Unit 3
Basics of superconductivity

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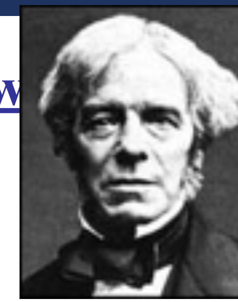
Scope of the course

- Basics of superconductivity
  1. History
  2. General principles
  3. Diamagnetism
  4. Type I and II superconductors
  5. Flux pinning and flux creep
  6. Critical surfaces for superconducting materials

References

- Wilson, "Superconducting Magnets"
- Mess, Schmueser, Wolff, "Superconducting Accelerator Magnets"
- Arno Godeke, thesis: "Performance Boundaries in Nb3Sn Superconductors"
- Alex Guerivich, Lectures on Superconductivity
- Roberto Casalbuoni: Lecture Notes on Superconductivity: Condensed Matter and QCD

History of Cryogenics



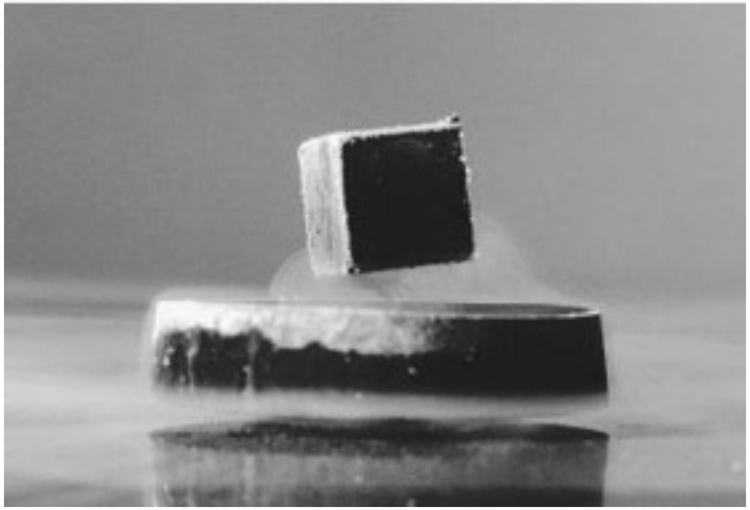
[Cryogenics is the science of producing temperatures below ~200K]()

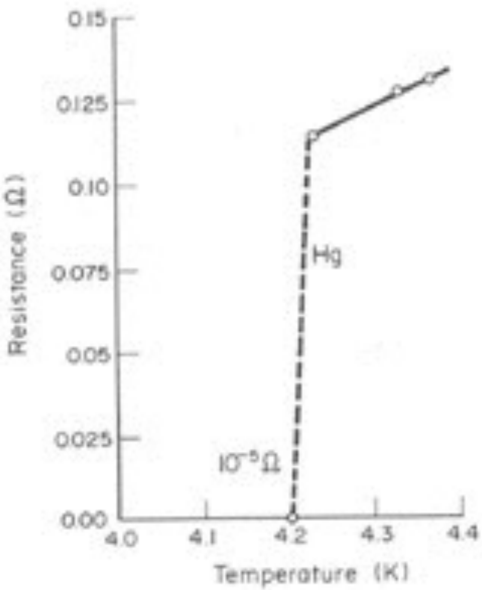
- Faraday (~1820's) demonstrates ability to liquify most known gases by first cooling with a bath of ether and dry ice, followed by pressurization
  - he was unable to liquify oxygen, hydrogen, nitrogen, carbon monoxide, methane, and nitric oxide
  - The noble gases, helium, argon, neon, krypton and xenon had not yet been discovered (many of these are critical cryogenic fluids today)
- In 1848 Lord Kelvin determined the existence of absolute zero:
  - 0K=-273C (=-459F)
- In 1877 Louis Caillettet (France) and Raoul-Pierre Pictet (Switzerland) succeed in liquifying air
- In 1883 Von Wroblewski (Cracow) succeeds in liquifying Oxygen
- In 1898 James Dewar succeeded in liquifying hydrogen (~20K!); he then went on to freeze hydrogen (14K).
- Helium remained elusive; it was first discovered in the spectrum of the sun
- 1908: Kamerlingh Onnes succeeded in liquifying Helium

History

- 1911: Kamerlingh Onnes discovery of mercury superconductivity: “Perfect conductors”
  - A few years earlier he had succeeded in liquifying Helium, a critical technological feat needed for the discovery
- 1933: Meissner and Ochsenfeld discover perfect ***diamagnetic*** characteristic of superconductivity

Kamerlingh Onnes, Nobel Prize 1913





History - theory

- A theory of superconductivity took time to evolve:
  - 1935: London brothers propose two equations for E and H
    - results in concept of penetration depth λ
  - 1950:Ginzburg and Landau propose a macroscopic theory (GL) for superconductivity, based on Landau's theory of second-order phase transitions
    - Results in concept of coherence length ξ

Heinz and Fritz London

Ginzburg and Landau (circa 1947)
Nobel Prize 2003: Ginzburg, Abrikosov, Leggett

History - theory

- 1957: Bardeen, Cooper, and Schrieffer publish microscopic theory (BCS) of Cooper-pair formation that continues to be held as the standard for low-temperature superconductors
- 1957: Abrikosov considered GL theory for case $\kappa=\lambda/\varepsilon>>1$
  - Introduced concept of Type II superconductor
  - Predicted flux penetrates in fixed quanta, in the form of a vortex array

Bardeen, Cooper and Schrieffer
Nobel Prize 1972

History - theory

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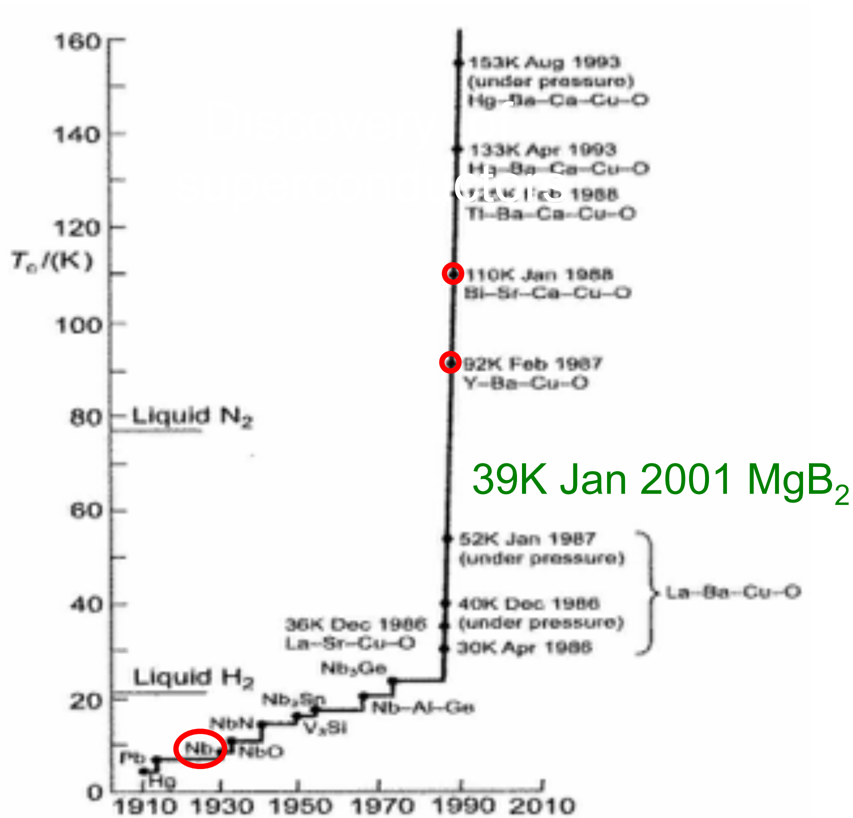
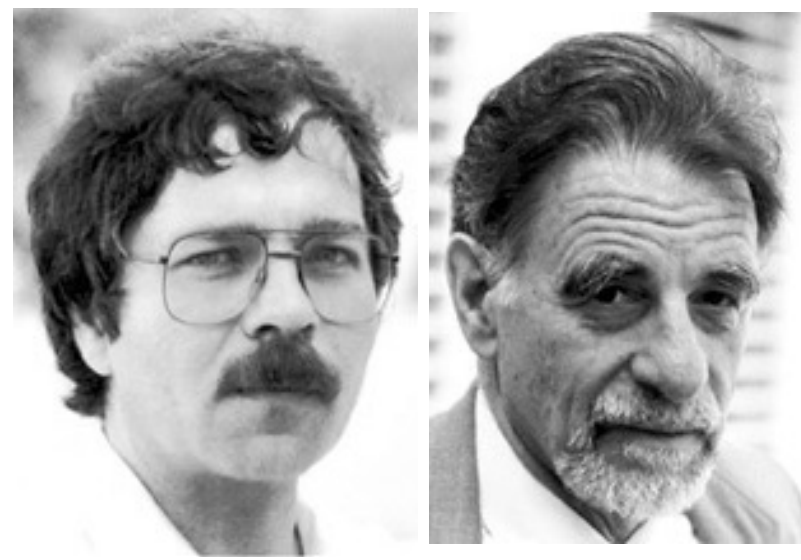
Nobel Prize 2003: Ginzburg, Abrikosov, Leggett (the GLAG members)

