Unit 3
Basics of superconductivity

Soren Prestemon
Lawrence Berkeley National Laboratory (LBNL)

Paolo Ferracin and Ezio Todesco
European Organization for Nuclear Research (CERN)
Scope of the course

Basics of superconductivity

1. History
2. General principles
3. Diamagnetism
4. Type I and II superconductors
5. Flux pinning and flux creep
6. Critical surfaces for superconducting materials
References

- Wilson, “Superconducting Magnets”
- Mess, Schmueser, Wolff, “Superconducting Accelerator Magnets”
- Arno Godeke, thesis: “Performance Boundaries in Nb3Sn Superconductors”
- Alex Guerivich, Lectures on Superconductivity
- Roberto Casalbuoni: Lecture Notes on Superconductivity: Condensed Matter and QCD
**History of Cryogenics**

Cryogenics is the science of producing temperatures below ~200K

- Faraday (~1820’s) demonstrates ability to liquify most known gases by first cooling with a bath of ether and dry ice, followed by pressurization
  - he was unable to liquify oxygen, hydrogen, nitrogen, carbon monoxide, methane, and nitric oxide
  - The noble gases, helium, argon, neon, krypton and xenon had not yet been discovered (many of these are critical cryogenic fluids today)
- In 1848 Lord Kelvin determined the existence of absolute zero:
  - 0K=-273C (=−459F)
- In 1877 Louis Cailletet (France) and Raoul-Pierre Pictet (Switzerland) succeed in liquifying air
- In 1883 Von Wroblewski (Cracow) succeeds in liquifying Oxygen
- In 1898 James Dewar succeeded in liquifying hydrogen (~20K!); he then went on to freeze hydrogen (14K).
- Helium remained elusive; it was first discovered in the spectrum of the sun
- 1908: Kamerlingh Onnes succeeded in liquifying Helium
History

1911: Kamerlingh Onnes discovery of mercury superconductivity: “Perfect conductors”
   - A few years earlier he had succeeded in liquifying Helium, a critical technological feat needed for the discovery

1933: Meissner and Ochsenfeld discover perfect \textit{diamagnetic} characteristic of superconductivity

Kamerlingh Onnes, Nobel Prize 1913
A theory of superconductivity took time to evolve:

- **1935**: London brothers propose two equations for E and H
  - results in concept of penetration depth $\lambda$

- **1950**: Ginzburg and Landau propose a macroscopic theory (GL) for superconductivity, based on Landau’s theory of second-order phase transitions
  - Results in concept of coherence length $\xi$

Nobel Prize 2003: Ginzburg, Abrikosov, Leggett

Heinz and Fritz London

Ginzburg and Landau (circa 1947)
1957: Bardeen, Cooper, and Schrieffer publish microscopic theory (BCS) of Cooper-pair formation that continues to be held as the standard for low-temperature superconductors

1957: Abrikosov considered GL theory for case $\kappa = \lambda / \xi >> 1$

- Introduced concept of Type II superconductor
- Predicted flux penetrates in fixed quanta, in the form of a vortex array

Bardeen, Cooper and Schrieffer
Nobel Prize 1972
1957: Abrikosov considered GL theory for case $\kappa = \lambda / \varepsilon >> 1$

- Introduced concept of Type II superconductor
- Predicted flux penetrates in fixed quanta, in the form of a vortex array

_Nobel Prize 2003: Ginzburg, Abrikosov, Leggett (the GLAG members)_
History – High temperature superconductors

1986: Bednorz and Muller discover superconductivity at high temperatures in layered materials comprising copper oxide planes

George Bednorz and Alexander Muller
Nobel prize for Physics (1987)
Superconductivity refers to a material state in which current can flow with no resistance

- Not just “little” resistance - truly ZERO resistance
- Resistance in a conductor stems from scattering of electrons off of thermally activated ions
  - Resistance therefore goes down as temperature decreases
- The decrease in resistance in normal metals reaches a minimum based on irregularities and impurities in the lattice, hence concept of RRR (Residual resistivity ratio)
- RRR is a rough measure of cold-work and impurities in a metal

