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- Overview of migration methods
- Spatial Aliasing
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- 2D data can be migrated only in the plane of the section

- All migration methods are based upon simplified models of the real Earth
3D migration

Post-stack time migration

model

Pre-stack depth migration
What is Migration?

Migration is the process which removes the effect of wave propagation from the seismic data!
Migration Purpose

- The recorded wave field is measured at the surface: $P(x, y, t, z=0)$

- What is the sub-surface Image $(x, y, z)$?

- The propagation of a wavefield $P(x, y, z, t)$ is defined as:

$$\frac{\partial^2 P(x, y, z, t)}{\partial x^2} + \frac{\partial^2 P(x, y, z, t)}{\partial y^2} + \frac{\partial^2 P(x, y, z, t)}{\partial z^2} = \frac{1}{V^2(x, y, z)} \frac{\partial^2 P(x, y, z, t)}{\partial t^2}$$

- But to access to the image, the *velocity model* $V(x, y, z)$ must be defined.
Migration Methods

- Classes of Migration based on algorithms
  - Kirchhoff Migration (KIRCH)
    - performed by diffraction summation
  - FD Migration (WEMIG)
    - uses the finite difference solution of the wave equation in T-X space
  - FX Migration (FXMIG, GTMIG)
    - uses the finite difference solution of the wave equation in F-X space
  - FK Migration (FKMIG)
    - based on the 2D Fourier Transform
Kirchhoff Migration

- Kirchhoff migration is performed by summation using either:
  - Wavefront method
  - Diffraction method

- The following is a graphical explanation of the wavefront and diffraction methods
Consider a wavefront impinging upon a dipping reflector. The actual reflection from P is assumed to be at P'. Both P and P' lie on the same wavefront.

**Method**

Construct a wavefront chart. In a constant velocity model, the wavefronts are semi-circular.
Wavefront Migration - 2

- Use the wavefront Chart in conjunction with a simple reflection model
  - The event is at P₁' on trace S₁
  - The event is at P₂' on trace S₂
  - The event is at P₃' on trace S₃
- Plot the wavefront which overlays reflection point P₁'

- Move chart to second trace
- Plot the wavefront which overlies point P₂'

- Repeat for the third trace
- The common tangent to the wavefronts is the true position of the reflector.

- A more realistic approach is to assume velocity increases with depth which gives this wavefront characteristic.
The wavefront is in fact, an isochron curve.

When velocity increases in depth and changes laterally, this isochron curve becomes more and more complex.

To take into account these time variations in the X, Y domain, we can build a table of travel time for each shot point and each receiver point. This method is in fact used for Pre-stack depth migration only.
Diffraction Curves Migration

- The diffracting point can be regarded as a new source point at depth
Diffraction Curves - 1

- Construct diffraction curves based on hyperbolic equation
  - Overlay these over the traces with a dipping reflection: slide along until the dipping event is a tangent
  - Note the position of the diffraction curve which is a tangent to the dipping event
  - Repeat for the other traces
The line which joins the diffraction curves apex is the true reflector position.

Note: These curves are sensitive to velocity.

The distance by which an event is spatially shifted is proportional to the SQUARE of the velocity.

- stacking velocity is unlikely to be correct for migration.
Consider the location of each seismic sample to be a ‘pigeon hole’

Superimpose a wavefront chart onto a trace
Migration by Integral Methods: Kirchhoff Migration - 2

- Each sample value is ‘copied’ into the pigeon holes through which the wavefront curves pass.

- This procedure is repeated on all traces: the net result is a build-up of energy at the migrated position.

- Note: Kirchhoff Method may also use Diffraction Curve summation.
Huygen's Principle: Every reflector location is a Secondary Point Source

- Image of Reflector: Integral of diffraction curve
- Recorded Reflection: Tangent to diffraction curves
- Extrapolation: Computation of diffraction curves (travel time)
- Imaging: Integration of the data along the diffraction curves
Wavefronts and Diffraction Curves

- Wavefronts and Diffraction curves can be used together.
- Migrated position at intersection of the two curves.
1- Apply static corrections

2- Sort traces into common offset gathers

In the case of non-zero offset traces the wavefronts are ellipses, (not circles as produced by zero offset traces).

