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$K \to \pi \ell^+ \ell^-$

Status and update

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Summary

1. Brief survey on rare decays
2. $K^+ \to \pi^+ \ell^+ \ell^-,\ K_S \to \pi^0 \ell^+ \ell^-$
3. $K_L \to \pi^0 \ell^+ \ell^-$
4. Overview

SM only

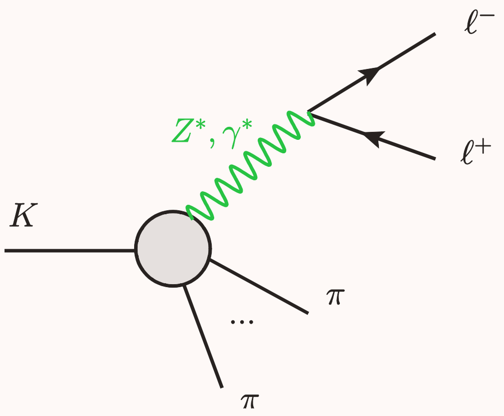
I. Brief survey on rare decays

Past, Present and Future Experiments

Experiment	Kaon Physics Main Goal
NA48 (CERN),KTeV (Fermilab)	$K_{\ell 3},\ K_{\ell 4},\ K \to \pi\pi/\pi\pi\pi,\ K \to \pi\gamma\gamma,$ $K \to \pi\ell^+\ell^-\ ,\ \varepsilon'$
NA62 (CERN)	$K^+ \to \pi^+\nu\bar{\nu}\ ,\ K^+ \to \pi^+\gamma\gamma$
K^0TO (J-PARC)	$K_L \to \pi^0\nu\bar{\nu}$
TREK (J-PARC)	$K^+ \to \pi^0\mu^+\nu_\mu$
KLOE-2 (KLOE) (DAΦNE)	CP issues, radiative decays
OKA (ISTRA+) (IHEP, Protvino)	Kaon decays (BR ~ $10^{-3} – 10^{-8}$)
KLOD (IHEP, Protvino) 👎	$K_L \to \pi^0\nu\bar{\nu}$
Project – X (Fermilab) 👎	$K \to \pi\nu\bar{\nu},\ \ K_L \to \pi^0\ell^+\ell^-$
LHCb	$K_S \to \mu^+\mu^-\ ,\ K_S \to \pi^0\mu^+\mu^-$

Outlook on rare decays

Type



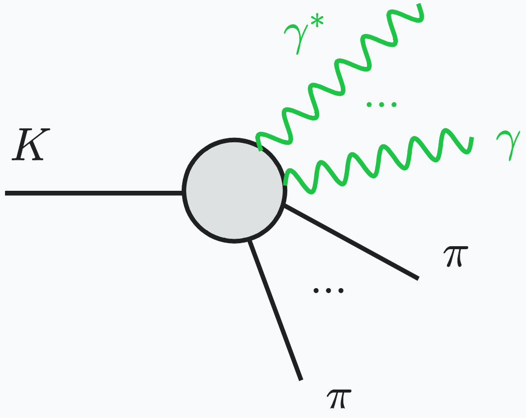
Processes

$$K \to \ell^+\ell^-, \quad K \to \pi\ell^+\ell^-$$
$$K \to \pi\pi\ell^+\ell^-$$

Main Features

Low-energy dominated, many FCNC

Type

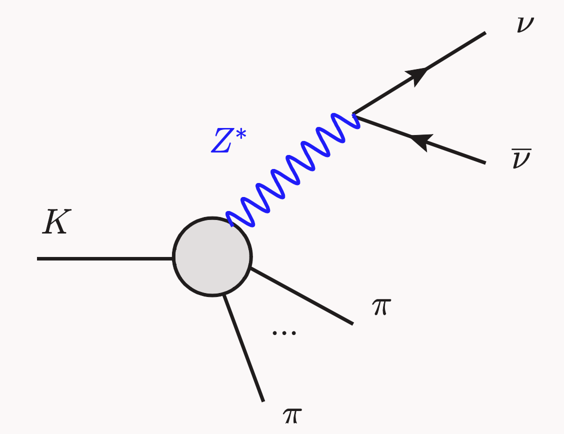


Processes

$$K \to \gamma\gamma^{(*)}, \quad K \to \pi\gamma\gamma^{(*)}$$
$$K \to \pi\pi\gamma^{(*)} \quad K \to \ell^+\ell^-\ell^+\ell^-$$

