

# Radiative corrections in kaon decays

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# OUTLINE

- Introduction

- Tools

Radiative corrections to kaon decays from the low-energy EFT point of view

NR-EFT and  $K \rightarrow \pi\pi\pi$

Dispersive constructions and IB corrections in  $K_{e4}$

(Lattice  $\longrightarrow$  cf. talk by G. Martinelli)

- Radiative corrections in real life: the  $K_{e4}^{00}$  case

- Conclusion

For more (radiative and other kaon decay modes), see

V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, J. Portolés, Rev. Mod. Phys. 84, 399 (2012) [arXiv:1107.6001 [hep-ph]]

# Introduction

Precision measurements on kaon decays allow to put constraints on physics beyond the standard model (tests of lepton flavour universality or of CKM unitarity, CP violation, admixture of right-handed currents,...)...

... but also provide information on low-energy strong interactions (e.g.  $\pi\pi$  scattering lengths, structure of form factors,...), that allow to test predictions or to determine non-perturbative parameters (low-energy constants) that occur also in other processes

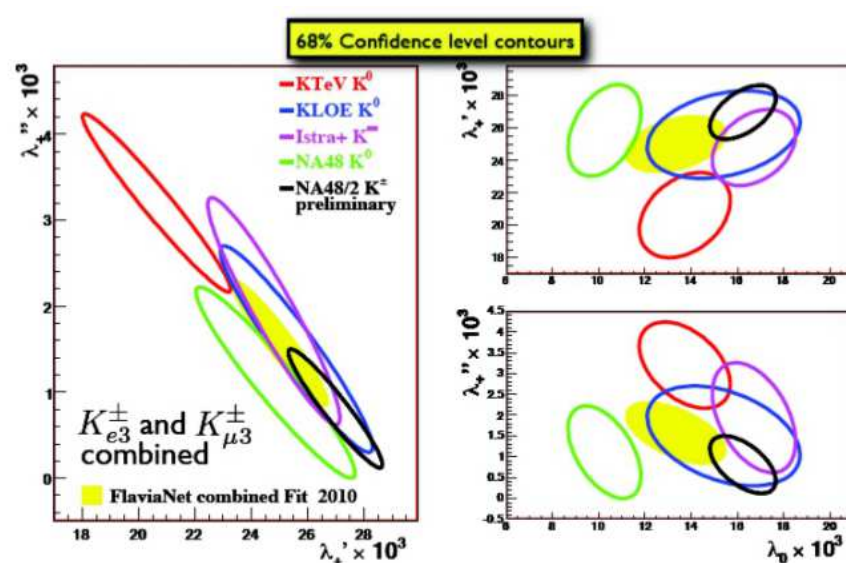
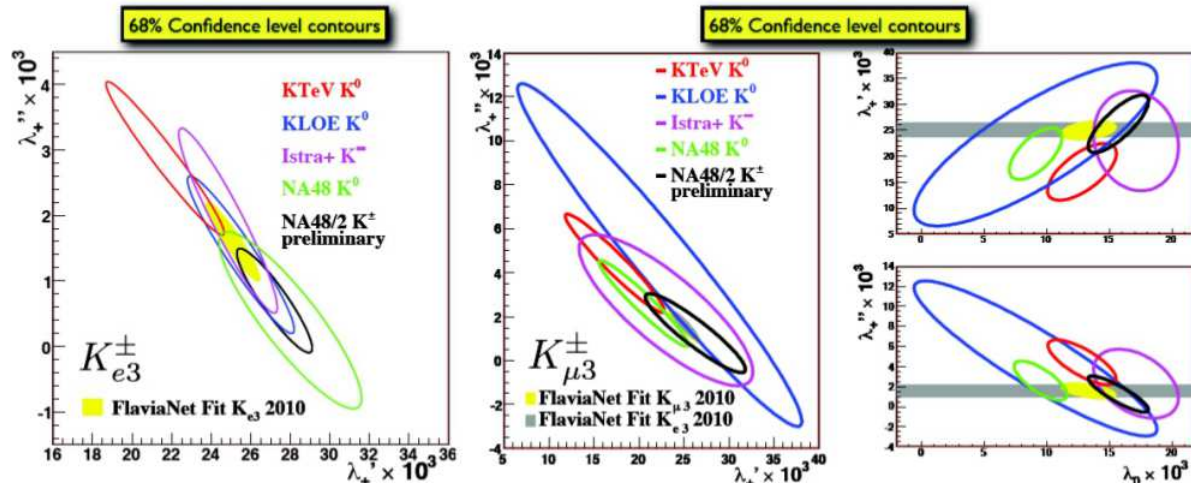
Predictions are often made in a world where  $\alpha = 0$  ( $M_\pi = M_{\pi^0}$ ), and even  $m_u = m_d$ . One has to connect this “theoretician’s paradise” (J. Gasser) to the real world, where  $\alpha \neq 0$  (which means in particular  $M_\pi \neq M_{\pi^0}$ )