RRR=$\frac{\rho(273K)}{\rho(4K)}$
Aside: Maxwell’s equations

\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon} \quad \text{Gauss’ law} \]

\[ \nabla \cdot \vec{B} = 0 \]

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{Faraday’s law} \]

\[ \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{Ampere’s law} \text{ (corrected by Maxwell)} \]

\[ \mu_0 = 4\pi \times 10^{-7} \quad \text{N/A}^2 \quad \text{Permeability of free space} \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \quad \text{C}^2/(N\text{m}^2) \quad \text{Permittivity of free space} \]
Some reminders of useful formulas

\[ \nabla \cdot (\nabla \times \vec{F}) = 0 \quad \forall \vec{F} \]
\[ \nabla \times (\nabla \times \vec{F}) = \nabla (\nabla \cdot \vec{F}) - \nabla^2 \vec{F} \quad \forall \vec{F} \]

\[ \nabla \times (\nabla u) = 0 \quad \forall u \quad \text{or} \quad \nabla \times \vec{F} = 0 \quad \Leftrightarrow \vec{F} = \nabla u \]

(F is conservative if curl \( \vec{F} \) is zero)

Volume Integral

\[ \int_{V} \vec{F} \cdot \vec{n} \, dS = \int_{V} \nabla \cdot \vec{F} \, dV \]

Divergence Theorem

Surface Integral (Flux)

\[ \oint_{C} \vec{F} \cdot d\vec{l} = \int_{S} (\nabla \times \vec{F}) \cdot \vec{n} \, dS \]

Curl Theorem (Stoke's Theorem)

Line Integral (Circulation)
Some direct results from Maxwell

Electric and magnetic fields are fundamentally linked
- dB/dt induces voltage (Faraday)
- Moving charge generates B (Ampere)

Amperes law applied to DC fields and flowing currents:
\[ \nabla \times \vec{B} = \mu_0 \vec{J} \Rightarrow \oint \vec{B} \cdot dl = \mu_0 I_{\text{enclosed}} \]

Gauss’s law: no magnetic monopoles
\[ \nabla \cdot \vec{B} = 0 \]

Magnetic field lines cannot emanate from a point; they “curl” around current

Equations admit wave solutions
- Take the curl of Faraday’s and Ampere’s laws; E and B admit waves with velocity
\[ \nu = \sqrt{\frac{1}{\mu_0 \varepsilon}} = c = \text{speed of light} \]
From a macroscopic perspective, critical insight can be gleaned from magnetization measurements.

Magnetization is the magnetic (dipole) moment generated in a material by an applied field.

\[ \nabla \times B = \mu_0 J \]

\[ J = J_{\text{free}} + J_{\text{bound}} \]

\[ J_{\text{bound}} = \nabla \times M \]

\[ \Rightarrow H = \frac{1}{\mu_0} B - M \]

\[ \Rightarrow \nabla \times H = J_{\text{free}} \Rightarrow \oint H \cdot \text{dl} = I_{\text{enclosed free current}} \]

**Amperes law**

**Arbitrary but useful distinction**

**Results in a practical definition: we know and control free currents**

Note:
We do not need M; every calculation could be performed using B and H.
Example: iron is ferromagnetic – it has a strong paramagnetic moment (i.e. the magnetization is parallel and additive to the applied field)

- Most materials are either diamagnetic or paramagnetic, but the moments are extremely small compared to ferromagnetism.
- In diamagnetic and paramagnetic materials, the magnetization is a function of the applied field, i.e. remove the field, and the magnetization disappears.
- In ferromagnetic materials, some of the magnetization remains “frozen in” => hysteretic behavior.
In a superconductor, when the temperature descends below the critical temperature, electrons find it energetically preferable to form Cooper pairs.

