_6

μ_{lat} =1.3GeV

Traditional Basis

α_s at 3 Loops





m_c at 2 Loops

Phenomenology

εpsilon'/εpsilon in the SM

$$\langle \pi^0\pi^0|\mathcal{H}_{\rm eff}|K^0\rangle = A_0 e^{i\delta_0} + A_2 e^{i\delta_2}/\sqrt{2}$$
$$\langle \pi^+\pi^-|\mathcal{H}_{\rm eff}|K^0\rangle = A_0 e^{i\delta_0} - A_2 e^{i\delta_2}/\sqrt{2}$$
$$\langle \pi^+\pi^0|\mathcal{H}_{\rm eff}|K^0\rangle = 3A_2^+ e^{i\delta_2^+}/2$$

A_0 & A_2 : Isospin amplitudes for isospin conservation

A_0, A_2 & A_2^+ from experiment

[Cirigliano, et. al. `11]

Normalise to K^+ decay (ω_+, a) and ε_K
expand in A_2/A_0 and CP violation

Lattice QCD gives us:

$$A_I = \sum_j f(V_{\rm CKM})\mathcal{C}_i \langle(\pi\pi)_I|\mathcal{O}_j|K\rangle$$

The CPV is parametrized as,

Adjusted to keep EW in Im(A0)

$$\mathcal{R}e\left(\frac{\varepsilon'}{\varepsilon}\right) \simeq \frac{\varepsilon'}{\varepsilon} = -\frac{\omega_+}{\sqrt{2}|\varepsilon_K|}\left[\frac{\mathcal{I}m(A_0)}{\mathcal{R}e(A_0)}(1-\Omega_{\rm eff}) - \frac{1}{a}\frac{\mathcal{I}m(A_2)}{\mathcal{R}e(A_2)}\right]$$

[Buras, Gorbahn, Jäger, Jamin `15]** **[Cirigliano, et.al. `11]

For I=2:

[Buras, Gorbahn, Jäger & Jamin `15]

$$\mathrm{Re}A_2 = \frac{G_F}{\sqrt{2}} V_{ud} V^*_{us} z_+ \langle \mathcal{O}_+ \rangle_0$$

For (V-A)x(V-A) structure

$$\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right)_{\text{V-A}} = \mathrm{Im}\tau \frac{3(y_9+y_{10})}{2z_+}$$

Free from hadronic uncertainties.

For (V-A)x(V+A) operators:

$$\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right)_{\text{V+A}} = -\frac{G_F}{\sqrt{2}} \mathrm{Im}\lambda_t \, y_8^{\mathrm{eff}} \frac{\langle \mathcal{O}_8 \rangle_2}{\mathrm{Re}A_2}$$

Small effects of ME Q7 I=2.

O-, O3, O5, O6 are pure I=1/2 operators → **In the isospin limit, ME for I=2 of these operators vanish**

For **I=0:**

[Buras, Gorbahn, Jäger & Jamin `15]

$$\mathrm{Re}A_0 = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* (z_+ \langle \mathcal{O}_+ \rangle_0 + z_\langle \mathcal{O}_- \rangle_0)$$

Fierz relations for (V-A)x(V-A) give, e.g.: $\langle \mathcal{O}_4 \rangle_0 = \langle \mathcal{O}_3 \rangle_0 + 2 \langle \mathcal{O}_- \rangle_0$

$$\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right)_{\text{V-A}} = \mathrm{Im}\tau \frac{(2y_4 - b[3y_9 - y_{10}])}{(1+q) z_-} + \mathrm{Im}\tau \ b \frac{3[y_9 + y_{10}]q}{2(1+q) z_+}$$

is only a function of the Wc's and the ratio

dominated by short distance

$$q \equiv \frac{z_+(\mu) \langle \mathcal{O}_+(\mu) \rangle_0}{z_-(\mu) \langle \mathcal{O}_-(\mu) \rangle_0}$$