These gathers contain non-zero offset traces.
Procedure 2

1. Apply static corrections

2. Sort traces into common offset gathers

These gathers contain non-zero offset traces.

Also the diffraction curves are flattened.
Procedure 2 bis

1- Apply static corrections

2- Sort traces into common offset gathers

3- Migration

The apparent reflection point P' migrates along the elliptical wavefront or along the flattened diffraction curve to the true reflection point P.

Each common offset gather is migrated separately.
Procedure 3

1- Apply static corrections

2- Sort traces into common offset gathers

3- Migration

Total travel Time, T is equal to:

Travel time S to P
+ Travel time P to R

For the non-coincident sources and receivers:

- Elliptical wavefront
- Flattened hyperbolic diffraction curve

Surface

Distance

Depth

Velocity = constant
Procedure 3 bis

1- Apply static corrections

2- Sort traces into common offset gathers

3- Migration

\[ \sqrt{\left( \frac{T_8}{2} \right)^2 + \left( \frac{L + \left( \frac{X}{2} \right)}{v} \right)^2} + \sqrt{\left( \frac{T_8}{2} \right)^2 + \left( \frac{L - \left( \frac{X}{2} \right)}{v} \right)^2} \]

Therefore, total travel time \( T \) is given by the formula below.
1- Apply static corrections
2- Sort traces into common offset gathers
3- Migration
4- Resort of traces

The migrated traces are sorted into Common CDP order.

1- Apply static corrections
2- Sort traces into common offset gathers
3- Migration
4- Resort of traces
5- Apply dynamic corrections (NMO)
6- Stack

Addition of migrated traces.
Finite Difference Migration - 1

Implemented in:
- t-x domain (WEMIG)
- f-x domain (FXMIG)

Consider how the shape of a diffraction curve changes with depth in a constant velocity depth model.

At the surface ($z = 0$) the curve is collapsed into a point.

$$Z_n = \left(\frac{v_0 T_n}{2}\right)$$
If all diffraction origin points could be placed at the surface then seismic energy is concentrated only at their points of origin (the apex of the curves) - the data is therefore migrated!

The finite difference method uses a wave equation to ‘strip off’ layers of the earth (z steps) effectively projecting the source and receivers down to each successive layer.
Consider a simple anticlinal model in depth and time

- Points $P_1$ and $P_2$ are used as reference points to check how the migration is progressing.

- Starting at $T=0$ the first layer $S_1$ is stripped from the section and stored.
- The receivers are next placed at depth $Z_1$ and the wavefield is recalculated: layer $S_2$ is then removed and stored.

- The receivers are placed at depth $Z_2$ and the wavefield recalculated: layer $S_3$ is then removed and stored.
- Repeat for depth $Z_3$ and layer $S_4$ ...

- ... and finally for depth $Z_4$ and layer $S_5$
Collect together the time slices $S_1, S_2, S_3, S_4, S_5$ together...

... and combine to form the migrated section
Frequency-Wavenumber (f-k) Migration

- f-k migration is based on the 2-D Fourier transform
- Migration performed in f-k space
Remapping the f-k Spectrum -

- Consider two time sections with the position of a dipping event pre- and post- migration

- It can be shown that \( \tan \theta = \sin \theta_m \)

\[ V = \text{Velocity (Constant)} \]
Consider what happens to the period of a wavelet during migration:

- The apparent spatial wavelength $\lambda$ (and thus the wavenumber $k$) is unchanged.
- The temporal period, $T$, is increased to $T'$,
- **thus the frequency $F$ is reduced**
An un-migrated event in the f-k domain will plot thus:

After migration, the event will plot thus:
Migration in the f-k domain is a downwards vertical frequency shift.
Aliasing and migration

At least 2 CMPs within the apparent wavelength of the emerging wavefield

In this example, if the trace spacing is equal to or greater than 25 metres, both humans and machines would migrate some energy incorrectly.

