Experimental Tests of Vacuum Energy

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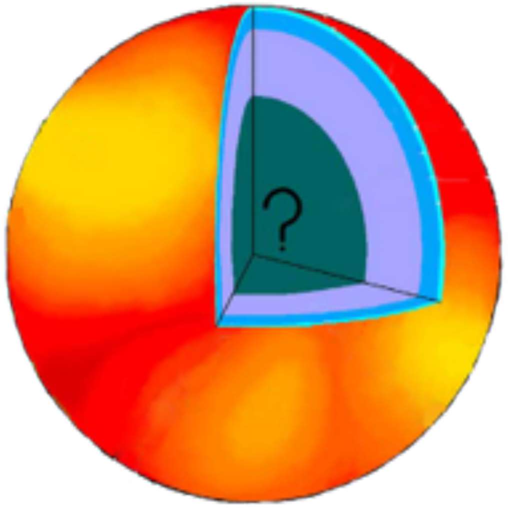
astro-ph/1502.04702

Outline

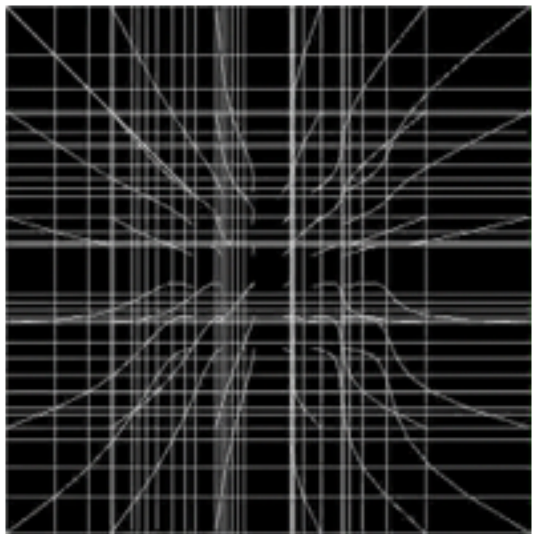
Motivation:
observe changing vacuum energy

Λ

Neutron Stars



Cosmological Phase Transitions
and Gravitational Waves



Conclusions

The Evolution of Vacuum Energy

Λ

The cosmological constant is very small today

$$\Lambda \sim (10^{-3}\ \mathrm{eV})^4$$

from quantum field theory we expect

$$(TeV)^4, M_{Pl}^4$$

Why so small? Why not zero?

Is there an adjustment mechanism?
Is it always very small?

Vacuum Energy and Electroweak PT Λ

$$\Delta\Lambda \sim -M_W^4 \qquad \Lambda + \Delta\Lambda \sim \mathcal{O}(\Lambda_{QCD}^4)$$

