New BaBar Results on CPT Symmetry Tests

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Overview

- “Tests of *CPT* symmetry in $B^0 - \overline{B}^0$ mixing and in $B^0 \to c\bar{c}K^0$ decays”

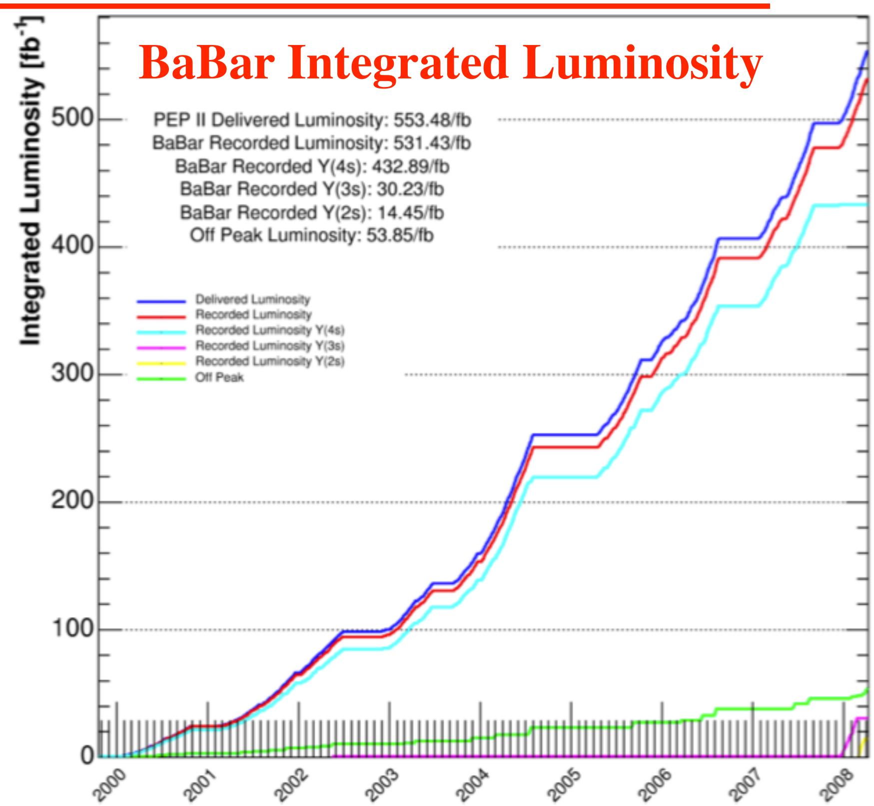
Phys. Rev. D 94, 011101(R) (2016)

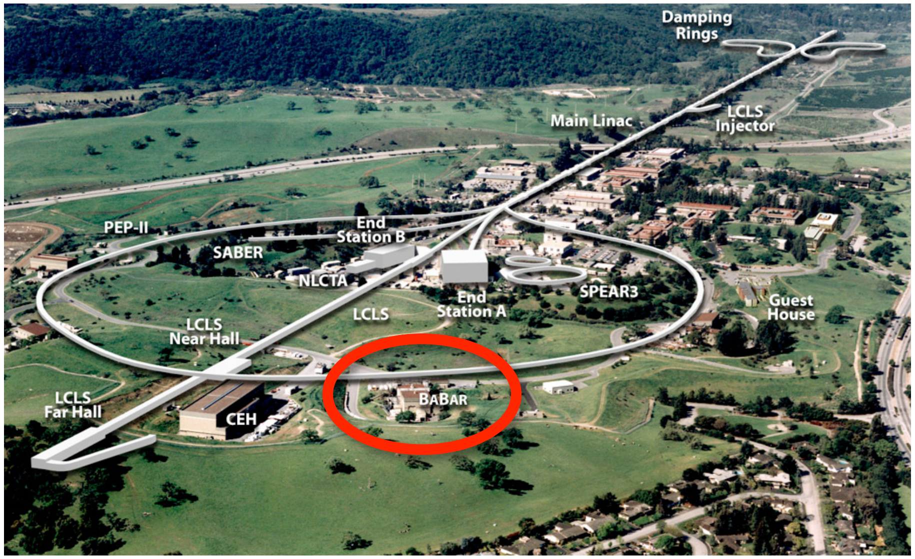
Using the eight time dependences $e^{-\Gamma t}(1+C_i \cos\Delta m t + S_i \sin\Delta m t)$ for the decays $\Upsilon(4S)\to B^0\overline{B}^0\to f_j f_k$, with the decay into a flavor-specific state $f_j = \ell^\pm X$ before or after the decay into a CP eigenstate $f_k = c\bar{c}K_{S,L}$ as measured by the *BABAR* experiment, we determine the three CPT-sensitive parameters Re (z) and Im (z) in B^0-$\overline{B}^0$ mixing and $|\overline{A}/A|$ in $B^0\to c\bar{c}K^0$ decays. We find Im (z) $= 0.010 \pm 0.030 \pm 0.013$, Re (z) $= -0.065 \pm 0.028 \pm 0.014$, and $|\overline{A}/A| = 0.999 \pm 0.023 \pm 0.017$, in agreement with CPT symmetry.

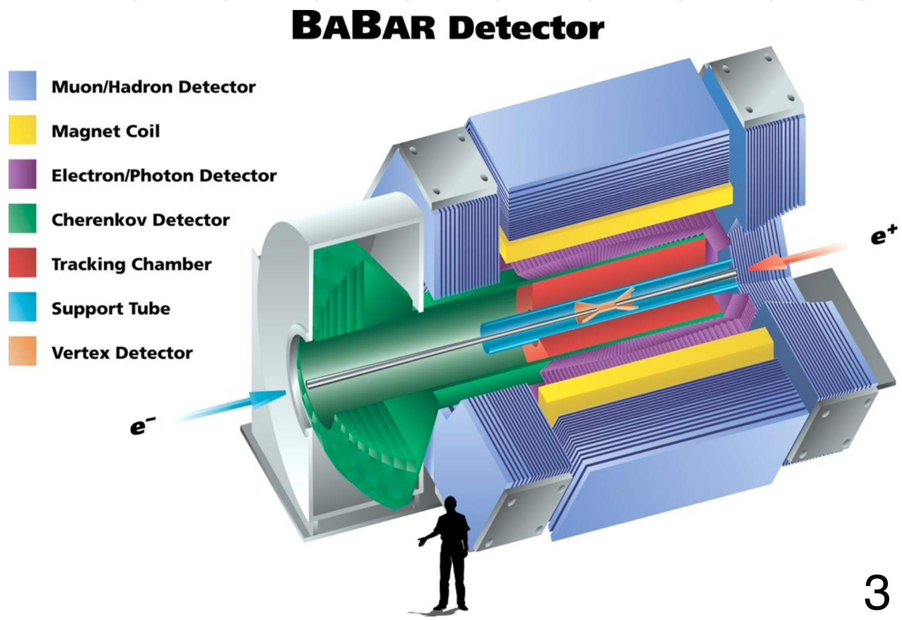
- Determines the Re and Im parts of the parameter z
  - z relates to *CPT* symmetry in mixing
- Also determines $|\overline{A}/A|$
  - $|\overline{A}/A|$ relates to *CPT* symmetry in decay amplitudes