—→ quantitative control over radiative, or more generally, isospin-breaking (IB) corrections has become mandatory

A lot of progress on the experimental side during the last decade or so on “traditional” (i.e. non rare) kaon decay modes (ISTRA+ @ IHEP, KTeV @ FNAL, KLOE and KLOE2 @ DAΦNE, NA48 and NA48/2 @ SPS), and more is to come, e.g. NA62

Illustration with structure of  $K_{\ell 3}$  form factors...

M. Antonelli et al., Eur Phys. J. C 69, 399 (2010)



D. Madigozhin [NA48/2 Coll.], PoS DIS2013, 135 (2013)

... or with  $K^\pm \rightarrow \pi^+\pi^-e^\pm\nu$

- Geneva-Saclay high-statistics experiment:  $3 \cdot 10^4$  events,  $a_0$  at 20%

L. Rosselet et al., Phys. Rev. D 15, 574 (1977)

- BNL-E865:  $4 \cdot 10^5$  events

S. Pislak et al., Phys. Rev. 67, 072004 (2003) [Phys. Rev. 81, 119903 (2010)] [hep-ex/0301040]

- NA48/2:  $1.1 \cdot 10^6$  events,  $a_0$  at 6%

J. R. Batley et al., Eur. Phys. J. C 70, 635 (2010)

The experimental values of the two S-wave scattering lengths

$$a_0 = 0.222(14) \quad a_2 = -0.0432(97)$$

compare quite well with the prediction from two-loop chiral perturbation theory

$$a_0 = 0.220(5) \quad a_2 = -0.0444(10)$$

G. Colangelo, J. Gasser, H. Leutwyler, Nucl. Phys. B 603, 125 (2001)

But taking isospin corrections ( $m_u \neq m_d$  and  $M_\pi \neq M_{\pi^0}$ ) into account turns out to be crucial in order to reach this agreement

J. Gasser, PoS KAON , 033 (2008), arXiv:0710.3048 [hep-ph]

Although it will not always be mentioned explicitly, only infrared finite radiatively-corrected observables will be considered [in particular, amplitudes include emission of one (soft) photon]

Radiative corrections to decay rates are typically at the level of a few %

$$\Gamma = \Gamma_0 \left[ 1 + \alpha \frac{\Delta\Gamma}{\Gamma_0} \right] \quad \alpha \frac{\Delta\Gamma}{\Gamma_0} \sim \pm(1 - 3)\%$$

Radiative corrections to *differential* decay rates can, locally, be more important, e.g.  $\sim \pm 10\%$

$$\frac{d^2\Gamma}{dx dy} = \frac{d^2\Gamma_0}{dx dy} [1 + \alpha\delta(x, y)] \quad \alpha\delta(x, y) \sim \pm(1 - 10)\%$$

[cf. also situations where there are cuts...]

Radiative corrections are often computed within a framework (ChPT, QED with point-like mesons,...) where the quantities one wishes to measure (slopes of form factors, scattering lengths,...) are actually **fixed** to their LO or NLO values

→ can produce biases...



