

## Unit 6

# Flux Jumps and Motion in Superconductors

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## ● Outline

- Review of flux penetration in type II superconductors
- Forces on fluxoids in type II superconductors
- Pinning strength
  - Definition and theoretical concepts
  - Most common pinning sites
  - Influence of other parameters, e.g. temperature
- Definition of stability
  - Adiabatic
  - Dynamic
- HTS specific: Flux creep and irreversibility field

## ● This lecture relies heavily on:

- Martin Wilson, “Superconducting Magnets”
- Alex Gurevich, Lectures on Superconductivity
- Ernst Helmut Brandt, “Electrodynamics of Superconductors exposed to high frequency fields”
- Marc Dhallé, “IoP Handbook on Superconducting Materials” (preprint)
- Arno Godeke, thesis: “Performance Boundaries in Nb<sub>3</sub>Sn Superconductors”
- Feynman “Lectures on Physics”

- A fluxoid is a “vortex” of normal material surrounded by circulating supercurrents generating a quantum of magnetic flux of amplitude:

$$\phi_0 = \frac{\pi \hbar}{|e|} = 2.07 \times 10^{-15} \text{ Vs (=Weber=T-m}^2\text{)}$$

- The fluxoids penetrate the superconductor so as to minimize the total free Gibbs energy: whereas Type I superconductors are diamagnetic, the fluxoid field is paramagnetic, with fluxoid penetration when

$$G < 0$$

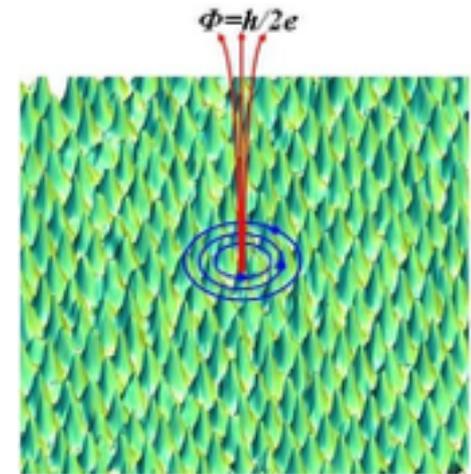
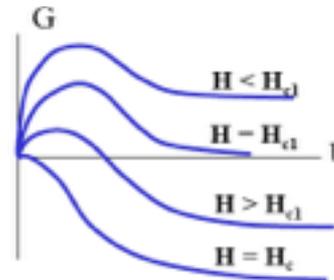
$$G = \varepsilon - H\phi_0; H_{c1} = \frac{\varepsilon}{\phi_0}; \varepsilon \text{ is the vortex self-energy}$$

$$H_{c1} = \frac{\phi_0}{4\pi\lambda^2} \left( \ln \frac{\lambda}{\xi} + 0.5 \right)$$

- The field in the sample is a function of the fluxoid

Cross-sectional density:

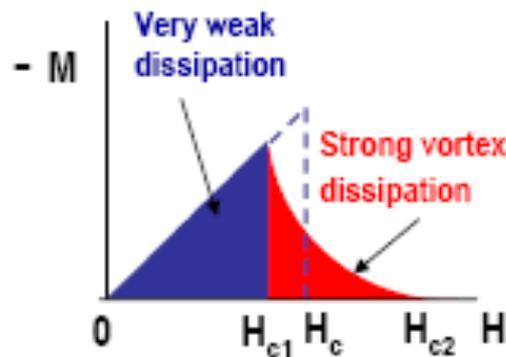
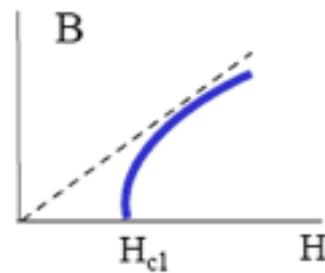
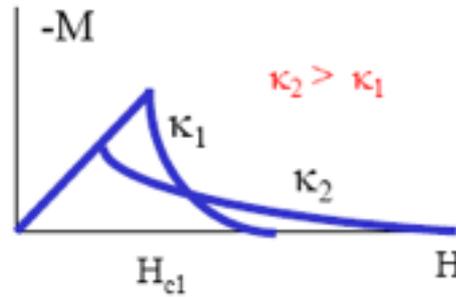
$$B = n\phi_0$$



Vortex flux lattice in  $V_3Si$   
 STMFermi-level conductance image  
 ,  $H=3T$ ,  $T=2.3K$   
 Center for Nanoscale Science, NIST

# Review of fundamentals

Gurevich lectures



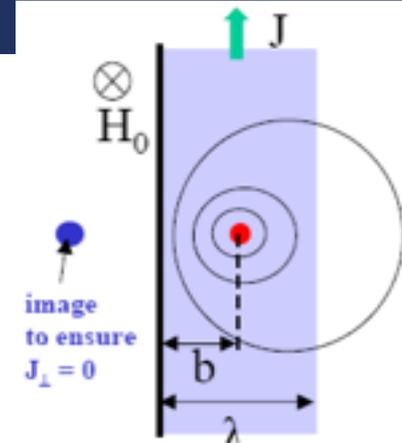
Material	T <sub>c</sub> (K)	H <sub>c</sub> (0) [T]	H <sub>c1</sub> (0) [T]	H <sub>c2</sub> (0) [T]	λ(0) [nm]
Pb	7.2	0.08	na	na	48
Nb	9.2	0.2	0.17	0.4	40
Nb <sub>3</sub> Sn	18	0.54	0.05	30	85
NbN	16.2	0.23	0.02	15	200
MgB <sub>2</sub>	40	0.43	0.03	3.5	140
YBCO	93	1.4	0.01	100	150

Material	T <sub>c</sub> (K)	λ(0), nm	ξ(0), nm	H <sub>c2</sub> (T)
Nb-Ti	9.5	240	4	13
Nb-N	16	200	5	15
Nb <sub>3</sub> Sn	18	65	3	30
MgB <sub>2</sub> (dirty)	32-39	140	6	35
YBa <sub>2</sub> Cu <sub>3</sub> O <sub>7</sub>	92	150	1.5	>100
Bi-2223	108	200	1.5	>100

The thermodynamic critical field  $H_c$  is defined as:

$$H_c = \frac{\phi_0}{2\sqrt{2}\pi\lambda\xi}$$

# Forces on the fluxoid



- The combination of Meissner current and interface forces results in a surface barrier energy that must be exceeded for flux to penetrate

- The surface barrier energy decreases to zero at  $H=H_c$ , the thermodynamic critical

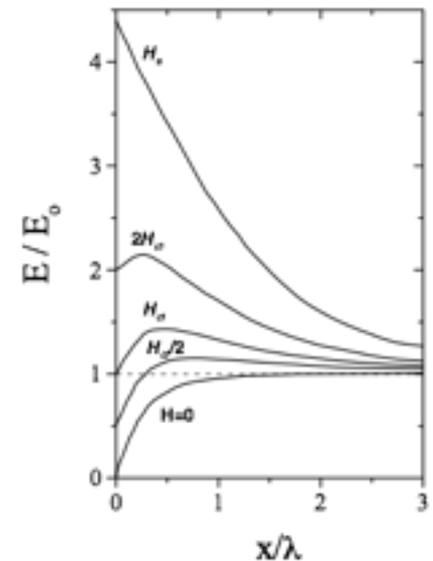
$$a_{\Delta}(H) = \left(\frac{4}{3}\right)^{1/4} \left(\frac{\phi_0}{\mu_0 H}\right)^{1/2}$$

- Fluxoids experience a variety of forces:

- Oriented parallel to each other, fluxoids are mutually repulsive, leading to a natural equal hexagonal spacing in bulk material that maximizes their separation
  - In the vicinity of current, the fluxoid (oriented by  $n$ ) sees a force

$$\vec{F}_v = \vec{J} \times \phi_0 \vec{n}$$

- Near a superconductor - normal metal interface, a fluxoid experiences an attractive force towards the surface (from its attractive image)
  - The resulting motion of a fluxoid is countered by:
    - Viscous drag associated with an effective normal resistivity
    - Pinning of the fluxoids by energetically preferable sites

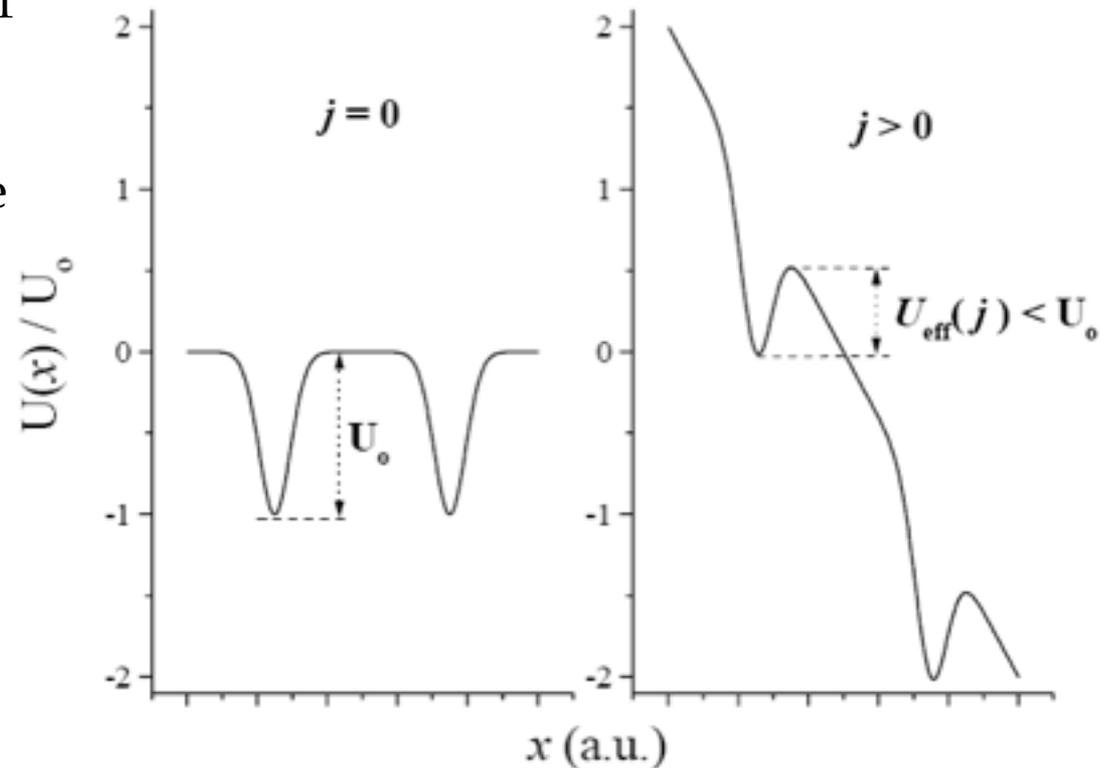


**Figure 22** : calculated Bean-Livingston energy barrier for flux - entry and - exit.  $x$  is the distance to the surface and  $E_0$  the energy per unit length of a flux line well inside the superconductor. Different curves correspond to different applied fields ( from [49] )

# Energy perspective

- Pinning sites effectively reduce the energy state of the system; transport current has the effect of reducing the depth of the potential well

From Dhalle, IOP Handbook of Superconducting Materials



**Figure 26 :** schematic description of Anderson-Kim flux creep. The left panel shows the undisturbed pinning-potential landscape, to which in the right panel the free energy contribution of the ‘Lorentz’ force exerted by a current density is added. The effect of the current density is to lower the energy barrier for thermally activated hopping of flux lines (or flux bundles) from one pinning site to the next.