The BaBar Experiment

- Data collected by BaBar detector at Stanford Linear Accelerator Center
- Asymmetric-energy e^+ and e^- beams
- Designed to be a B factory, operating primarily at $\Upsilon(4S)$ resonance, producing 470×10^6 $B\overline{B}$ pairs
- This analysis uses the full dataset of $\sim 430\ \mathrm{fb}^{-1}$ collected at the $\Upsilon(4S)$ resonance







Theory I

- Transitions in the $B-\overline{B}^0$ system are well described by the evolution of the two-state wave function

$$|\Psi\rangle = \psi_1 |B^0\rangle + \psi_2 |\overline{B}^0\rangle$$

using an effective Hamiltonian composed of two constant Hermitian matrices describing mass and decay-rate components:

$$i\frac{\partial}{\partial t}\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \left[\begin{pmatrix} m_{11} & m_{12} \\ m_{12}^* & m_{22} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix}\right]\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

Theory II

- If we do not assume *CPT* symmetry then the physical *B* meson states may be expressed as

$$|B_L\rangle = p\sqrt{1-z}|B^0\rangle + q\sqrt{1+z}|\bar{B}^0\rangle,$$
$$|B_H\rangle = p\sqrt{1+z}|B^0\rangle - q\sqrt{1-z}|\bar{B}^0\rangle,$$

where $\frac{q}{p} = \sqrt{\frac{m_{12}^* - \frac{i}{2}\Gamma_{12}^*}{m_{12} - \frac{i}{2}\Gamma_{12}}}, \quad \mathbf{z} = \frac{(m_{11} - m_{22}) - \mathrm{i}\,(\Gamma_{11} - \Gamma_{22})/2}{\Delta m - \mathrm{i}\,\Delta\Gamma/2}$

$$\Delta m = m(B_H) - m(B_L)$$
$$\Delta\Gamma = \Gamma(B_H) - \Gamma(B_L)$$

  - The complex parameter z vanishes under *CPT* symmetry
  - *T* invariance requires |q/p|=1
  - *CP* invariance requires |q/p|=1 and z=0

- When $\Delta\Gamma \ll \Delta m$, the *CPT* violating parameter z relates to neutral *B* mass and width differences according to:

$$\mathrm{Re}\,\mathbf{z} \equiv (\mathrm{m}_{\mathrm{B}^0} - \mathrm{m}_{\bar{\mathrm{B}}^0})/\Delta\mathrm{m}$$
$$\mathrm{Im}\,\mathbf{z} \equiv (\Gamma_{\bar{\mathrm{B}}^0} - \Gamma_{\mathrm{B}^0})/(2\Delta\mathrm{m})$$

Previous BaBar Analysis I

- The present analysis is an extension of a 2012 BaBar analysis:

  "Observation of Time-Reversal Violation in the B^0 meson system"

  Phys. Rev. Lett. 109, 211801 (2012)

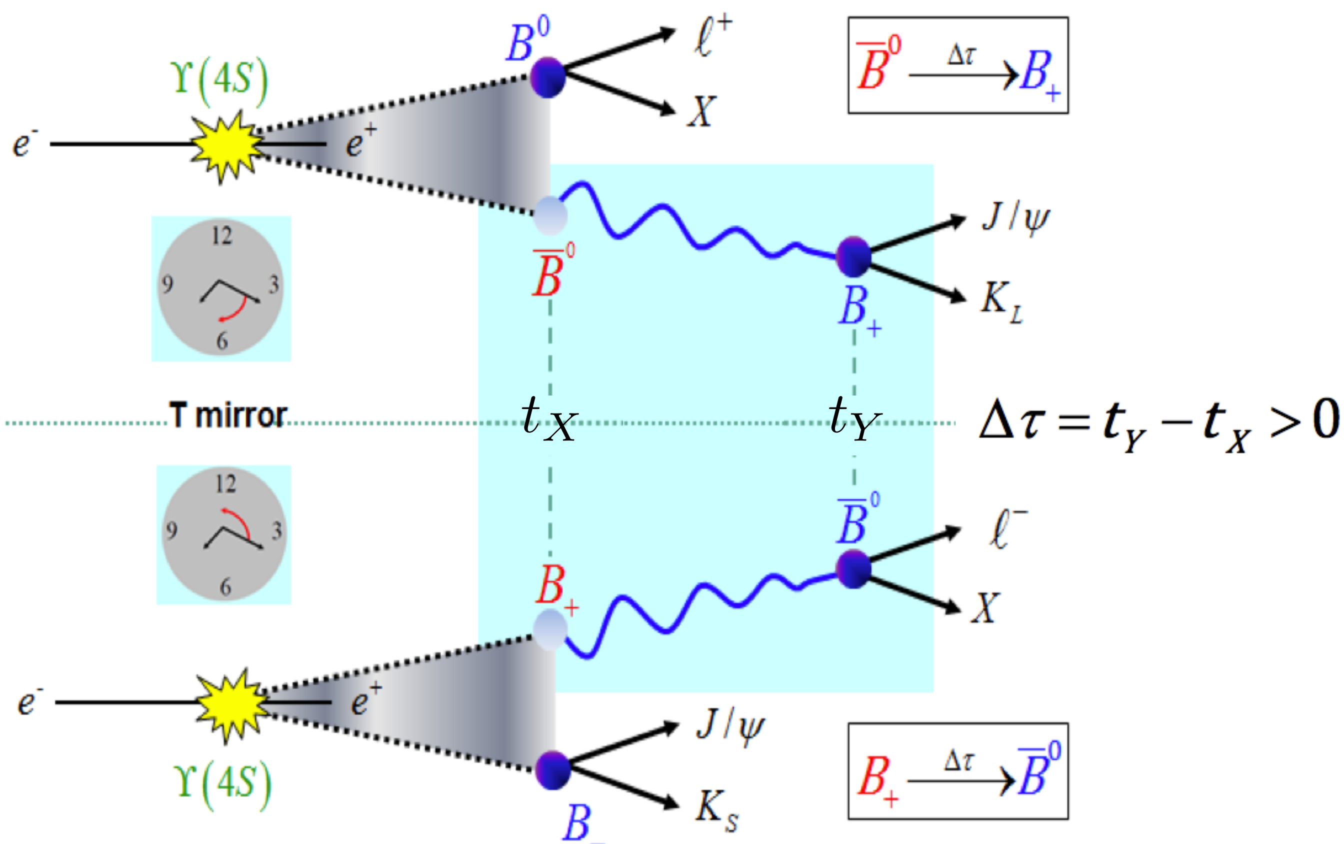
Although *CP* violation in the *B* meson system has been well established by the *B* factories, there has been no direct observation of time-reversal violation. The decays of entangled neutral *B* mesons into definite flavor states (B^0 or $\bar{B}^0$), and $J/\psi K_L^0$ or $c\bar{c}K_S^0$ final states (referred to as B_+ or B_-), allow comparisons between the probabilities of four pairs of *T*-conjugated transitions, for example, $\bar{B}^0 \to B_-$ and $B_- \to \bar{B}^0$, as a function of the time difference between the two *B* decays. Using 468×10^6 $B\bar{B}$ pairs produced in $\Upsilon(4S)$ decays collected by the *BABAR* detector at SLAC, we measure *T*-violating parameters in the time evolution of neutral *B* mesons, yielding $\Delta S_T^+ = -1.37 \pm 0.14(\text{stat}) \pm 0.06(\text{syst})$ and $\Delta S_T^- = 1.17 \pm 0.18(\text{stat}) \pm 0.11(\text{syst})$. These nonzero results represent the first direct observation of *T* violation through the exchange of initial and final states in transitions that can only be connected by a *T*-symmetry transformation.