Tools

## Tool 1 : chiral lagrangian

At energies well below the electroweak scale, the weak interactions are described by effective lagrangians involving four-fermion operators

- For the  $\Delta S = 1$  non-leptonic transitions:

$$\mathcal{L}_{\text{eff}}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) Q_i(\mu)$$

- $C_i(\mu) \longrightarrow$  perturbative QCD corrections from  $M_W$  down to  $\mu \lesssim m_c$

- For the semi-leptonic transitions:

$$\mathcal{L}_{\text{eff}}^{SL} = -\frac{G_F}{\sqrt{2}} [\bar{\ell} \gamma_\mu (1 - \gamma_5) \nu_\ell] \{V_{ud} [\bar{u} \gamma^\mu (1 - \gamma_5) d] + V_{us} [\bar{u} \gamma^\mu (1 - \gamma_5) s]\} + \text{h. c.}$$

- No QCD corrections in  $\mathcal{L}_{\text{eff}}^{SL} \longrightarrow$  factorized form, the description of semi-leptonic decays amounts to the evaluation of the relevant form factors

For  $\mu \ll \Lambda_{\text{had}} \sim 1\text{GeV}$  (where kaon physics takes place), the relevant degrees of freedom are no longer quarks, but the lightest pseudoscalar mesons that become the Goldstone bosons of the spontaneous breaking of chiral symmetry in the limit of massless light quarks  $m_{u,d,s} \rightarrow 0$

—→ construct an effective lagrangian that describes the interactions among these pseudoscalar mesons in a systematic low-energy expansion

S. Weinberg, Physica A 96, 327 (1979)

J. Gasser, H. Leutwyler, Annals Phys. 158, 142 (1984); Nucl. Phys. B 250, 465 (1985)

- strong interactions among mesons at low-energies

$$\mathcal{L}^{\text{str}} = \mathcal{L}_2^{\text{str}}(2) + \mathcal{L}_4^{\text{str}}(10 + 0) + \mathcal{L}_6^{\text{str}}(90 + 23) + \dots$$

- $\Delta S = 1$  transitions

$$\mathcal{L}_{\text{eff}}^{\Delta S=1} \longrightarrow \mathcal{L}_2^{\Delta S=1}(1 + 1) + \mathcal{L}_4^{\Delta S=1}(22 + 28) + \dots$$

J. A. Cronin, Phys. Rev. 161, 1483 (1967)

J. Kambor, J. H. Missimer, D. Wyler, Nucl. Phys. B 346, 17 (1990)

G. Esposito-Farese, Z. Phys. C 50, 255 (1991)

G. Ecker, J. Kambor, D. Wyler, Nucl. Phys. B 394, 101 (1993)

Adding electromagnetic interactions requires to include the photon as a low-energy degree of freedoms (loops involving virtual photons will produce their own divergences, which require additional low-energy constants)...

$$\mathcal{L}^{\text{str};EM} = \mathcal{L}_2^{\text{str};EM}(1) + \mathcal{L}_4^{\text{str};EM}(13 + 0) + \dots$$

$$\mathcal{L}_2^{\text{str};EM} = e^2 C \langle QU^\dagger QU \rangle \quad \mathcal{L}_4^{\text{str};EM}(13 + 0) = \sum_{i=1}^{13} K_i \mathcal{O}_i^{\text{str};EM}$$

G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B 321, 311 (1989)

R. Urech, Nucl. Phys. B 433, 234 (1995)

H. Neufeld, H. Rupertsberger, Z. Phys. C 71, 131 (1996)

$$\mathcal{L}^{\Delta S=1;EM} = \mathcal{L}_2^{\Delta S=1;EM}(1) + \mathcal{L}_4^{\Delta S=1;EM}(14+?) + \dots$$

$$\mathcal{L}_2^{\Delta S=1;EM} = e^2 G_8 F_0^6 g_{\text{weak}} \langle \lambda_{23} U^\dagger QU \rangle \quad \mathcal{L}_4^{\Delta S=1;EM} = e^2 G_8 F_0^4 \sum_{i=1}^{14} Z_i \mathcal{O}_i^{\Delta S=1;EM}$$