Abrikosov with Princess Madeleine

History – High temperature superconductors

- 1986: Bednorz and Muller discover superconductivity at high temperatures in layered materials comprising copper oxide planes

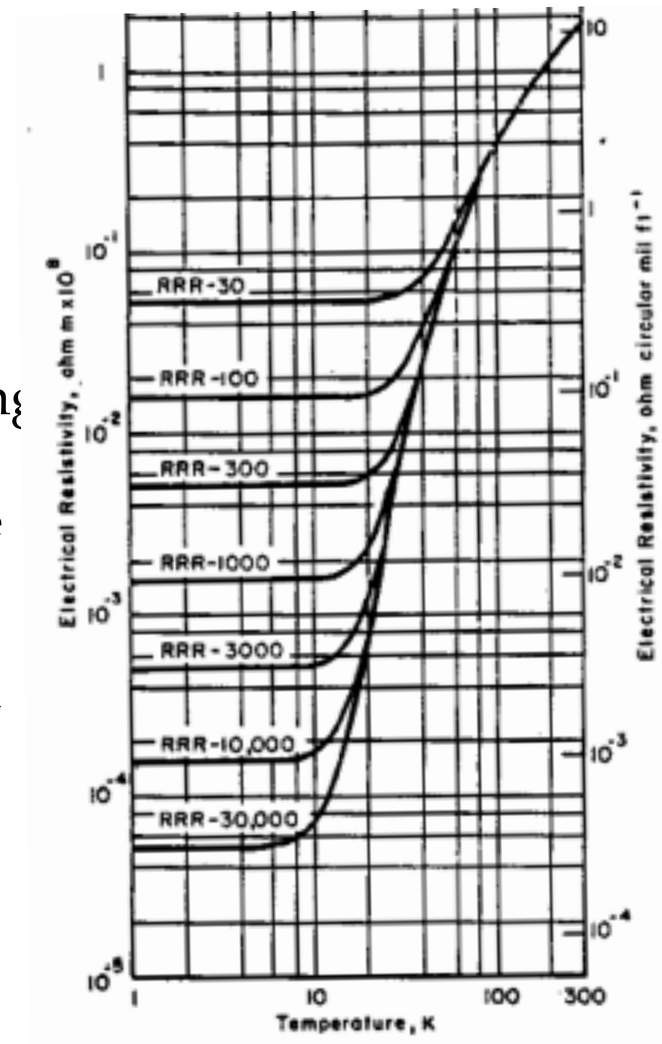




George Bednorz and Alexander Muller
Nobel prize for Physics (1987)

General Principals

- Superconductivity refers to a material state in which current can flow with no resistance
  - Not just “little” resistance - truly ZERO resistance
  - Resistance in a conductor stems from scattering of electrons off of thermally activated ions
    - Resistance therefore goes down as temperature decreases
    - The decrease in resistance in normal metals reaches a minimum based on irregularities and impurities in the lattice, hence concept of RRR (Residual resistivity ratio)
  - RRR is a rough measure of cold-work and impurities in a metal



RRR=ρ(273K)/ ρ(4K))

Aside: Maxwell's equations

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$$ Gauss' law

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ Faraday's law

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$ Ampere's law (corrected by Maxwell)

$$\mu_0 = 4\pi \times 10^{-7} \quad N/A^2$$ Permeability of free space

$$\epsilon_0 = 8.85 \times 10^{-12} \quad C^2/(Nm^2)$$ Permittivity of free space

Some reminders of useful formulas

$$\nabla \cdot \left(\nabla \times \overline{F} \right) = 0 \quad \forall \overline{F}$$

$$\nabla \times \left(\nabla \times \overline{F} \right) = \nabla \left(\nabla \cdot \overline{F} \right) - \nabla^2 \overline{F} \quad \forall \overline{F}$$

$$\nabla \times \left(\nabla u \right) = 0 \qquad \forall u$$

or

$$\nabla \times \overline{F} = 0 \quad \Leftrightarrow \quad \overline{F} = \nabla u$$

(***F*** is conservative if curl ***F*** is zero)

Volume Integral

$$\int_S \overline{F} \cdot \overline{n} \; dS = \int_V \nabla \cdot \overline{F} \; dV$$

Divergence Theorem

Surface Integral (Flux)

$$\oint_l \overline{F} \cdot d\overline{l} = \int_S \left(\nabla \times \overline{F} \right) \overline{n} \, dS$$

Curl Theorem (Stoke’s Theorem)

Line Integral (Circulation)

Some direct results from Maxwell

- Electric and magnetic fields are fundamentally linked
  - dB/dt induces voltage (Faraday)
  - Moving charge generates B (Ampere)
- Amperes law applied to DC fields and flowing currents:

$$\nabla \times \vec{B} = \mu_0 \vec{J} \Rightarrow \oint \vec{B} \cdot dl = \mu_0 I_{enclosed}$$

- Gauss's law: no magnetic monopoles

$$\nabla \cdot \vec{B} = 0$$

Magnetic field lines cannot emanate from a point; they "curl" around current

- Equations admit wave solutions
  - Take the curl of Faraday's and Ampere's laws; E and B admit waves with velocity

$$v = \sqrt{\frac{1}{\mu_0 \varepsilon}} = c = \text{speed of light}$$

Magnetization

- From a macroscopic perspective, critical insight can be gleaned from magnetization measurements
  - Magnetization is the magnetic (dipole) moment generated in a material by an applied field

$$\nabla \times B = \mu_0 J$$

Amperes law

$$J = J_{free} + J_{bound}$$

$$J_{bound} = \nabla \times M$$

Arbitrary but useful distinction

$$\Rightarrow H = \frac{1}{\mu_0} B - M$$

$$\Rightarrow \nabla \times H = J_{free} \Rightarrow \oint H \bullet dl = I_{\text{enclosed free current}}$$

Results in a practical definition: we know and control free currents

Note:
We do not *need* M; every calculation could be performed using B and H

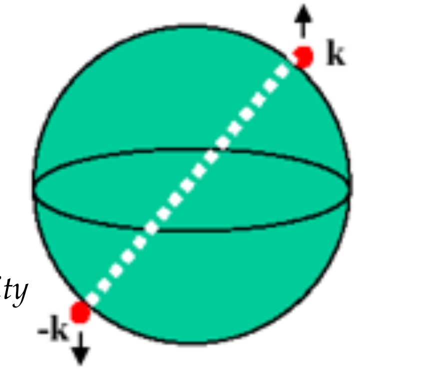
magnetization in superconductors

- Example: iron is ferromagnetic – it has a strong paramagnetic moment (i.e. the magnetization is parallel and additive to the applied field)
  - Most materials are either diamagnetic or paramagnetic, but the moments are extremely small compared to ferromagnetism
  - In diamagnetic and paramagnetic materials, the magnetization is a function of the applied field, i.e. remove the field, and the magnetization disappears.
  - In ferromagnetic materials, some of the magnetization remains "frozen in" => hysteretic behavior

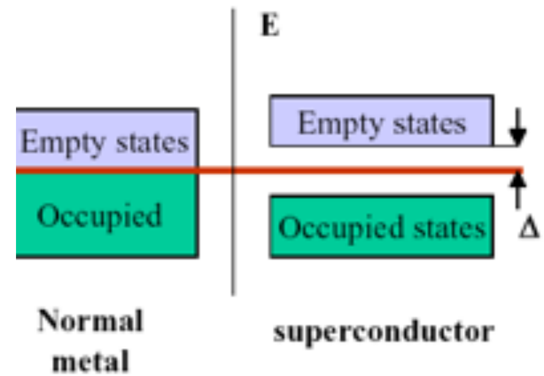
Basics of superconductivity

- In a superconductor, when the temperature descends below the critical temperature, electrons find it energetically preferable to form Cooper pairs
  - The Cooper pairs interact with the positive ions of the lattice
  - Lattice vibrations are often termed "phonons"; hence the coupling between the electron-pair and the lattice is referred to as electron-phonon interaction
  - The balance between electron-phonon interaction forces and Coulomb (electrostatic) forces determines if a given material is superconducting