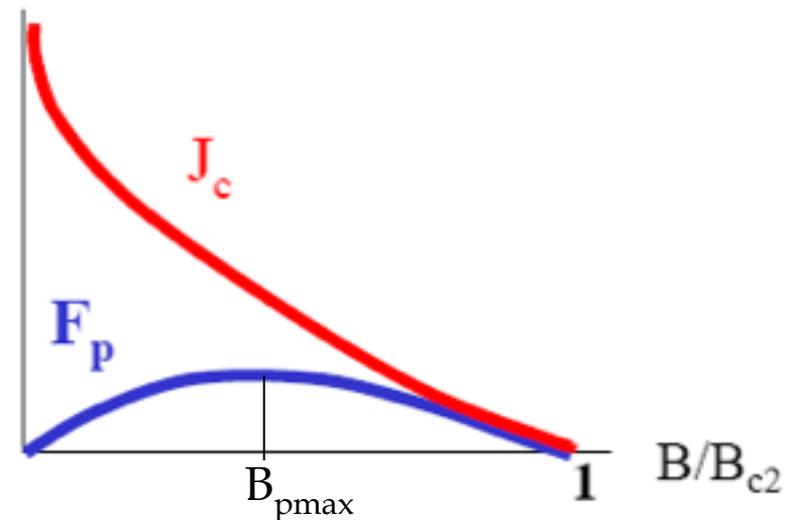
# Pinning of fluxoids

- Useful type II superconductors require that no significant viscous flux-flow occur
  - Flux-flow results in heating, changing current with time, etc (see AC losses)
- The pinning must therefore balance the Lorentz force:

$$J_c(B, T) \times B = F_p(B, T)$$

- The pinning force must be zero at  $B=0$  and at  $B=B_{c2}$ ; it has a maximum at some intermediate field  $B_{pmax}$ .

*Note: the pinning force defines  $J_c$ ; whereas other critical values ( $H_{c1}$ ,  $H_c$ ,  $H_{c2}$ ,  $T_c$ ) are intrinsic to the material, the  $J_c$  can vary strongly with specific material characteristics (defects, etc)*



# Flux pinning – general comments

- First key question: what is the maximum current density that can be obtained?

- “Unlimited” pinning force would result in  $J_c = J_d$ , the de-pairing current density (Dhalle) :

$$J_d(T) = \frac{4}{3\sqrt{6}} \frac{H_c(T)}{\lambda(T)}$$

- Unfortunately (or fortunately!?),  $J_d$  is usually orders of magnitude larger than  $J_c$

- Real pinning sites differ in strength, distribution, etc

- Key pinning sites:

- Precipitates of non-superconducting materials
- Dislocations
- Grain boundaries and other planar defects

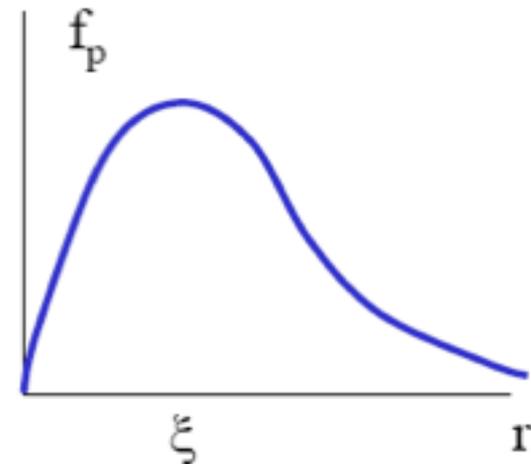
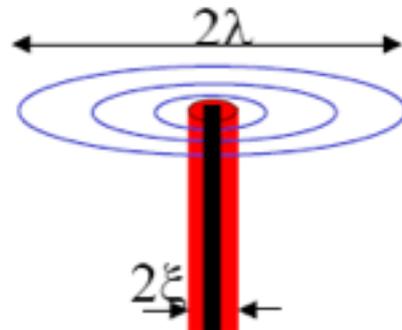
- Details of pinning are complex:

- Attraction to pinning sites competes with vortex-vortex repulsion

- Pinning a fluxoid “saves” a fraction of the vortex core energy:

$$\varepsilon_0 = \frac{1}{2} \pi \mu_0 \xi^2 H_c^2$$

- If the site has a characteristic dimension  $r \ll \xi$ , only a small fraction of the core energy is saved, and the pinning strength is weak;
- If the site has a characteristic dimension  $r \gg \xi$ , the *relative* strength is again weak
- The optimum pinning occurs for  $r \sim \xi$ .



- Furthermore, the pinning strength scales with the fraction of the fluxoid length subjected to pinning
  - Optimal sized pinning sites may not be all that effective if they are point sources
  - Interface sites (i.e. planar sites) often provide very strong pinning
  - Some of the strongest pinning sites are the result of image-vortex energies at insulating boundaries
  - Examples:  $\alpha$ -Ti in NbTi, Nb<sub>3</sub>Sn grain boundaries
- Note that large (insulating) planar defects inhibit normal transport current
  - *The best superconductors are characterized by poor normal conductivity! (This is important from a stability point of view)*

# Flux motion and Heat

- The movement of flux through a superconductor (in the absence of pinning) is accompanied by an electric field and hence a dissipative E-J relationship

- Simple model:

$$\vec{E} = \vec{B} \times \vec{v}_f$$

$$\dot{w} = nF_L \cdot \vec{v}_f = (\vec{J} \times \vec{B}) \cdot \vec{v}_L$$

$$= \vec{J} \cdot \vec{E} = JE$$

*Note: when pinned, the fluxoids generally exhibit no resistivity.*

*Exceptions:*

- *flux jumps (coming soon)*
- *Thermally induced flux-flow (mainly seen in HTS materials)*
- *High-frequency applied fields: fluxoids are not infinitely rigid => can vibrate, yielding losses despite pinning*

# Flux flow – critical current E-J relations



- Macroscopically, the nature of the E-J relation at the onset of flux-flow when transport current reaches critical current can be modeled using a power law or exponential relation:

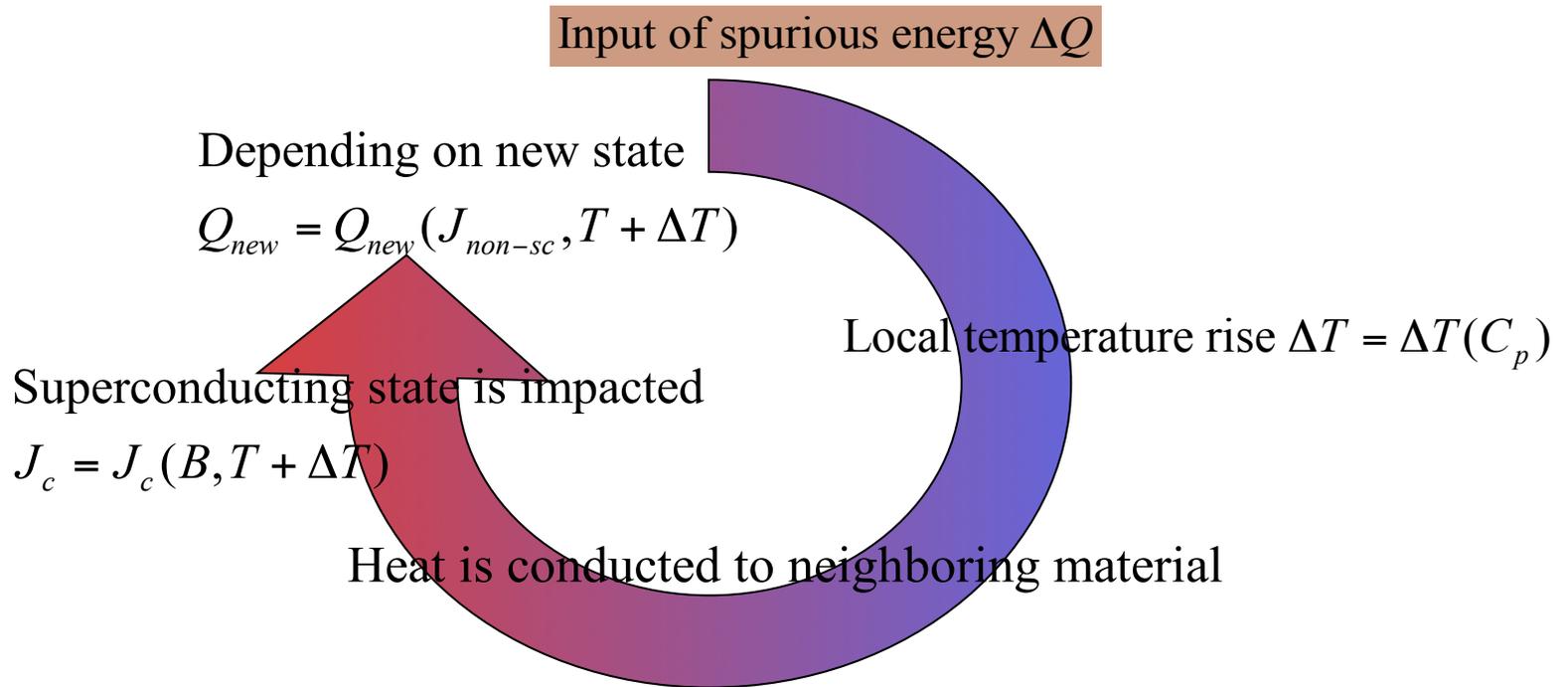
$$E = E_c \left[ \frac{J}{J_c(H, T)} \right]^{n(H, T)}$$

$$E = E_c \exp \left[ -n(H, T) \left( 1 - \frac{J}{J_c(H, T)} \right) \right]$$

- Both formulations provide reasonable fits to data
- Both forms stem from a physical model of the depinning of fluxoids
  - Assumes flux depinning from thermal excitations
  - Assumes Maxwell-Boltzmann statistics for probability of depinning
  - Difference only in the  $J$ -dependence of the activation energy
- The parameter  $n(H, T)$ , characterizes the transition between normal and superconducting states; a higher  $n$ -value implies a more homogeneous pinning within the superconductor
  - The  $n$ -value is experimentally determined by measuring V-I transitions on short-samples and fitting the measured data (preferably in log-log plots!)

# Concept of stability

- The concept of stability concerns the interplay between the following elements:
  - The addition of a (small) thermal fluctuation local in time and space
  - The heat capacities of the neighboring materials, determining the local temperature rise
  - The thermal conductivity of the materials, dictating the effective thermal response of the system
  - The critical current dependence on temperature, impacting the current flow path
  - The current path taken by the current and any additional resistive heating sources stemming from the initial disturbance



# Source of initial disturbance: Flux jump



- We have seen that fluxoid motion induces heat
  - For LTS materials:
    - The heat capacity at low temperatures is low
    - The critical current decreases strongly with increasing temperature
- ⇒ For sufficient flux-flow motion, the cycle described previously avalanches towards a cascade of local fluxoids (vortex bundle)
- ⇒ Depending on the ability of neighboring material to accommodate the resulting heat, the conductor may become normal, and possibly *quench*.