J. Bijnens, M. B. Wise, Phys. Lett. B 137, 245 (1984)

G. Ecker, G. Isidori, Müller, H. Neufeld, A. Pich, Nucl. Phys. 591, 1419 (2000)

... as well as the light leptons (for the description of radiative corrections to semi-leptonic decays)

$$\mathcal{L}^{\text{lept}} = \mathcal{L}_2^{\text{lept}}(0) + \mathcal{L}_4^{\text{lept}}(5) + \dots \quad \mathcal{L}_4^{\text{lept}} = \sum_{i=1}^5 X_i \mathcal{O}_i^{\text{lept}}$$

M. K., H. Neufeld, H. Rupertsberger, P. Talavera, Eur. Phys. J. C 12, 469 (2000)

## Determination of low-energy constants

- $K_i$

- identify the corresponding QCD correlators (two-, three- and four-point functions), convoluted with the free photon propagator
- study their short-distance behaviour
- write spectral sum rules
- saturate with lowest-lying narrow-width resonances

B. Moussallam, Nucl. Phys. B 504, 391 (1997) [hep-ph/9701400]

B. Ananthanarayan, B. Moussallam, JHEP06, 047 (2004) [hep-ph/0405206]

Analogous to the DGMLY sum-rule for  $C$

$$C = -\frac{1}{16\pi^2} \frac{3}{2\pi} \int_0^\infty ds s \ln \frac{s}{\mu^2} [\rho_{VV}(s) - \rho_{AA}(s)]$$

T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low and J. E. Young, Phys. Rev. Lett. 18, 759 (1967)

B. Moussallam, Eur. Phys. J. C 6, 681 (1999) [hep-ph/9804271]

## Determination of low-energy constants

- $X_i$

- two-step matching procedure:

- compute radiative corrections to  $\bar{q}q' \rightarrow \ell\nu$  in the SM and in the four-fermion theory
- match the radiatively corrected four-fermion theory to the chiral lagrangian, by identifying the QCD correlators (convoluted with the free photon propagator) that describe the  $X_i$ 's  
Saturate the resulting spectral sum rules with lowest-lying resonance states

S. Descotes-Genon, B. Moussallam, Eur. Phys. J. C 42, 403 (2005) [hep-ph/0505077]

- $g_{\text{weak}}$  and  $Z_i$

Have been estimated in the large- $N_c$  limit

V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, Eur. Phys. J. C 33, 269 (2004)

For instance

$$(g_8 e^2 g_{\text{weak}})^\infty = - \left( \frac{\langle \bar{\psi}\psi \rangle}{F_0^3} \right)^2 \left[ 3C_8(\mu) + \frac{16}{3} e^2 C_6(\mu) (K_9 - 2K_{10}) \right]$$

## Determination of low-energy constants

The dependence on the short-distance scale vanishes at leading-order in the large- $N_c$  limit. A scale dependence remains at subleading order in  $1/N_c$ .

The (subleading order) contribution of  $Q_7$  can also be computed,

$$(g_8 e^2 g_{\text{weak}})^{1/N_c; Q_7} = -\frac{9}{8\pi^2} C_7(\mu) \frac{M_\rho^2}{F_0^2} \left[ \ln \frac{\mu^2}{M_\rho^2} + \frac{1}{3} - 2 \ln 2 \right]$$

