Istituto Nazionale di Fisica Nucleare
Sezione di Napoli

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Rare and radiative kaon decays

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Collaboration with Cappiello,Luigi Cata, Oscar and Gao, Droneng
EPJ . C72 (2012) 1872

Collaboration with Coluccio-Leskow, Tifi, Greynat,David and Nath,Atanu
Phys.Rev. D 2016

Collaboration with Greynat,David and Nath,Atanu
in progress

Outline

- BBG approach: can we test it in rare decays? I like to understand the method!
- $K^+ \to \pi^+ l^+ l^-$ / $K_S \to \pi^0 l^+ l^-$ form factors
- L9 in progress
- $K^+ \to \pi^+ \pi^0 l^+ l^-$ and other decays channels
- Conclusions

QCD at work: Short Distance expansion for weak interaction

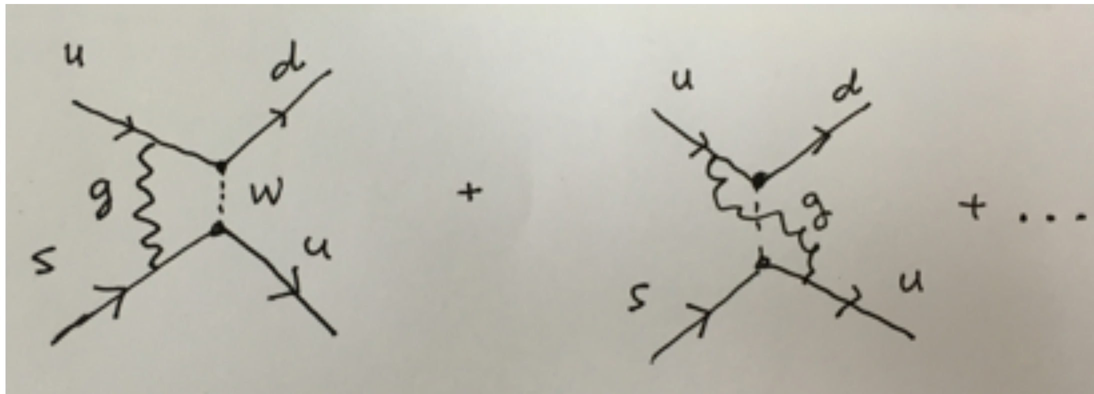
- Fermi lagrangian: description of the ΔS=1 weak lagrangian, in particular the explanation of ΔI =1/2 rule

$$\frac{A(K^+ \to \pi^+\pi^0)}{A(K_S \to \pi^+\pi^-)} \sim \frac{1}{22}$$

- Wilson suggestion (Feynman) , short distance expansion

$$-\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* C_- (\overline{s}_L \gamma^\mu u_L)(\overline{u}_L \gamma_\mu d_L)$$

- Gaillard Lee, Altarelli Maiani: right direction but not fully understood (Long distance?)



QCD at work, theoretical tools

- analytic calculation 't Hooft, large Nc (it explains basic phenomenological facts of QCD, i.e. Zweig's rule) many implications: Skyrme model, VMD, Maldacena

- G. Parisi, `80s lattice: can we predict from QCD the proton mass at 10% level?

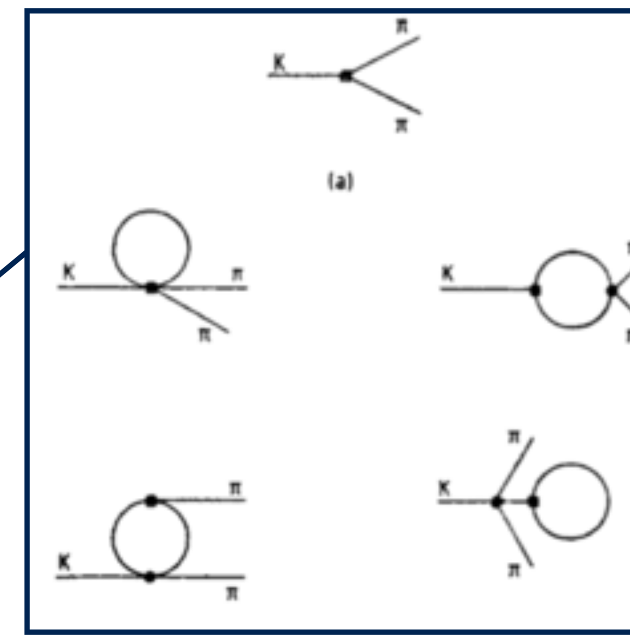
- Precise calculation of low energy QCD?

Bardeen Buras Gerard approach to K->ππ

Also evaluated ΔS=2 transitions, epsilon′ (Buras) and $\pi^+ - \pi^0$ mass diff.

Main idea: phys. amplitudes scale independent

Match SD with LD with a precise prescription for CT



CHPT+Large Nc

$$H_{\rm eff} = \sum_i C_i(\mu)\; Q_i(\mu)$$

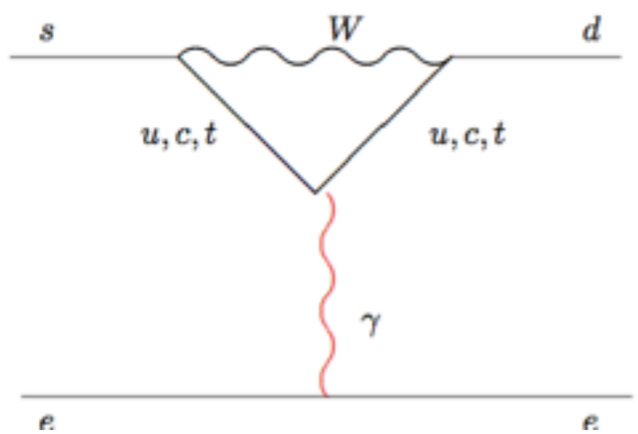
SD

Can we test somewhereelse the Bardeen Buras Gerard (BBG) approach?

