Recommended Further Reading,
List of Symbols,
Additional Material for Individual Study

Recommended Further Reading – Basic Concepts

- T. Weiland, R. Wanzenberg: *Wake Fields and Impedances* (1991)
- U. van Rienen: *Numerical Methods in Computational Electrodynamics* (2001)
- U. van Rienen: *Zur numerischen Berechnung ….*, DESY-M-89-04 (1989)
- K. Bane, P.B. Wilson, T. Weiland: *Wake Fields and Wake Field Acceleration* (1984)
- P.B. Wilson: *Introduction to Wakefields and Wake Potentials* (1989)
- A. Chao: *Physics of Collective Beam Instabilities in High Energy Accelerators* (1993)
- T. Wangler: *RF Linear Accelerators* (2008)
- P. Tenenbaum: *Fields in Waveguides – a Guide for Pedestrians* (2003)
- R.E. Collin: *Field Theory of Guided Waves, Second Edition* (1991)
- B.W. Zotter, S.A. Kheifets: *Impedances and Wakes in High-Energy Particle Accelerators* (1998)

Recommended Further Reading – Application of Wakefields

- F. Reimann; U. van Rienen, P. Michel, U. Lehnert: *A dielectrically lined rectangular waveguide as a wakefield dechirper for ELBE* (2015)
- K. Bane, *G. Stupakov: Corrugated Pipe As a Beam Dechirper* (2012)
- S. Antipov et al.: *Passive Momentum Spread Compensation By a „Wakefield Silencer“* (2012)
- K. Bane, A. Novokhatskii: *The Resonator Impedance Model of Surgace Roughness Applied to the LCLS Parameters* (1999)
- K. Bane, G. Stupakov: *Terahertz Radiation from a Pipe With Small Corrugations* (2012)
- B.M. Bolotovskii: *Theory of Cerenkov Radiation* (1961)

List of Symbols (I)

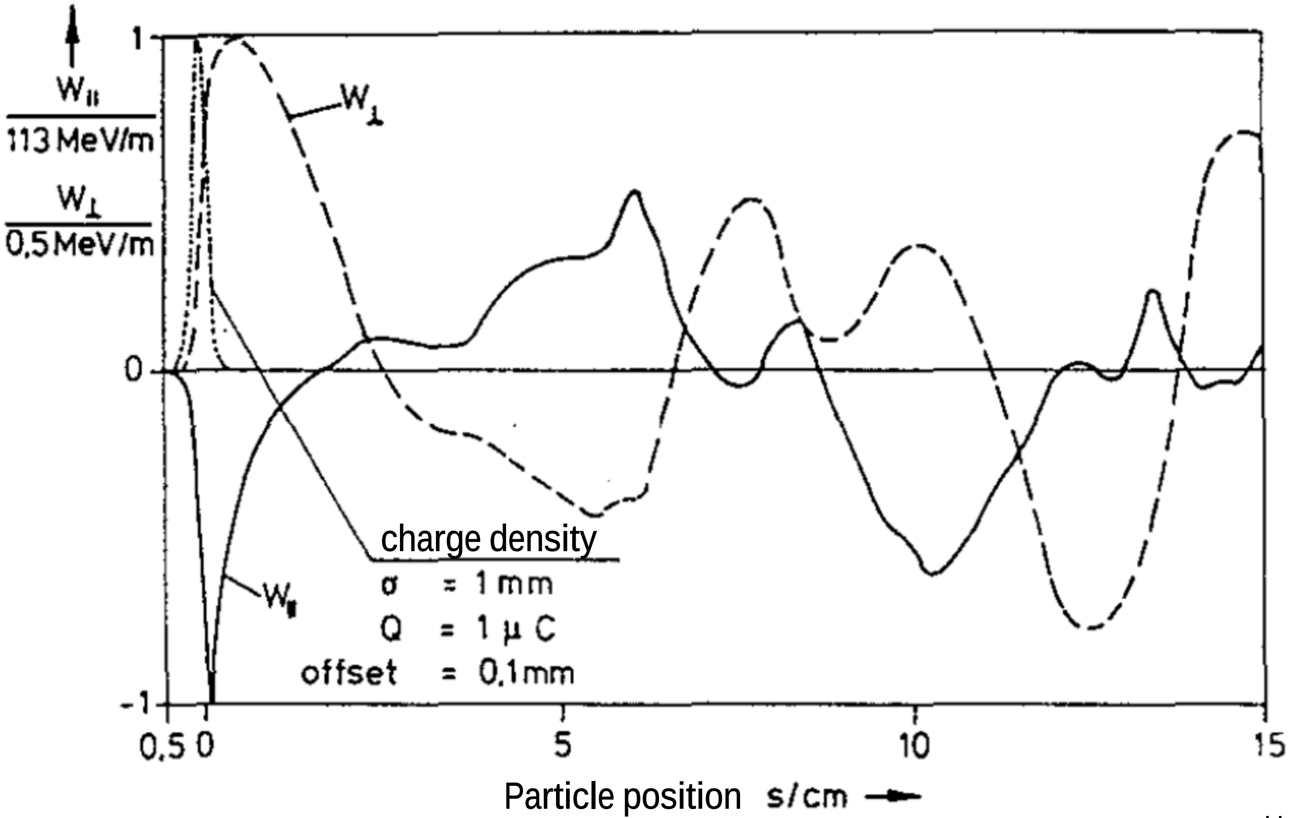
- Basic quantities:
  q charge
  $\boldsymbol{r}$ position of the charge (x,y,z)
  t time
  c speed of light
  v velocity of the charge
  p momentum
  U energy
  V voltage
  $\boldsymbol{E}(\boldsymbol{r},t)$ electric field
  $\boldsymbol{B}(\boldsymbol{r},t)$ magnetic flux
  $\boldsymbol{H}(\boldsymbol{r},t)$ magnetic field
  ω angular frequency
  λ wave length