Previous BaBar Analysis II

- Measurement of *T*, *CP*, and *CPT* violation
- Takes advantage of the fact that B-mesons are produced as entangled pairs in $\Upsilon(4S)$ decays
  - Can be expressed in terms of either flavor-eigenstates, B^0 and $\overline{B}^0$, or the states B_+ and B_-
- The states B_+ and B_- are tagged by decays to $J/\psi K_L$ (*CP*-even) and $J/\psi K_S$ (*CP*-odd), respectively
- Flavor eigenstates can be tagged by semileptonic B decays to $\ell^+ X$ and $\ell^- X$
- Search for *T* violation by comparing rates for transitions between flavor and *CP* states with the rates for the time-reversed processes

Previous BaBar Analysis III

- Example decay sequence:



$$\Delta t = t_{CP} - t_{\text{flav}}$$

Reference (X, Y)	***T-Transformed (X, Y)***
$B^0 \rightarrow B_+$ $(l^-, J/\psi K_L)$	$B_+ \rightarrow B^0$ $(J/\psi K_S, l^+)$
$B^0 \rightarrow B_-$ $(l^-, J/\psi K_S)$	$B_- \rightarrow B^0$ $(J/\psi K_L, l^+)$
$\bar{B}^0 \rightarrow B_+$ $(l^+, J/\psi K_L)$	$B_+ \rightarrow \bar{B}^0$ $(J/\psi K_S, l^-)$
$\bar{B}^0 \rightarrow B_-$ $(l^+, J/\psi K_S)$	$B_- \rightarrow \bar{B}^0$ $(J/\psi K_L, l^-)$

Reference: Physical Process
(X,Y): Reconstructed Final States

B_+, B_- —tagged by→ $J/\psi K_L$, $J/\psi K_S$

Previous BaBar Analysis IV

- Extract 4 pairs of S and C parameters by fitting the expression

$$R_i = N_i e^{-\Gamma t}\left[1 + C_i \cos(\Delta m \cdot t)\Delta m t + S_i \sin(\Delta m \cdot t)\right]$$

  to the four observed rates with the $\ell^{\pm}\nu X$ decay first and the $c\bar{c}K_{S,L}$ second, where:

$$t = \Delta t = t(c\bar{c}K) - t(\ell \nu X)$$

- Extract another 4 pairs of S and C parameters by fitting the same expressions to the four rates with the $c\bar{c}K_{S,L}$ decay first and the $\ell^{\pm}\nu X$ decay second, where the evolution time is now given by:

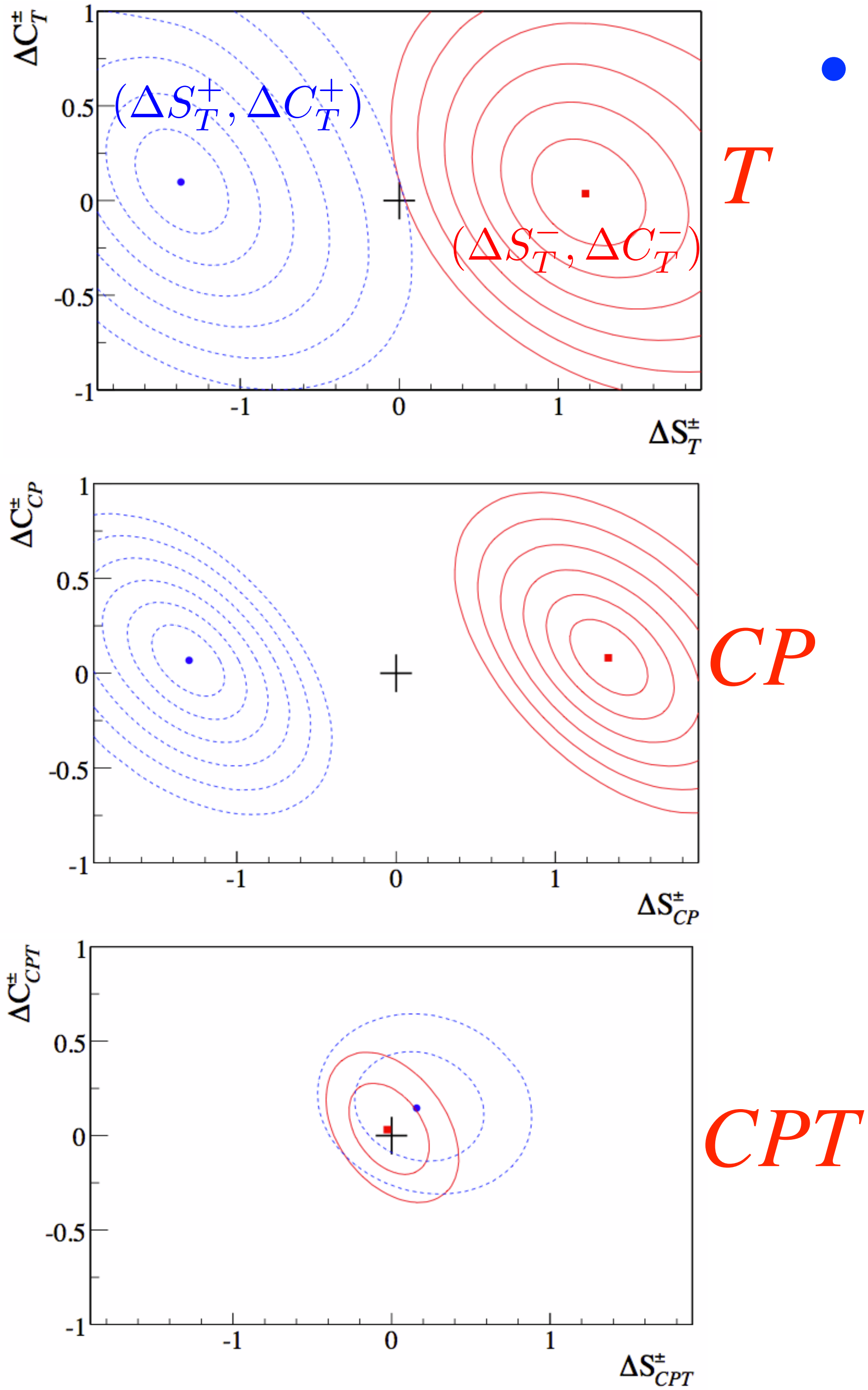
$$t = t(\ell \nu X) - t(c\bar{c}K) = -\Delta t$$

- Note: Due to entanglement of the B pairs, we must have $C_i = C_{i-4}$ and $S_i = -S_{i-4}$ for $i = 5...8$

Previous BaBar Analysis V

- In total, we can build:
  - 4 Independent T comparisons (e.g. $R(B^0 \to B_+) - R(B_+ \to B^0)$)
  - 4 Independent CP comparisons (e.g. $R(B^0 \to B_+) - R(\overline{B}^0 \to B_+)$)
  - 4 Independent CPT comparisons (e.g. $R(B^0 \to B_+) - R(B_+ \to \overline{B}^0)$)
- Analysis performed using the five assumptions:
  - $A = A(B^0 \to c\bar{c}K^0)$ and $\overline{A} = A(\overline{B}^0 \to c\bar{c}\overline{K}^0)$ have a single weak phase
  - Assume B^0 does not decay to $c\bar{c}\overline{K}^0$ and $\overline{B}^0$ does not decay to $c\bar{c}K^0$
  - CP violation in $K^0 - \overline{K}^0$ mixing is negligible $\Longrightarrow K_S = \frac{K^0 + \overline{K}^0}{\sqrt{2}}, \; K_L = \frac{K^0 - \overline{K}^0}{\sqrt{2}}$
  - Assume that $\Delta\Gamma = 0$
  - In order to use the B_+ and B_- states to test T-symmetry, it is necessary to assume that $\langle B_+ | B_- \rangle = 0$, which means that $|\overline{A}/A| = 1$
- Note: CPT symmetry requires that $|\overline{A}/A| = 1$ when A and $\overline{A}$ have a single weak phase

Previous BaBar Analysis VI



- Results:

i	decays	S_i	stat.err	sys.err	C_i	stat.err	sys.err
1	ℓ^-, K_L	0.51	0.17	0.11	-0.01	0.13	0.08
2	ℓ^+, K_L	-0.69	0.11	0.04	-0.02	0.11	0.08
3	ℓ^-, K_S	-0.76	0.06	0.04	0.08	0.06	0.06
4	ℓ^+, K_S	0.55	0.09	0.06	0.01	0.07	0.05
5	K_L, ℓ^-	-0.83	0.11	0.06	0.11	0.12	0.08
6	K_L, ℓ^+	0.70	0.19	0.12	0.16	0.13	0.06
7	K_S, ℓ^-	0.67	0.10	0.08	0.03	0.07	0.04
8	K_S, ℓ^+	-0.66	0.06	0.04	-0.05	0.06	0.03

Symmetry	**Significance of Violation**
T	14 sigma
CP	17 sigma
CPT	0.3 sigma

Present BaBar Analysis

- For the present analysis, we still make the first four assumptions mentioned above, and use as our starting point the 8 pairs of S and C parameters measured by the previous analysis (along with their correlations)

- Because we do not need to use the concept of the states B_+ and B_-, we no longer need to make the fifth assumption (that $|\overline{A}/A| = 1$) and so in this extension of the 2012 BaBar analysis, we extract the parameter $|\overline{A}/A|$, which relates to *CPT* violation in decay amplitudes

- We also extract the Re and Im parts of z, which relate to *CPT* violation in mixing

- As in the 2012 analysis, we use $\Delta\Gamma = 0$, but we perform a study to demonstrate that the final results are independent of this constraint

Time-Dependent Rates I

- Setting $\lambda_f = q\overline{A}_f/(pA_f)$ and neglecting higher order terms in z, we obtain

$$R(B^0 \to f) \;=\; \frac{|A_f|^2\,\mathrm{e}^{-\Gamma t}}{4}\left|(1-\mathsf{z}+\lambda_f)\,\mathrm{e}^{\mathrm{i}\Delta m t}\,\mathrm{e}^{\Delta\Gamma t/4} + (1+\mathsf{z}-\lambda_f)\,\mathrm{e}^{-\Delta\Gamma t/4}\right|^2,$$

$$R(\overline{B}^0 \to f) \;=\; \frac{|\overline{A}_f|^2\,\mathrm{e}^{-\Gamma t}}{4}\left|(1+\mathsf{z}+1/\lambda_f)\,\mathrm{e}^{\mathrm{i}\Delta m t}\,\mathrm{e}^{\Delta\Gamma t/4} + (1-\mathsf{z}-1/\lambda_f)\,\mathrm{e}^{-\Delta\Gamma t/4}\right|^2$$

for decays into final states *f* with amplitudes $A_f = A(B^0 \to f)$ and $\overline{A}_f = A(\overline{B}^0 \to f)$