Main Features

Low-energy dominated, some FCNC

Type	Processes
	$K \to \pi\nu\overline{\nu}, \quad K_L \to \gamma\nu\overline{\nu}$ $K \to \pi\pi\nu\overline{\nu}$
	Main Features
	High-energy dominated, FCNC

V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, J.P. **Kaon Decays in the Standard Model**, Rev. Mod. Phys. 84 (2012) 399

$$BR \geq 10^{-11}$$

Non- Rare versus Rare Decays

BR > 10^{-5}

Decay	BR
$K^+ \to \pi^+ \nu_\mu$	0.6355 (11)
$K^+ \to \pi^0 \mu^+ \nu_\mu$	0.03353 (34)
$K_L \to \pi^\pm e^\mp \nu_e$	0.4055 (12)
$K^+ \to \pi^+ \pi^0$	0.2066 (8)
$K^+ \to \pi^+ \pi^+ \pi^-$	0.0559 (4)
$K_S \to \pi^0 \pi^0$	0.3069 (5)
$K_S \to \pi^+ \pi^-$	0.6920 (5)
$K_L \to \pi^0 \pi^0 \pi^0$	0.1952 (12)
$K_L \to \pi^+ \pi^- \pi^0$	0.1254 (5)
$K^+ \to \pi^+ \pi^0 \gamma$	2.75 (15) x 10^{-4}
$K_L \to \gamma\gamma$	5.47 (4) x 10^{-4}
$K_L \to \pi^+ \pi^- \gamma$	4.15 (15) x 10^{-5}
$K_S \to \pi^+ \pi^- \gamma$	1.79 (5) x 10^{-3}

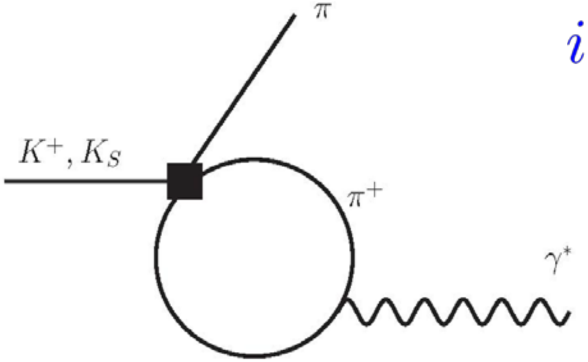
BR < 10^{-5}

Decay	BR x 10^{5}
$K^+ \to \pi^+ \gamma\gamma$	0.1003 (56)
$K^+ \to \pi^+ e^+ e^- \gamma$	1.19 (13) x 10^{-3}
$K^+ \to \pi^+ e^+ e^-$	0.0300 (9)
$K_S \to \gamma\gamma$	0.263 (17)
$K_S \to \pi^0 \mu^+ \mu^-$	2.9 (1.5) x 10^{-4}
$K_S \to \mu^+ \mu^-$	< 9 x 10^{-4} (90% C.L.)
$K_L \to \pi^0 \gamma\gamma$	0.1274 (34)
$K_L \to e^+ e^-$	9 $(^{+6}_{-4})$ x 10^{-7}
$K_L \to \pi^+ \pi^- e^+ e^-$	0.0311 (19)
$K_L \to \mu^+ \mu^- e^+ e^-$	2.69 (27) x 10^{-4}
$K_L \to \pi^0 \mu^+ \mu^-$	< 3.8 x 10^{-5} (90% C.L.)
$K^+ \to \pi^+ \nu\bar{\nu}$	1.7 (1.1) x 10^{-5}
$K_L \to \pi^0 \nu\bar{\nu}$	< 6.7 x 10^{-3} (90% C.L.)