Alex Guerivich, lecture on superconductivity

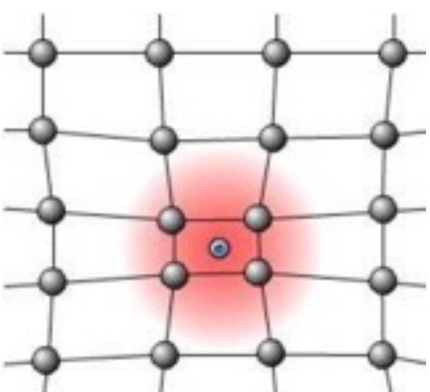


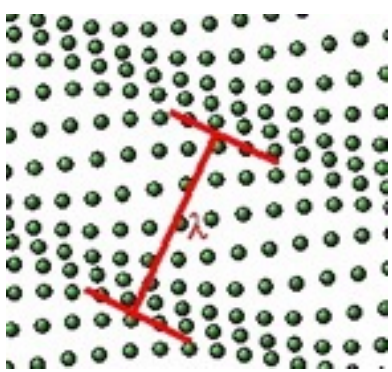
Cooper pair on the Fermi surface



Electron-phonon interaction can occur over long distances; Cooper pairs can be separated by many lattice spacings

BCS breakthrough:
Fermi surface is unstable to bound states of electron-pairs





~meV~10^{-22}J!!

$$\Delta_0 \cong 2\hbar\omega_D \exp\left[-\frac{1}{\lambda_{ep}}\right]$$

$$T_c \cong \frac{e^{\gamma}}{\pi k_b}\Delta_0$$

k_B=Boltzmann constant =1.38x10^{-23}

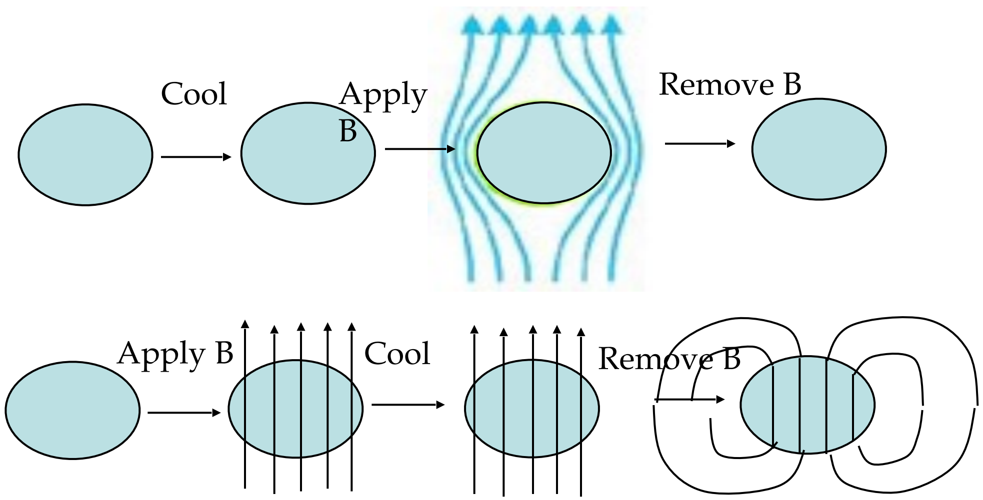
ω_D=Debye frequency

λ_{ep} =electron-phonon coupling

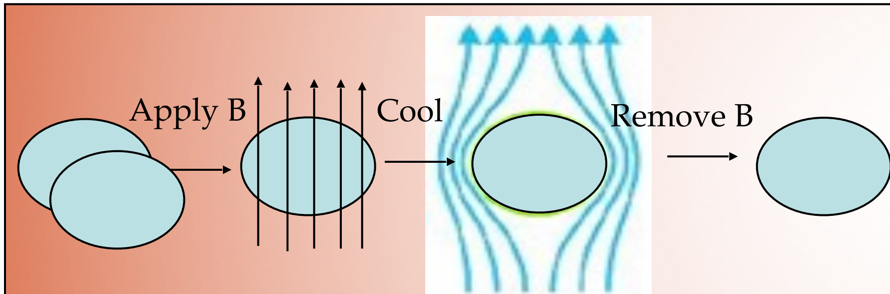
γ=euler constant=0.577

Diamagnetic behavior of superconductors

- What differentiates a “perfect” conductor from a diamagnetic material?



A perfect conductor apposes any change to the existing magnetic state



Superconductors exhibit diamagnetic behavior: flux is always expulsed - Meissner effect

The London equations

- Derive starting from the classical Drude model, but adapt to account for the Meissner effect:
  - The Drude model of solid state physics applies classical kinetics to electron motion
    - ✓ Assumes static positively charged nucleus, electron gas of density n.
    - ✓ Electron motion damped by collisions

"Frictional drag" on "normal" conduction electrons

$$
\begin{aligned}
m\frac{d\vec{v}}{dt} &= e\vec{E} - \gamma\vec{v} \\
\vec{J}_s &= -en_s\vec{v} \\
&\Rightarrow \frac{\partial}{\partial t}\left(\frac{m}{n_s e^2}\nabla\times\vec{J}_s + \vec{B}\right) = 0 \implies \nabla^2\vec{B} = \frac{\mu_0 n_s e^2}{m}\vec{B} = \frac{1}{\lambda_L^2}\vec{B}
\end{aligned}
$$

- The penetration depth λ_L is the characteristic depth of the supercurrents on the surface of the material.

Concept of coherence length

- The density of superconducting electrons n_s decreases to zero near a superconducting / normal interface, with a characteristic length ξ (coherence length, first introduced by Pippard in 1953). The two length scales ξ and λ_L define much of the superconductors behavior.
  - The coherence length is proportional to the mean free path of conduction electrons; e.g. for pure metals it is quite large, but for alloys (and ceramics…) it is often very small. Their ratio, the GL parameter, determines flux penetration:

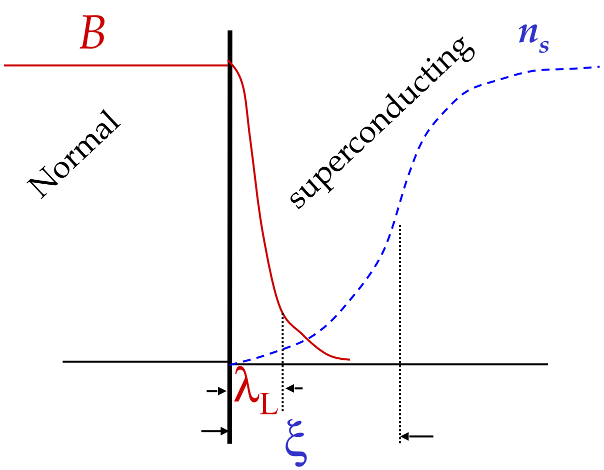
$$\kappa = \lambda_L / \xi$$

- From "GLAG" theory, if:

$\kappa < 1/\sqrt{2}$ Type I superconductor

$\kappa > 1/\sqrt{2}$ Type II superconductor

Note: in reality ξ and λ_L are functions of temperature



Thermodynamic critical field

- The Gibbs free energy of the superconducting state is lower than the normal state. As the applied field *B* increases, the Gibbs free energy increases by $B^2/2\mu_0$.
- The thermodynamic critical field at $T=0$ corresponds to the balancing of the superconducting and normal Gibbs energies:

$$G_n = G_s + \frac{H_c}{2}$$

- The BCS theory states that $H_c(0)$ can be calculated from the electronic specific heat (Sommerfeld coefficient):

$$H_c(0) = 7.65\times10^{-4}\frac{\gamma^{1/2}T_c}{\mu_0}$$

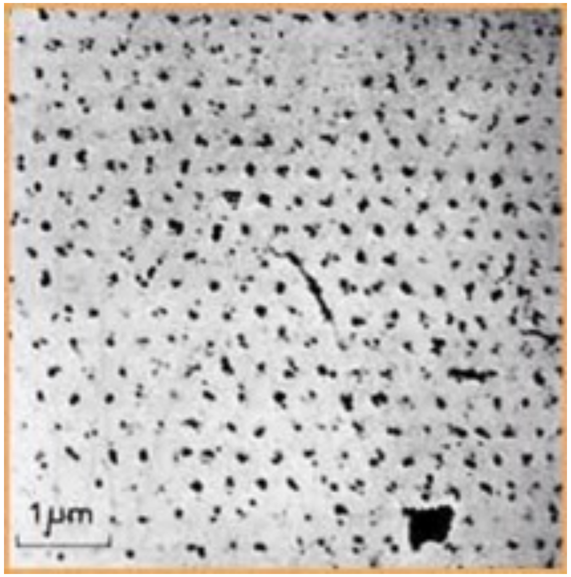
Table 2.2. Coefficient of the Electronic Specific Heat for Various Metallic Elements of Technical Interest[a]

Element	γ (mJ/mol · K^2)
Ag	0.646
Al	1.35
Au	0.729
Cd	0.688
Cr	1.40
Cu	0.695
Fe	4.98
Ga	0.596
Hf	2.16
Hg	1.79
In	1.69
Nb	7.79
Ni	7.02
Pb	2.98
Sn	1.78
Ti	3.35
V	9.26
Zn	0.64
Zr	2.80

[a] From Kittel.[1]

Type I and II superconductors

- Type I superconductors are characterized by the Meissner effect, i.e. flux is fully expulsed through the existence of supercurrents over a distance λ_L.
- Type II superconductors find it energetically favorable to allow flux to enter via normal zones of fixed flux quanta: "fluxoids" or vortices.
  - The fluxoids or flux lines are vortices of normal material of size $\sim\pi\xi^2$ "surrounded" by supercurrents shielding the superconducting material.