M. K., S. Peris, E. de Rafael, Phys. Lett. B 457, 227 (1999)

but this does not completely remove the residual scale dependence

## Examples of applications (non-exhaustive list)

- $\pi \rightarrow \ell \nu_\ell(\gamma)$  and  $K \rightarrow \ell \nu_\ell(\gamma)$       M. K., H. Neufeld, H. Rupertsberger, P. Talavera, Eur. Phys. J. C 12, 469 (2000)  
V. Cirigliano, I. Rosell, JHEP 0710, 005 (2007)  
J. Gasser, G. R. S. Zarnauskas, Phys. Lett. B 693, 122 (2010)  
V. Cirigliano, H. Neufeld, Phys. Lett. B 700, 7 (2011)
- $K \rightarrow \pi \ell \nu_\ell(\gamma)$       V. Cirigliano, M. K., H. Neufeld, H. Rupertsberger and P. Talavera, Eur. Phys. J. C 23, 121 (2002)  
A. Kastner, H. Neufeld, Eur. Phys. J. C 57, 541 (2008)  
V. Cirigliano, M. Giannotti, H. Neufeld, JHEP 0811, 006 (2008)  
J. Gasser, B. Kubis, N. Paver, M. Verbeni, Eur. Phys. J. C 40, 205 (2005)
- $\pi^+ \rightarrow \pi^0 e \nu_e$       V. Cirigliano, M. K., H. Neufeld, H. Pichl, Eur. Phys. J. C 27, 255 (2003)
- $K^+ \rightarrow \pi^+ \pi^- \ell \nu_\ell$       V. Cuplov, PhD thesis (2004); V. Cuplov, A. Nehme, hep-ph/0311274  
A. Nehme, Nucl. Phys. B 682, 289 (2004)  
P. Stoffer, Eur. Phys. J. C 74, 2749 (2014)
- $K \rightarrow \pi \pi$       V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, Phys. Rev. Lett. 91, 162001 (2003)  
V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, Eur. Phys. J. C 33, 269 (2004)  
V. Cirigliano, G. Ecker, A. Pich, Phys. Lett. B 679, 445 (2009)
- $K \rightarrow \pi \pi \pi$       J. Bijnens, F. Borg, Nucl Phys. B 697, 319 (2004); Eur. Phys. J. C 39, 347 (2005); C 40, 383 (2005)
- ...      V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, J. Portolés, Rev. Mod. Phys. 84, 399 (2012)



## Tool 2 : non-relativistic effective field theory

$K \rightarrow \pi\pi^0\pi^0$ : Important experimental feature: cusp at  $M_{00} = 2M_\pi$  in the invariant mass distribution of the two neutral pions

First observed by NA48/2 in a sample of  $2.3 \cdot 10^7$   $K^\pm \rightarrow \pi^\pm\pi^0\pi^0$

J. R. Batley et al., Phys. Lett. B 633, 176 (2006)

Correctly interpreted as a rescattering effect  $\pi^+\pi^- \rightarrow \pi^0\pi^0$  ( $M_\pi \neq M_{\pi^0}$ ), corresponding to the combination  $a_0 - a_2$  of S-wave scattering lengths

N. Cabibbo, Phys. Rev. Lett. 93, 121801 (2004)

But simple phenomenological parametrizations

N. Cabibbo and G. Isidori, JHEP0503, 021 (2005)

E. Gamiz, J. Prades and I. Scimemi, Eur. Phys. J. C 50, 405 (2007)

or one-loop ChPT calculations including isospin breaking

J. Bijnens, F. Borg, Nucl Phys. B 697, 319 (2004); Eur. Phys. J. C 39, 347 (2005); Eur. Phys. J. C 40, 383 (2005)

either do not give the correct analyticity properties or do not give a sufficiently accurate description of the cusp

## Tool 2 : non-relativistic effective field theory

Better description obtained by combining a non relativistic EFT framework

$$|\mathbf{p}|/M_\pi \sim \mathcal{O}(\epsilon)$$

and a *systematic* expansion in powers of the *scattering lengths* (treated as free parameters), including orders  $\epsilon^2$ ,  $a\epsilon^3$ ,  $a^2\epsilon^2$

G. Colangelo, J. Gasser, B. Kubis and A. Rusetsky, Phys. Lett. B 638, 187 (2006)

J. Gasser, B. Kubis and A. Rusetsky, Nucl. Phys. B 850, 96 (2011)

Radiative corrections were also included

M. Bissegger, A. Fuhrer, J. Gasser, B. Kubis and A. Rusetsky, Nucl. Phys. B 806, 178 (2009)

$$\longrightarrow a_0 - a_2 = 0.2571 \pm 0.0056$$

J. R. Batley et al, Eur. Phys. J. C 64, 589 (2009)

Later also observed by KTeV in a sample of  $6.8 \cdot 10^7 K_L \rightarrow \pi^0\pi^0\pi^0$  events but the rescattering effect is quite smaller

$$a_0 - a_2 = 0.215 \pm 0.031$$

E. Abouzaid et al., Phys. Rev. D 78, 032009 (2008)

## Tool 3: Dispersive constructions of amplitudes and form factors

Illustration:  $M_\pi \neq M_{\pi^0}$  effects in the phases of  $K_{\ell 4}$  form factors