- For the *CP* eigenstates $c\bar{c}K^0_L$ and $c\bar{c}K^0_S$, with $A_{S(L)} = A[B^0 \to c\bar{c}K^0_{S(L)}]$ and $\overline{A}_{S(L)} = A[\overline{B}^0 \to c\bar{c}K^0_{S(L)}]$, our assumptions yield:

$$A_S = A_L = A/\sqrt{2}$$

$$\overline{A}_S = -\overline{A}_L = \overline{A}/\sqrt{2}$$

so we can use $\lambda = \lambda_S = -\lambda_L$

Time-Dependent Rates II

- Setting $\Delta\Gamma = 0$ and keeping only first-order terms in the small quantities z, $|\lambda| - 1$, and $r = |q/p| - 1$, we obtain:

$$R_i(t) = N_i\ \mathrm{e}^{-\Gamma t}\ (1 + C_i \cos \Delta m t + S_i \sin \Delta m t)$$

where

$$
\begin{aligned}
S_1 = S(\ell^- X, c\bar{c}K_L) &= \frac{2\,\mathrm{Im}\,(\lambda)}{1+|\lambda|^2} - \mathrm{Re}\,(\mathrm{z})\mathrm{Re}\,(\lambda)\mathrm{Im}\,(\lambda) + \mathrm{Im}\,(\mathrm{z})[\mathrm{Re}\,(\lambda)]^2\ , && S_5 = -\,S_1\\
C_1 &= +\frac{1-|\lambda|^2}{2} - \mathrm{Re}\,(\lambda)\,\mathrm{Re}\,(\mathrm{z}) - \mathrm{Im}\,(\lambda)\,\mathrm{Im}\,(\mathrm{z})\,, && C_5 = C_1\\
S_2 = S(\ell^+ X, c\bar{c}K_L) &= -\frac{2\,\mathrm{Im}\,(\lambda)}{1+|\lambda|^2} - \mathrm{Re}\,(\mathrm{z})\mathrm{Re}\,(\lambda)\mathrm{Im}\,(\lambda) - \mathrm{Im}\,(\mathrm{z})[\mathrm{Re}\,(\lambda)]^2\ , && S_6 = -\,S_2\\
C_2 &= -\frac{1-|\lambda|^2}{2} + \mathrm{Re}\,(\lambda)\,\mathrm{Re}\,(\mathrm{z}) - \mathrm{Im}\,(\lambda)\,\mathrm{Im}\,(\mathrm{z})\,, && C_6 = C_2\\
S_3 = S(\ell^- X, c\bar{c}K_S) &= -\frac{2\,\mathrm{Im}\,(\lambda)}{1+|\lambda|^2} - \mathrm{Re}\,(\mathrm{z})\mathrm{Re}\,(\lambda)\mathrm{Im}\,(\lambda) + \mathrm{Im}\,(\mathrm{z})[\mathrm{Re}\,(\lambda)]^2\ , && S_7 = -\,S_3\\
C_3 &= +\frac{1-|\lambda|^2}{2} + \mathrm{Re}\,(\lambda)\,\mathrm{Re}\,(\mathrm{z}) + \mathrm{Im}\,(\lambda)\,\mathrm{Im}\,(\mathrm{z})\,, && C_7 = C_3\\
S_4 = S(\ell^+ X, c\bar{c}K_S) &= \frac{2\,\mathrm{Im}\,(\lambda)}{1+|\lambda|^2} - \mathrm{Re}\,(\mathrm{z})\mathrm{Re}\,(\lambda)\mathrm{Im}\,(\lambda) - \mathrm{Im}\,(\mathrm{z})[\mathrm{Re}\,(\lambda)]^2\ , && S_8 = -\,S_4\\
C_4 &= -\frac{1-|\lambda|^2}{2} - \mathrm{Re}\,(\lambda)\,\mathrm{Re}\,(\mathrm{z}) + \mathrm{Im}\,(\lambda)\,\mathrm{Im}\,(\mathrm{z}) && C_8 = C_4
\end{aligned}
$$

Fitting

- The relationship between the 16 S and C observables and the 4 parameters

$$p_1 = \frac{1-|\lambda|^2}{2} = 1-|\lambda|, \quad p_2 = \frac{2\,\mathrm{Im}(\lambda)}{1+|\lambda|^2} = \mathrm{Im}\left(\frac{\lambda}{|\lambda|}\right), \quad p_3 = \mathrm{Im}(z), \quad p_4 = \mathrm{Re}(z)$$

  is approximately linear

- Therefore, we extract the 4 parameters in a two-step linear χ^2 fit using matrix algebra

  - The fit step determines p_1 and p_2 by fixing $\mathrm{Re}(\lambda)$ and $\mathrm{Im}(\lambda)$ in the products:

$$\mathrm{Re}(z)\,\mathrm{Re}(\lambda)\,, \quad \mathrm{Im}(z)\,\mathrm{Im}(\lambda)\,, \quad \mathrm{Im}(z)\,[\mathrm{Re}(\lambda)]^2\,, \quad \mathrm{Re}(z)\,\mathrm{Re}(\lambda)\,\mathrm{Im}(\lambda)$$

  - After fixing these terms, the relationship between the vectors $y = (S_1, ..., C_8)^T$ and $p = (p_1, p_2, p_3, p_4)^T$ is linear:

$$y = M_1\ p$$

where M_1 uses $\mathrm{Im}(\lambda) = 0.67$ and $\mathrm{Re}(\lambda) = -0.74$, based on the results of analyses assuming *CPT* symmetry

Chin. Phys C 38, 090001 (2014)

- For the second step of the iterative fit, we fix $\mathrm{Re}(\lambda)$ and $\mathrm{Im}(\lambda)$ to the results of the first step and follow the same procedure as before, but replacing M_1 with M_2