First photograph of vortex lattice,
U. Essmann and H. Trauble
Max-Planck Institute, Stuttgart
Physics Letters 24A, 526 (1967)

Fluxoids

- Fluxoids, or flux lines, are continuous thin tubes characterized by a normal core and shielding supercurrents.
- The flux contained in a fluxoid is quantized:

$$\phi_0 = h/(2e)$$

$$h = \text{Planck's constant} = 6.62607 \times 10^{-34}\ \text{Js}$$

$$e = \text{electron charge} = 1.6022 \times 10^{-19}\ \text{C}$$

  - The fluxoids in an idealized material are uniformly distributed in a triangular lattice so as to minimize the energy state
  - Fluxoids in the presence of current flow (e.g. transport current) are subjected to Lorentz force:

$$\vec{F}_L = \vec{J} \times \vec{B}$$

⇒ Concept of flux-flow and associated heating

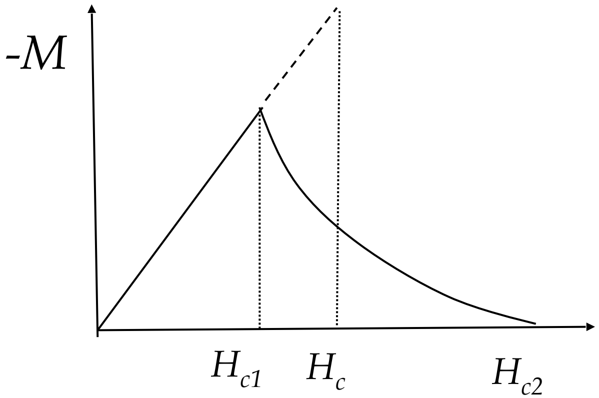
Solution for real conductors: provide mechanism to ***pin*** the fluxoids

See flux flow movies…

Critical field definitions
T=0

- H_{c1}: critical field defining the transition from the Meissner state

$$H_{c1} \approx \frac{\phi_0}{4\sqrt{2}\pi\mu_0\lambda^2} Ln\left(\kappa + \frac{1}{2}\right); \quad \kappa >> 1$$



- H_c: Thermodynamic critical field
  - *Hc*=*Hc1* for type I superconductors

$$H_c = \frac{\phi_0}{2\sqrt{2}\mu_0\kappa\pi\xi^2}$$

- H_{c2}: Critical field defining the transition to the normal state

$$H_{c2} = \frac{\phi_0}{2\pi\mu_0\xi^2}$$

Examples of Superconductors

- Many elements are superconducting at sufficiently low temperatures
- None of the pure elements are useful for applications involving transport current, i.e. they do not allow flux penetration
- Superconductors for transport applications are characterized by alloy/composite materials with κ>>1

Material	T_c (K)	λ(0), nm	ξ(0), nm	H_{c2} (T)
Nb-Ti	9.5	240	4	13
Nb-N	16	200	5	15
Nb_3Sn	18	65	3	30
MgB_2 (dirty)	32-39	140	6	35
$YBa_2Cu_3O_7$	92	150	1.5	>100
Bi-2223	108	200	1.5	>100

Table 2.9. Critical Temperature and Critical Field of Type I Superconductors

Material	T_c (K)	$\mu_0 H_0$ (mT)
Aluminum	1.2	9.9
Cadmium	0.52	3.0
Gallium	1.1	5.1
Indium	3.4	27.6
Iridium	0.11	1.6
Lanthanum α	4.8	
β	4.9	
Lead	7.2	80.3
Lutecium	0.1	35.0
Mercury α	4.2	41.3
β	4.0	34.0
Molybdenum	0.9	
Osmium	0.7	~6.3
Rhenium	1.7	20.1
Rhodium	0.0003	4.9
Ruthenium	0.5	6.6
Tantalum	4.5	83.0
Thalium	2.4	17.1
Thorium	1.4	16.2
Tin	3.7	30.6
Titanium	0.4	
Tungsten	0.016	0.12
Uranium α	0.6	
β	1.8	
Zinc	0.9	5.3
Zirconium	0.8	4.7

Aside – uses for type I superconductors

- Although type I superconductors cannot serve for large-scale transport current applications, they can be used for a variety of applications
  - Excellent electromagnetic shielding for sensitive sensors (e.g. lead can shield a sensor from external EM noise at liquid He temperatures
  - Niobium can be deposited on a wafer using lithography techniques to develop ultra-sensitive sensors, e.g. transition-edge sensors
    - Using a bias voltage and Joule heating, the superconducting material is held at its transition temperature;
    - absorption of a photon changes the circuit resistance and hence the transport current, which can then be detected with a SQUID (superconducting quantum interference device)

    See for example research by J. Clarke, UC Berkeley;

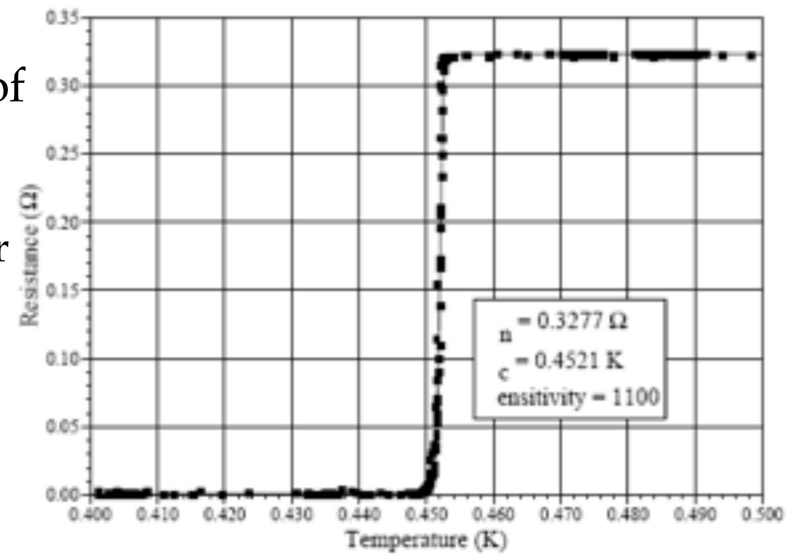


Figure 2. Resistance vs. temperature for a high-sensitivity TES bilayer.

Mo/Au bilayer TES detector
Courtesy Benford and Moseley, NASA Goddard

Flux Flow

- The Lorentz force acting on a fluxoid will, in the absence of pinning, result in motion of the fluxoid
- Fluxoid motion generates a potential gradient (i.e. voltage) and hence heating
  - This can be modeled using Faraday's law of induction:

$$\begin{aligned}\vec{E} &= \vec{B} \times \vec{v}_f \\ \dot{w} &= nF_L \cdot \vec{v}_f = (\vec{J} \times \vec{B}) \cdot \vec{v}_L = \vec{J} \cdot \vec{E} = JE\end{aligned}$$

$\Rightarrow$ ***$E=\varrho_{eff}J$ => "ideal" superconductors can support no transport current beyond H_{c1}!***

- Real superconductors have defects that can prevent the flow of fluxoids
  - The ability of real conductors to carry transport current depends on the number, distribution, and strength of such pinning centers

Flux pinning

- Fluxoids can be pinned by a wide variety of material defects
  - Inclusions
    - Under certain conditions, small inclusions of appropriate materials can serve as pinning site locations; this suggests tailoring the material artificially through manufacturing
  - Lattice dislocations / grain boundaries
    - These are known to be primary pinning sites. Superconductor materials for wires are severely work hardened so as to maximize the number and distribution of grain boundaries.
  - Precipitation of other material phases
    - In NbTi, mild heat treatment can lead to the precipitation of an a-phase Ti-rich alloy that provides excellent pinning strength.