II. $K^+ \to \pi^+ \ell^+ \ell^-\,,\; K_S \to \pi^0 \ell^+ \ell^-$

$K^\pm, K_S \to \pi \ell^+ \ell^-$

Long-distance dominated



$$i \int d^4x\, e^{iqx} \langle \pi(p) | T \left\{ J^\mu_{em} \mathcal{L}_{\Delta S=1}(0) \right\} | K_j(k) \rangle =$$

$$= \frac{G_F M_K^2}{(4\pi)^2} V_j(z) \left[z(k+p)^\mu - (1 - r_\pi^2) q^\mu \right]$$

$q = k - p$, $z = q^2/M_K^2$, $r_\pi = M_\pi / M_K$ $\qquad j = +, S$

[Ecker et al, 1987][D'Ambrosio et al, 1998]

.....up to $\mathcal{O}(p^6)$ in χPT

$$\boxed{V_j(z) = a_j + b_j z + V_j^{\pi\pi}(z)}$$

$$V_j^{\pi\pi}(z) = \underbrace{\frac{\alpha_j r_\pi^2 + \beta_j (z - z_0)}{G_F M_K^2 r_\pi^4}}_{K \to \pi\pi\pi} \underbrace{\left[1 + \frac{z}{r_V^2} \right]}_{F_V(z)} \underbrace{\left[\Phi\left(z/r_\pi^2 \right) + \frac{1}{6} \right]}_{\text{loop}}$$

$z_0 = r_\pi^2 + 1/3$, $\quad r_V = M_V / M_K$

Analysis

$$V_j(z) = a_j + b_j z + \overbrace{\frac{\alpha_j r_\pi^2 + \beta_j (z - z_0)}{G_F M_K^2 r_\pi^4}}^{K \to \pi\pi\pi} \overbrace{\left[1 + \frac{z}{r_V^2}\right]}^{F_V(z)} \overbrace{\left[\Phi\left(z/r_\pi^2\right) + \frac{1}{6}\right]}^{\text{loop}}$$

$$z = q^2/M_K^2$$

a_j, b_j

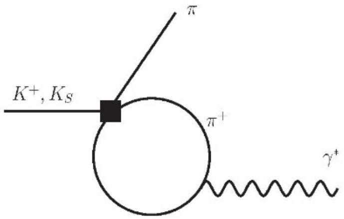
Not fully determined by chiral symmetry (LEC’s)

$$a_+, a_S \sim \mathcal{O}(p^4) + \ldots$$

$$b_+, b_S \sim \mathcal{O}(p^4) + \mathcal{O}(p^6) + \ldots$$

Tiny Kaon loop contribution

Higher energy contributions



α_j, β_j

Devlin and Dickey parameterization of $K \to \pi\pi\pi$

$$\alpha_+, \beta_+ \sim \Delta I = 1/2 \ (G_8), \quad \Delta I = 3/2 \ (G_{27})$$

$$\alpha_S, \beta_S \sim \Delta I = 3/2 \ (G_{27})$$

[Ananthanarayan et al, 2012]

$$F_V(z) = \frac{M_V^2}{M_V^2 - q^2} \simeq 1 + \frac{q^2}{M_V^2} \ldots$$

Analysis

$$V_j(z) = a_j + b_j z + \overbrace{\frac{\alpha_j r_\pi^2 + \beta_j (z-z_0)}{G_F M_K^2 r_\pi^4}}^{K \to \pi\pi\pi} \overbrace{\left[1 + \frac{z}{r_V^2}\right]}^{F_V(z)} \overbrace{\left[\Phi\left(z/r_\pi^2\right) + \frac{1}{6}\right]}^{\text{loop}}$$

$$z = q^2/M_K^2$$

a_j, b_j

Not fully determined by chiral symmetry (LEC’s)

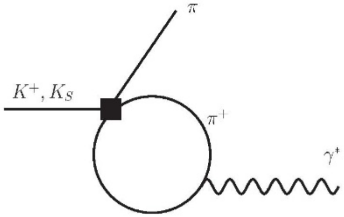
$$a_+, a_S \sim \mathcal{O}(p^4) + \ldots$$

$$b_+, b_S \sim \mathcal{O}(p^4) + \mathcal{O}(p^6) + \ldots$$

Higher energy contributions

$$\frac{b_j}{a_j} = \frac{M_K^2}{M_V^2} \sim 0.4$$

$$a_j\, b_j > 0$$



α_j, β_j

Devlin and Dickey parameterization of $K \to \pi\pi\pi$

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[Ananthanarayan et al, 2012]