Standard angular analysis of the  $K_{e4}^{+-}$  form factors provides information on low-energy  $\pi\pi$  scattering (Watson's theorem) through the phase difference

$$[\delta_S(s) - \delta_P(s)]_{\text{exp}}$$

N. Cabibbo, A. Maksymowicz, Phys. Rev. B 137, 438 (1965); Erratum-ibid 168, 1926 (1968)

F.A. Berends, A. Donnachie, G.C. Oades, Phys. Rev. 171, 1457(1968)

measurable in the interference of the  $F^{+-}$  and  $G^{+-}$  form factors.

Comparison with solutions of the Roy equations

$$[\delta_S(s) - \delta_P(s)]_{\text{exp}} = f_{\text{Roy}}(s; a_0^0, a_0^2)$$

allows to extract the values of the  $\pi\pi$   $S$ -wave scattering lengths in the isospin channels  $I = 0, 2$

$f_{\text{Roy}}(s; a_0^0, a_0^2)$  follows from:

- dispersion relations (analyticity, unitarity, crossing, Froissard bound)
- $\pi\pi$  data at energies  $\sqrt{s} \geq 1$  GeV
- isospin symmetry

S.M. Roy, Phys. Lett. B 36, 353 (1971)

Solutions can be constructed for  $(a_0^0, a_0^2) \in$  Universal Band

B. Ananthanarayan, G. Colangelo, J. Gasser, H. Leutwyler, Phys. Rep. 353, 207 (2001)

### Tool 3: Dispersive constructions of amplitudes and form factors

Once standard radiative corrections have been taken care of (more below), it is still important to take **isospin-breaking corrections due to  $M_\pi \neq M_{\pi^0}$  [also an  $\mathcal{O}(\alpha)$  effect!]** into account before analyzing data

J. Gasser, PoS KAON, 033 (2008)

Evaluation of IB corrections in ChPT

G. Colangelo, J. Gasser, A. Rusetsky, Eur. Phys. J. C 59, 777 (2009)

$$\longrightarrow a_0^0 = 0.2220(128)_{\text{stat}}(50)_{\text{syst}}(37)_{\text{th}} \quad a_0^2 = -0.0432(86)_{\text{stat}}(34)_{\text{syst}}(28)_{\text{th}}$$

However, IB corrections were **evaluated at fixed values** of the scattering lengths

$$[\delta_S(s) - \delta_P(s)]_{\text{exp}} = f_{\text{Roy}}(s; a_0^0, a_0^2) + \delta f_{\text{IB}}(s; (a_0^0)_{\text{ChPT}}^{\text{LO}}, (a_0^2)_{\text{ChPT}}^{\text{LO}})$$

Drawback shared by other studies devoted to isospin breaking in ChPT (QCD+QED)

V. Cuplov, PhD thesis (2004); V. Cuplov, A. Nehme, hep-ph/0311274

A. Nehme, Nucl. Phys. B 682, 289 (2004)

P. Stoffer, Eur. Phys. J. C 74, 2749 (2004)

Is it possible to obtain

$$[\delta_S(s) - \delta_P(s)]_{\text{exp}} = f_{\text{Roy}}(s; a_0^0, a_0^2) + \delta f_{\text{IB}}(s; a_0^0, a_0^2) \quad ?$$

What is the quantitative effect in the determination of the scattering lengths?