- The Cooper pairs interact with the positive ions of the lattice.
- Lattice vibrations are often termed “phonons”; hence the coupling between the electron-pair and the lattice is referred to as electron-phonon interaction.
- The balance between electron-phonon interaction forces and Coulomb (electrostatic) forces determines if a given material is superconducting.

Electron-phonon interaction can occur over long distances; Cooper pairs can be separated by many lattice spacings.

**BCS breakthrough:**
Fermi surface is unstable to bound states of electron-pairs.

$$\Delta_0 \approx 2\hbar \omega_D \exp \left[ -\frac{1}{\lambda_{ep}} \right]$$

$$T_c \approx \frac{e^\gamma}{\pi k_b} \Delta_0$$

- $k_b$ = Boltzmann constant $= 1.38 \times 10^{-23}$
- $\omega_D$ = Debye frequency
- $\lambda_{ep} =$ electron-phonon coupling
- $\gamma$ = Euler constant $= 0.577$

$$\sim \text{meV} \sim 10^{-22} \text{J}!!$$
Diamagnetic behavior of superconductors

What differentiates a “perfect” conductor from a diamagnetic material?

A perfect conductor apposes any change to the existing magnetic state.

Superconductors exhibit diamagnetic behavior: flux is always expelled - Meissner effect.
Derive starting from the classical Drude model, but adapt to account for the Meissner effect:

- The Drude model of solid state physics applies classical kinetics to electron motion
  - Assumes static positively charged nucleus, electron gas of density $n$.
  - Electron motion damped by collisions

\[
m \frac{d\vec{v}}{dt} = e\vec{E} - \gamma \vec{v}
\]

\[
\vec{J}_s = -en_s\vec{v}
\]

\[
\frac{\partial}{\partial t} \left( \frac{m}{n_se^2} \nabla \times \vec{J}_s + \vec{B} \right) = 0 \implies \nabla^2 \vec{B} = \frac{\mu_0 n_se^2}{m} \vec{B} = \frac{1}{\lambda_L^2} \vec{B}
\]

The penetration depth $\lambda_L$ is the characteristic depth of the supercurrents on the surface of the material.
The density of superconducting electrons $n_s$ decreases to zero near a superconducting / normal interface, with a characteristic length $\xi$ (coherence length, first introduced by Pippard in 1953). The two length scales $\xi$ and $\lambda_L$ define much of the superconductors behavior.

The coherence length is proportional to the mean free path of conduction electrons; e.g. for pure metals it is quite large, but for alloys (and ceramics...) it is often very small. Their ratio, the GL parameter, determines flux penetration:

$$\kappa = \frac{\lambda_L}{\xi}$$

From “GLAG” theory, if:

- $\kappa < \frac{1}{\sqrt{2}}$ Type I superconductor
- $\kappa > \frac{1}{\sqrt{2}}$ Type II superconductor

Note: in reality $\xi$ and $\lambda_L$ are functions of temperature.
The Gibbs free energy of the superconducting state is lower than the normal state. As the applied field $B$ increases, the Gibbs free energy increases by $B^2/2\mu_0$.

The thermodynamic critical field at $T=0$ corresponds to the balancing of the superconducting and normal Gibbs energies:

$$G_n = G_s + \frac{H_c}{2}$$

The BCS theory states that $H_c(0)$ can be calculated from the electronic specific heat (Sommerfeld coefficient):

$$H_c(0) = 7.65 \times 10^{-4} \frac{\gamma^{1/2} T_c}{\mu_0}$$
Type I and II superconductors

- Type I superconductors are characterized by the Meissner effect, i.e. flux is fully expelled through the existence of supercurrents over a distance $\lambda_L$.
- Type II superconductors find it energetically favorable to allow flux to enter via normal zones of fixed flux quanta: “fluxoids” or vortices.
  - The fluxoids or flux lines are vortices of normal material of size $\sim \pi \xi^2$ “surrounded” by supercurrents shielding the superconducting material.

First photograph of vortex lattice, 
U. Essmann and H. Trauble
Max-Planck Institute, Stuttgart
Fluxoids, or flux lines, are continuous thin tubes characterized by a normal core and shielding supercurrents.