# Basics of stability analysis

- To analyze the stability of a conductor, we need to know:
  1. The amplitude and distribution  $(t, \mathbf{x})$  of the initiating heat source
  2. The heat capacity and thermal conductivity of the adjoining materials
  3. The relative volumes of the adjoining materials (or cross sections for typical conductors)
  4. The critical surface  $(J_c(B, t, \dots))$  of the superconducting material

- In most cases the analyses can be done with a 1D model

- Insulation between conductors in a coilpack results in

$$K_{\perp} \ll K_{\parallel}$$

- Material properties through the conductor cross section can be lumped together as appropriate (e.g. specific heat averaged, thermal and electrical conductivity modeled as parallel circuits, etc.)

# Characteristic diffusion times

- In the case of a flux jump, do we need to include the temporal evolution of the flux motion in the stability analysis?
  - Consider characteristic times of
    1. thermal diffusion (time for thermal transport to neighboring materials)
    2. Magnetic diffusion (time for flux jump to proceed through avalanche)

Following Wilson:

$$\left. \begin{array}{l} \frac{\partial \theta}{\partial t} = D_{\theta} \nabla^2 \theta; \quad D_{\theta} = \frac{k}{\gamma C_p} \\ \frac{\partial B}{\partial t} = D_m \nabla^2 B; \quad D_m = \frac{\rho}{\mu_0} \end{array} \right\} \tau_{\theta} \sim \frac{L^2}{D_{\theta}}, \quad \tau_m \sim \frac{L^2}{D_m}$$

- Here  $L$  is the characteristic length of interest (the definitions can be made more precise for specific cases via direct solution of the PDE)

*Typically  $\tau_m \ll \tau_{\theta}$ , so the flux jump can be viewed as a point source in time for stability purposes*

## 5. Energy deposited quenches

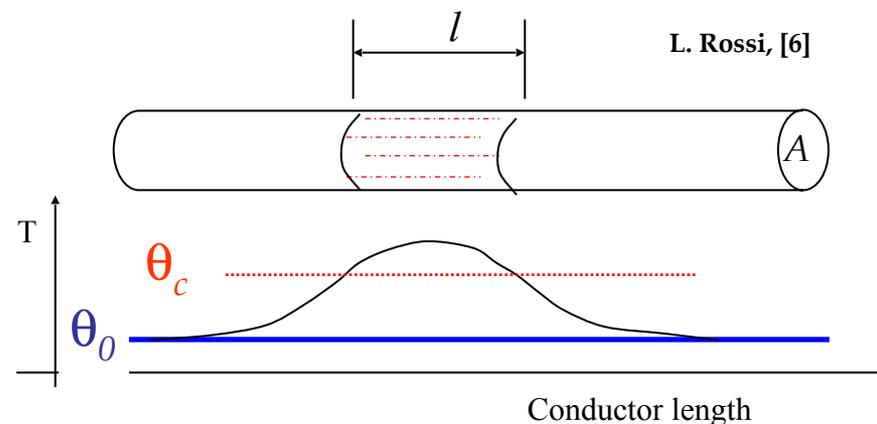
### Point disturbances & dynamic stability

- We start considering a wire made purely of superconductor.
- Let's assume that a certain amount of energy  $E$  increased the temperature of the superconductor beyond  $\theta_c$  over a length  $l$ . The segment  $l$  of superconductor is dissipating power given by  $J_c^2 \rho A l$  [W].
- Part (or all) of the heat is conducted out of the segment because of the thermal gradient, which can be approximated as  $(\theta_c - \theta_0)/(l/2)$ . Therefore, when the power dissipated equals the power conducted away

$$\frac{2kA(\theta_c - \theta_0)}{l} = J_c^2 \rho A l$$

which results in

$$l = \sqrt{\frac{2k(\theta_c - \theta_0)}{J_c^2 \rho}}$$



# 5. Energy deposited quenches

## Point disturbances

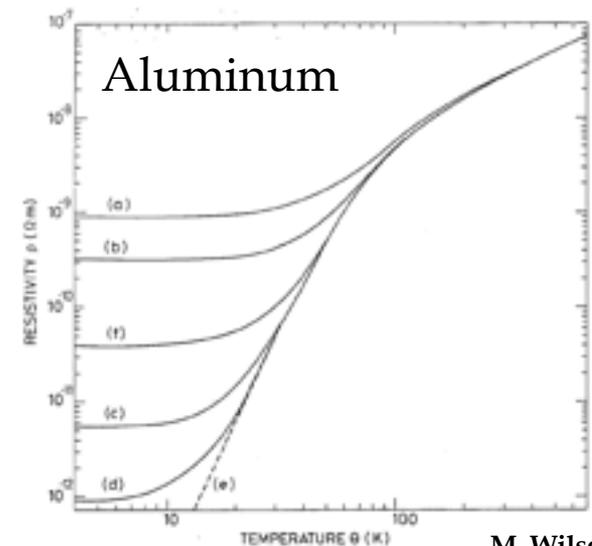
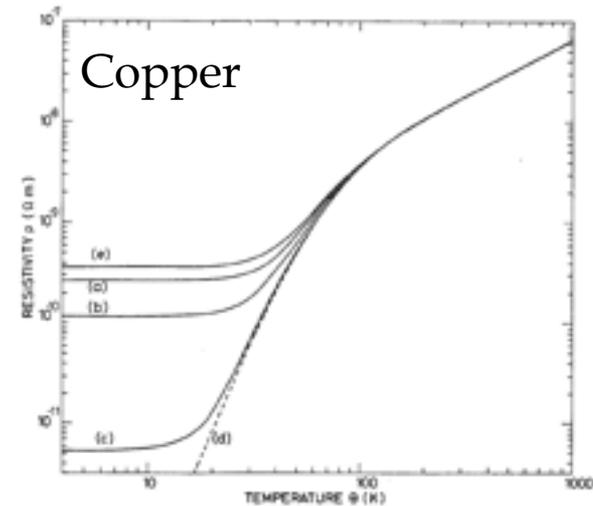


- The length  $l$  defines the MPZ (and MQE).
    - A normal zone longer than  $l$  will keep growing (quench). A normal zone shorter than  $l$  will collapse.
  - An example [2]
    - A typical NbTi 6 T magnet has the following properties
      - $J_c = 2 \times 10^9 \text{ A m}^{-2}$
      - $\rho = 6.5 \times 10^{-7} \text{ } \Omega \text{ m}$
      - $k = 0.1 \text{ W m}^{-1} \text{ K}^{-1}$
      - $\theta_c = 6.5 \text{ K}$
      - $\theta_o = 4.2 \text{ K}$
- $$l = \sqrt{\frac{2k(\theta_c - \theta_o)}{J_c^2 \rho}}$$
- In this case,  $l = 0.5 \text{ } \mu\text{m}$  and, assuming a 0.3 mm diameter, the required energy to bring to  $\theta_c$  is  $10^{-9} \text{ J}$ .
  - A wire made purely of superconductor, without any stabilizer (like copper) around, would quench with nJ of energy.
    - In order to increase  $l$ , since we do not want to reduce  $J_c$ , we have to increase  $k/\rho$ : we need a composite conductor!

# 5. Energy deposited quenches

## Point disturbances

- We now consider the situation where the superconductor is surrounded by material with low resistivity and high conductivity.
- Copper can have at 4.2 K
  - resistivity  $\rho = 3 \times 10^{-10} \Omega \text{ m}$  (instead of  $6.5 \times 10^{-7} \Omega \text{ m}$  for NbTi)
  - $k = 350 \text{ W m}^{-1} \text{ K}^{-1}$  (instead of  $0.1 \text{ W m}^{-1} \text{ K}^{-1}$  for NbTi).
- We can therefore increase  $k/\rho$  by almost a factor  $10^7$ .
- A significant improvement was achieved in the early years of superconducting magnet development after the introduction of composite conductor
  - Both for flux jump and stability viewpoint



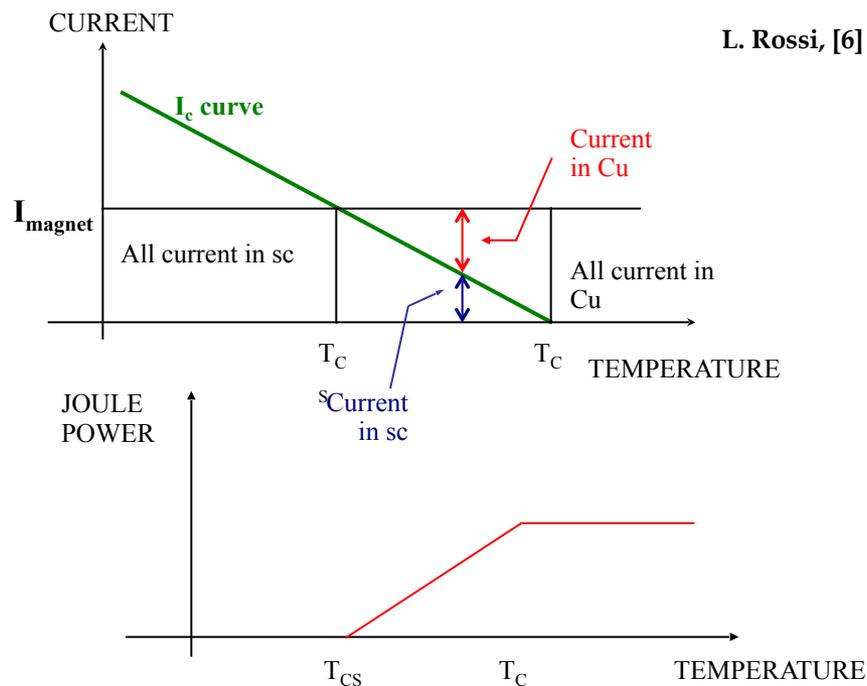
M. Wilson, [2]

# 5. Energy deposited quenches

## Point disturbances

- In a composite superconductor, the heat dissipated when a transition from normal to superconducting state occurs can be subdivided in three parts:
  - All the current flows in the superconductor
  - The current is shared by the superconductor and the stabilizer
  - All the current flows in the stabilizer.

$$\left\{ \begin{array}{ll}
 G = 0 & T \leq T_{cs} \\
 G = G_c \frac{T - T_{cs}}{T_c - T_{cs}} & T_{cs} < T \leq T_c \\
 G = G_c & T_c < T \\
 G_c = \rho_{stab} \frac{\lambda^2 J_m^2}{1 - \lambda} & \lambda = \frac{A_{sc}}{A_{tot}} \\
 & J_m = \frac{I_{magnet}}{A_{sc}}
 \end{array} \right.$$



# Calculation of the bifurcation point for superconductor instabilities

Heat Balance Equation in 1D, without coolant: [W/m<sup>3</sup>]

Thanks to Matteo Allesandrini, Texas Center for Superconductivity, for these calculations and slides