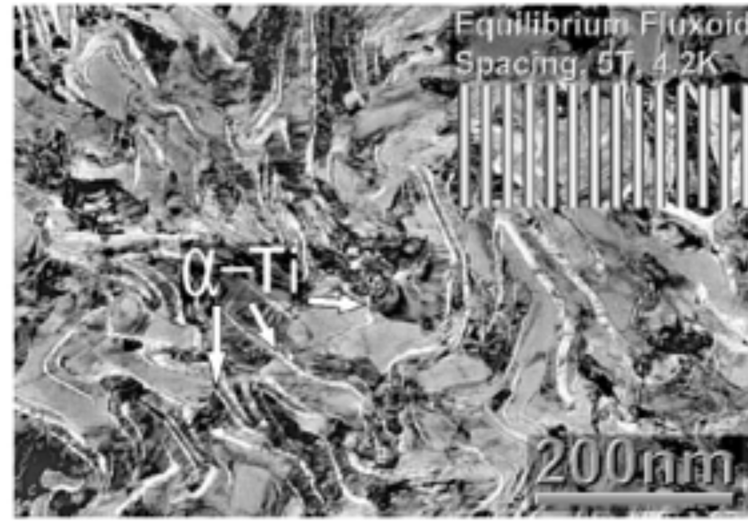


Fig. 1: Microstructure of a NbTi filament (Courtesy of P.J. Lee, University of Wisconsin at Madison).

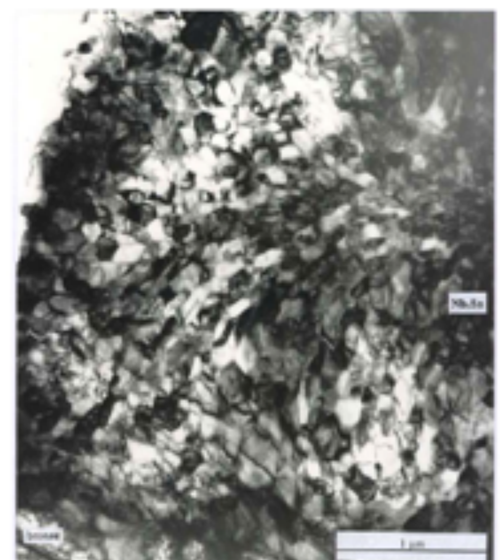
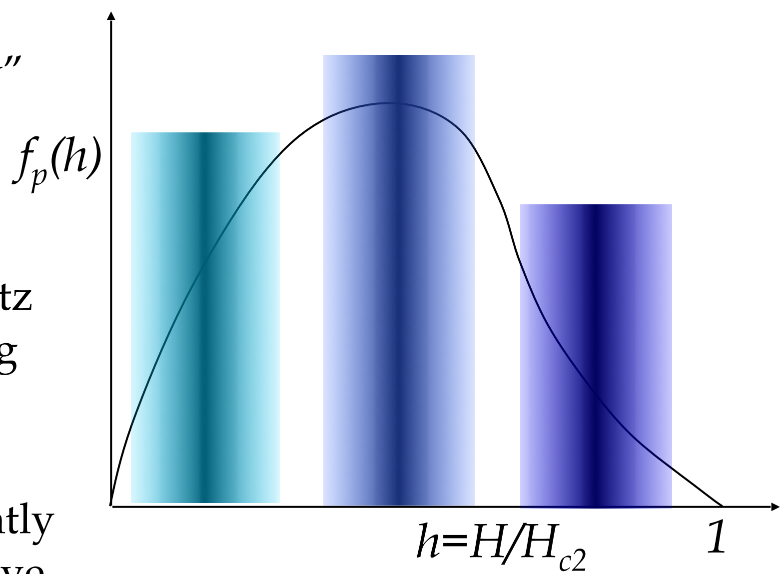


Fig. 6: Microstructure of a Nb_3Sn filament (Courtesy of C. Verwaerde, Alstom/MSA).

Pinning strength

- The distribution and pinning of fluxoids depends on the operating regime:
  - At low field (but>Hc1) the distribution is governed mainly by interaction between flux-lines, i.e. the fluxoids find it energetically advantageous to distribute themselves "evenly" over the volume (rather weak)
  - At intermediate fields, the pinning force is provided by the pinning sites, capable of hindering flux flow by withstanding the Lorentz force acting on the fluxoids. Ideally, the pinning sites are uniformly distributed in the material (very strong)
  - At high field, the number of fluxoids significantly exceeds the number of pinning sites; the effective pinning strength is a combination of defect pinning strength and shear strength of the fluxlines (rather weak)



High-Temperature superconductors

- Much of HTS behavior can be understood in terms of the BCS and GLAG theory parameters
- The new features of HTS have to do with:
  1) highly two-dimensional domains of superconductor, separated by regions of “inert” material
     - Macroscopic behavior is therefore highly anisotropic
     - Different layers must communicate (electrically) via tunneling, or incur Joule losses
  2) a much larger range of parameter space in which multiple effects compete
     - The coherence lengths for HTS materials are far smaller than for LTS materials
     - Critical fields are ~10 times higher

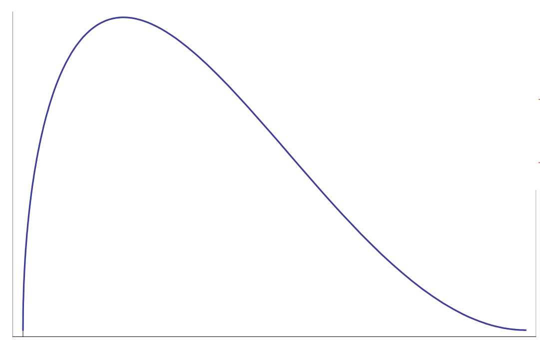
     => Thermal excitations play a much larger role in HTS behavior

Modeling pinning

- Precise first-principles physical descriptions of overall pinning strength (and hence critical current) of real superconductors is difficult due to ambiguities intrinsic in pinning
- Nevertheless, models based on sound physics minimize free parameters needed to fit measured data and provide reliable estimates for classes of materials
- One of the most cited correlations is that of Kramer:

$$F_p = F_{max} f(h) \propto \frac{H_{c2}^{\nu}}{\kappa^{\gamma}} f(h)$$

$$f(h) = h^{1/2}(1-h)^2; \quad h = H/H_{c2}$$



"Kramer plot"

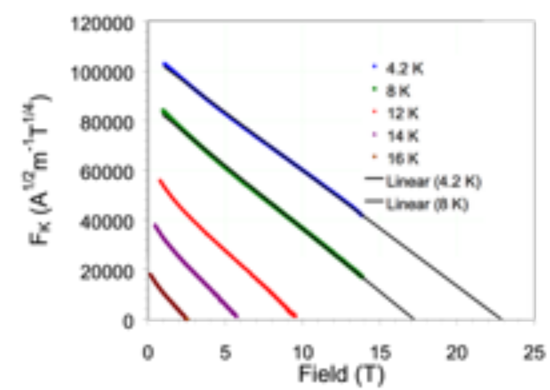
$F_p = J_c B \propto b^{1/2}(1-b)^2$

$F_K = J_c^{1/2} B^{1/4} \propto (1-b)$

From L. Cooley, USPAS

The fitting coefficients ν and γ depend on the type of pinning. Furthermore, it is experimentally verifies that