tuning

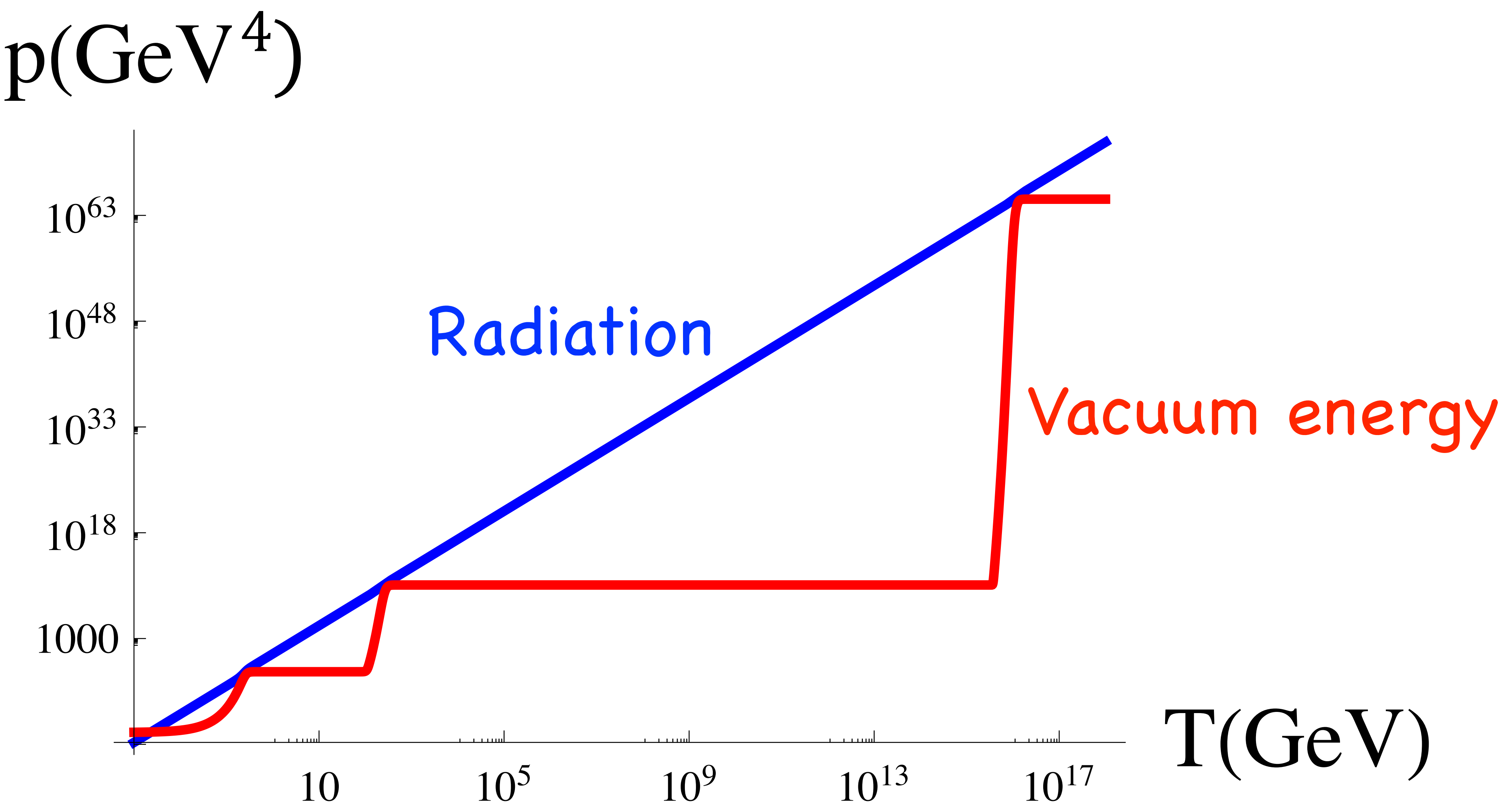
At one phase transition Universe already "knows" where the next phase transition will be

At least QCD, EW PT, potentially also SUSY and/or GUT phase transitions

previously Λ was much larger than now, but never dominated previously!