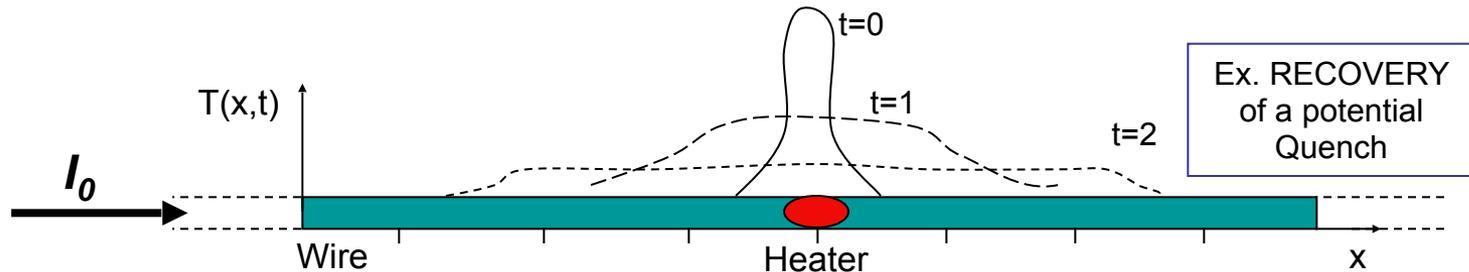
$$\frac{d}{dx} \left( k(T) \cdot \frac{dT}{dx} \right) + \rho(T) \cdot J^2 + Q_{initial\_pulse} - C(T)_{volume} \cdot \frac{dT}{dt} = 0$$

Heat conduction

Joule effect

Quench trigger

Heat stored in the material



Assumption on  $\rho(T) \cdot J^2$  :

- CASE 1.  $T < T_{sc\_crit}$ 
  - 1.a)  $T < T_{current\ sharing} \longrightarrow I_{sc} = I_0, I_{metal} = 0$
  - 1.b)  $T > T_{current\ sharing} \longrightarrow I_{sc} = I_{crit}(T), I_{metal} = I_0 - I_{sc}$
- CASE 2.  $T > T_{sc\_crit} \quad -- \quad I_{sc} = 0, I_{metal} = I_0$

# Quench Propagation Rate

Heat Balance Equation in 1D, without coolant:  $[W/m^3]$

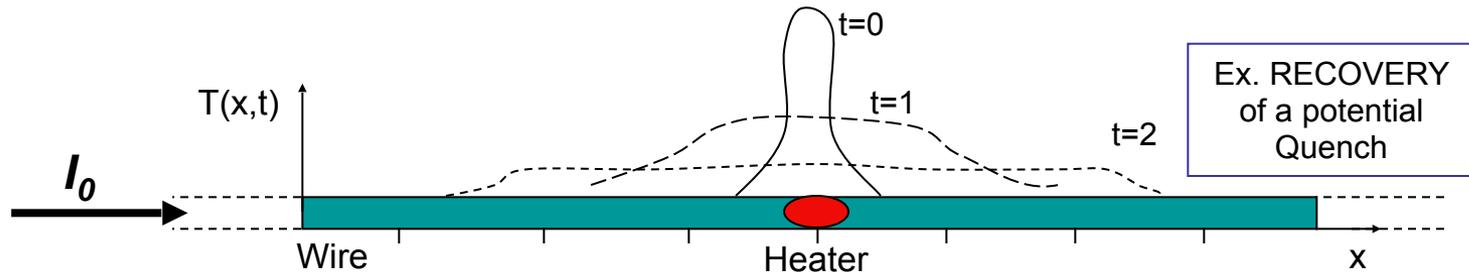
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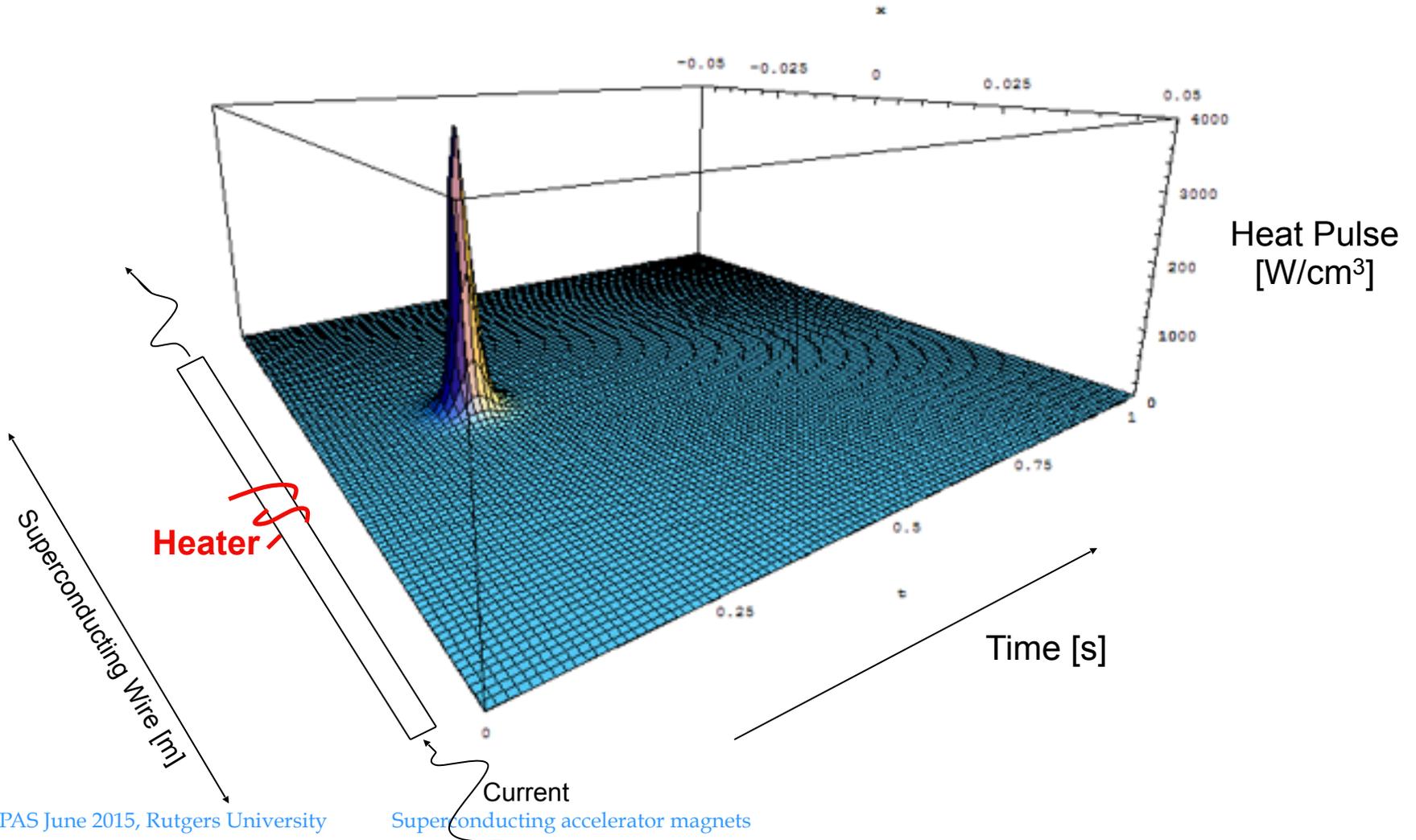
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- CASE 2.  $T > T_{sc\_crit} \quad -- \quad I_{sc} = 0, I_{metal} = I_0$

# Heater

Energy deposited on the wire

EXAMPLE: material properties are not realistic here.

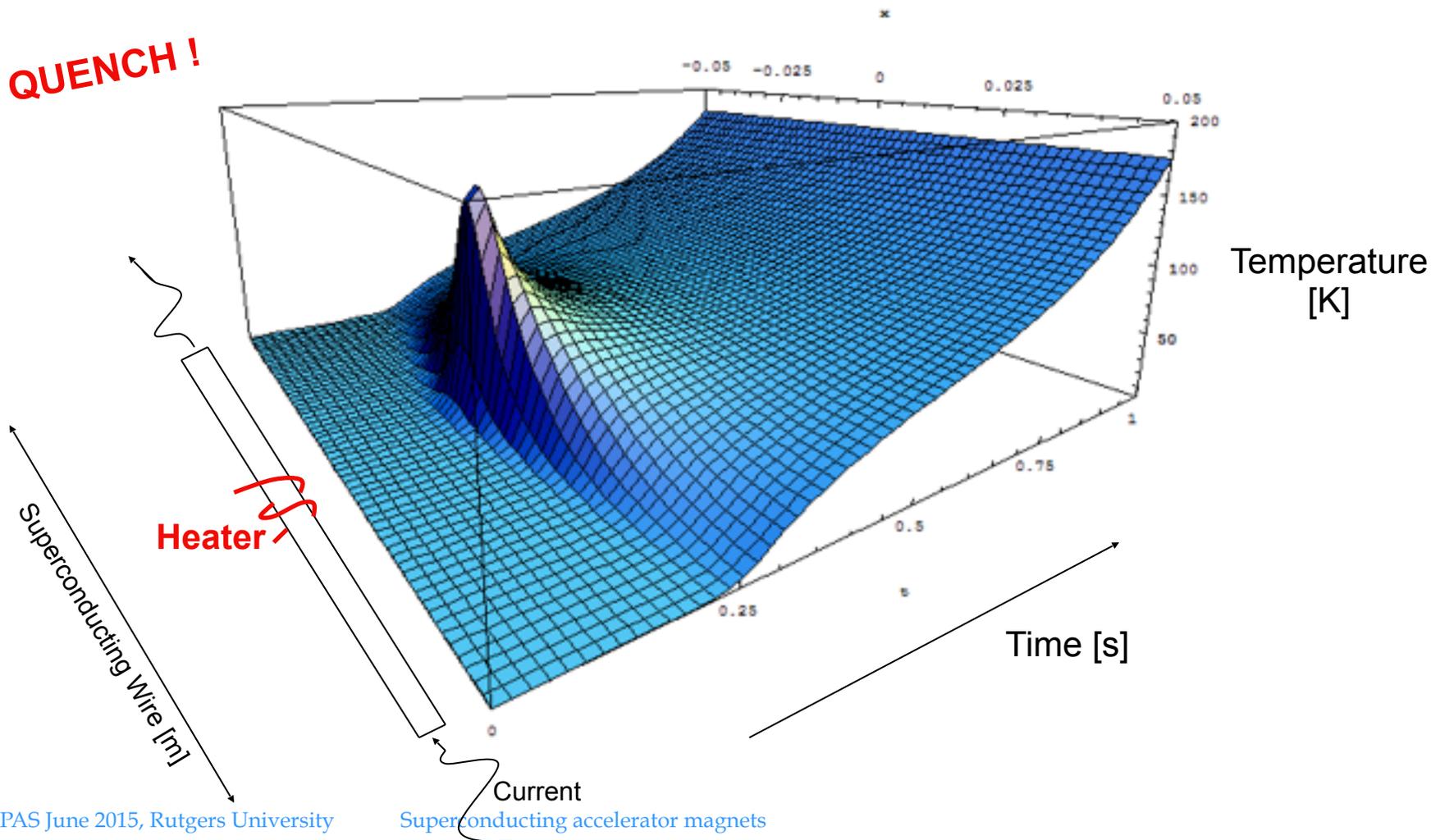


# Temperature profile

EXAMPLE: material properties are not realistic here.