$$H_c(T) \simeq H_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$



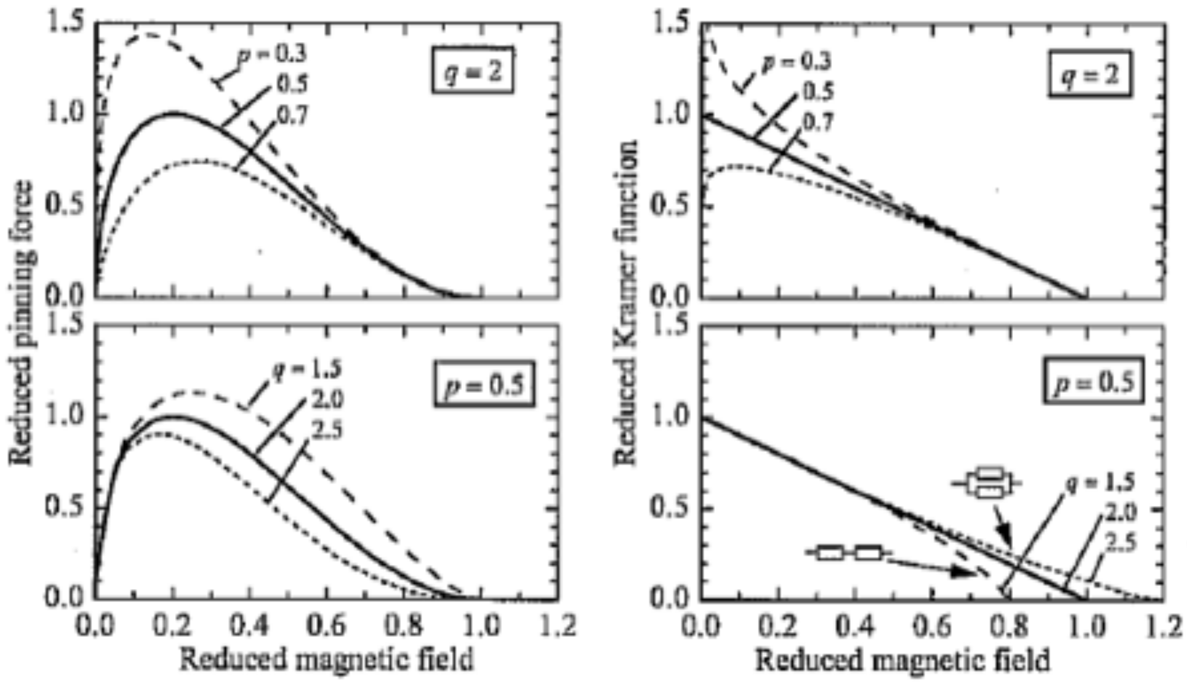
Scaling of critical current: field dependence

- The Kramer formulation provides excellent fits in the region 0.2<h<0.6 for Nb_3Sn; it is appropriate for regimes where the number of fluxoids exceeds the number of pinning sites
- Outside this region, a variety of effects (e.g. inhomogeneity averaging) can alter the pinning strength behavior, so the pinning strength is often fitted with the generalization

$$f_p(h) \propto h^p \left(1-h\right)^q ; \quad h = H / H_{c2}$$

- It is preferable to stay with the Kramer formulation, yielding:

$$J_c^{1/2} B^{1/4} \simeq \frac{1.1\times 10^5}{\kappa} \mu_0 \left(H_{c2} - H\right)$$



Scaling of critical current: temperature dependence

- The temperature dependence of J_c stems from the term

$$\frac{\left[\mu_0 H_{c2}(T)\right]^{\nu}}{\kappa^{\gamma}(T)}$$

- Scalings are typically generated by considering the normalized thermodynamic critical field and the the normalized GL parameter (here $t=T/T_c$):

$$\frac{H_c(T)}{H_c(0)} = 1 - t^2$$

$$\frac{\kappa(T)}{\kappa(0)} = \begin{cases} 1 - 0.31t^2\left(1 - 1.77\ln(t)\right) & \text{Summers} \\ 1 - 0.33t & \text{Summers (reduced)} \\ \dfrac{1 - t^{1.52}}{1 - t^2} & \text{Godeke / De Gennes} \end{cases}$$

Scaling of critical current, Nb_3Sn Empirical Strain dependence

- The critical current of Nb_3Sn is strain dependent, particularly at high field
- The strain dependence is typically modeled in terms of the normalized critical temperature:

$$\frac{H_{c2}(4.2,\varepsilon)}{H_{c2m}(0)} \simeq \left[\frac{T_c(\varepsilon)}{T_{cm}}\right]^3 = s(\varepsilon)$$

- The term T_{cm} and H_{c2m} refer to the peaks of the strain-dependent curves
- A "simple" strain model proposed by Ekin yields

$$s(\varepsilon) = 1 - a\left|\varepsilon_{axial}\right|^{1.7}$$

$$a = \begin{cases} 900 & \varepsilon_{axial} < 0 \\ 1250 & \varepsilon_{axial} > 0 \end{cases}$$

Strain dependence of Jc in Nb_3Sn: physics-based model

- A physics-based model of strain dependence has been developed using the frequency-dependent electron-phonon coupling interactions (Eliashberg; Godeke , Markiewitz)

Phonon density of states

$$\lambda_{ep}(\varepsilon) = 2\int \frac{\alpha^2(\omega)F(\omega)}{\omega} d\omega$$

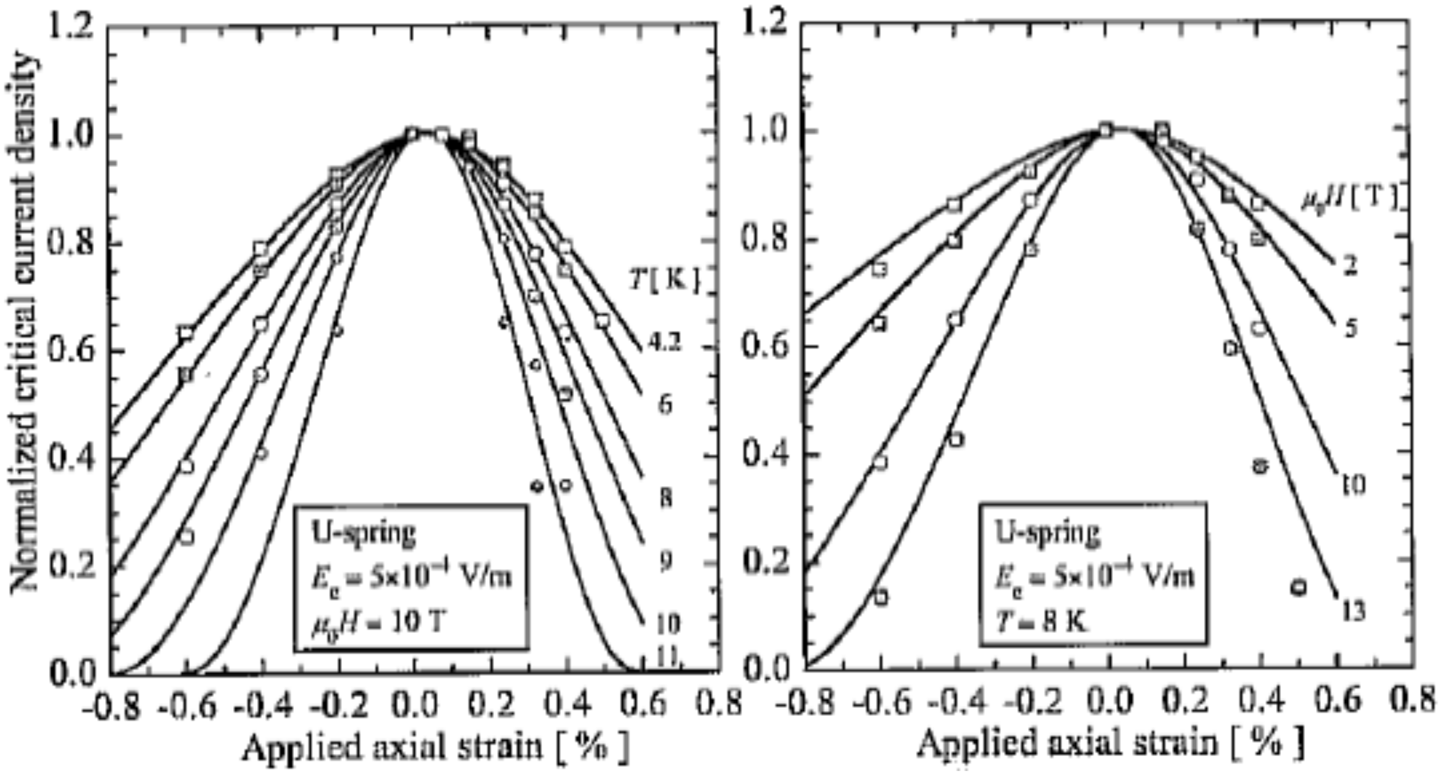
- From the interaction parameter the strain dependence of T_c can be derived
- Experimentally, the strain dependence of Hc2 behaves as

$$\frac{H_{c2}(4.2,\varepsilon)}{H_{c2m}(4.2)} \cong \frac{T_c(\varepsilon)}{T_{cm}}$$

- The theory predicts strain dependence of J_c for all LTS materials, but the amplitude of the strain effects varies (e.g. very small for NbTi)
- The resulting model describes quite well the asymmetry in the strain dependence of B_{c2}, and the experimentally observed strong dependence on the deviatoric strain

Strain dependence of J_c in Nb_3Sn

- The strain dependence is a strong function of the applied field and of temperature



Critical surface: Example fit for NbTi

- NbTi parameterization
  - Temperature dependence of B_{C2} is provided by Lubell's formulae:

$$B_{C2}(T) = B_{C20}\left[1 - \left(\frac{T}{T_{C0}}\right)^{1.7}\right]$$

where B_{C20} is the upper critical flux density at zero temperature (~14.5 T)

  - Temperature and field dependence of *Jc* can be modeled, for example, by Bottura's formula

$$\frac{J_C(B,T)}{J_{C,ref}} = \frac{C_{NbTi}}{B}\left[\frac{B}{B_{C2}(T)}\right]^{\alpha_{NbTi}}\left[1 - \frac{B}{B_{C2}(T)}\right]^{\beta_{NbTi}}\left[1 - \left(\frac{T}{T_{C0}}\right)^{1.7}\right]^{\gamma_{NbTi}}$$

where $J_{C,Ref}$ is critical current density at 4.2 K and 5 T (e.g. ~3000 A/mm^2) and C_{NbTi} (~30 T), α_{NbTi} (~0.6), β_{NbTi} (~1.0), and γ_{NbTi} (~2.3) are fitting parameters.