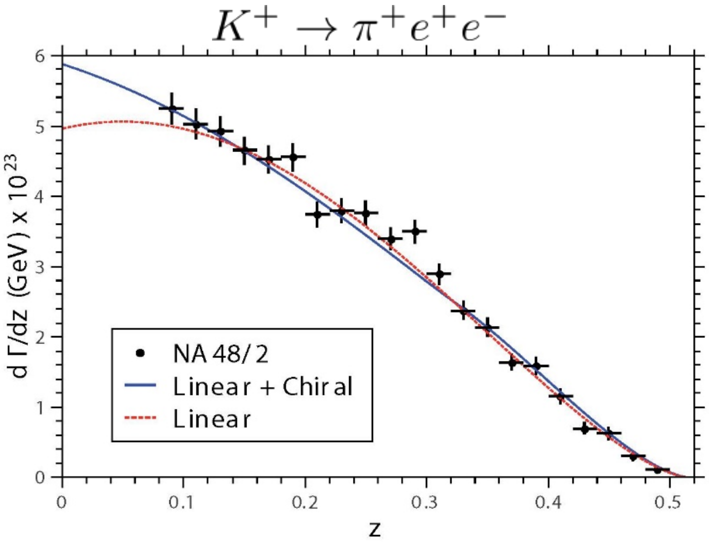
$$F_V(z) = \frac{M_V^2}{M_V^2 - q^2} \simeq 1 + \frac{q^2}{M_V^2} \ldots$$

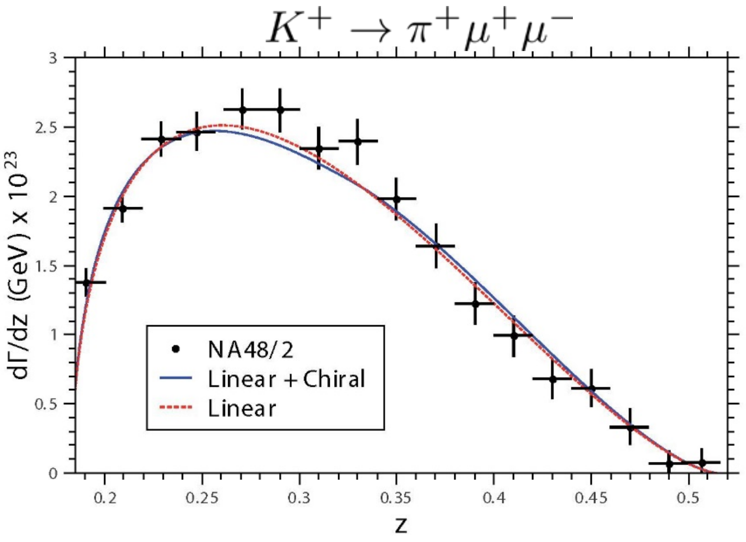
Vector Meson Dominance

Phenomenology

[NA48/1, 2003,2004][NA48/2,2009,2011]

Process	Br x 10^9	*a*	*b*	*b/a*
$K^+ \to \pi^+ e^+ e^-$	314 ± 10	-0.578 ± 0.016	-0.779 ± 0.066	~ 1.35
$K^+ \to \pi^+ \mu^+ \mu^-$	96.2 ± 2.5	-0.575 ± 0.039	-0.813 ± 0.145	~ 1.41
$K_S \to \pi^0 e^+ e^-$	$5.8^{+2.9}_{-2.4}$	$\lvert 1.06 \rvert^{+0.26}_{-0.21}$	$a_S\, M_K^2/M_\rho^2$	-
$K_S \to \pi^0 \mu^+ \mu^-$	$2.0^{+1.5}_{-1.2}$	$\lvert 1.54 \rvert^{+0.40}_{-0.32}$	$a_S\, M_K^2/M_\rho^2$	-





Models...