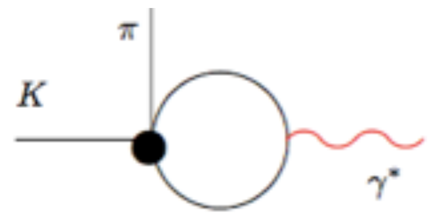
Coluccio-Leskow, Estefania, GD, Greynat,David and Nath,Atanu

$K^+ \to \pi^+ e^+ e^-$ $K_S \to \pi^0 e^+ e^-$

SD



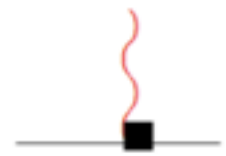
CHPT



CHPT

$$W^i = G_F m_K^2 (a_i + b_i z) + W^i_{\pi\pi}(z)$$

$$i = \pm, S$$



$$a_i, b_i \sim O(1), \qquad z = \frac{q^2}{m_K^2}$$

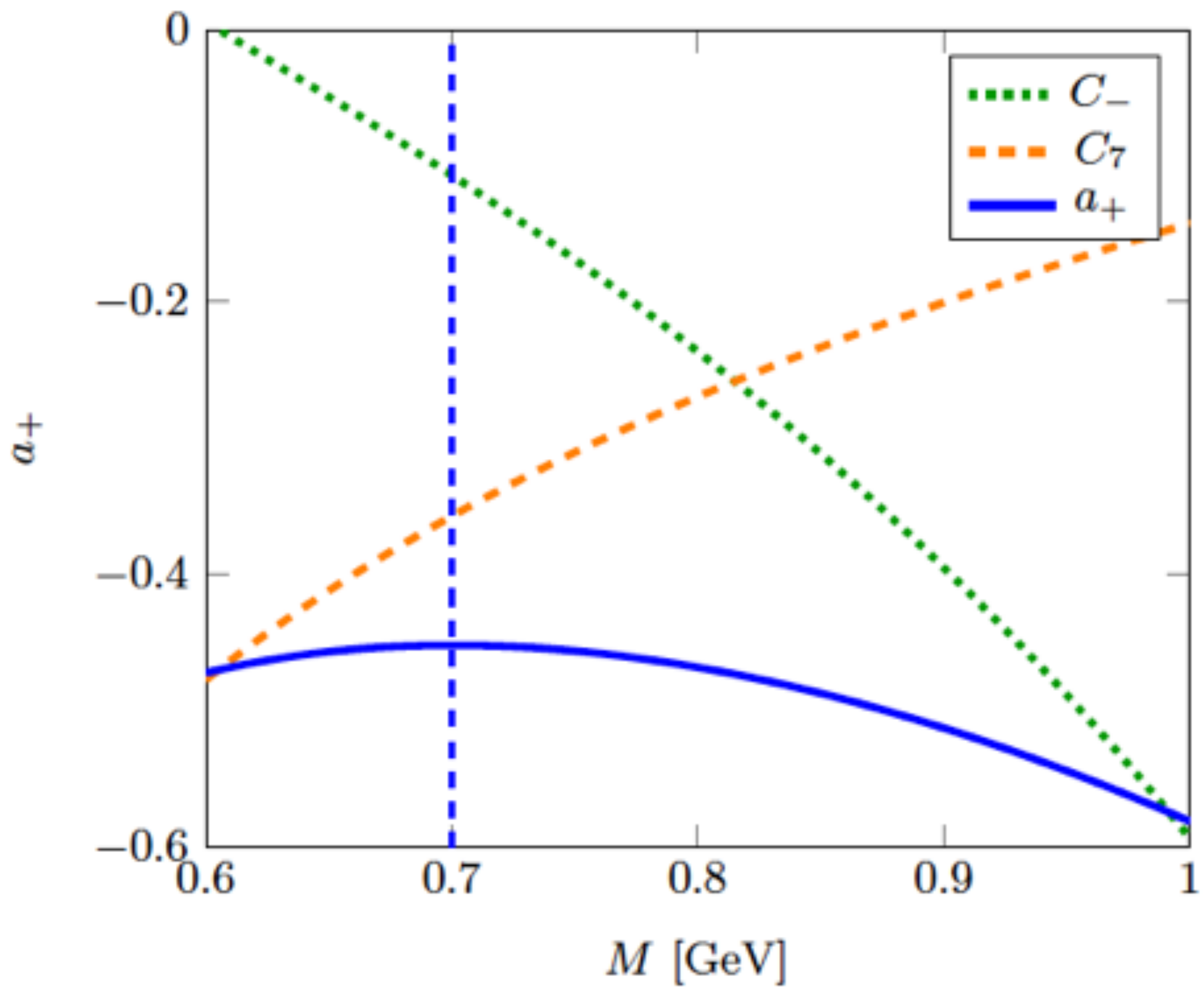
- Observables $\Gamma(K^+ \to \pi^+ e^+ e^-)$, $\Gamma(K^+ \to \pi^+ \mu\bar{\mu})$, slopes
- a_i $O(p^4)$ $a_+ \sim N_{14} - N_{15}$, $a_S \sim 2N_{14} + N_{15}$ Ecker, Pich, de Rafae
- b_i $O(p^6)$ G.D., Ecker, Isidori, Portole
- a_+, b_+ in general not related to a_S, b_S

averaging flavour

$$a_+^{\mathrm{exp.}} = -0.578 \pm 0.016$$
$$b_+^{\mathrm{exp.}} = -0.779 \pm 0.066$$