Fit Results

- Results:

$$
\begin{aligned}
|\lambda| &= 1 - p_1 = 0.999 \pm 0.023 \pm 0.017\,, \\
\mathrm{Im}\,(\lambda) &= (1 - p_1)\, p_2 = 0.689 \pm 0.034 \pm 0.019\,, \\
\mathrm{Re}\,(\lambda) &= -(1 - p_1)\sqrt{1 - p_2^2} \\
&= -0.723 \pm 0.043 \pm 0.028\,,
\end{aligned}
$$

$$
\begin{aligned}
p_3 = \mathrm{Im}\,(\mathrm{z}) &= 0.010 \pm 0.030 \pm 0.013 \\
p_4 = \mathrm{Re}\,(\mathrm{z}) &= -0.065 \pm 0.028 \pm 0.014
\end{aligned}
$$

- The $\mathrm{Re}\,(z)$ result deviates from 0 by 2.1σ

- The result for λ can be converted into $|\overline{A}/A|$ by using the world average of measurements for $|q/p|$: $|q/p| = 1.0008 \pm 0.0008$. This yields:

$$|\overline{A}/A| = 0.999 \pm 0.023 \pm 0.017$$

- Correlation coefficients are calculated to be:

	$\lvert\overline{A}/A\rvert$	$\mathrm{Im}(z)$	$\mathrm{Re}(z)$
$\lvert\overline{A}/A\rvert$	1.00	0.03	0.44
$\mathrm{Im}(z)$	0.03	1.00	0.03
$\mathrm{Re}(z)$	0.44	0.03	1.00

Statistical

	$\lvert\overline{A}/A\rvert$	$\mathrm{Im}(z)$	$\mathrm{Re}(z)$
$\lvert\overline{A}/A\rvert$	1.00	0.03	0.48
$\mathrm{Im}(z)$	0.03	1.00	-0.15
$\mathrm{Re}(z)$	0.48	-0.15	1.00

Systematic

Estimating The Influence of $\Delta\Gamma$

- As mentioned earlier, we assume $\Delta\Gamma = 0$
- We test the validity of our approximation by generating two toy MC samples: one with $\Delta\Gamma = 0$, and one with $\Delta\Gamma$ set to approximately one standard deviation from the world average
- Fitting each sample with a model which assumes $\Delta\Gamma = 0$, we find that the fit results for the two simulations agree to within 0.002 for C and 0.008 for S
  - Therefore, we conclude that the omission of the $\Delta\Gamma$ term has a negligible impact on the result

Conclusions

- Using 470M $B^0\overline{B}^0$ events from the BaBar experiment, we have determined:

$$\begin{aligned} \mathrm{Im}\,(\mathsf{z}) &= 0.010 \pm 0.030 \pm 0.013\,, \\ \mathrm{Re}\,(\mathsf{z}) &= -0.065 \pm 0.028 \pm 0.014\,, \\ |\overline{A}/A| &= 0.999 \pm 0.023 \pm 0.017\,, \end{aligned}$$

- All three results are compatible with CPT symmetry in $B^0 - \overline{B}^0$ mixing and in $B^0 \to c\bar{c}K$ decays

- The uncertainties on $\mathrm{Re}\,(z)$ are comparable to those obtained by Belle in 2012 with 535M $B^0\overline{B}^0$ events: $\mathrm{Re}\,(z) = -0.019 \pm 0.037 \pm 0.033$

Phys. Rev. D 85, 071105 (2012)

- As expected, the uncertainties on $\mathrm{Im}\,(z)$ are much larger than those obtained by BaBar in 2006 using di-lepton decays from 232M $B^0\overline{B}^0$ events: $\mathrm{Im}\,(z) = -0.014 \pm 0.007 \pm 0.003$

Phys. Rev. Lett. 96, 251802 (2006)

- Our new result supersedes the BaBar result of 2004

Phys. Rev. D 70, 012007 (2004)

Backup Slides

Selected Results

- Present BaBar Analysis **Phys. Rev. D 94, 011101(R) (2016)**

$$\mathrm{Im}\,(\mathsf{z}) = 0.010 \pm 0.030 \pm 0.013\,,$$
$$\mathrm{Re}\,(\mathsf{z}) = -0.065 \pm 0.028 \pm 0.014\,,$$
$$|\overline{A}/A| = 0.999 \pm 0.023 \pm 0.017\,,$$

- 2012 Belle Analysis **Phys. Rev. D 85, 071105 (2012)**

$$\mathrm{Re}\,(z) = 0.019 \pm 0.037 \pm 0.033$$
$$\mathrm{Im}\,(z) = -0.0057 \pm 0.0033 \pm 0.0033$$