The flux contained in a fluxoid is quantized:

\[ \phi_0 = \frac{h}{2e} \]

\[ h = \text{Planck's constant} = 6.62607 \times 10^{-34} \text{ Js} \]

\[ e = \text{electron charge} = 1.6022 \times 10^{-19} \text{ C} \]

The fluxoids in an idealized material are uniformly distributed in a triangular lattice so as to minimize the energy state.

Fluxoids in the presence of current flow (e.g. transport current) are subjected to Lorentz force:

\[ \vec{F}_L = \vec{J} \times \vec{B} \]

⇒ Concept of flux-flow and associated heating
Solution for real conductors: provide mechanism to \textit{pin} the fluxoids

See flux flow movies…
Critical field definitions

$T=0$

- $H_{c1}$: Critical field defining the transition from the Meissner state

$$H_{c1} \approx \frac{\phi_0}{4\sqrt{2}\pi\mu_0\lambda^2} \ln(\kappa + \frac{1}{2}); \quad \kappa \gg 1$$

- $H_c$: Thermodynamic critical field
  - $H_c=H_{c1}$ for type I superconductors

$$H_c = \frac{\phi_0}{2\sqrt{2}\mu_0\kappa\pi\xi^2}$$

- $H_{c2}$: Critical field defining the transition to the normal state

$$H_{c2} = \frac{\phi_0}{2\pi\mu_0\xi^2}$$
Examples of Superconductors

- Many elements are superconducting at sufficiently low temperatures
- None of the pure elements are useful for applications involving transport current, i.e. they do not allow flux penetration
- Superconductors for transport applications are characterized by alloy/composite materials with $\kappa >> 1$

### Table 2.9. Critical Temperature and Critical Field of Type I Superconductors

<table>
<thead>
<tr>
<th>Material</th>
<th>$T_c$ (K)</th>
<th>$\lambda(0)$, nm</th>
<th>$\xi(0)$, nm</th>
<th>$H_{c2}$ (T)</th>
<th>$\mu_0H_0$ (mT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>1.2</td>
<td></td>
<td></td>
<td></td>
<td>9.9</td>
</tr>
<tr>
<td>Cadmium</td>
<td>0.52</td>
<td></td>
<td></td>
<td></td>
<td>3.0</td>
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<tr>
<td>Gallium</td>
<td>1.1</td>
<td></td>
<td></td>
<td></td>
<td>5.1</td>
</tr>
<tr>
<td>Indium</td>
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<td></td>
<td></td>
<td></td>
<td>27.6</td>
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<tr>
<td>Iridium</td>
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<td></td>
<td></td>
<td></td>
<td>1.6</td>
</tr>
<tr>
<td>Lanthanum $\alpha$</td>
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<td></td>
<td></td>
<td></td>
<td>4.9</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Molybdenum</td>
<td>0.9</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Osmium</td>
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<td></td>
<td>17.1</td>
</tr>
<tr>
<td>Thorium</td>
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<td></td>
<td>16.2</td>
</tr>
<tr>
<td>Tin</td>
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<td></td>
<td></td>
<td>30.6</td>
</tr>
<tr>
<td>Titanium</td>
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<td></td>
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<td></td>
<td></td>
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<tr>
<td>Tungsten</td>
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<td></td>
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<td></td>
<td>0.12</td>
</tr>
<tr>
<td>Uranium $\alpha$</td>
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<td></td>
<td></td>
<td></td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zinc</td>
<td>0.9</td>
<td></td>
<td></td>
<td></td>
<td>5.3</td>
</tr>
<tr>
<td>Zirconium</td>
<td>0.8</td>
<td></td>
<td></td>
<td></td>
<td>4.7</td>
</tr>
</tbody>
</table>
Aside – uses for type I superconductors

Although type I superconductors cannot serve for large-scale transport current applications, they can be used for a variety of applications:

- Excellent electromagnetic shielding for sensitive sensors (e.g. lead can shield a sensor from external EM noise at liquid He temperatures)
- Niobium can be deposited on a wafer using lithography techniques to develop ultra-sensitive sensors, e.g. transition-edge sensors
  - Using a bias voltage and Joule heating, the superconducting material is held at its transition temperature;
  - absorption of a photon changes the circuit resistance and hence the transport current, which can then be detected with a SQUID (superconducting quantum interference device)

See for example research by J. Clarke, UC Berkeley;

![Figure 2. Resistance vs. temperature for a high-sensitivity TES bilayer. Mo/Au bilayer TES detector

Courtesy Benford and Moseley, NASA Goddard](image-url)
Flux Flow

The Lorentz force acting on a fluxoid will, in the absence of pinning, result in motion of the fluxoid.

Fluxoid motion generates a potential gradient (i.e. voltage) and hence heating.

This can be modeled using Faraday’s law of induction:

\[
\vec{E} = \vec{B} \times \vec{\nu}_f
\]

\[
\dot{\nu} = nF_L \cdot \vec{\nu}_f = (\vec{J} \times \vec{B}) \cdot \vec{\nu}_L = \vec{J} \cdot \vec{E} = JE
\]

\[E = \rho_{\text{eff}} J \Rightarrow \text{“ideal” superconductors can support no transport current beyond } H_{c1}!\]

Real superconductors have defects that can prevent the flow of fluxoids.

The ability of real conductors to carry transport current depends on the number, distribution, and strength of such pinning centers.
Flux pinning

Fluxoids can be pinned by a wide variety of material defects

- **Inclusions**
  - Under certain conditions, small inclusions of appropriate materials can serve as pinning site locations; this suggests tailoring the material artificially through manufacturing.

- **Lattice dislocations / grain boundaries**
  - These are known to be primary pinning sites. Superconductor materials for wires are severely work hardened so as to maximize the number and distribution of grain boundaries.

- **Precipitation of other material phases**
  - In NbTi, mild heat treatment can lead to the precipitation of an a-phase Ti-rich alloy that provides excellent pinning strength.
The distribution and pinning of fluxoids depends on the operating regime:

- At low field (but > $H_{c1}$) the distribution is governed mainly by interaction between flux-lines, i.e. the fluxoids find it energetically advantageous to distribute themselves “evenly” over the volume (rather weak).

- At intermediate fields, the pinning force is provided by the pinning sites, capable of hindering flux flow by withstanding the Lorentz force acting on the fluxoids. Ideally, the pinning sites are uniformly distributed in the material (very strong).

- At high field, the number of fluxoids significantly exceeds the number of pinning sites; the effective pinning strength is a combination of defect pinning strength and shear strength of the fluxlines (rather weak).
High-Temperature superconductors

Much of HTS behavior can be understood in terms of the BCS and GLAG theory parameters

The new features of HTS have to do with:

1) highly two-dimensional domains of superconductor, separated by regions of “inert” material
   - Macroscopic behavior is therefore highly anisotropic
   - Different layers must communicate (electrically) via tunneling, or incur Joule losses

2) a much larger range of parameter space in which multiple effects compete
   - The coherence lengths for HTS materials are far smaller than for LTS materials
   - Critical fields are ~10 times higher

=> Thermal excitations play a much larger role in HTS behavior
Modeling pinning

Precise first-principles physical descriptions of overall pinning strength (and hence critical current) of real superconductors is difficult due to ambiguities intrinsic in pinning.

Nevertheless, models based on sound physics minimize free parameters needed to fit measured data and provide reliable estimates for classes of materials.