[Ecker et al, 1990]

Weak Deformation Model $u_\mu \longrightarrow u_\mu + G_8 F_\pi^2 \left\{u_\mu, u\lambda_6 u^\dagger\right\} - \frac{2}{3} G_8 F_\pi^2 \left\langle u_\mu u\lambda_6 u^\dagger\right\rangle \mathbb{1}$

[Friot et al, 2004]

Large-N$_C$ QCD, Minimal Hadronic Approximation: $a_j + b_j\, z = f\left(z, M_\rho^2, M_{K^*}^2\right)$

[Dubnickova et al, 2008]

Padé-type approximation involving resonances and chiral loops

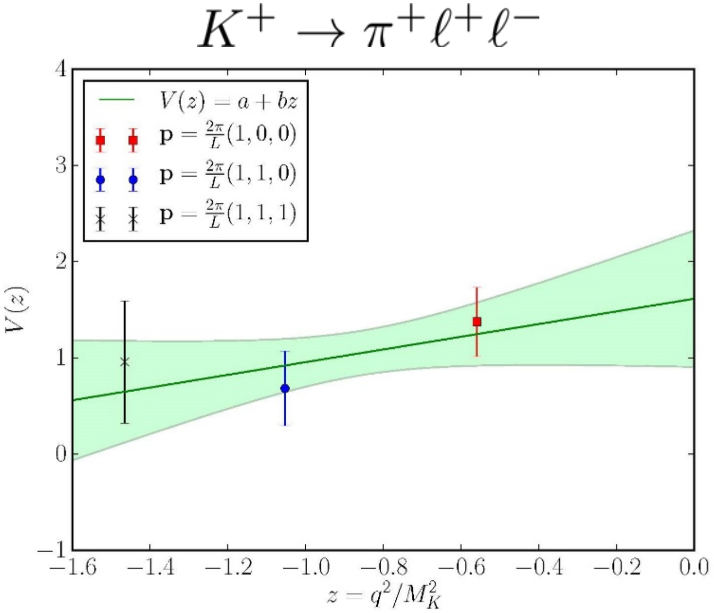
[Coluccio Leskow et al, 2016]

Bardeen-Buras-Gérard model, Large-N$_C$, $M = 0.7\,\mathrm{GeV}$, a_+, b_+

✓

… and the Lattice

[RBC and UKQCD, 2015, 2016][Isidori et al, 2006]



$$V_+(z) = a_+ + b_+ z$$

- Unphysically heavy pion and kaon masses.
- Unphysically light charm mass (GIM).
- Extrapolation to the physical point: $\mathcal{O}(p^4)$ χPT results.

$$a_+ = 1.4 \pm 0.7, \quad b_+ = 0.7 \pm 0.8$$

Process	Br x 10^8	*a*	*b*	*b/a*
$K^+ \to \pi^+ e^+ e^-$	31.4 ± 1.0	-0.578 ± 0.016	-0.779 ± 0.066	~ 1.35
$K^+ \to \pi^+ \mu^+ \mu^-$	9.62 ± 0.25	-0.575 ± 0.039	-0.813 ± 0.145	~ 1.41

Charge asymmetry

[D'Ambrosio et al, 1998]

Interference between $K \to \pi\gamma^*$ and

$$Q_{7V} = [\bar{s}\gamma^\mu(1-\gamma_5)d] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu \ell]$$

$$\Gamma(K^+ \to \pi^+\ell^+\ell^-) - \Gamma(K^- \to \pi^-\ell^+\ell^-) \propto$$

$$\Gamma(K^+ \to \pi^+\ell^+\ell^-) - \Gamma(K^- \to \pi^-\ell^+\ell^-) \propto \int dz\, \mathrm{Im}\,(a_+ + b_+ z)\, \mathrm{Im}\,(V_+(z))$$

$$\mathrm{Im}\, a_+ = \frac{4\pi}{\sqrt{2}} \frac{y_{7V}}{\alpha} \mathrm{Im}\, \lambda_t \qquad \frac{\mathrm{Im}\, b_+}{\mathrm{Im}\, a_+} \simeq \frac{M_K^2}{M_V^2}$$

$$\lambda_t = V_{td} V_{ts}^*$$

$$\mathrm{Im}\, \lambda_t \simeq \eta\, \lambda^5 A^2$$

$$\frac{|\Gamma(K^+\to\pi^+e^+e^-) - \Gamma(K^-\to\pi^-e^+e^-)|}{\Gamma(K^+\to\pi^+e^+e^-) + \Gamma(K^-\to\pi^-e^+e^-)} \simeq 10^{-5} \times \left(\frac{|\mathrm{Im}\, \lambda_t|}{10^{-4}}\right)$$