Matching a la BBG for $K^+ \to \pi^+ e^+ e^-$

Coluccio-Leskow,E. G.D ,Greynat, D and Nath, A



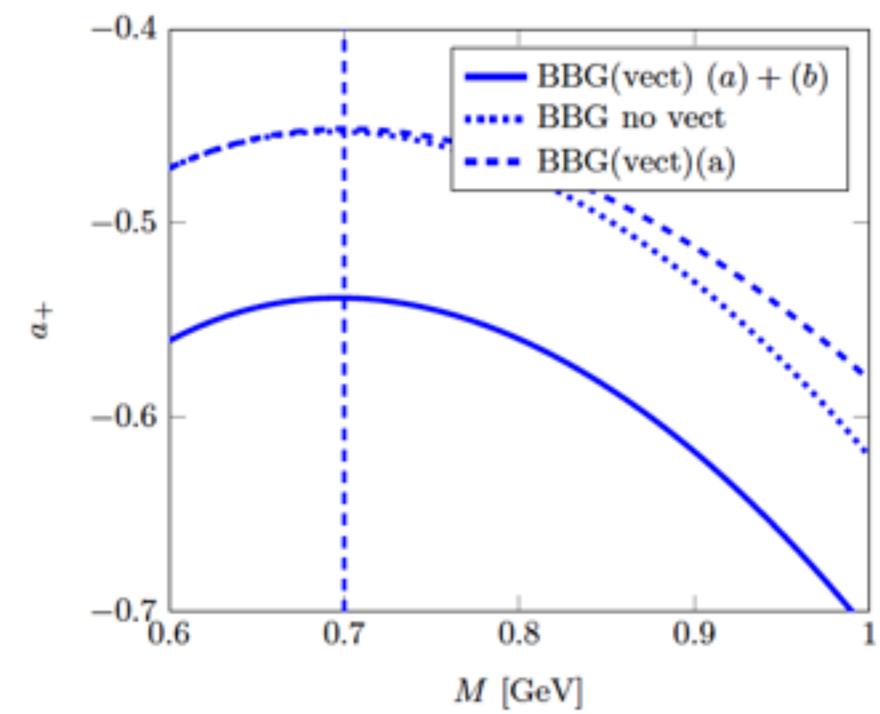


FIG. 5. a_+ as a function of M in the three different frameworks:'BBG no vect.' where vectors are not included, 'BBG(vect)(a)' represents the contribution coming only from diagrams (a) in Fig. 4 and 'BBG(vect) $(a)+(b)$' is the case where both (a) and (b) diagrams were included. The vertical line indicates the value $M = 0.7$ GeV.

L9

Collaboration with Greynat,David and Nath,Atanu

in progress

$$\mathcal{L} \sim -i \; L_9 \; F_{\mu\nu} \;\; < Q \; D^\mu U \; D^\nu U >$$

- No OPE: DIFFERENT from the weak decays!!
- However it must appear at O(p4) to improve matching with QCD

VMD '88: DEGLP, Donoghue et al

Matching the BBG form factor,
M-dependent with phenomenology

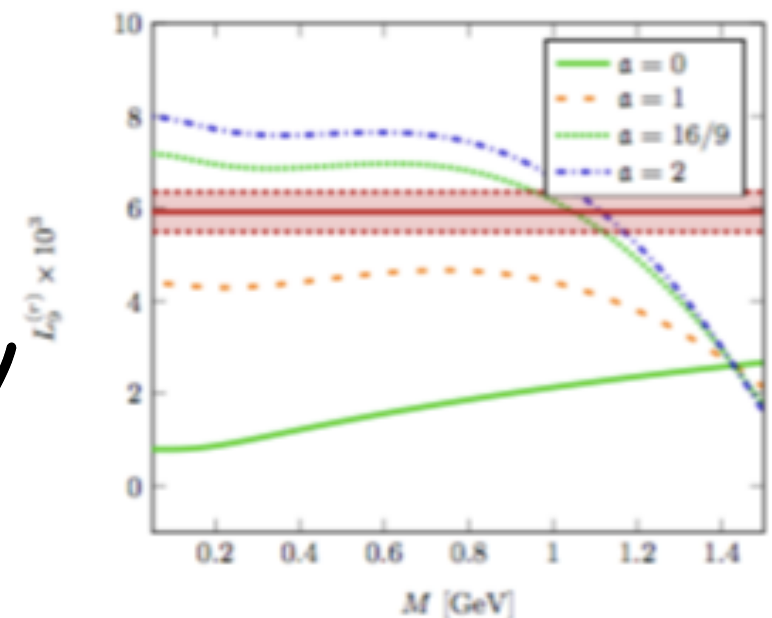


FIG. 9. Variation of $L_9^{(r)+Vec}(M)$ with scale M (in GeV) (calculated in BBG scheme including vectors through hidden local symmetry) given by Eq. (17). Area shaded in pink represents the uncertainty of $\pm 0.43 \times 10^{-3}$ around the phenomenological value 5.93×10^{-3} measured [28] at $m_\rho =$ 0.77 GeV.

$$K(p_K) \to \pi(p_1)\pi(p_2)\gamma(q)$$

- Lorentz + gauge invariance $\Rightarrow$ Electric (E) and Magnetic(M) amplitude

$$A(K \to \pi\pi\gamma) = F^{\mu\nu}\left[E\partial_\mu K\partial_\nu\pi + M\varepsilon_{\mu\nu\rho\sigma}\partial^\rho K\partial^\sigma\pi\right]$$

- Unpolarizated photons

$$\frac{d^2\Gamma}{dz_1dz_2} \sim |E|^2 + |M|^2$$

$$|E^2| = |E_{IB}|^2 + 2Re(E^*_{IB}E_D) + |E_D|^2$$

$$\downarrow$$

Low Theorem $\Rightarrow E_{IB} \sim \frac{1}{E^*_\gamma} +$ c $\qquad E_D$, M chiral tests

We need FIGHT DE/IB$\sim 10^{-3}$

	IB	DE_{exp}	
$K_S \to \pi^+\pi^-\gamma$	10^{-3}	$< 9 \cdot 10^{-5}$	$E1$
$K^+ \to \pi^+\pi^0\gamma$	10^{-4} $(\Delta I = \frac{3}{2})$	$(0.44 \pm 0.07)10^{-5}$ PDG	$M1$,$E1$
$K_L \to \pi^+\pi^-\gamma$	10^{-5} (CPV)	$(2.92 \pm 0.07)10^{-5}$ KTeVnew	$M1$, VMD

CPV is only from IB K_L (also measured in $K_L \to \pi^+\pi^-e^+e^-$)

BUT IB suppressed in K^+ and K_L.

$$K^+ \to \pi^+\pi^0\gamma$$

$$A(K \to \pi\pi\gamma) = F^{\mu\nu}\left[E\partial_\mu K\partial_\nu\pi + M\varepsilon_{\mu\nu\rho\sigma}\partial^\rho K\partial^\sigma\pi\right]$$