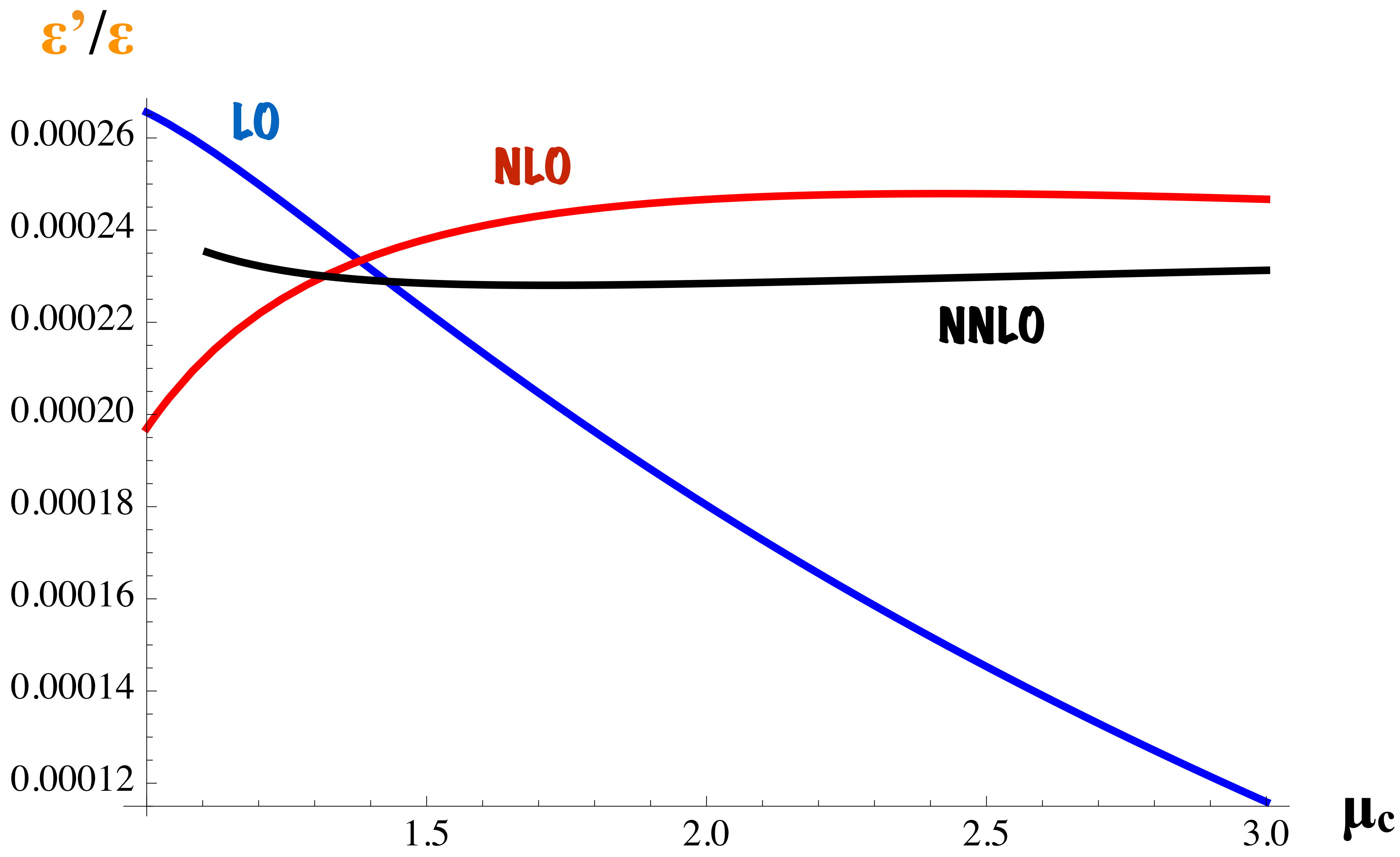
For (V-A)x(V+A) operators:

dominated by long distance

$$\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right)_{\text{V+A}} = - \frac{G_F}{\sqrt{2}} \mathrm{Im}\lambda_t \ y_6 \frac{\langle \mathcal{O}_6 \rangle_0}{\mathrm{Re}A_0}$$

ε'/ε & μ_c

Residual μ_c scale dependence originating from the QCD penguins



Outlook

- **Perturbation theory gives consistent results for QCD penguins at NNLO**

NEXT STEPS

- **Inclusion of the EW penguin and the CC contributions**
- **Inclusion of QED corrections**
- **Extending the formalism to four flavor** **[Jäger's talk]**
- **Combining perturbation theory with Lattice. Renormalisation scheme**

Thanks!!!