With  $T_{cr} = 39\text{ K}$

**QUENCH !**

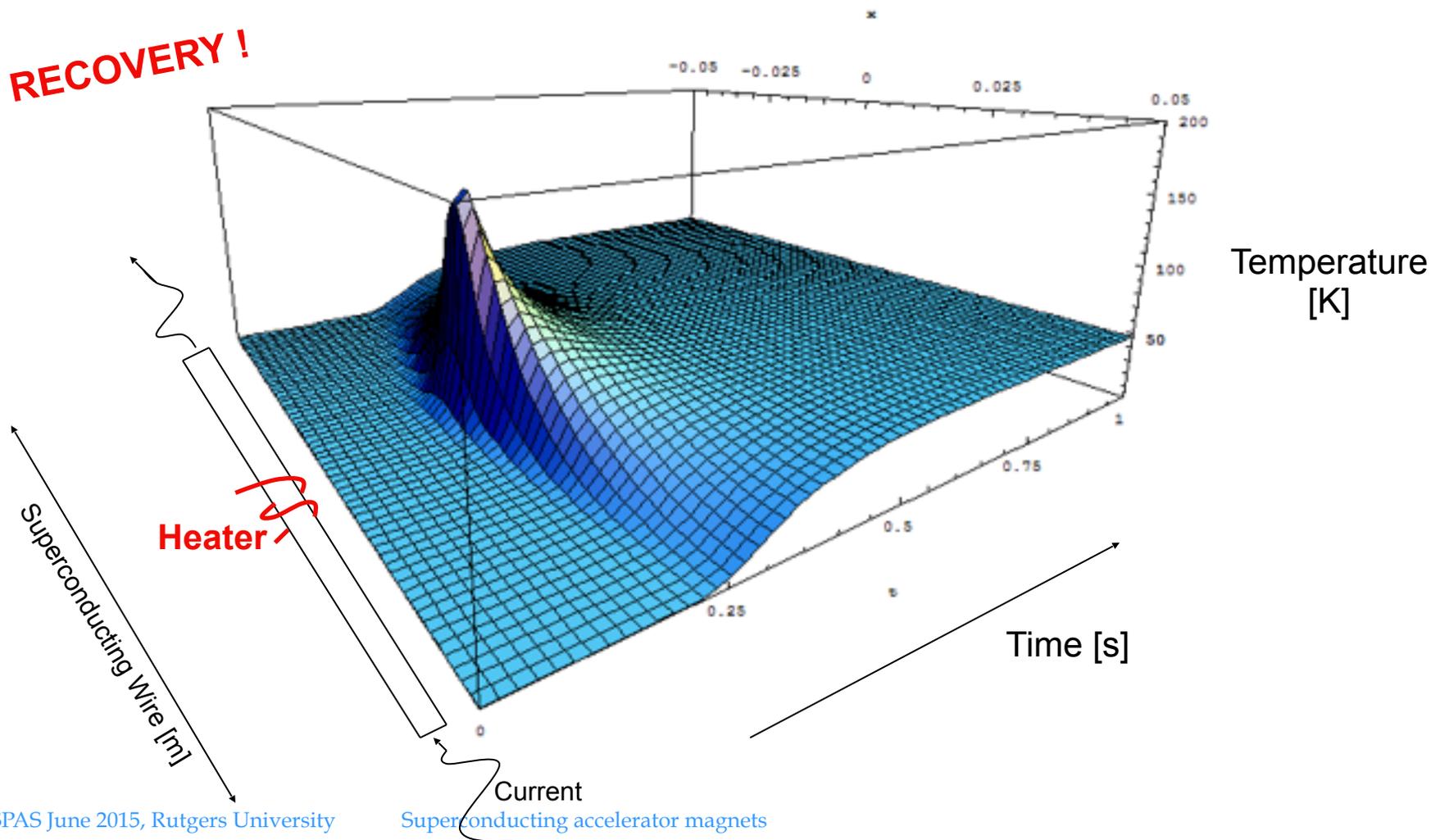


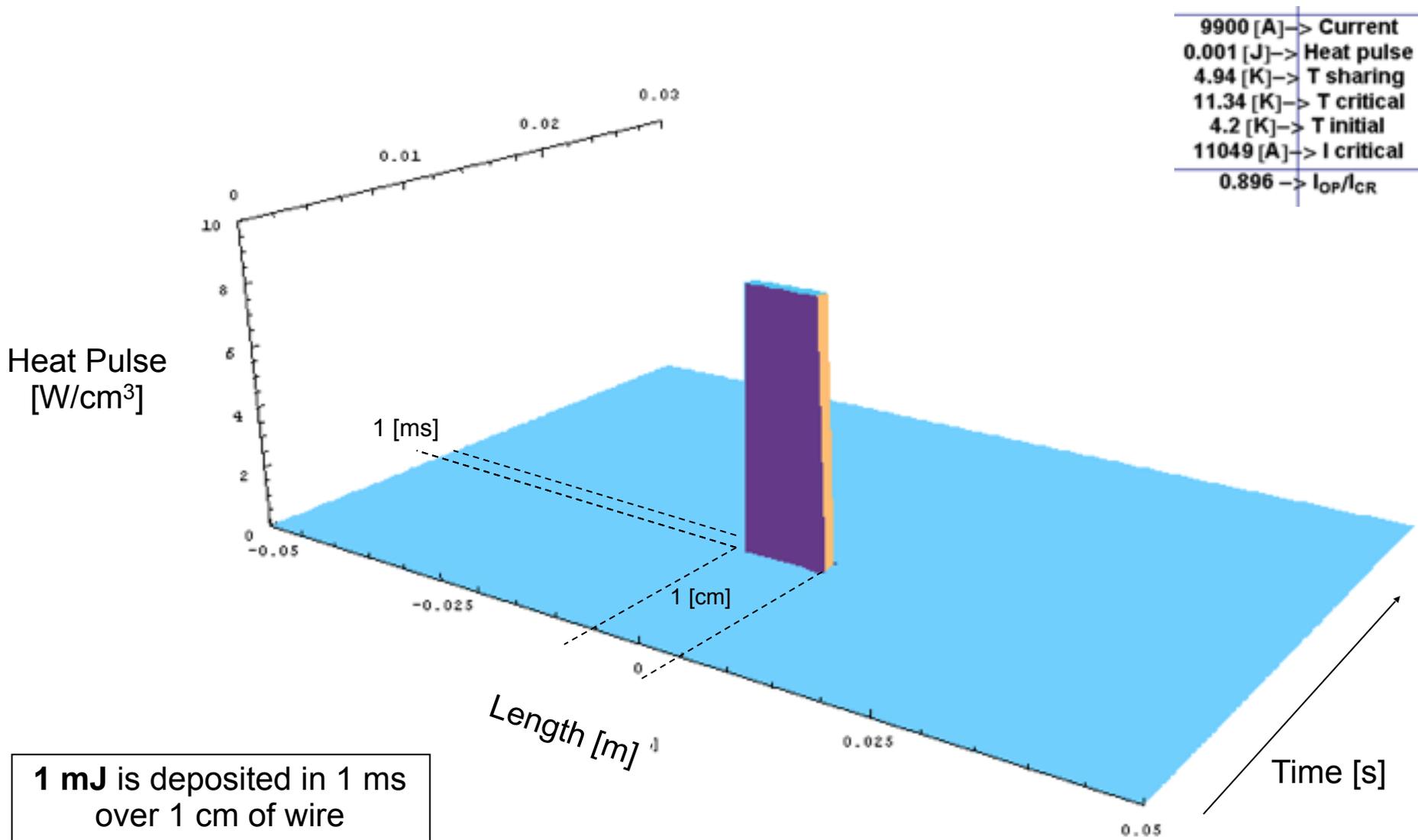
# Temperature profile

EXAMPLE: material properties are not realistic here.