## Tool 3: Dispersive constructions of amplitudes and form factors

Adapt the approach (“reconstruction theorem”) described in

J. Stern, H. Sazdjian, N. H. Fuchs, Phys. Rev. D 47, 3814 (1993)

for the  $\pi\pi$  scattering amplitude, and implemented in

M. Knecht, B. Moussallam, J. Stern, N.H. Fuchs, Nucl. Phys. B 457, 513 (1995)

Rests on very general principle [cf. talk by P. Stoffer]

a) Relativistic invariance

b) Analyticity, unitarity, crossing

c) Chiral counting

Note: isospin symmetry not required

→  $\delta f_{\text{IB}}(s; a_0^0, a_0^2)$  worked out at NLO

S. Descotes-Genon, M. K., Eur. Phys. J. C 72, 1962 (2012)

V. Bernard, S. Descotes-Genon, M. K., Eur. Phys. J. C 73, 2478 (2013)

Re-analysis of NA48/2 data

$$a_0^0 = 0.221 \pm 0.018 \quad a_0^2 = -0.0453 \pm 0.0106$$

to be compared to

$$a_0^0 = 0.2220(128)_{\text{stat}}(50)_{\text{syst}}(37)_{\text{th}} \quad a_0^2 = -0.0432(86)_{\text{stat}}(34)_{\text{syst}}(28)_{\text{th}}$$

Radiative corrections in real life:  
A case study with  $K_{e4}$

NA48/2 has measured the two  $K_{e4}^{\pm}$  channels:

$K_{e4}^{+-}$  [i.e.  $K^{\pm} \rightarrow \pi^+ \pi^- e^{\pm} \nu_e$ ], about  $10^6$  events

J. R. Batley et al. [NA48/2 Coll.], Phys. Lett. B 715, 105 (2012)

$K_{e4}^{00}$  [i.e.  $K^{\pm} \rightarrow \pi^0 \pi^0 e^{\pm} \nu_e$ ], about  $6.5 \cdot 10^4$  events (unitarity cusp in  $M_{\pi^0 \pi^0}$  seen)

J. R. Batley et al. [NA48/2 Coll.], JHEP 1408, 159 (2014)

The two matrix elements have a form factor ( $F^{+-} = F^{00}$ ) in common in the isospin limit

$$(1 + \delta_{EM}) \frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]} = 1.065 \pm 0.010$$

Can one understand this 6.5% effect in terms of isospin breaking?

About two thirds of the effect can be ascribed to isospin breaking in the quark masses

$$\left. \frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]} \right|_{LO} = 1.039 \pm 0.002 \quad \left[ R = \frac{m_s - m_{ud}}{m_d - m_u} = 38.2(1.1)(0.8)(1.4) \right]$$

V. Cuplov, PhD Thesis (2004); A. Nehme, Nucl. Phys. B 682, 289 (2004)

Z. Fodor et al. [BMW Coll.], Phys. Rev. Lett. 117, 082001 (2016)

Radiative corrections?  $\delta_{EM} \sim 2.5\%$ ?

→ need to understand how radiative corrections were treated in the  $K_{e4}^{+-}$  mode...

## Treatment of radiative corrections in the data analyses:

$K_{e4}^{00}$ : no radiative corrections whatsoever applied (hence the factor  $\delta_{EM}$ !)

$K_{e4}^{+-}$ :

- Sommerfeld-Gamow-Sakharov factors applied to each pair of charged legs
- Corrections induced by emission of real photons treated with PHOTOS