Unintegrated asymmetry

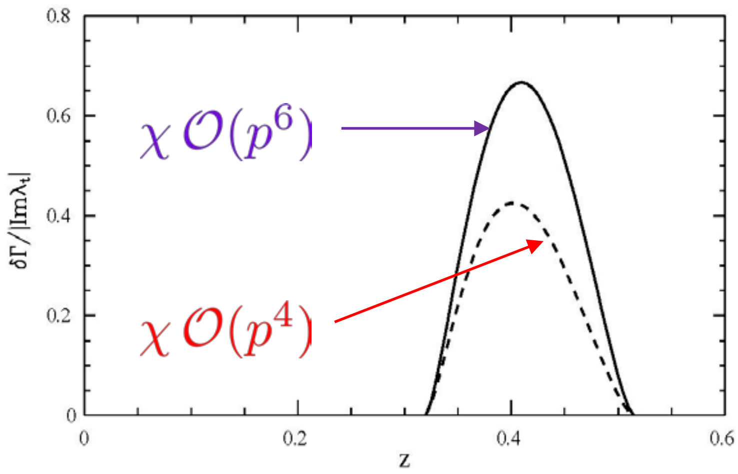
$$\delta\Gamma(z) = \frac{\left|\frac{d\Gamma}{dz}(K^+\to\pi^+e^+e^-) - \frac{d\Gamma}{dz}(K^-\to\pi^-e^+e^-)\right|}{\Gamma(K^+\to\pi^+e^+e^-) + \Gamma)K^-\to\pi^-e^+e^-)}$$

… far away of the dominant background

$$K^\pm \to \pi^\pm \pi^0_D$$

$$\hookrightarrow e^+e^-\gamma$$

$$z = (M_{ee}/M_K)^2 > 0.08$$



III. $K_L \to \pi^0 \ell^+ \ell^-$

$K_L \to \pi^0 \ell^+ \ell^-$

Short-distance contributions
Long-distance contributions

$$K_L \to \pi^0 \gamma^* \qquad \cancel{CP}$$

$$S(z,y), P(z,y), V(z,y), A(z,y),$$

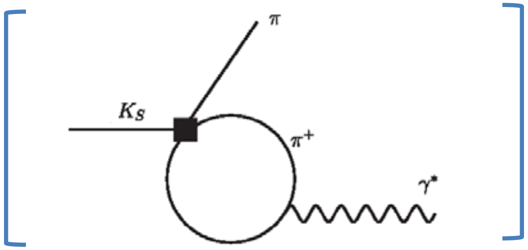
$$z \sim (p+p')^2, \quad y \sim p_K \cdot (p-p')$$

1. CP-violating contributions

[Flynn et al, 1989][Heiliger et al, 1993][D’Ambrosio et al, 1998][Buchalla et al, 2003] [Isidori et al, 2004]

- Indirect CP-violating transition due to $K^0 - \overline{K}^0$ oscillation

$$A^{ind}_{\mathrm{CPV}}\left(K_L \to \pi^0 \ell^+ \ell^-\right) = \varepsilon \times$$



$$V^{ind}(z) = \pm \varepsilon \left[a_S + b_S z + V_S^{\pi\pi}(z) \right]$$

- Direct CP-violating transition

$$\mathcal{L}_{\rm eff}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud}V_{us}^* \left[C_{7V}(\mu)\, [\bar{s}\gamma^\mu(1-\gamma_5)d] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu \ell] + C_{7A}(\mu)\, [\bar{s}\gamma^\mu(1-\gamma_5)d] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu\gamma_5 \ell] \right]$$

$$C_i(\mu) = z_i(\mu) - \frac{\lambda_t}{\lambda_u}\, y_i(\mu)$$

$$V^{dir}(z) = i\,\frac{4\pi}{\sqrt{2}\alpha}\, y_{7V}\, {\rm Im}\,\lambda_t\, f_+^{K\pi}(z)$$