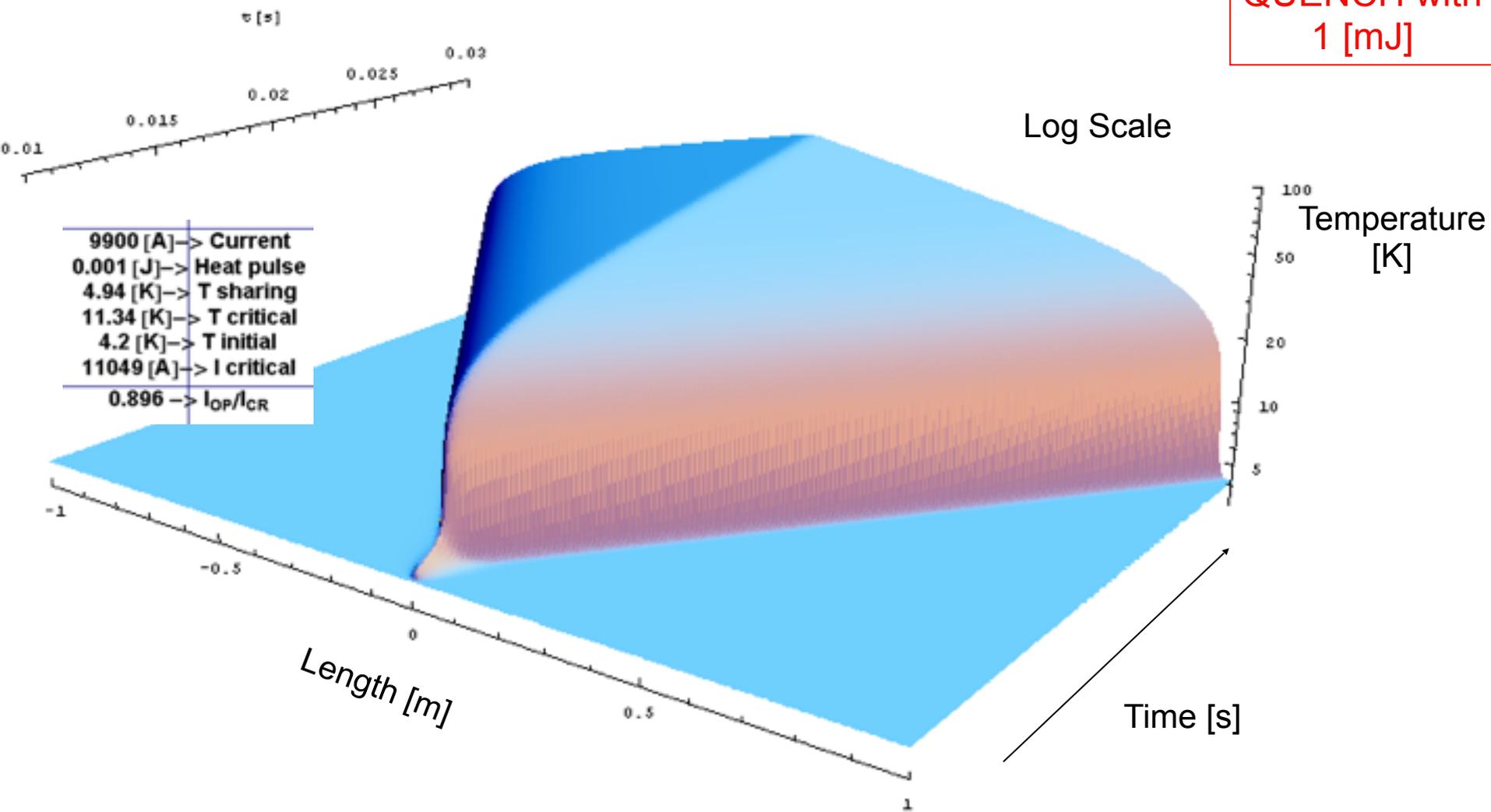
With  $T_{cr} = 70\text{ K}$

**RECOVERY !**

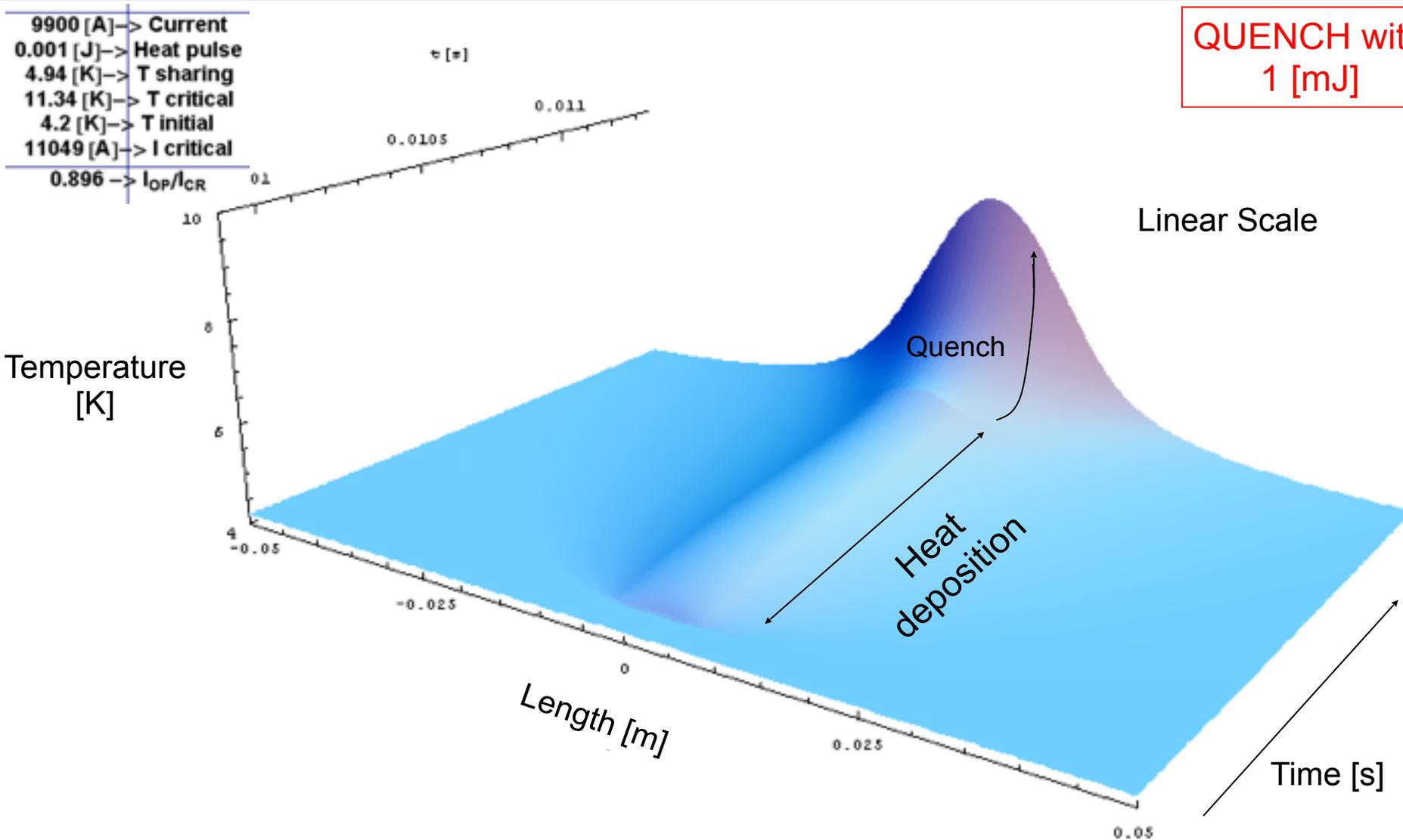


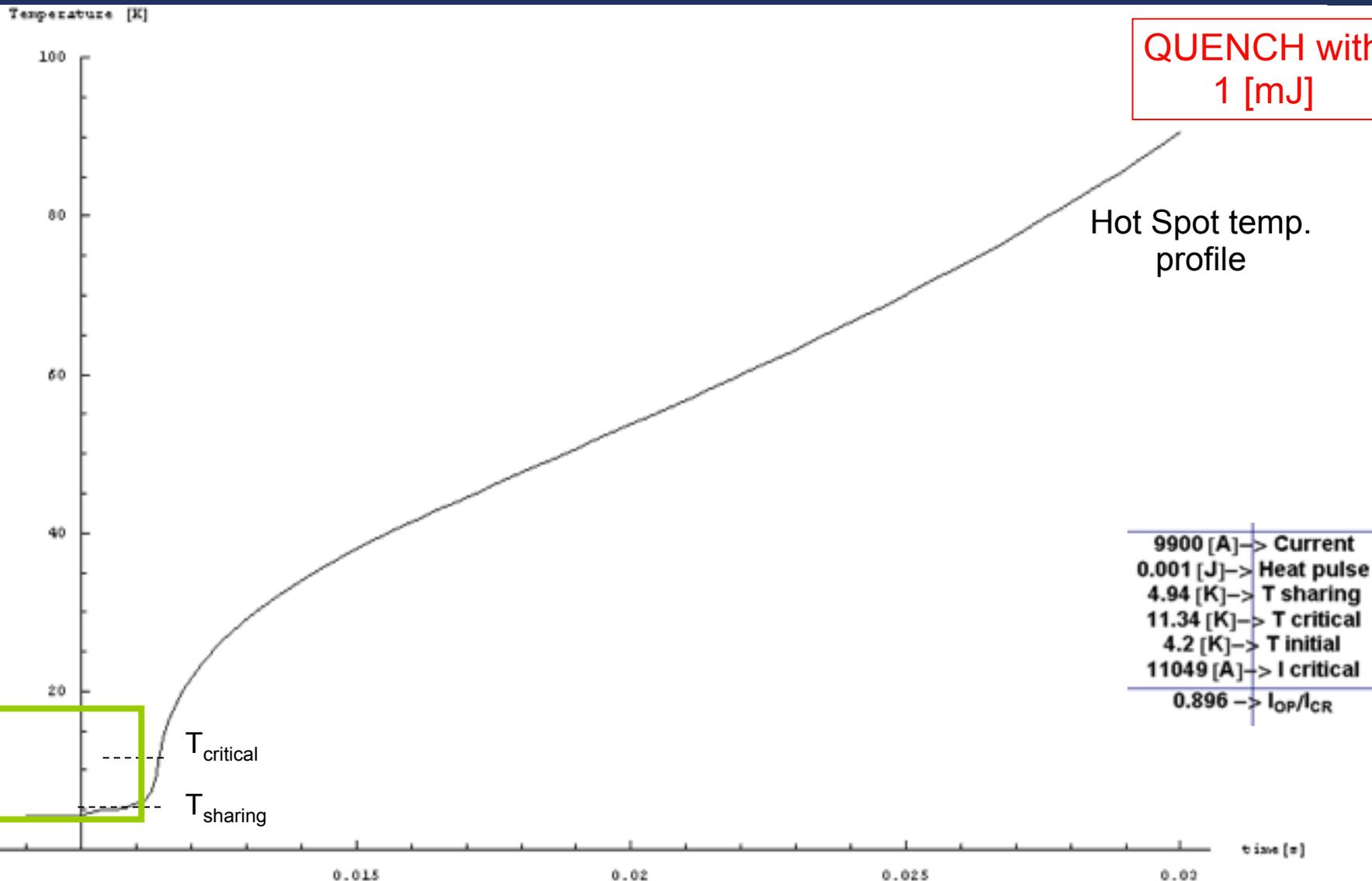


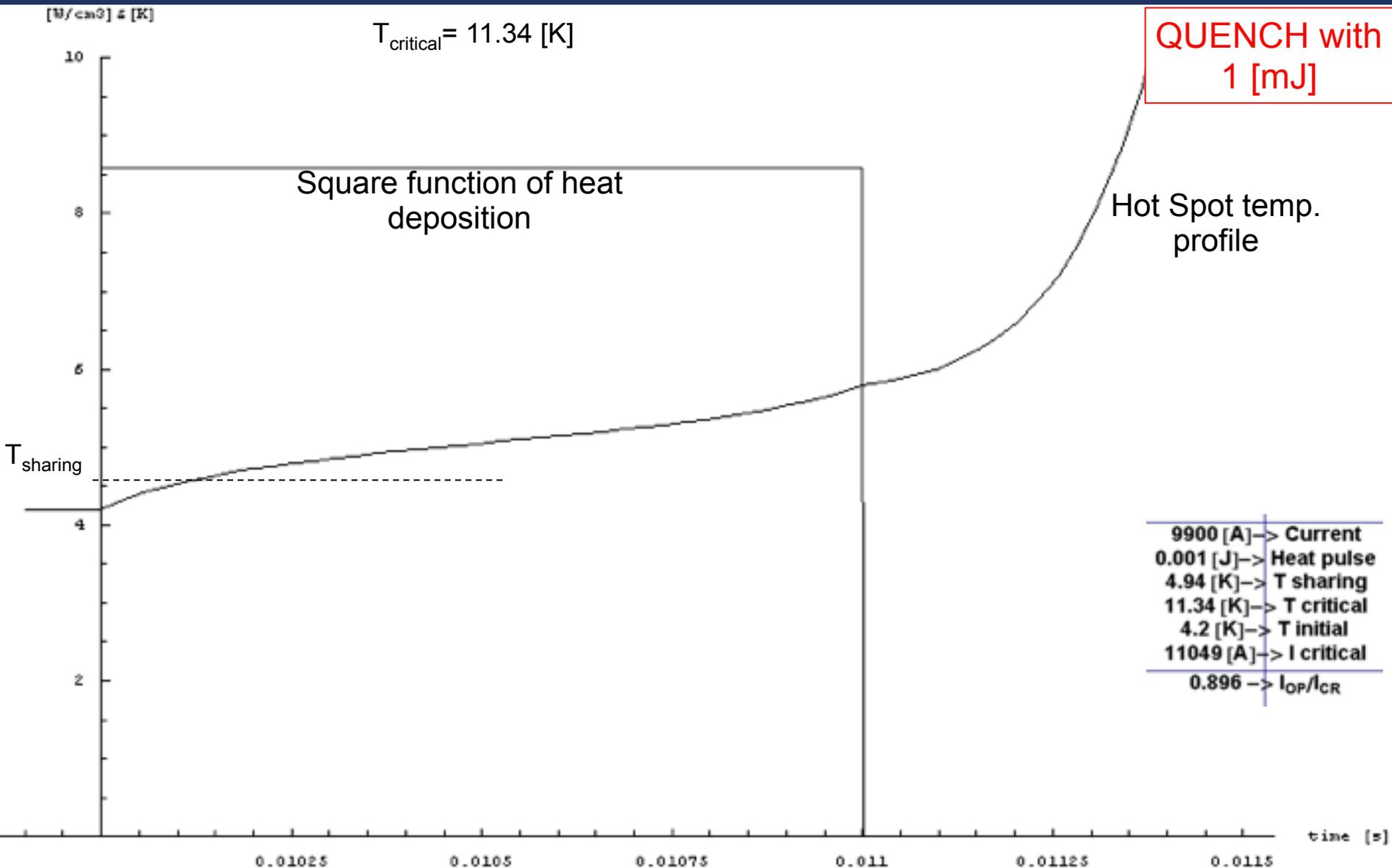
**QUENCH with  
1 [mJ]**

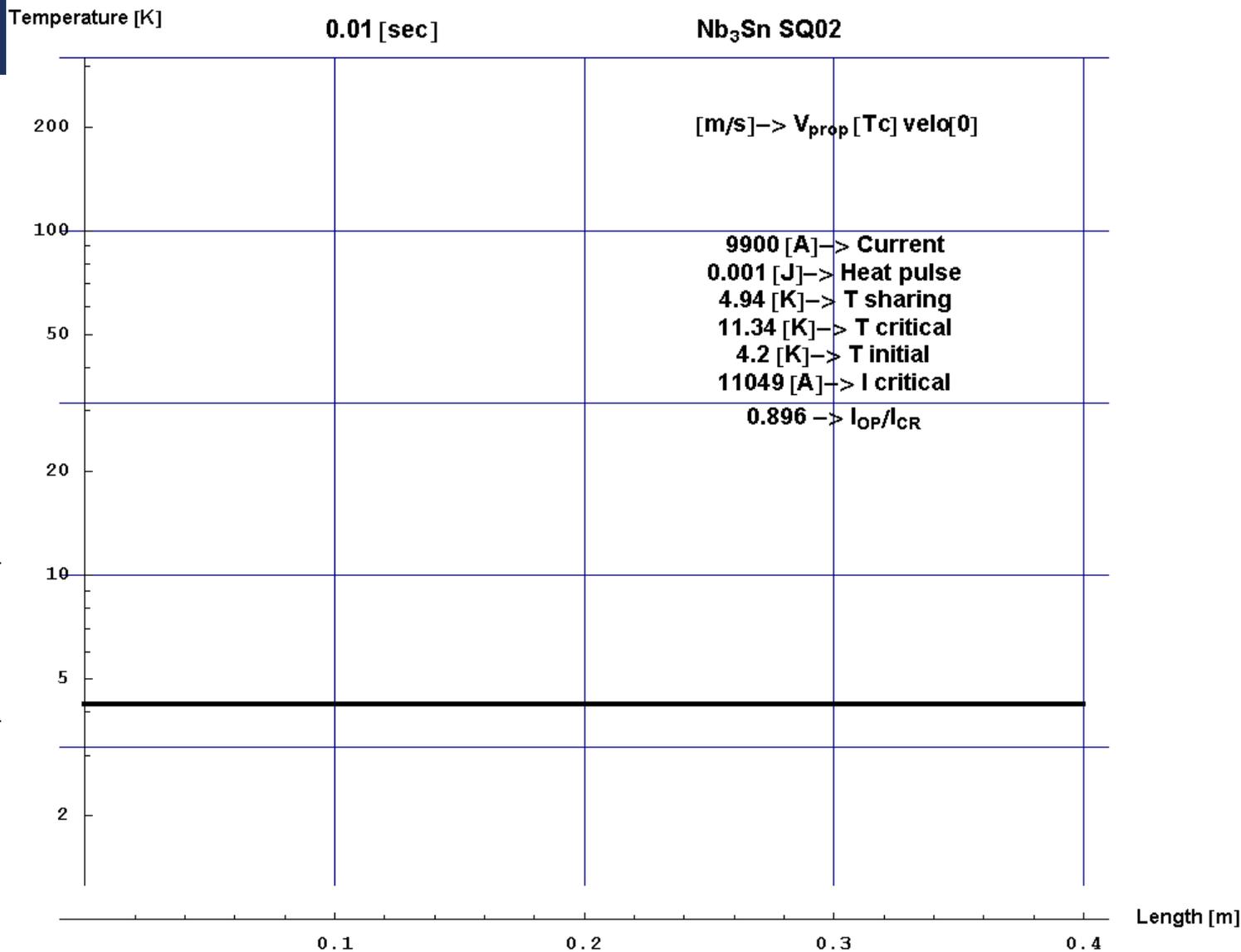


**QUENCH with  
1 [mJ]**









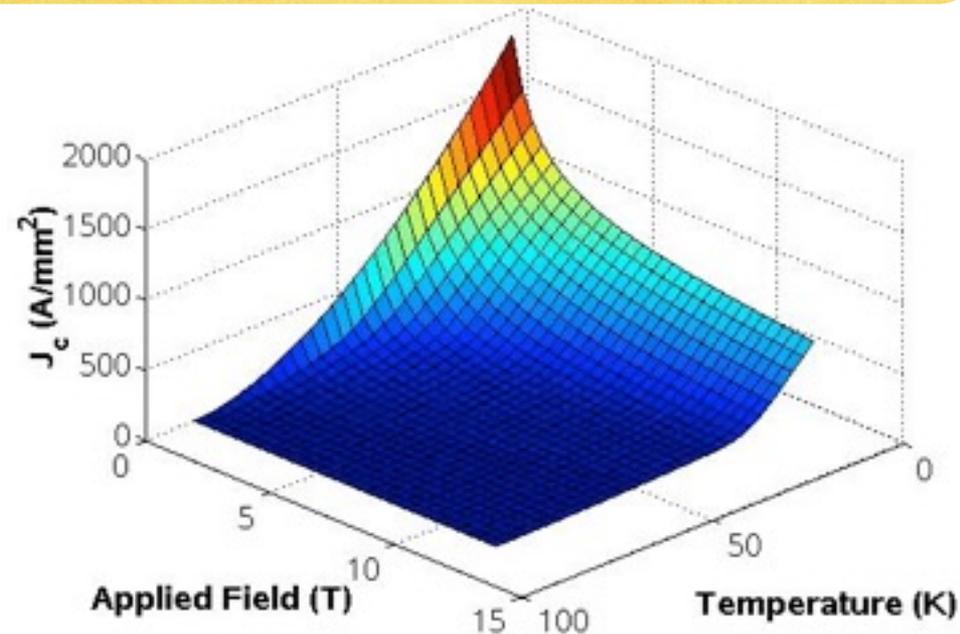
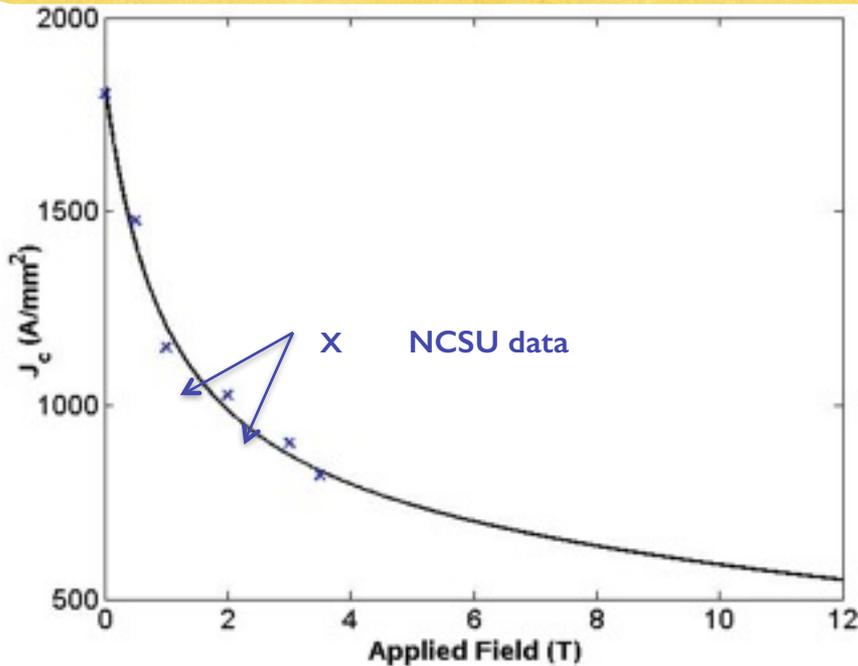