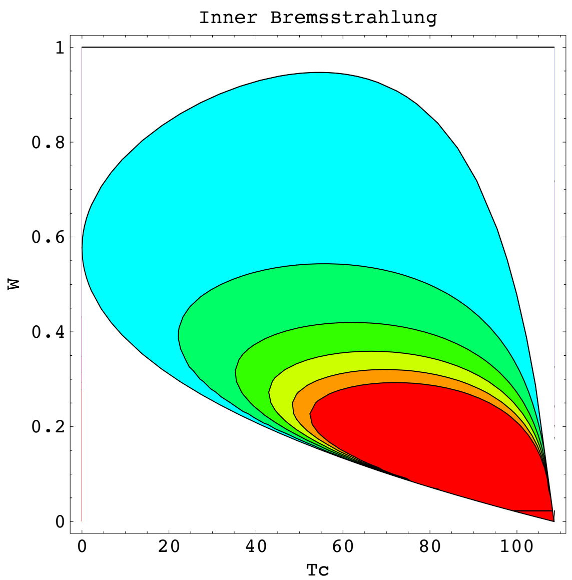
$E1$ and $M1$ are measured with Dalitz plot

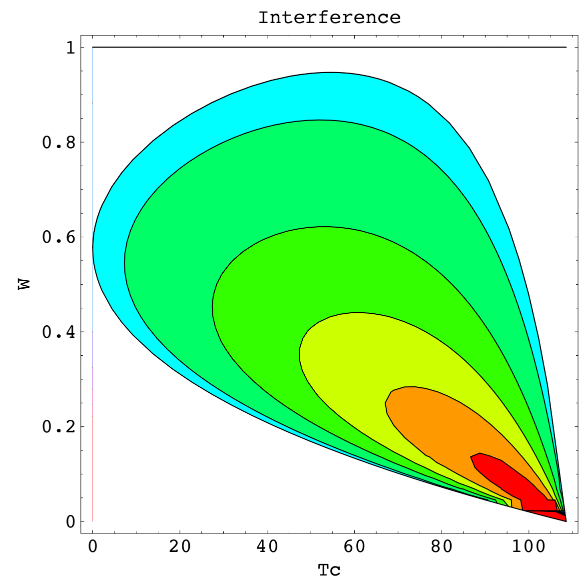
$$\frac{\partial^2\Gamma}{\partial T_c^*\partial W^2} = \frac{\partial^2\Gamma_{IB}}{\partial T_c^*\partial W^2}\left[1 + \frac{m_{\pi^+}^2}{m_K}2Re\left(\frac{E1}{eA}\right)W^2 + \frac{m_{\pi^+}^4}{m_K^2}\left(\left|\frac{E1}{eA}\right|^2 + \left|\frac{M1}{eA}\right|^2\right)W^4\right]$$

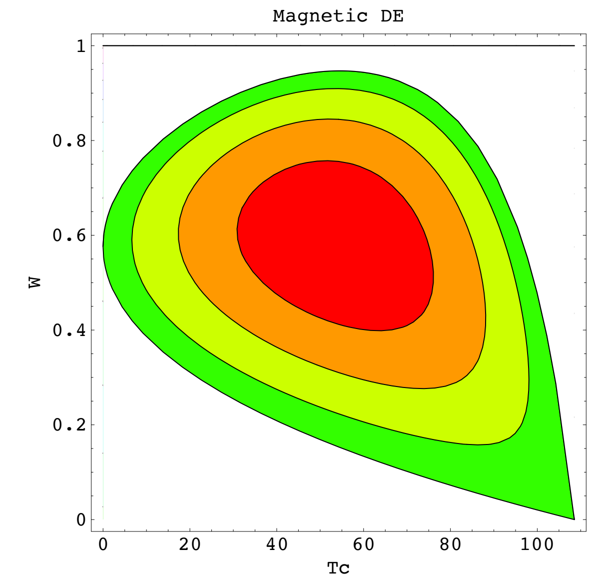
$$W^2 = (q \cdot p_K)(q \cdot p_+)/(m_\pi^2 m_K^2)$$