Scaling Jc for NbTi & Nb3Sn
(Courtesy Arno Godeke)

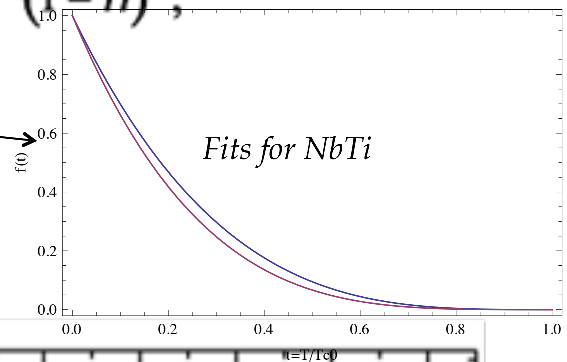
$$J_c(H,T,\varepsilon) \cong \frac{C_1}{\mu_0 H} s(\varepsilon)\underbrace{\left(1-t^{n_1}\right)\left(1-t^{n_2}\right)} h^p (1-h)^q,$$

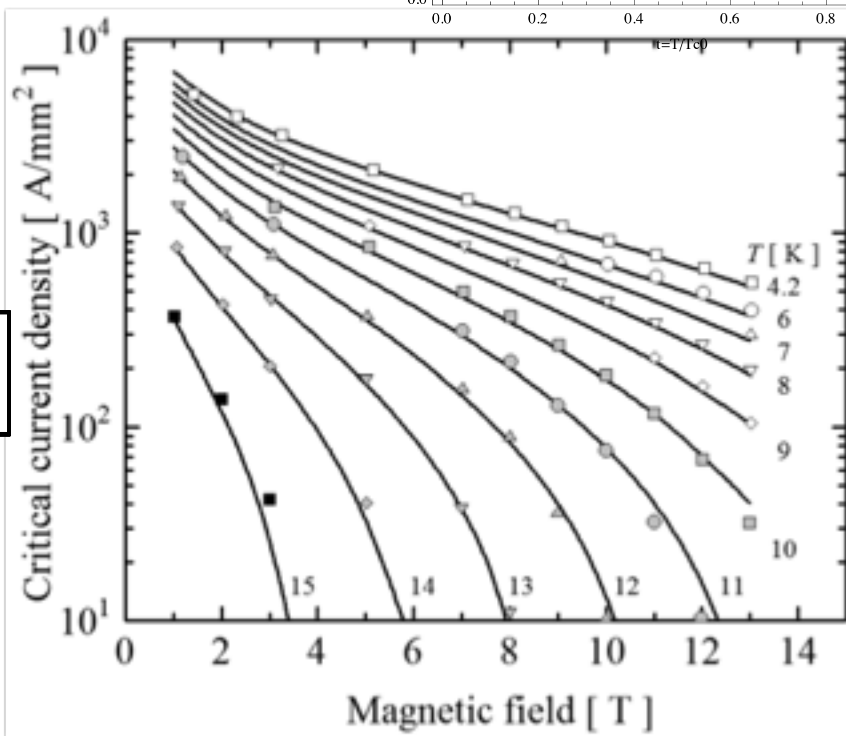
with

$$t \equiv T/T_c^*(\varepsilon), \quad h \equiv H/H_{c2}^*(T,\varepsilon),$$

$$H_{c2}^*(T,\varepsilon) \cong H_{c2m}^*(0)\, s(\varepsilon)\left(1-t^{n_1}\right),$$

$$T_c^*(\varepsilon) = T_{cm}^*\, s(\varepsilon)^{\frac{1}{3}}$$





Godeke et al., Supercond. Sci. Technol. ***19 (2006)***

Nb_3Sn

Godeke, SuST **19**

- $n_1 \cong 1.52$
- $n_2 = 2$
- $p = 0.5$
- $q = 2$
- $s(\varepsilon)$ = strain dependence

NbTi

Bottura, TAS **19**

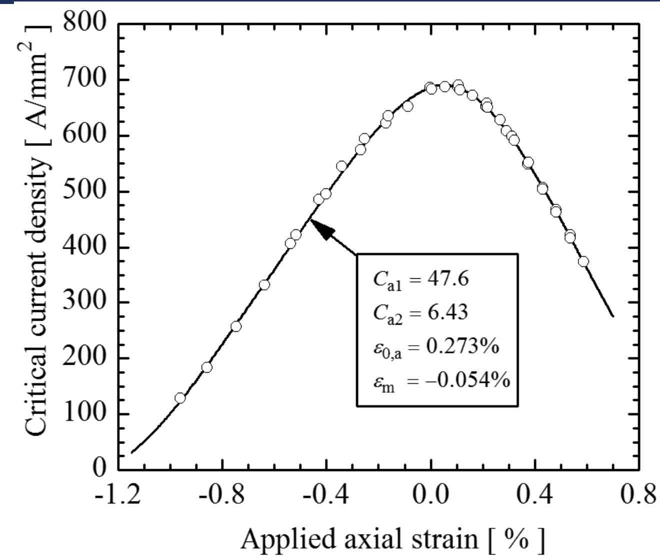
- $n_1 = n_2 \cong 1.7$
- $p \cong 0.73$
- $q \cong 0.9$
- $s(\varepsilon) \cong 1$

Nb_3Sn: Strain and temp. dependence

(Courtesy Arno Godeke)

$$s\left(\varepsilon_{\text{axial}}\right)=\frac{C_{\text{a1}}\left[\sqrt{\left(\varepsilon_{\text{shift}}\right)^2+\left(\varepsilon_{0,\text{a}}\right)^2}-\sqrt{\left(\varepsilon_{\text{axial}}-\varepsilon_{\text{shift}}\right)^2+\left(\varepsilon_{0,\text{a}}\right)^2}\right]-C_{\text{a2}}\varepsilon_{\text{axial}}}{1-C_{\text{a1}}\varepsilon_{0,\text{a}}}+1,$$

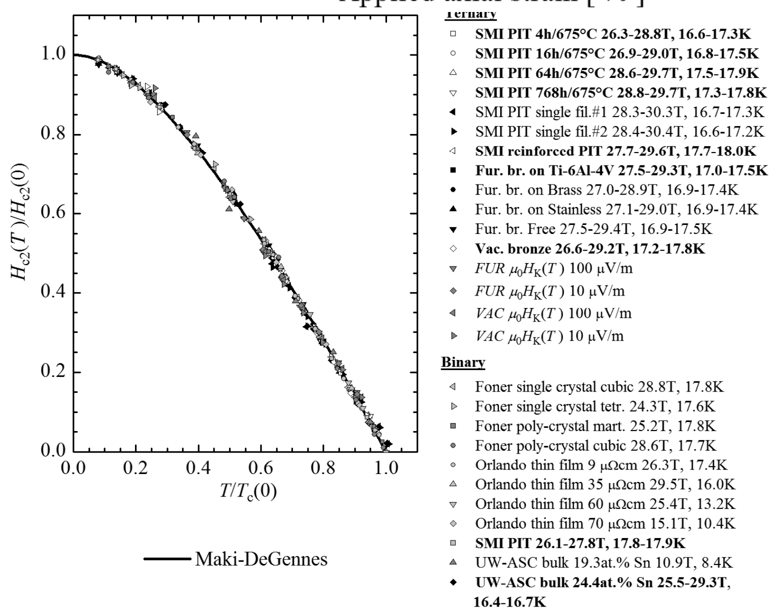
$$\varepsilon_{\text{shift}}=\frac{C_{\text{a2}}\varepsilon_{0,\text{a}}}{\sqrt{\left(C_{\text{a1}}\right)^2-\left(C_{\text{a2}}\right)^2}}$$



$$J_{\text{c}}\left(H,T,\varepsilon\right)\cong\frac{C_1}{\mu_0 H}s\left(\varepsilon\right)\left(1-t^{1.52}\right)\left(1-t^2\right)h^{0.5}\left(1-h\right)^2$$

$$H_{\text{c2}}^*\left(T,\varepsilon\right)\cong H_{\text{c2m}}^*\left(0\right)s\left(\varepsilon\right)\left(1-t^{1.52}\right)$$

$$\ln\left(\frac{T}{T_{\text{c}}\left(0\right)}\right)=\psi\left(\frac{1}{2}\right)-\psi\left(\frac{1}{2}+\frac{\hbar D\mu_0 H_{\text{c2}}\left(T\right)}{2\phi_0 k_{\text{B}}T}\right)$$