ICH with [mJ]

- T<sub>critical</sub>  
 - T<sub>sharing</sub>

# Bi2212 Model Properties

- $J_c(B,T)$  parameterization (Bottura, CryoSoft library) used to fit data from NCSU

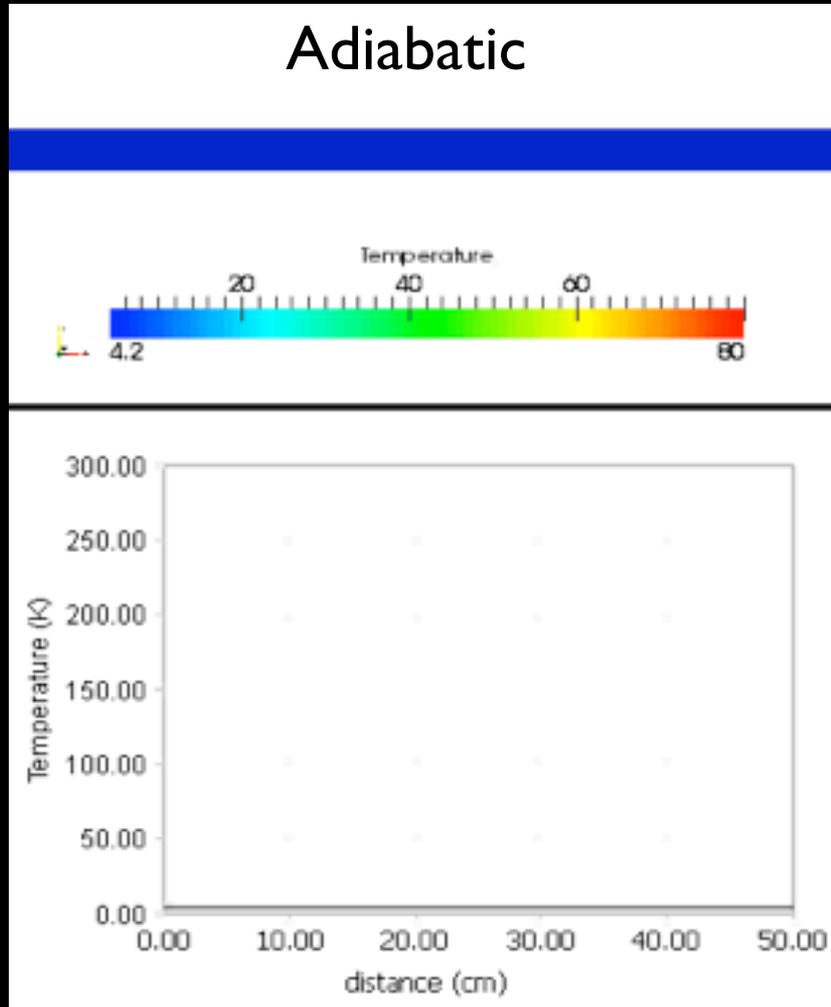
$$J_c(T,B) = J_0 \left(1 - \frac{T}{T_c}\right)^\gamma \left[ (1 - \chi) \frac{B_0}{B_0 + B} + \chi \exp\left(-\frac{\beta B}{B_{c0} \exp(-\alpha T/T_c)}\right) \right]$$