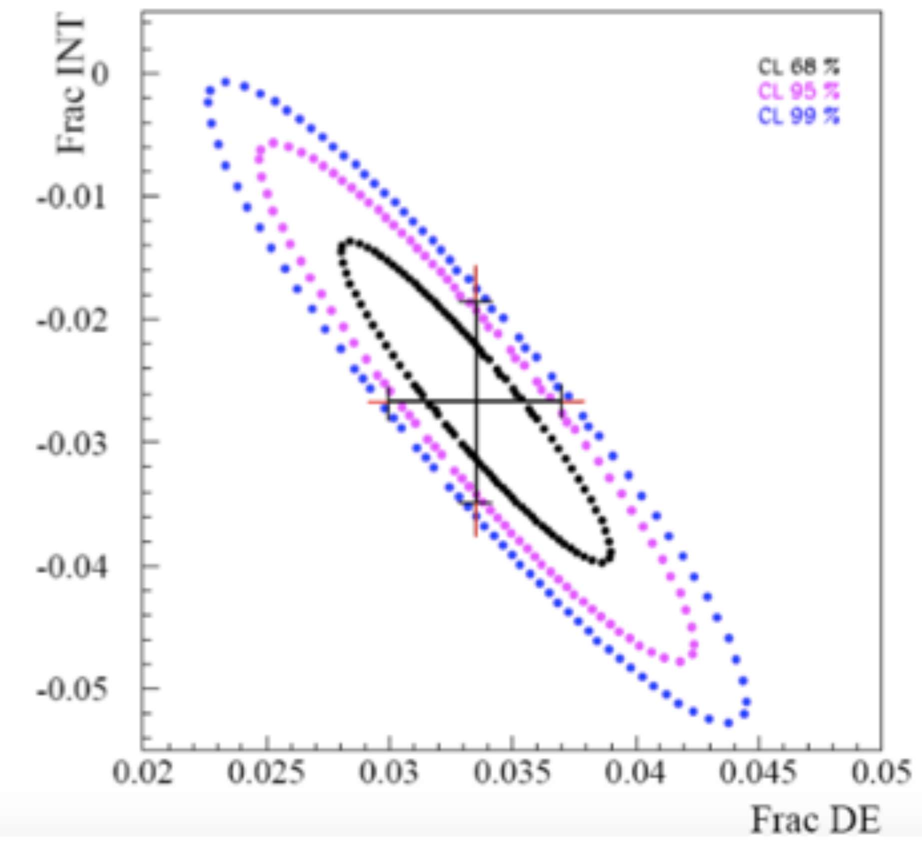
$$A = A(K^+ \to \pi^+\pi^0)$$

Dalitz plot NA48/2

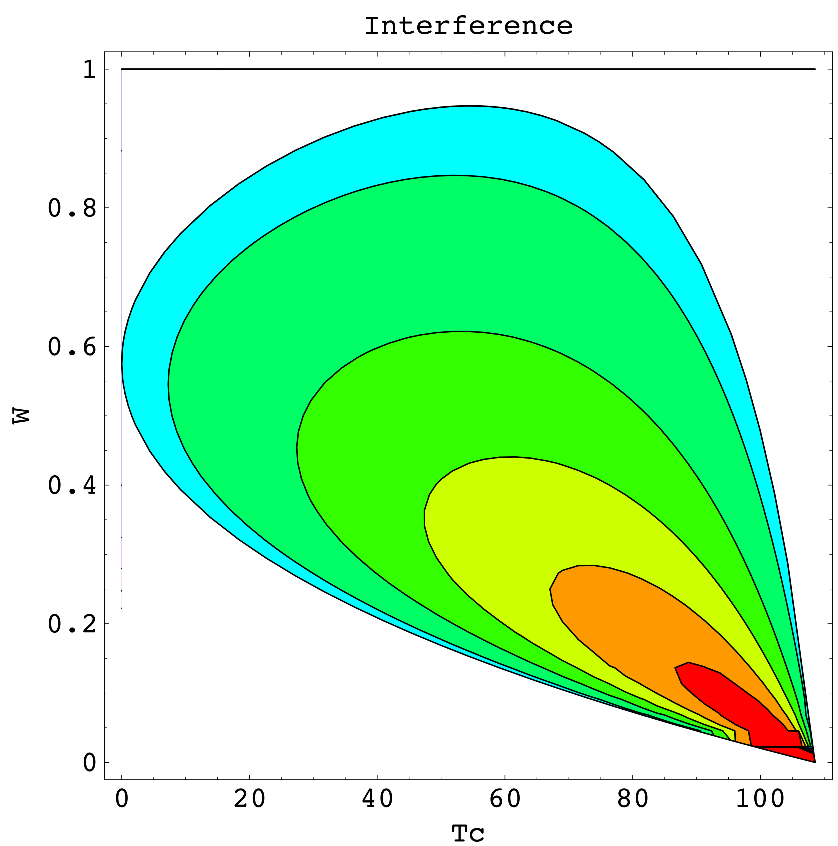








NA48/2 CP violation



Dalitz plot analysis crucial

SM $\leq \mathcal{O}(10^{-5})$ Paver et al.

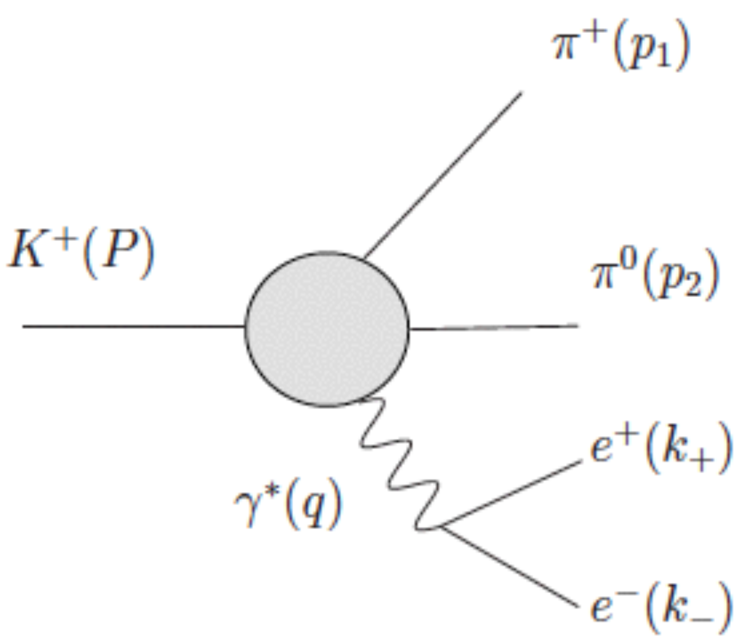
NP $\leq \mathcal{O}(10^{-4})$ Colangelo et al.

NA48/2 $< 1.5 \cdot 10^{-3}$ at 90% CL

BUT NOT in the interesting interf. kin. region (statistics)

$K_L \to \pi^+\pi^-\gamma^* \to \pi^+\pi^- e^+ e^-$

Sehgal et al; Savage,Wise et al



- $\mathcal{M}_{LD} = \frac{e}{q^2}\, \bar{e}\gamma^\mu(1-\gamma^5)\ e\ H_\mu$
- $H^\mu = F_1 p_1^\mu + F_2 p_2^\mu + F_3\ \varepsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\alpha} q_\beta$
- $F_{1,2} \sim E \quad F_3 \sim M$