At least 2 CMPs within the apparent wavelength of the emerging wavefield
Spatial Aliasing - 1

Consider the following emerging wavefield at surface:

\[
\sin \theta = \frac{v \Delta t}{2 \Delta x}
\]
Spatial Aliasing - 2

At the alias point, the distance BC corresponds to half the alias period $T_a$

$$BC = \frac{v \Delta t}{2}$$

$$\Delta t = \frac{T_a}{2}$$

$$\sin \theta = \frac{v \Delta t}{2 \Delta x} = \frac{v T}{4 \Delta x}$$

Since $f_{max} = \frac{1}{T_a}$

$$\sin \theta = \frac{v}{4 f_{max} \Delta x}$$

Therefore:

$$f_{max} = \frac{v}{4 \Delta x \sin \theta}$$

To avoid spatial aliasing:

$$2 \Delta X < \frac{v}{2 \sin \theta} f_{max}$$
Horizontally stratified media

Travel times of reflection move out on CMP gathers can be expressed as (Taner and Koehler 1969):

\[ t^2(x) = C_0 + C_1 x^2 + C_2 x^4 + \ldots + C_k x^{2k} + \ldots \]

With x offset and

\[ C_1 = \frac{1}{v_{rms}^2} \]

\[ t^2(x) = t^2_0 + \frac{x^2}{v_{rms}^2} \]

If x \( \ll \) depth
Dipping interface

\[ \sin \alpha = \frac{v \Delta t_d}{2 \Delta x} \]
Move Out for a Dipping Layer

\[ t^2(x) = t^2_0 + \frac{x^2}{v^2} - \frac{x^2 \sin^2 \alpha}{v^2} \]

\[ t^2(x) = t^2_0 + \frac{x^2 \cos^2 \alpha}{v^2} \]

\[ v^2 \Rightarrow \frac{v^2}{\cos^2 \alpha} \]

\[ v \Rightarrow \frac{v}{\cos \alpha} \]

If more than 1 layer:

\[ V_{\text{stack}} = \frac{V_{\text{rms}}}{\cos \alpha} \]
Normal Move Out without dip correction

A flat layer model NMO velocity is too slow for a dipping horizon
NMO will over correct the dipping horizon
Effect of dip on Move Out

Moveout of reflection events:

- hyperbola (offset<depth)
- asymptote $1/v_{\text{rms}}$
- $v_{\text{rms}}$ to time interval velocity
  \[ V_{\text{Dix}} = \frac{(V_{i+1}t_{i+1} - V_it_i)}{\Delta t} \]
- higher apparent velocity for dipping events
Dipping Reflectors

Common mid-point gather

Reflection point smearing
Moveout (NMO+DMO)

DMO correction removes:

- dip-dependency in velocity field
- reflection point smearing in the gather

NMO+DMO corrections transforms non-zero offset data into zero offset data (TZO)
DMO+NMO corrections and post stack migration

**D.M.O.** is a *dynamic correction* which takes into account *structural dip*. It is a partial migration carried out before stack on groups of constant offset traces.
It can be shown that...

\[ \Delta = \frac{x^2}{D} \cos \phi \sin \phi \]

Where...

\( \Delta = \text{moveout up dip} \)
\( x = \text{half source-receiver offset} \)
\( D = \text{depth to reflector at the midpoint (along normal)} \)
\( \phi = \text{local dip of reflector} \)
DMO operator – same dip, different depths

Here is the effect of a DMO operator on the events recorded on a trace with a short offset $S_1$ to $R_1$.

And here the effect of a DMO operator on the events recorded on a trace with a long offset $S_2$ to $R_2$. 