One of the most cited correlations is that of Kramer:

\[ F_p = F_{\text{max}} f(h) \propto \frac{H^{\nu}}{K^\gamma} f(h) \]

\[ f(h) = h^{1/2} (1-h)^2 ; \quad h = H / H_{c2} \]

The fitting coefficients \( \nu \) and \( \gamma \) depend on the type of pinning. Furthermore, it is experimentally verifies that

\[ H_c(T) \approx H_c(0) \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right] \]

\[ F_K = J_c^{1/2} B^{1/4} \propto (1-b) \]

From L. Cooley, USPAS
Scaling of critical current: field dependence

The Kramer formulation provides excellent fits in the region $0.2 < h < 0.6$ for $\text{Nb}_3\text{Sn}$; it is appropriate for regimes where the number of fluxoids exceeds the number of pinning sites.

Outside this region, a variety of effects (e.g. inhomogeneity averaging) can alter the pinning strength behavior, so the pinning strength is often fitted with the generalization

$$f_p(h) \propto h^p (1 - h)^q; \quad h = \frac{H}{H_{c2}}$$

It is preferable to stay with the Kramer formulation, yielding:

$$J_c^{1/2} B^{1/4} \simeq \frac{1.1 \times 10^5}{\kappa} \mu_0 (H_{c2} - H)$$
Scaling of critical current: temperature dependence

The temperature dependence of \( J_c \) stems from the term

\[
\frac{\mu_0 H_{c2}(T)}{\kappa^\gamma(T)}
\]

Scalings are typically generated by considering the normalized thermodynamic critical field and the the normalized GL parameter (here \( t = T/T_c \)):

\[
\frac{H_c(T)}{H_c(0)} = 1 - t^2
\]

\[
\frac{\kappa(T)}{\kappa(0)} = \begin{cases} 
1 - 0.31t^2 \left( 1 - 1.77 \ln(t) \right) & \text{Summers} \\
1 - 0.33t & \text{Summers (reduced)} \\
1 - t^{1.52} & \text{Godeke / De Gennes}
\end{cases}
\]
The critical current of Nb$_3$Sn is strain dependent, particularly at high field.

The strain dependence is typically modeled in terms of the normalized critical temperature:

$$\frac{H_{c2}(4.2, \varepsilon)}{H_{c2m}(0)} \simeq \left[ \frac{T_c(\varepsilon)}{T_{cm}} \right]^3 = s(\varepsilon)$$

The term $T_{cm}$ and $H_{c2m}$ refer to the peaks of the strain-dependent curves.

A “simple” strain model proposed by Ekin yields

$$s(\varepsilon) = 1 - a |\varepsilon_{axial}|^{1.7}$$

$$a = \begin{cases} 900 & \varepsilon_{axial} < 0 \\ 1250 & \varepsilon_{axial} > 0 \end{cases}$$
A physics-based model of strain dependence has been developed using the frequency-dependent electron-phonon coupling interactions (Eliashberg; Godeke, Markiewitz)

\[ \lambda_{ep} (\varepsilon) = 2 \int \frac{\alpha^2(\omega)F(\omega)}{\omega} d\omega \]

From the interaction parameter the strain dependence of \( T_c \) can be derived

Experimentally, the strain dependence of \( H_{c2} \) behaves as

\[ \frac{H_{c2}(4.2, \varepsilon)}{H_{c2m}(4.2)} \approx \frac{T_c(\varepsilon)}{T_{cm}} \]

The theory predicts strain dependence of \( J_c \) for all LTS materials, but the amplitude of the strain effects varies (e.g. very small for NbTi)

The resulting model describes quite well the asymmetry in the strain dependence of \( B_{c2} \), and the experimentally observed strong dependence on the deviatoric strain
Strain dependence of $J_c$ in Nb$_3$Sn

The strain dependence is a strong function of the applied field and of temperature.
Critical surface:
Example fit for NbTi

NbTi parameterization

- Temperature dependence of $B_{C2}$ is provided by Lubell’s formulae:

$$B_{C2}(T) = B_{C20} \left[ 1 - \left( \frac{T}{T_{C0}} \right)^{1.7} \right]$$

where $B_{C20}$ is the upper critical flux density at zero temperature ($\sim 14.5$ T)