- Wakefield-specific quantities:
  q_1 field-generating charge
  q_2 test charge
  $s = ct - z$ distance between field-generating charge and test charge
  r_1 transversal offset of the charges
  $\boldsymbol{W}$ wakefield
  $W_\parallel$ longitudinal wakefield
  $\boldsymbol{W}_\perp$ transversal wakefield
  $W_{\parallel,0}$ point charge longitudinal wakefield
  $Z_\parallel$ longitudinal impedance
  k_n loss parameter of a mode

List of Symbols (II)

- Other quantities:
  $\chi_n(t)$ expansion coefficients
  $\lambda(s)$ bunch shape function
  σ Gaussian pulse width
  l flat top pulse width
  $J_n(x)$ Bessel function of order n
  j_{0n} nth zero of $J_0(x)$
  R radius of pillbox cavity
  g height of pillbox cavity

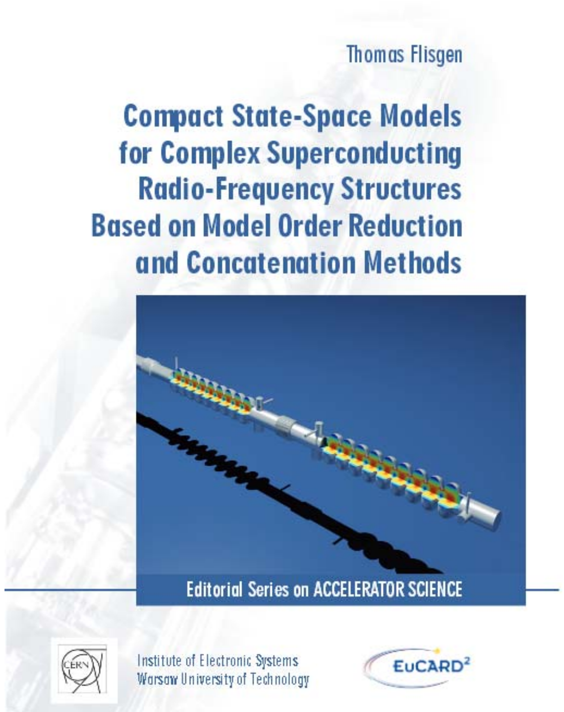
Wakefield for the Pillbox-like 3-cell Cavity



U. van Rienen, DESY-M-89-04

State Space Concatenations (SSC)

- Find more details here:



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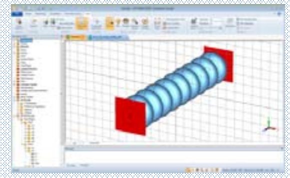
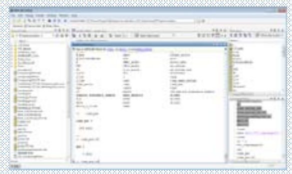


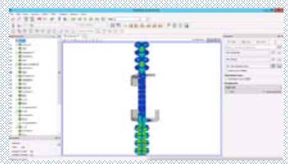
- geometrical modelling
- discretization

Matlab®



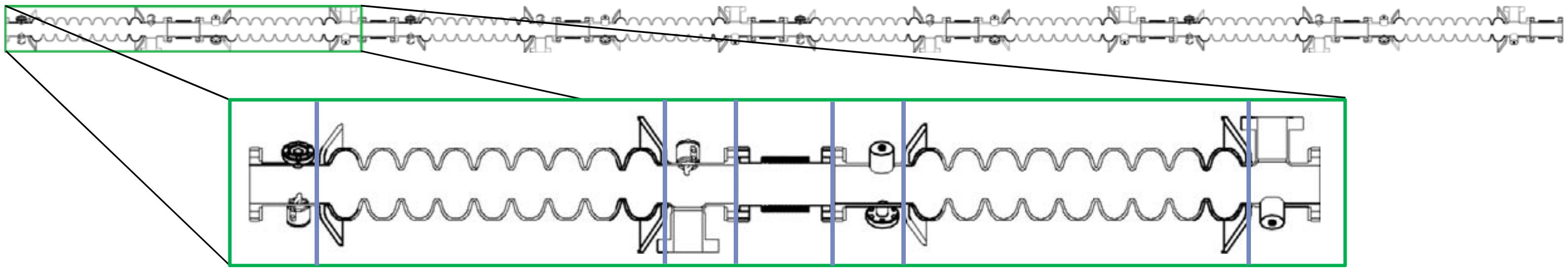
- model-order reduction
- concatenation
- computation of derived quantities

ParaView



- visualization of field distributions

Chain of Eight SC 3rd Harmonic Cavities in XFEL



	HOM Coupler	Cavity	HOM Coupler with Power Coupler	Bellow	HOM Coupler (rotated)	HOM Coupler with Power Coupler (rotated)
N_{dof}	411,015	1,119,963	585,915	427,119	411,015	585,915
$N_{dof,red}$	138	258	164	145	138	164

- Full model of eight cavity chain is estimated to have N_{dof} = 20,239,977 degrees of freedom
- SSC: Reduced-order models of segments concatenated to reduced-order model of full chain with only $N_{dof,red}$ = 2,931 degrees of freedom

Figures of cavities E. Vogel et al.: "Status of the 3rd harmonic systems for FLASH and XFEL in summer 2008", Proc. LINAC 2008

Courtesy of
T. Flisgen, J. Heller

Appendix: Proof of Panofsky-Wenzel-Theorem (I)

- Transversal wakefield

$$\boldsymbol{W}_{\perp}(\boldsymbol{r_1}, s) = \frac{1}{\mathrm{q}_1} \int_{-\infty}^{\infty} [\boldsymbol{E}_{\perp}(\boldsymbol{r_1}, z, t) + c\boldsymbol{e_z} \times \boldsymbol{B}(\boldsymbol{r_1}, z, t)]_{t=(z+s)/c}\, dz$$

- Derivative with respect to s (with $s = ct - z$ and $\partial s = c\partial t$)

$$\frac{\partial}{\partial s}\boldsymbol{W}_{\perp}(\boldsymbol{r_1}, s) = \frac{1}{\mathrm{q}_1} \int_{-\infty}^{\infty} \left[\underbrace{\frac{\partial}{c\partial t}\boldsymbol{E}_{\perp}(\boldsymbol{r_1}, z, t)}_{T_1} + \underbrace{\boldsymbol{e_z} \times \frac{\partial}{\partial t}\boldsymbol{B}(\boldsymbol{r_1}, z, t)}_{T_2}\right]_{t=(z+s)/c} dz$$