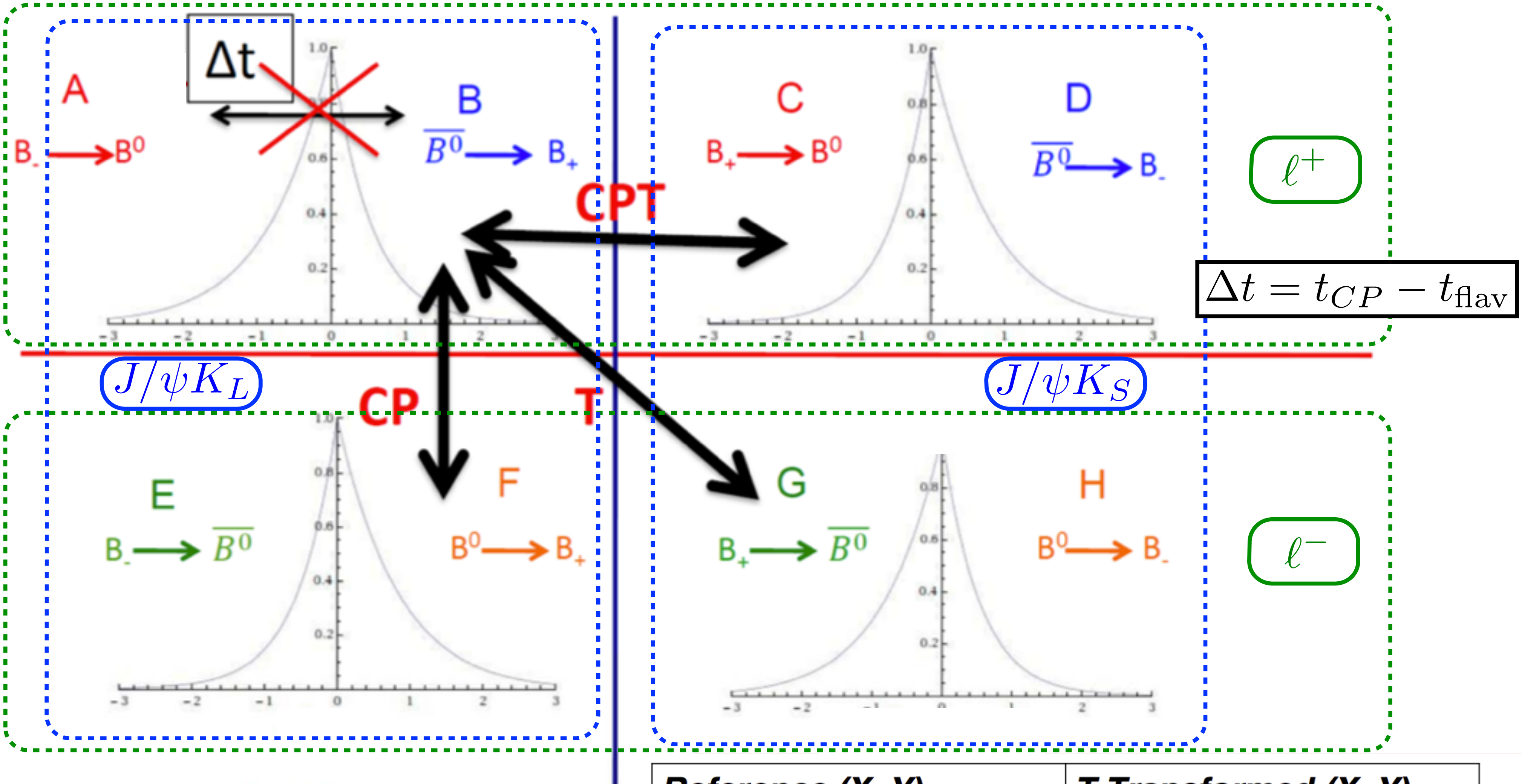
- 2006 BaBar Analysis **Phys. Rev. Lett. 96, 251802 (2006)**

$$|q/p| - 1 = -0.0008 \pm 0.0027 \pm 0.0019$$
$$\mathrm{Im}\,(z) = -0.0139 \pm 0.0073 \pm 0.0032$$
$$\Delta\Gamma \times \mathrm{Re}\,(z) = -0.0071 \pm 0.0039 \pm 0.0020\ \mathrm{ps}^{-1}$$

- 2004 BaBar Analysis **Phys. Rev. D 70, 012007 (2004)**

$$|q/p| = 1.029 \pm 0.013 \pm 0.011$$
$$\mathrm{Im}\,(z) = 0.038 \pm 0.029 \pm 0.025$$

Previous BaBar Analysis I



In total we can build:

- 4 Independent T comparisons.
- 4 Independent CP comparisons.
- 4 Independent CPT comparisons.

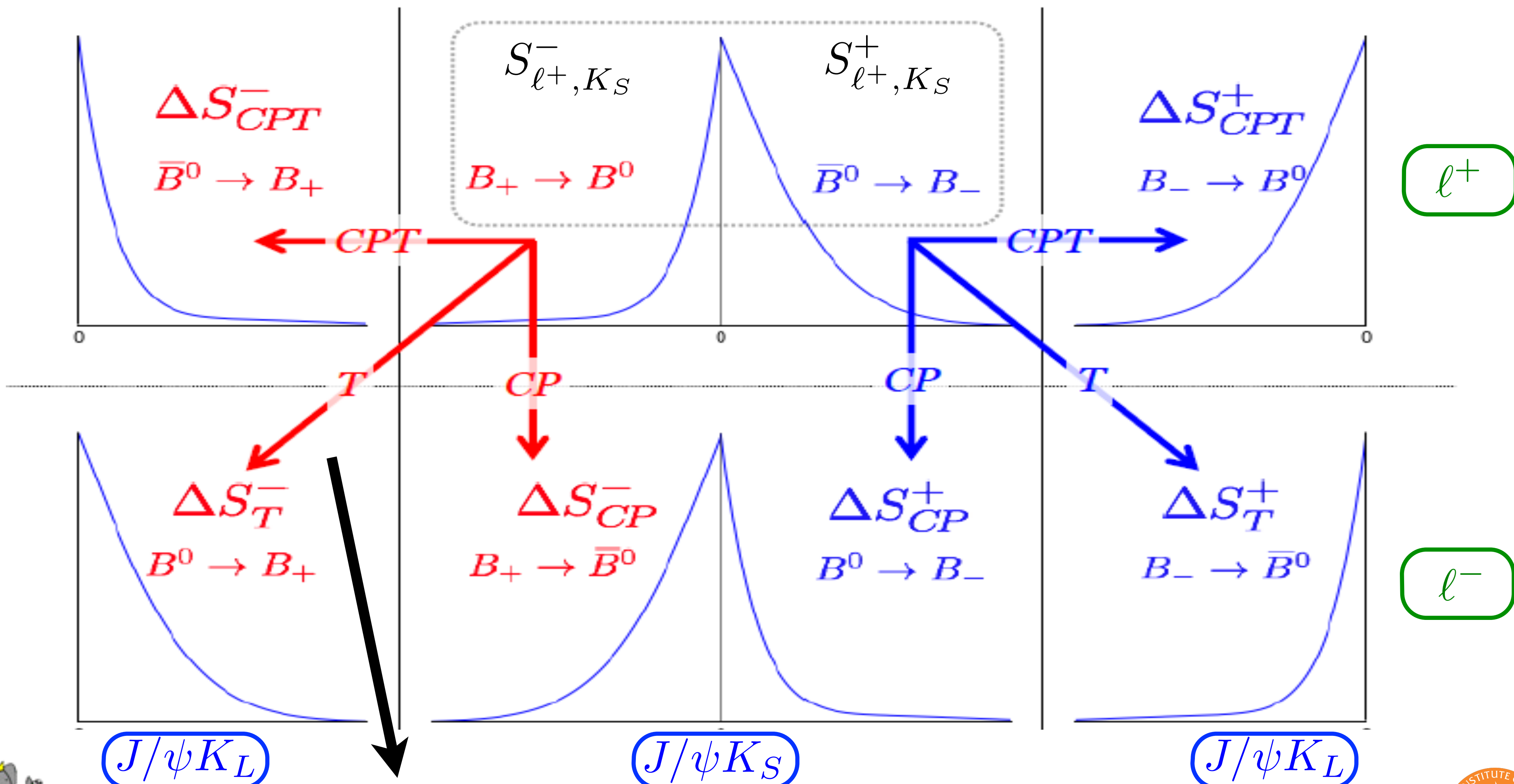
Reference (X, Y)	***T-Transformed (X, Y)***
$B^0 \rightarrow B_+$ (l^- , $J/\psi\ K_L$)	$B_+ \rightarrow B^0$ ($J/\psi\ K_S$, l^+)
$B^0 \rightarrow B_-$ (l^- , $J/\psi\ K_S$)	$B_- \rightarrow B^0$ ($J/\psi\ K_L$, l^+)
$\bar{B}^0 \rightarrow B_+$ (l^+ , $J/\psi\ K_L$)	$B_+ \rightarrow \bar{B}^0$ ($J/\psi\ K_S$, l^-)
$\bar{B}^0 \rightarrow B_-$ (l^+ , $J/\psi\ K_S$)	$B_- \rightarrow \bar{B}^0$ ($J/\psi\ K_L$, l^-)