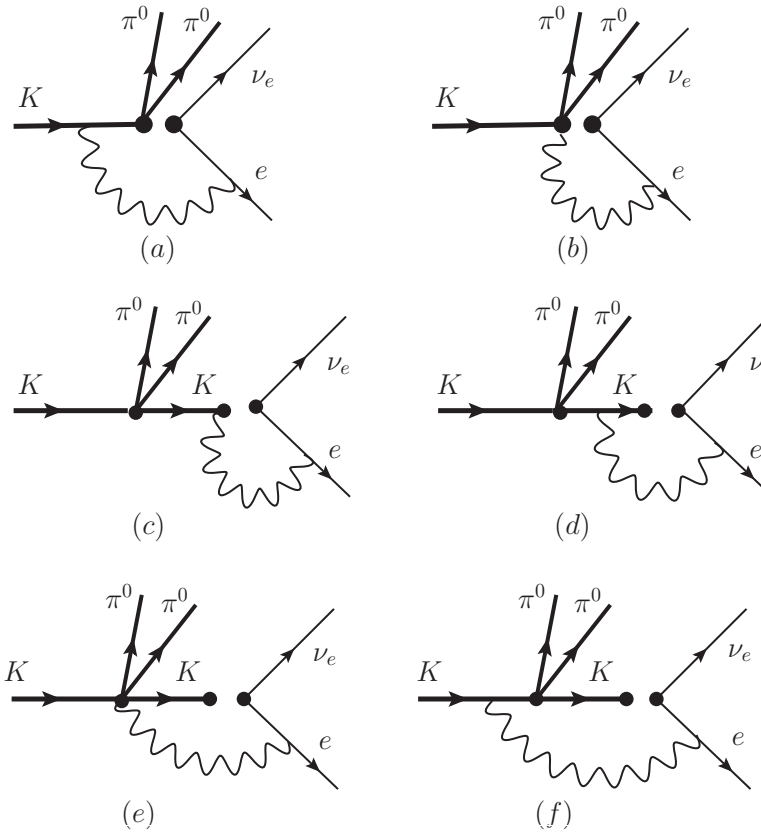
Z. Was et al., *Comp. Phys. Comm.* 79, 291 (1994); *Eur. Phys. J. C* 45, 97 (2006); *C* 51, 569 (2007); *Chin. Phys. C* 34, 889 (2010)

- PHOTOS also implements (1 loop QED) w.f.r. on the external charged legs [ $\longrightarrow$  no IR divergences], based on

Y. M. Bystritskiy, S. R. Gevorkian, E. A. Kuraev, *Eur. Phys. J. C* 67, 47 (2009)

- All structure-dependent corrections are discarded (gauge invariant truncation)
- $R^{00}$  form factor neglected (appears multiplied by  $m_e^2$  in the differential decay rate), but there is a contribution to  $\Delta F^{00}$  of the type  $\mathcal{O}(\alpha) \times R^{00}$  ( $m_e = 0$  is not equivalent to  $R^{00} = 0$  in the presence of radiative corrections)
- UV divergences not treated





Non factorizable radiative corrections

Besides w.f. factors of QED, only diagram (a) is considered in a PHOTOS-like treatment of radiative corrections [diagrams (b), (c), and (d) vanish for  $m_e \rightarrow 0$ ]

Adding the diagrams for the emission of a soft photon, one obtains

$$\Gamma^{\text{tot}} = \Gamma(K_{e4}^{00}) + \bar{\Gamma}^{\text{soft}}(K_{e4\gamma}^{00}) = \Gamma_0(K_{e4}^{00}) \times (1 + 2\delta_{EM})$$

with  $\delta_{EM} = 0.018 \rightarrow \frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]} = 1.065 \pm 0.010 - 0.018 \sim (1 + \frac{3}{2R})$

# Conclusions

High precision reached by the data concerning non-leptonic and semi-leptonic decay modes of the kaons has made the treatment of isospin-breaking effects ( $m_u \neq m_d$  and  $\alpha \neq 0$ ) unavoidable

A lot of activity has been going on, extending the scope of the low-energy EFT in order to meet this necessity (inclusion of photons, leptons). Only a fraction of the many applications has been mentioned here

The issue of additional low-energy constants has been dealt with in a rather satisfactory manner (progress on estimates of the  $Z_i$ 's would be welcome, though)

The effects due to  $M_\pi \neq M_{\pi^0}$  are important (especially for  $K \rightarrow \pi\pi\pi$  and for  $K_{e4}$ ).  
ChPT at NLO is not always sufficient.

→ This issue can be dealt with through more elaborate/adapted approaches, like NREFT, dispersive representations,...

Watch out for possible biases if the radiative corrections to form factors and/or decay distributions are given for fixed values of the parameters one actually wants to extract from data

→ Not the case for  $\delta_0(s) - \delta_1(s)$  extracted from  $K_{e4}$

Treatment of radiative corrections in  $K_{e4}$  rather rudimentary, does not match the quality of the data

→ Improvements should be possible

Thank you!