- Now: substitute T_1 and T_2 by more convenient expressions

Appendix: Proof of Panofsky-Wenzel-Theorem (II)

- Term T_1: use Maxwell's equations

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial}{\partial t}\boldsymbol{B}$$

  - Apply cross product with $\boldsymbol{e_z}$ (beam direction)

$$\boldsymbol{e_z} \times \frac{\partial}{\partial t}\boldsymbol{B} = \frac{\partial}{\partial z}\boldsymbol{E}_\perp - \boldsymbol{\nabla}_\perp \boldsymbol{E}_z$$

- Term T_2: total derivative with respect to z of transversal electric field (with $s = ct - z$)

$$\frac{d}{dz}\boldsymbol{E}_\perp\left(x, y, z, \frac{\mathrm{z} + \mathrm{s}}{\mathrm{c}}\right) = \left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)\boldsymbol{E}_\perp\left(x, y, z, \frac{\mathrm{z} + \mathrm{s}}{\mathrm{c}}\right)$$

  - Can be reformulated into:

$$\frac{1}{\mathrm{c}}\frac{\partial}{\partial t}\boldsymbol{E}_\perp\left(x, y, z, \frac{\mathrm{z} + \mathrm{s}}{\mathrm{c}}\right) = \left(\frac{d}{dz} - \frac{\partial}{\partial z}\right)\boldsymbol{E}_\perp\left(x, y, z, \frac{\mathrm{z} + \mathrm{s}}{\mathrm{c}}\right)$$

Appendix: Proof of Panofsky-Wenzel-Theorem (III)

- Inserting T_1 and T_2 leads to:

$$\frac{\partial}{\partial s}\boldsymbol{W}_{\perp} = \frac{1}{\mathrm{q}_1}\int_{-\infty}^{\infty}\left[\left(\frac{d}{dz}-\frac{\partial}{\partial z}\right)\boldsymbol{E}_{\perp} + \left(\frac{\partial}{\partial z}\boldsymbol{E}_{\perp} - \boldsymbol{\nabla}_{\perp}\boldsymbol{E}_{z}\right)\right]_{t=(z+s)/c} dz$$

- Reformulating:

$$\frac{\partial}{\partial s}\boldsymbol{W}_{\perp}(\boldsymbol{r}_1, s) = \frac{1}{\mathrm{q}_1}\int_{-\infty}^{\infty}\left[\frac{d}{dz}\boldsymbol{E}_{\perp}(\boldsymbol{r}_1, z, t) - \boldsymbol{\nabla}_{\perp}\boldsymbol{E}_{z}(\boldsymbol{r}_1, z, t)\right]_{t=(z+s)/c} dz$$

Appendix: Proof of Panofsky-Wenzel-Theorem (IV)

- If $\boldsymbol{E}_{\perp}(\boldsymbol{r_1}, z, t)$ vanishes at the boundaries, the Panofsky-Wenzel-Theorem results:

$$\frac{\partial}{\partial s}\boldsymbol{W}_{\perp}(\boldsymbol{r_1}, s) = -\frac{1}{\mathrm{q}_1}\boldsymbol{\nabla}_{\perp}\int_{-\infty}^{\infty}\boldsymbol{E}_z\left(\boldsymbol{r_1}, z, \frac{z+s}{c}\right)dz$$

- Thus

$$\frac{\partial}{\partial s}\boldsymbol{W}_{\perp}(\boldsymbol{r_1}, s) = -\boldsymbol{\nabla}_{\perp}W_{\parallel}(\boldsymbol{r_1}, s)$$

- Integrating over s leads to

$$\boldsymbol{W}_{\perp}(x, y, s) = -\boldsymbol{\nabla}_{\perp}\int_{-\infty}^{s}W_{\parallel}(x, y, s')ds'$$

Addendum: Space Charge Effects

- So far: wakefields
  - Only valid for ultrarelativistic particles, i.e. $v \cong c$
- For $v < c$:
  - Space charge effects due to the Coulomb interaction of charged particles
  - Need to be taken into account as well
  - Are usually of larger influence for the following particles than the actual wakefields
- Effects of space charges include:
  - Deflection of charged particles
  - Beam instabilities
  - High (unwanted) field intensities
  - Material breakdown
  - etc.