Previous BaBar Analysis II

$\Delta t = t_{CP} - t_{\rm flav}$

$$g^{\pm}_{\alpha,\beta}(\Delta\tau) \propto e^{-\Gamma\Delta\tau}\{1 + S^{\pm}_{\alpha,\beta}\sin(\Delta m_d \Delta\tau) + C^{\pm}_{\alpha,\beta}\cos(\Delta m_d\Delta\tau)\}$$

$\alpha \in \{\ell^+,\ \ell^-\}$, $\beta \in \{K_S,\ K_L\}$



$$\Delta S^{-}_{T} = S^{+}_{\ell-,K^0_L} - S^{-}_{\ell+,K^0_S}$$

Previous BaBar Analysis III

$$g^{\pm}_{\alpha,\beta}(\Delta\tau) \propto e^{-\Gamma\Delta\tau}\{1 + S^{\pm}_{\alpha,\beta}\sin(\Delta m_d \Delta\tau) + C^{\pm}_{\alpha,\beta}\cos(\Delta m_d \Delta\tau)\}$$

$$\alpha \in \{\ell^+,\ \ell^-\} \quad , \quad \beta \in \{K_S,\ K_L\}$$

- Note: The +/- superscript indicates whether the decay to the flavor final state occurs before or after the decay to the *CP* final state

Parameter	Result
$\Delta S_T^+ = S^-_{\ell^-,K_L^0} - S^+_{\ell^+,K_S^0}$	$-1.37 \pm 0.14 \pm 0.06$
$\Delta S_T^- = S^+_{\ell^-,K_L^0} - S^-_{\ell^+,K_S^0}$	$1.17 \pm 0.18 \pm 0.11$
$\Delta C_T^+ = C^-_{\ell^-,K_L^0} - C^+_{\ell^+,K_S^0}$	$0.10 \pm 0.14 \pm 0.08$
$\Delta C_T^- = C^+_{\ell^-,K_L^0} - C^-_{\ell^+,K_S^0}$	$0.04 \pm 0.14 \pm 0.08$
$\Delta S_{CP}^+ = S^+_{\ell^-,K_S^0} - S^+_{\ell^+,K_S^0}$	$-1.30 \pm 0.11 \pm 0.07$
$\Delta S_{CP}^- = S^-_{\ell^-,K_S^0} - S^-_{\ell^+,K_S^0}$	$1.33 \pm 0.12 \pm 0.06$
$\Delta C_{CP}^+ = C^+_{\ell^-,K_S^0} - C^+_{\ell^+,K_S^0}$	$0.07 \pm 0.09 \pm 0.03$
$\Delta C_{CP}^- = C^-_{\ell^-,K_S^0} - C^-_{\ell^+,K_S^0}$	$0.08 \pm 0.10 \pm 0.04$
$\Delta S_{CPT}^+ = S^-_{\ell^+,K_L^0} - S^+_{\ell^+,K_S^0}$	$0.16 \pm 0.21 \pm 0.09$
$\Delta S_{CPT}^- = S^+_{\ell^+,K_L^0} - S^-_{\ell^+,K_S^0}$	$-0.03 \pm 0.13 \pm 0.06$
$\Delta C_{CPT}^+ = C^-_{\ell^+,K_L^0} - C^+_{\ell^+,K_S^0}$	$0.14 \pm 0.15 \pm 0.07$
$\Delta C_{CPT}^- = C^+_{\ell^+,K_L^0} - C^-_{\ell^+,K_S^0}$	$0.03 \pm 0.12 \pm 0.08$
$S^+_{\ell^+,K_S^0}$	$0.55 \pm 0.09 \pm 0.06$
$S^-_{\ell^+,K_S^0}$	$-0.66 \pm 0.06 \pm 0.04$
$C^+_{\ell^+,K_S^0}$	$0.01 \pm 0.07 \pm 0.05$
$C^-_{\ell^+,K_S^0}$	$-0.05 \pm 0.06 \pm 0.03$

Fitting

- After fixing the parameters, the χ^2 is given by

$$\chi^2 = (M_1\ p - \hat{y})^T\ G\ (M_1\ p - \hat{y})$$

$\dim[M_1] = 16 \times 4$

where $\hat{y}$ is the vector of measured observables and the weight matrix G is:

$$G = [C_{\mathrm{stat}}(y) + C_{\mathrm{sys}}(y)]^{-1}$$

$\dim[G] = 16 \times 16$

- The χ^2 is minimized when:

$$\hat{p} = \mathcal{M}_1\ \hat{y} \quad \text{with} \quad \mathcal{M}_1 = (M_1^T\ G\ M_1)^{-1}\ M_1^T\ G$$

and the uncertainties on $\hat{p}$ are given by the covariance matrices:

$$\begin{aligned} C_{\mathrm{stat}}(p) &= \mathcal{M}_1\ C_{\mathrm{stat}}(y)\ \mathcal{M}_1^T \\ C_{\mathrm{sys}}(p) &= \mathcal{M}_1\ C_{\mathrm{sys}}(y)\ \mathcal{M}_1^T \end{aligned}$$

$\dim[C(p)] = 4 \times 4$

with the property

$$C_{\mathrm{stat}}(p) + C_{\mathrm{sys}}(p) = (M_1^T\ G\ M_1)^{-1}$$