- Temperature and field dependence of $J_c$ can be modeled, for example, by Bottura’s formula

$$\frac{J_C(B,T)}{J_{C,\text{Ref}}} = \frac{C_{\text{NbTi}}}{B} \left[ \frac{B}{B_{C2}(T)} \right]^\alpha_{\text{NbTi}} \left[ 1 - \frac{B}{B_{C2}(T)} \right]^\beta_{\text{NbTi}} \left[ 1 - \left( \frac{T}{T_{C0}} \right)^{1.7} \right]^\gamma_{\text{NbTi}}$$

where $J_{C,\text{Ref}}$ is critical current density at 4.2 K and 5 T (e.g. $\sim 3000$ A/mm$^2$) and $C_{\text{NbTi}}$ ($\sim 30$ T), $\alpha_{\text{NbTi}}$ ($\sim 0.6$), $\beta_{\text{NbTi}}$ ($\sim 1.0$), and $\gamma_{\text{NbTi}}$ ($\sim 2.3$) are fitting parameters.
Scaling $J_c$ for NbTi & Nb$_3$Sn
(Courtesy Arno Godeke)

$J_c(H, T, \varepsilon) \approx \frac{C_1}{\mu_0 H} s(\varepsilon)(1 - t^n)(1 - t^p) h^p (1 - h)^q$, 

with 
$t \equiv \frac{T}{T_c}(\varepsilon), \quad h \equiv \frac{H}{H_{c2}}(T, \varepsilon), 
H_{c2}^*(T, \varepsilon) \equiv H_{c2m}^*(0) s(\varepsilon)(1 - t^n), 
T_c^*(\varepsilon) = T_{cm}^* s(\varepsilon)^{1/3}$


Fits for NbTi

Godeke, SuST 19
- $n_1 = 1.52$
- $n_2 = 2$
- $p = 0.5$
- $q = 2$
- $s(\varepsilon) = \text{strain dependence}$

NbTi
Bottura, TAS 19
- $n_1 = n_2 \approx 1.7$
- $p \approx 0.73$
- $q \approx 0.9$
- $s(\varepsilon) \approx 1$
\[
J_c(H, T, \varepsilon) \equiv \frac{C_1}{\mu_0 H} s(\varepsilon) (1 - t^{1.52}) (1 - t^2) h^{0.5} (1 - h)^2
\]

\[
H_{c2}^*(T, \varepsilon) \equiv H_{c2m}^*(0) s(\varepsilon) (1 - t^{1.52})
\]

\[
\ln \left( \frac{T}{T_c(0)} \right) = \psi \left( \frac{1}{2} \right) - \psi \left( \frac{1}{2} + \frac{\hbar D \mu_0 H_{c2}(T)}{2 \phi_0 k_B T} \right)
\]
Using magnetization data

We have seen that the Meissner state corresponds to perfect diamagnetic behavior.

We have seen that beyond $H_{c1}$, flux begins to penetrate and can be pinned at defects => hysteretic behavior

⇒ Much can be understood by measuring the effective magnetization of superconducting material.

The measured magnetization provides insight into flux pinning and flux motion, key concepts governing the performance of superconducting materials.

$$\Delta M \cdot B \propto F_p(T, B)$$

Often used to evaluate $J_c(B,T)$!
Example material: Nb$_3$Sn

Phase diagram, A15 lattice...
Recent developments in Tc and Jc are quite impressive
- Improvements in material processing has lead to
  - enhanced pinning
  - Enhanced Tc
  - Smaller superconducting filaments

Expect, *and participate in*, new and dramatic developments as fundamental understanding of superconductivity evolves and improvements in nanoscale fabrication processes are leveraged
- A basic theory of superconductivity for HTS materials has yet to be formulated!

Some understanding of the fundamentals of superconductivity are critical to appropriately select and apply these materials to accelerator magnets
- Superconductors can be used to generate very high fields for state-of-the-art facilities, but they are *not* forgiving materials – *in accelerator applications they operate on a precarious balance of large stored energy and minute stability margin!*