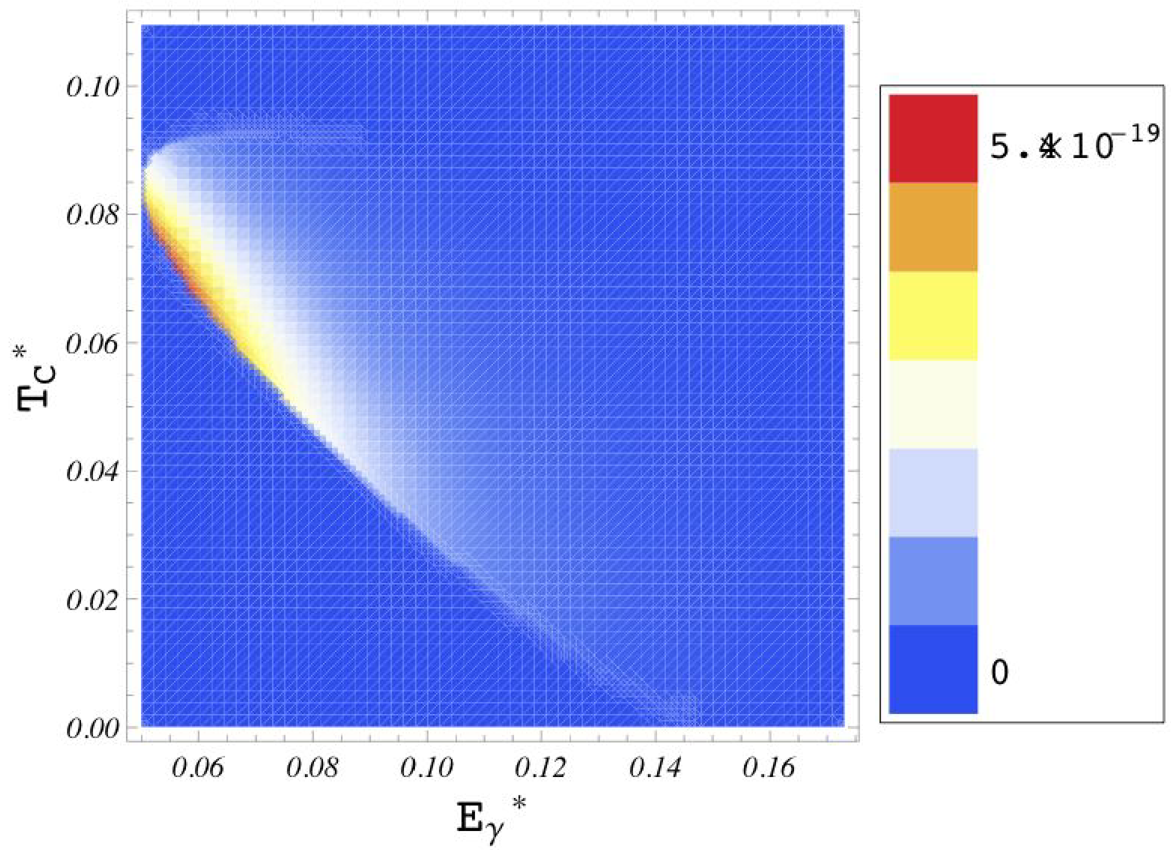
- Interference $E \quad M$ novel compared to $K_L \to \pi^+\pi^-\gamma$
- $E \quad M$ known from $K_L \to \pi^+\pi^-\gamma$ (IB and DE)

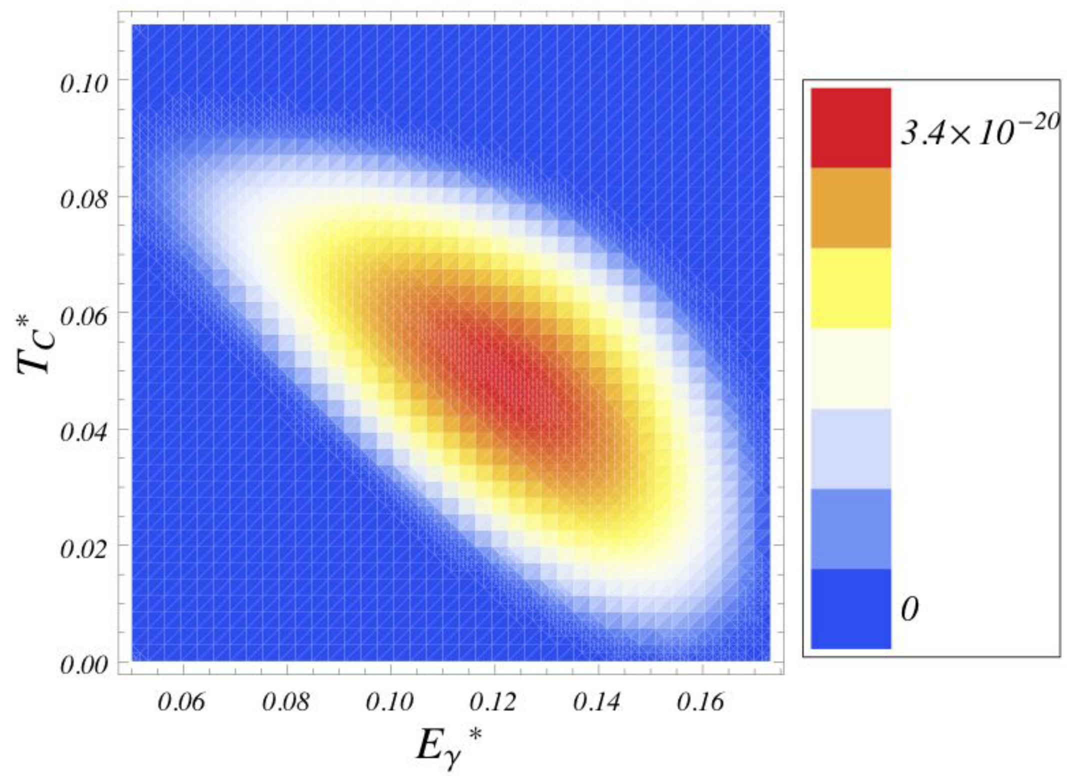
$K^+ \to \pi^+\pi^0\gamma^* \to \pi^+\pi^0 e^+ e^-$ Cappiello, Cata,G.D. and Gao,

- the asymm. , $\frac{\Re(E_B\ M^*)}{|E_B|^2+|M|^2}$, not as lucky $E_B >> M$:
- $B(K^+)_{IB} \sim 3.3 \times 10^{-6}\ \sim\ 50\ B(K^+)_M$
- Short distance info without having simultaneously K^+ and K^-, asymm. in phase space, (P-violation) interesting! No ϵ-contamination
- interesting Dalitz plots (at fixed q^2) to disentangle M from E_B
- at $q^2 = 50\mathrm{MeV}$ IB only 10 times larger than DE