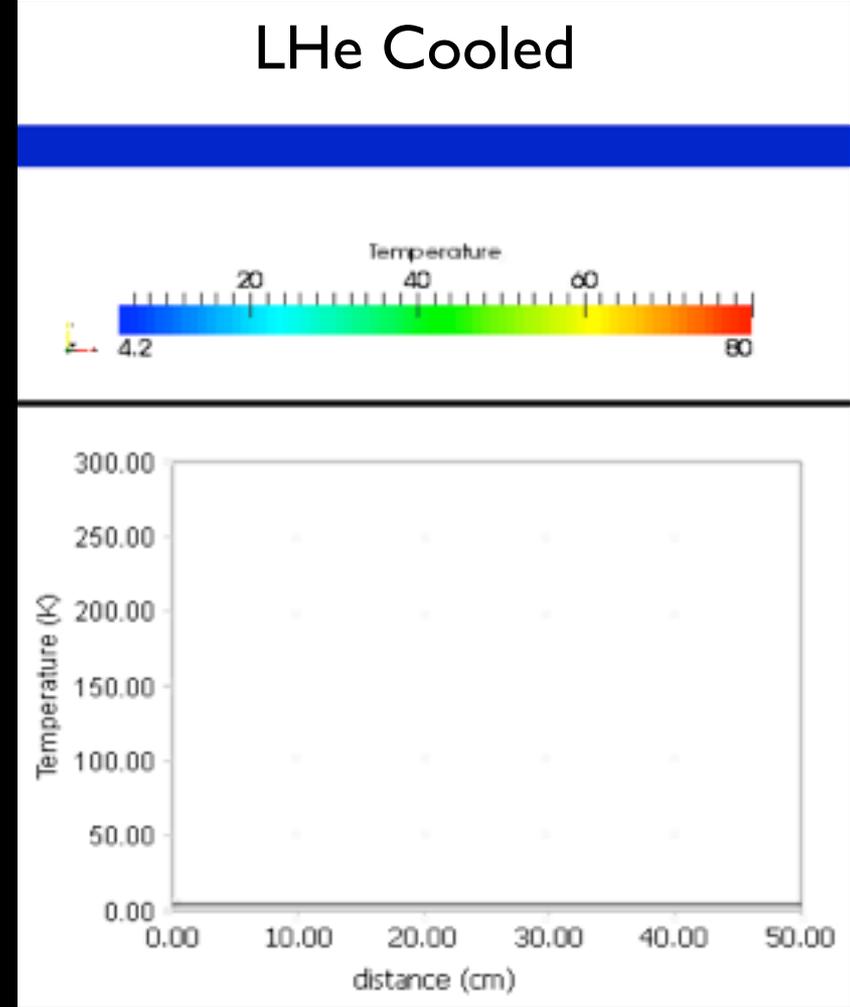
**Note: need more measurements for material properties input**

# Quench Simulation Examples

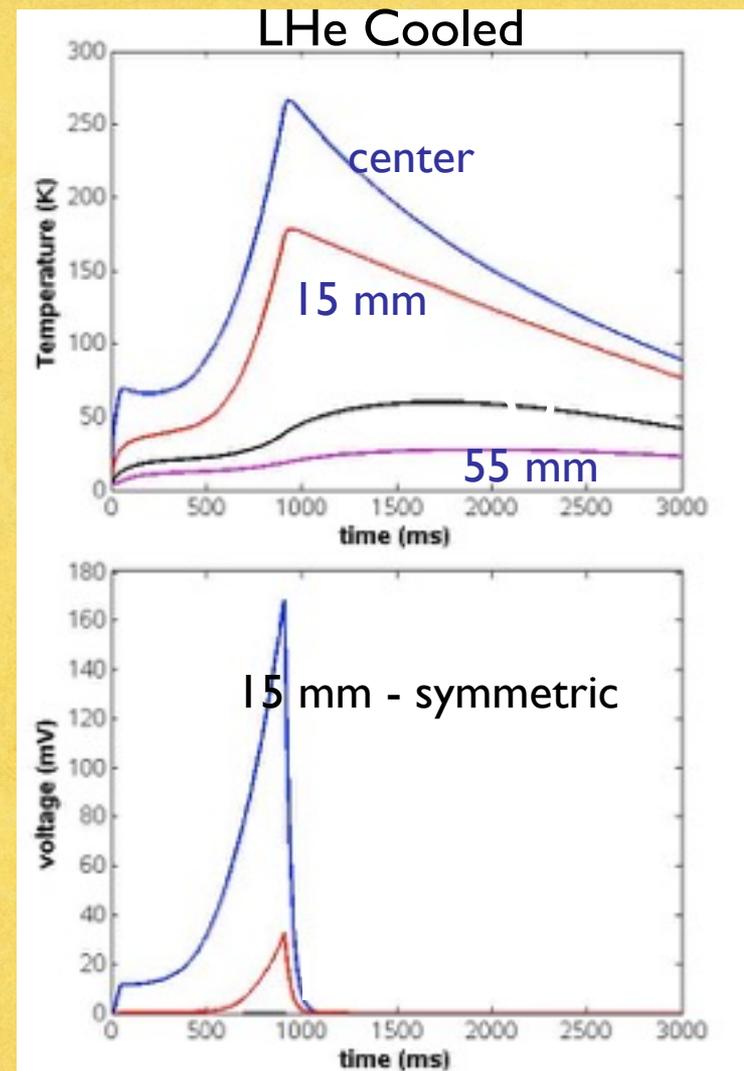
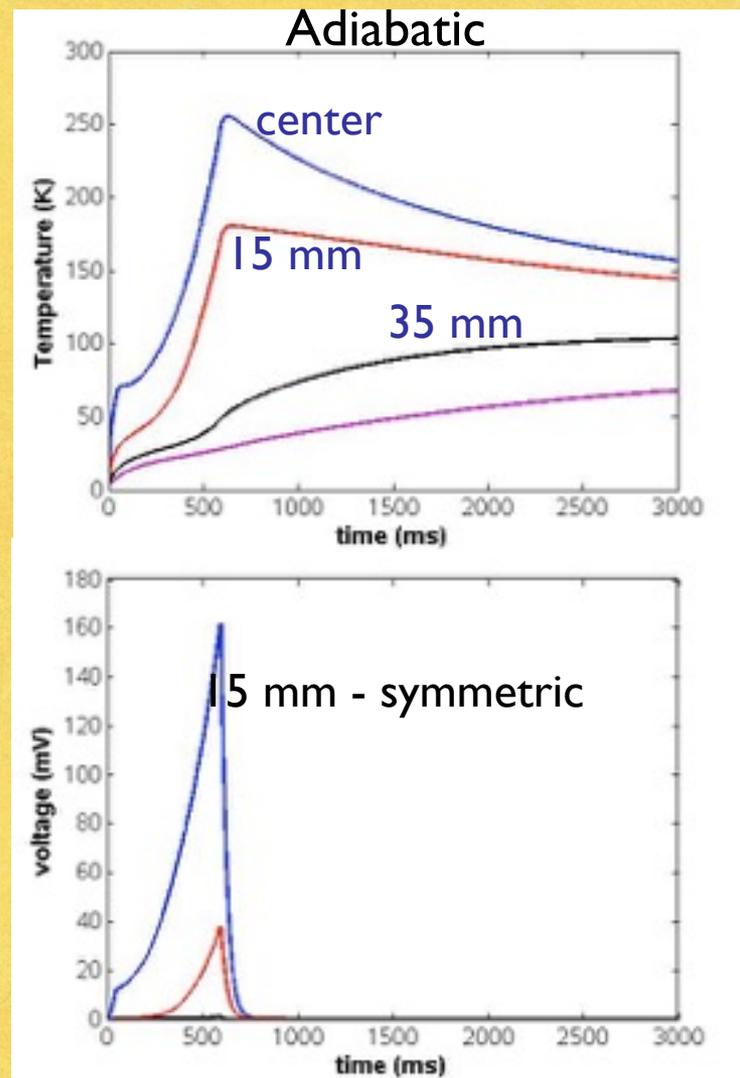
## Adiabatic



## LHe Cooled

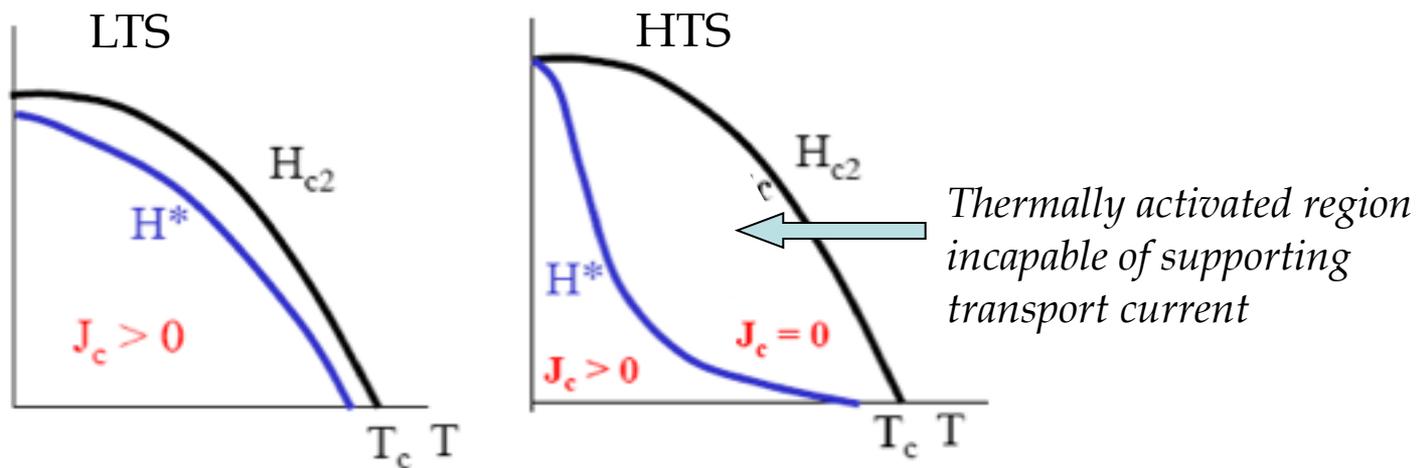


# Quench Simulation Examples



# HTS specific stability issues

- Whereas flux flow usually results in a flux jump condition in LTS materials, the far higher critical temperature of HTS materials provides significant heat capacity to mitigate the avalanche scenario
- The superconducting parameters for HTS materials and the (typically) higher operating temperature tend to increase the possibility of thermally induced flux motions: “melting” of the fluxoid lattice



# Quench propagation in Bi2212

- Quench initiation in HTS requires larger (10-100) amplitude thermal sources (i.e. higher stability)
- BUT: high temperature margin results in slow quench propagation – risk of high hot-spot temperature before quench is detected



- Fluxoids in a type II superconductor carrying transport current are subjected to forces which, if not countered, will result in flux flow and associated heating.
- Useful conductors (i.e. capable of carrying transport current) have pinning sites that counter the forces on the fluxoid.
- In LTS materials, flux motion due to a breakdown of a pinning site will often initiate an avalanche of flux flow – a flux jump.
- Depending on the details of the conductor composition and the amplitude of the heat induced by a flux jump, the conductor will either recover or quench.
- The recovery or quench of a superconducting wire subjected to a thermal disturbance can be reasonably analyzed using simple 1D analytic models (with a little help from numerics to accommodate more accurate material property models).