Using magnetization data

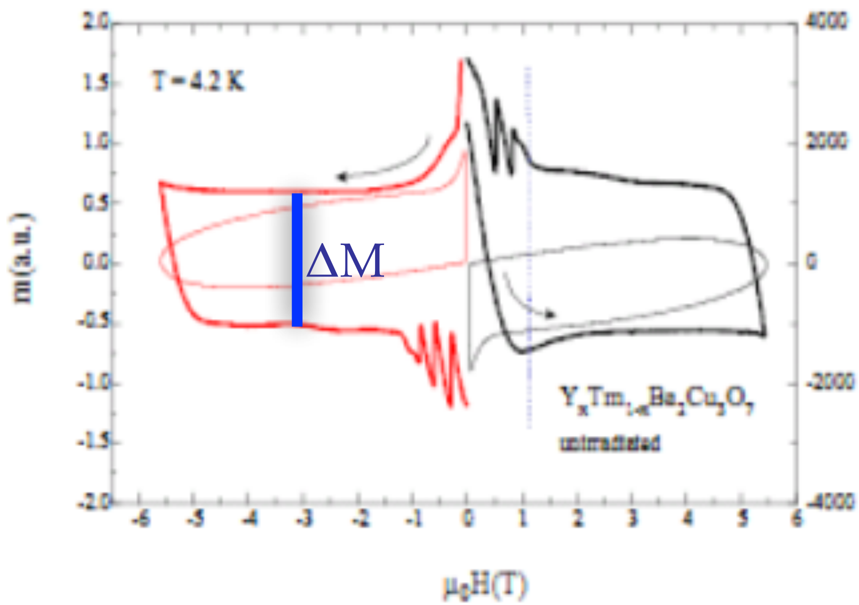
- We have seen that the Meissner state corresponds to perfect diamagnetic behavior
- We have seen that beyond H_{c1}, flux begins to penetrate and can be pinned at defects => hysteretic behavior

⇒ *Much can be understood by measuring the effective magnetization of superconducting material*

The measured magnetization provides insight into flux pinning and flux motion, key concepts governing the performance of superconducting materials.

$$\Delta M \cdot B \propto F_p(T, B)$$

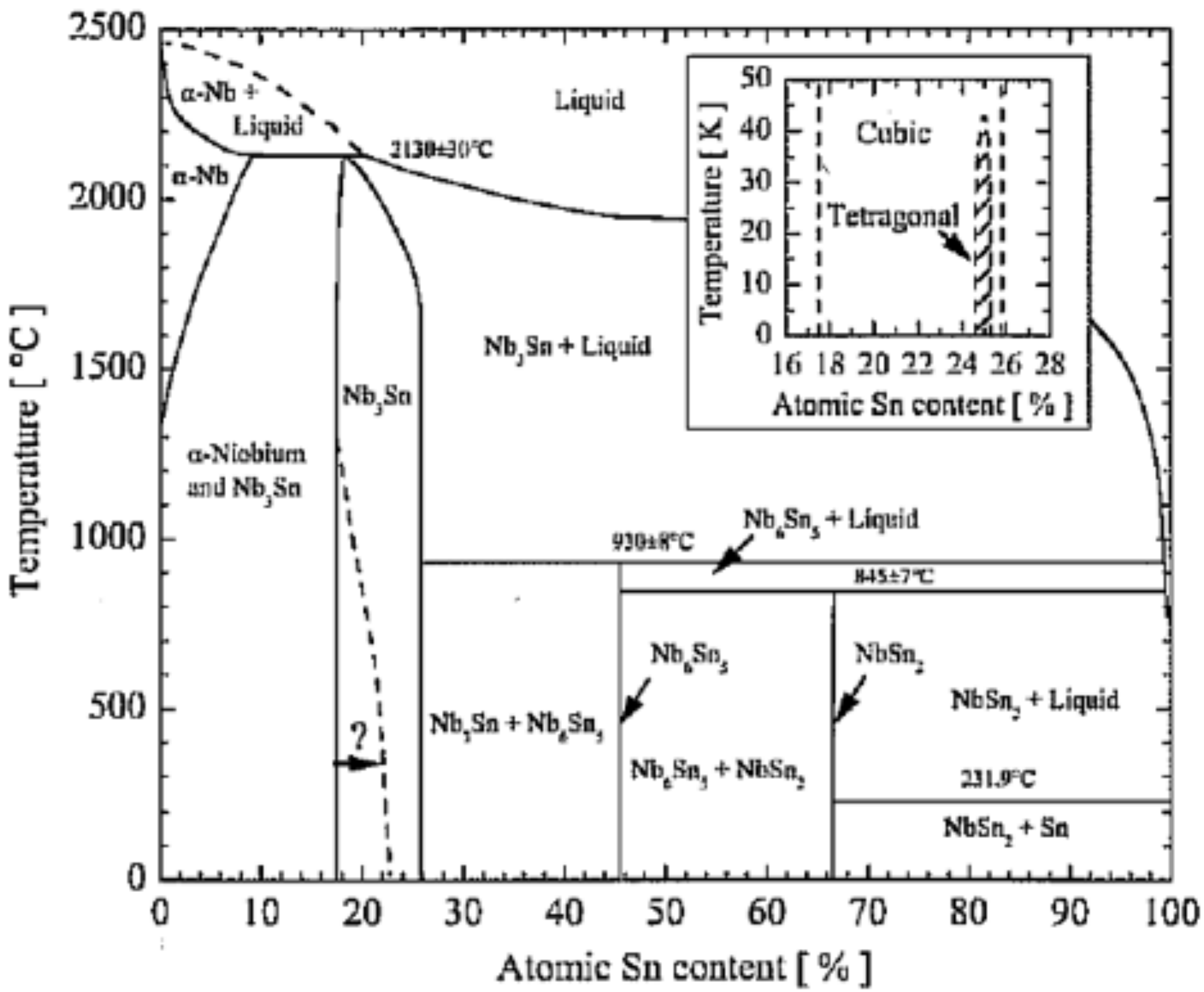
Often used to evaluate Jc(B,T)!

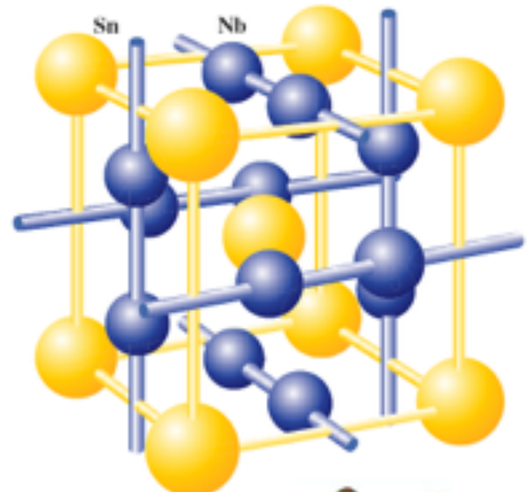


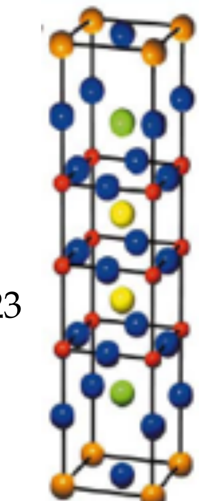
J. Vanacken, et. al, 1999.

Example material: Nb_3Sn

- Phase diagram, A15 lattice…







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Final comments

- Recent developments in Tc and Jc are quite impressive
  - Improvements in material processing has lead to
    - enhanced pinning
    - Enhanced Tc
    - Smaller superconducting filaments

- Expect, ***and participate in,*** new and dramatic developments as fundamental understanding of superconductivity evolves and improvements in nanoscale fabrication processes are leveraged
  - A basic theory of superconductivity for HTS materials has yet to be formulated!

- Some understanding of the fundamentals of superconductivity are critical to appropriately select and apply these materials to accelerator magnets
  - Superconductors can be used to generate very high fields for state-of-the-art facilities, but they are ***not*** forgiving materials – ***in accelerator applications they operate on a precarious balance of large stored energy and minute stability margin!***