Starting from CP conserving IB, DE

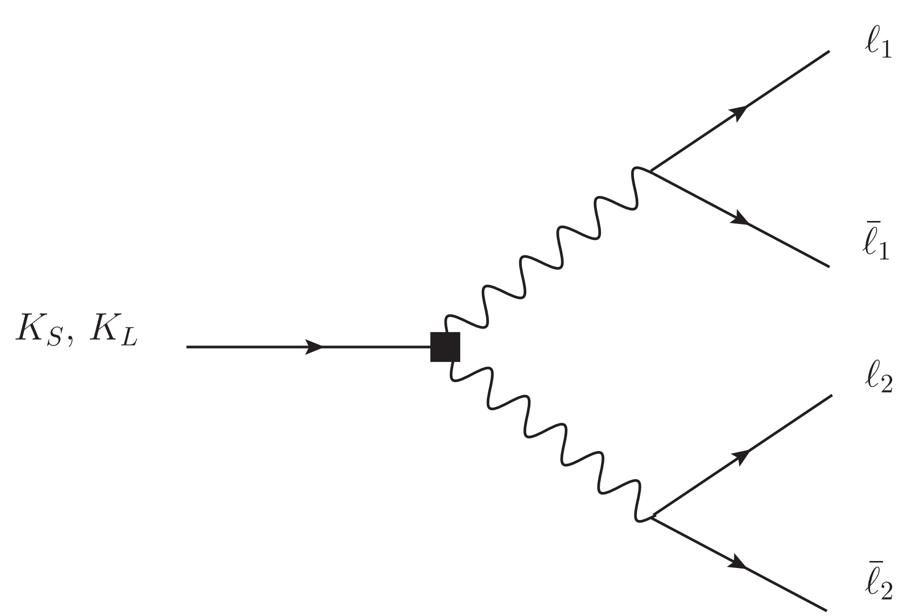
q_c (MeV)	B $[10^{-8}]$	B/M	B/E	B/BE	B/BM
$2m_l$	418.27	71	4405	128	208
55	5.62	12	118	38	44
100	0.67	8	30	71	36
180	0.003	12	5	-19	44