$$A(z) = i\,\frac{4\pi}{\sqrt{2}\alpha}\, y_{7A}\, {\rm Im}\,\lambda_t\, f_+^{K\pi}(z)$$

$$P(z) = -i\,\frac{8\pi}{\sqrt{2}\alpha}\, y_{7A}\, {\rm Im}\,\lambda_t\, f_-^{K\pi}(z)$$

$$\langle \pi(p_\pi) |\, \bar{s}\,\gamma_\mu\, u\, | K(p_K) \rangle = (p_K+p_\pi)_\mu\, f_+^{K\pi}(t) + (p_K-p_\pi)_\mu\, f_-^{K\pi}(t) \qquad t=(p_K-p_\pi)^2$$

All $\cancel{CP}$ contributions

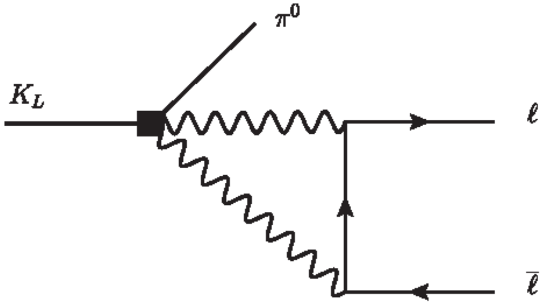
$$\text{BR}\left(K_L\to\pi^0 e^+e^-\right)\Big|_{\rm CPV} = 10^{-12}\times\left[15.7\,|a_S|^2 \pm 6.2\,|a_S|\left(\frac{{\rm Im}\,\lambda_t}{10^{-4}}\right) + 2.4\left(\frac{{\rm Im}\,\lambda_t}{10^{-4}}\right)^2\right]$$

$$\text{BR}\left(K_L\to\pi^0 \mu^+\mu^-\right)\Big|_{\rm CPV} = 10^{-12}\times\left[3.7\,|a_S|^2 \pm 1.6\,|a_S|\left(\frac{{\rm Im}\,\lambda_t}{10^{-4}}\right) + 1.0\left(\frac{{\rm Im}\,\lambda_t}{10^{-4}}\right)^2\right]$$

$$K_S\to\pi^0\ell^+\ell^- \quad\longrightarrow\quad |a_S| \sim 1$$

2. CP-conserving contribution from $K_L \to \pi^0\gamma\gamma \to \pi^0\ell^+\ell^-$

[Heiliger et al, 1993][Isidori et al, 2004]



Assuming positive interference between the CP-V contributions (theoretically preferred) …

	$\mathrm{BR}\left(K_L \to \pi^0 e^+ e^-\right)$	$\mathrm{BR}\left(K_L \to \pi^0 \mu^+ \mu^-\right)$
CP-V	$(3.1 \pm 0.9) \times 10^{-11}$	$(1.4 \pm 0.5) \times 10^{-11}$
CP-C	~ 0	$(5.2 \pm 1.6) \times 10^{-12}$
KTeV (90% C.L.)	$< \ 2.8 \times 10^{-10}$	$< \ 3.8 \times 10^{-10}$

[KTeV, 2000,2004]]

IV. Overview

1. Rare Kaon decays provide an excellent framework to settle SM predictions and, consequently, might foresee hints of BSM effects.

2. Most of long-distance dominated rare decays can also be predicted reasonably well within a 30 % in the branching ratios. This is not precision physics. In general, it will be difficult to increase the accuracy in the theoretical predictions of these processes….but

3. LATTICE (RBC and UKQCD) has started the study of several Rare Kaon decays: $K \to \pi \ell^+ \ell^-$ and $K \to \pi \nu \bar{\nu}$.

4. From a SM theoretical point of view our comprehension of $K \to \pi \ell^+ \ell^-$ decays is satisfactory. We can only proceed further by the use of models or LATTICE.

5. The experimental measurement (K^{0}TO, JPARC) of $K_L \to \pi^0 \ell^+ \ell^-$ would provide valuable information on the SM contributions.

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Stubbornly

Testing QCD

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