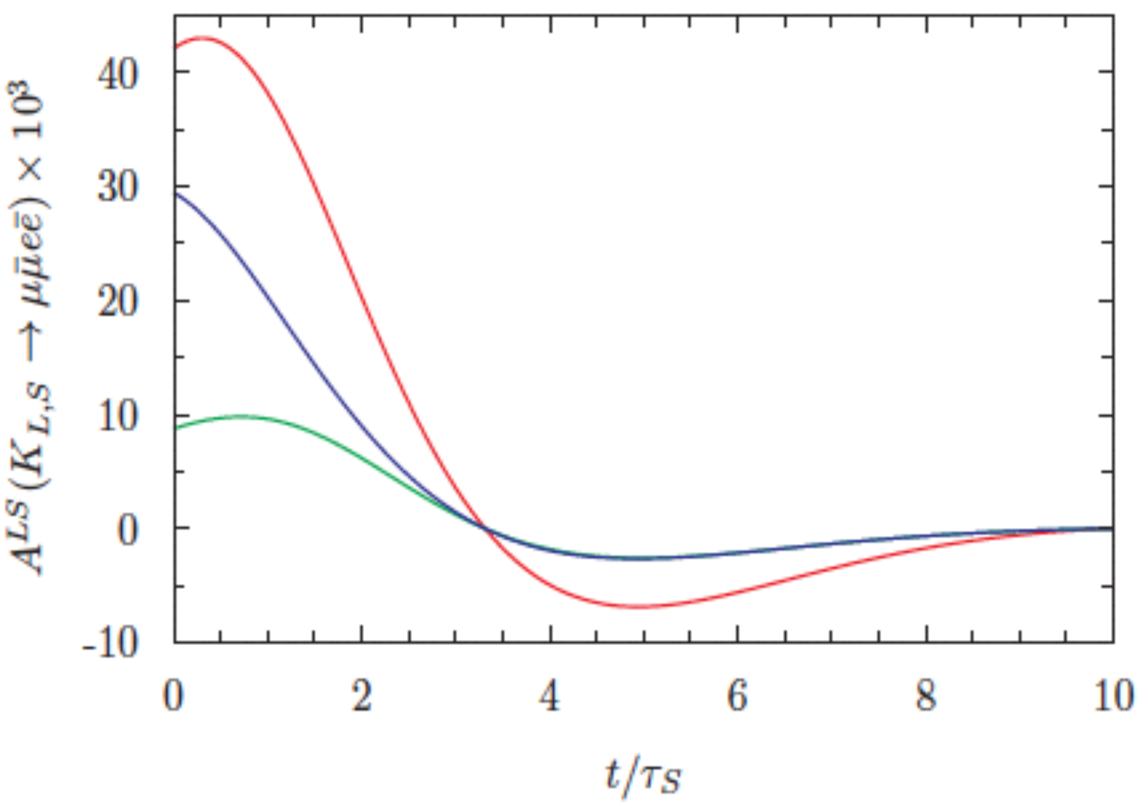
Other interesting channels

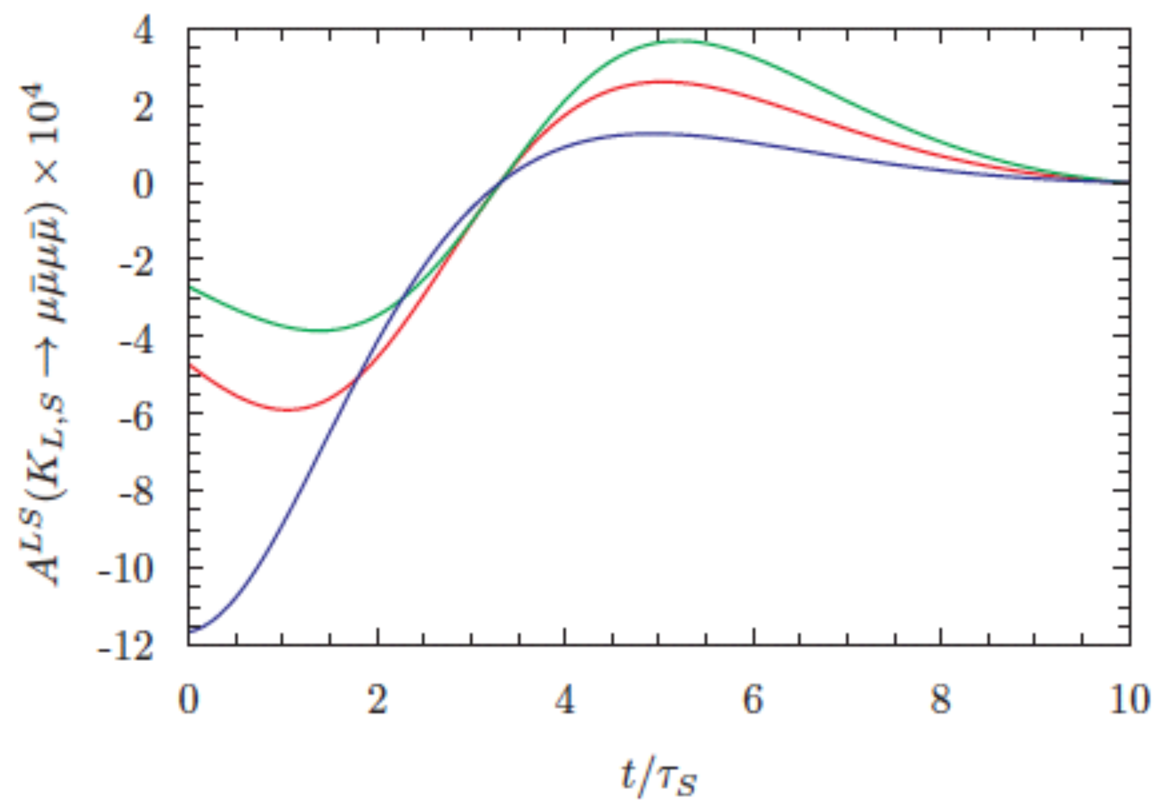
$$K_S \to \mu\mu\mu\mu \quad - \quad \text{SM LD} \;\sim 2\times 10^{-14}$$
$$K_S \to ee\mu\mu \quad - \quad \sim 10^{-11}$$
$$K_S \to eeee \quad - \quad \sim 10^{-10}$$



GD,Greynat, Vulvert

Time interference effects





Interferences between K_L and $K_S \to \ell_1\bar{\ell}_1\ell_2\bar{\ell}_2$. The red line corresponds to the case $\alpha_S = 0$, the green line is $\alpha_S = -3$ while the blue line is $\alpha_S = 3$. As explained in the text we assume the sign $K_L \to \gamma\gamma$. For 4μ's 10^{14} K_S needed , $ee\mu\mu$ 10^{12}

Conclusion

- BBG interesting and positive news from our calculations
- $K^+ \to \pi^+ \pi^0 l^+ l^-$ hard fight but good perspectives, more CP violating observables
- $K_S \to \pi^+ \pi^- \mu^+ \mu^-$ Br ~ 10^{-14}

Kaon physics

Tests of CPV already among most stringent (ε_K, ε')

Near future improvements mostly due to theory (Lattice)

More progress foreseen in rare decays

$\Rightarrow$ $K^+ \to \pi^+ \nu\bar{\nu}, K_L \to \pi^0 \nu\bar{\nu}$

$\Rightarrow$ rare K decays at HL-LHCb?

d'Ambrosio, PoS(FPCP2015)018

	PDG	Prospects	
$K_S \to \mu\mu$	$< 9 \times 10^{-9}$ at 90% CL	$(\mathrm{LD})(5.0 \pm 1.5) \cdot 10^{-12}$ NP $< 10^{-11}$	
$K_L \to \mu\mu$	$(6.84 \pm 0.11) \times 10^{-9}$	difficult : SD << LD	
$K_S \to \mu\mu\mu\mu$	–	SM LD $\sim 2 \times 10^{-14}$	NP?
$K_S \to ee\mu\mu$	–	$\sim 10^{-11}$	NP?
$K_S \to eeee$	–	$\sim 10^{-10}$	NP?
$K_S \to \pi^+\pi^-\mu^+\mu^-$	–	SM LD $\sim 10^{-14}$	

Vector Meson Dominance in the strong sector

Ecker, Gasser, de Rafael, Pich

L_i	L_i expts	V	A	Total (Scalar incl.)	Total QCD rel. incl.
L_1	0.4 ± 0.3	0,6	0	0,6	0,9
L_2	1.4 ± 0.3	1,2	0	1,2	1,8
L_3	-3.5 ± 1.1	-3,6	0	-3,0	-4,9
L_4	-0.3 ± 0.5	0	0	0	0
L_5	1.4 ± 0.5	0	0	1,4	1,4
L_6	-0.2 ± 0.3	0	0	0	0
L_7	-0.4 ± 0.2	0	0	-0,3	-0,3
L_8	0.9 ± 0.3	0	0	0,9	0,9
L_9	6.9 ± 0.7	6,9	0	6,9	7,3
L_{10}	-5.5 ± 0.7	-10	4	-6,0	-5,5

QCD inspired relations relations

$$F_V = 2G_V = \sqrt{2} f_\pi$$

$$F_A = f_\pi$$

$$M_A = \sqrt{2} M_V$$

KSFR: $G_V = \sqrt{2}\ F_\pi$ determined by dominance of pion, V,A to recover QCD short distance constraints

$$L_1^V = \frac{L_2^V}{2} = -\frac{L_3^V}{6} = \frac{G_V^2}{8M_V^2}, \qquad L_9^V = \frac{F_V G_V}{2M_V^2}, \qquad L_{10}^{V+A} = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}$$

QCD inspired relations relations

$$L_1^V = L_2^V/2 = -L_3^V/6 = L_9^V/8 = -L_{10}^{V+A}/6 = f_\pi^2/(16M_V^2)$$