Sketch of vacuum energy evolution

Λ

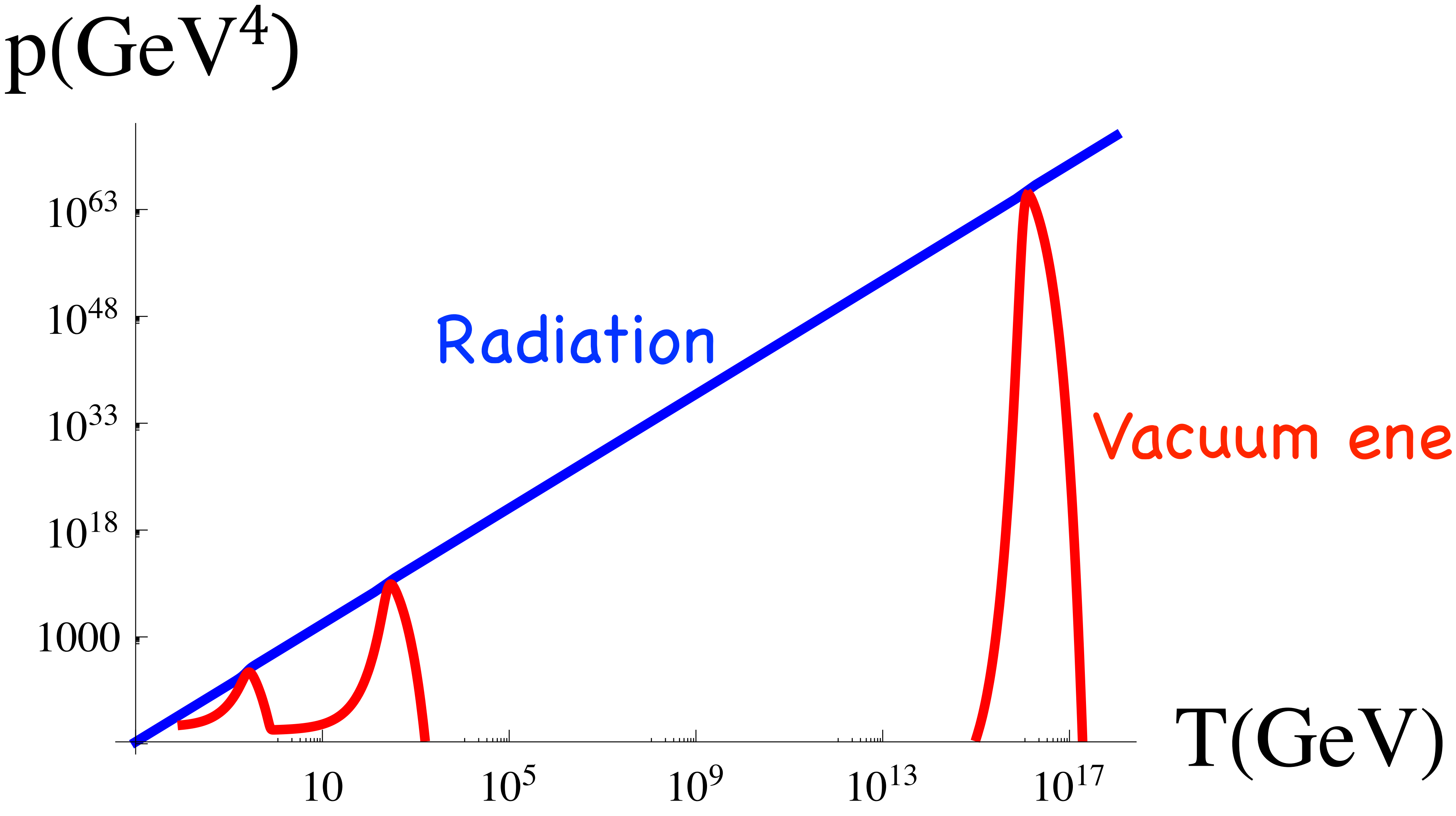


Size of step of order $T_{c,i}^4$

Amount of tuning: $T_{c,i+1}^4$

Evolution with adjustment

Λ



Heights and time-scales depend on details of adjustment mechanism

Steps or adjustment?

Λ

If steps: lends more credence to anthropics

If adjustment: need to find mechanism

Difficulty: Λ always sub-dominant

Last transition at Λ_{QCD}

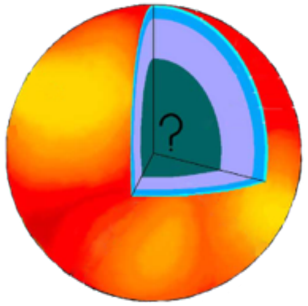
Above CMB, BBN, No direct tests...

Where should we look?

Λ

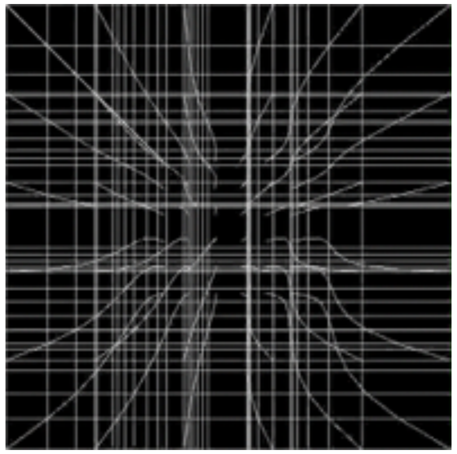
System where vacuum energy can be a significant fraction of total energy

Neutron stars

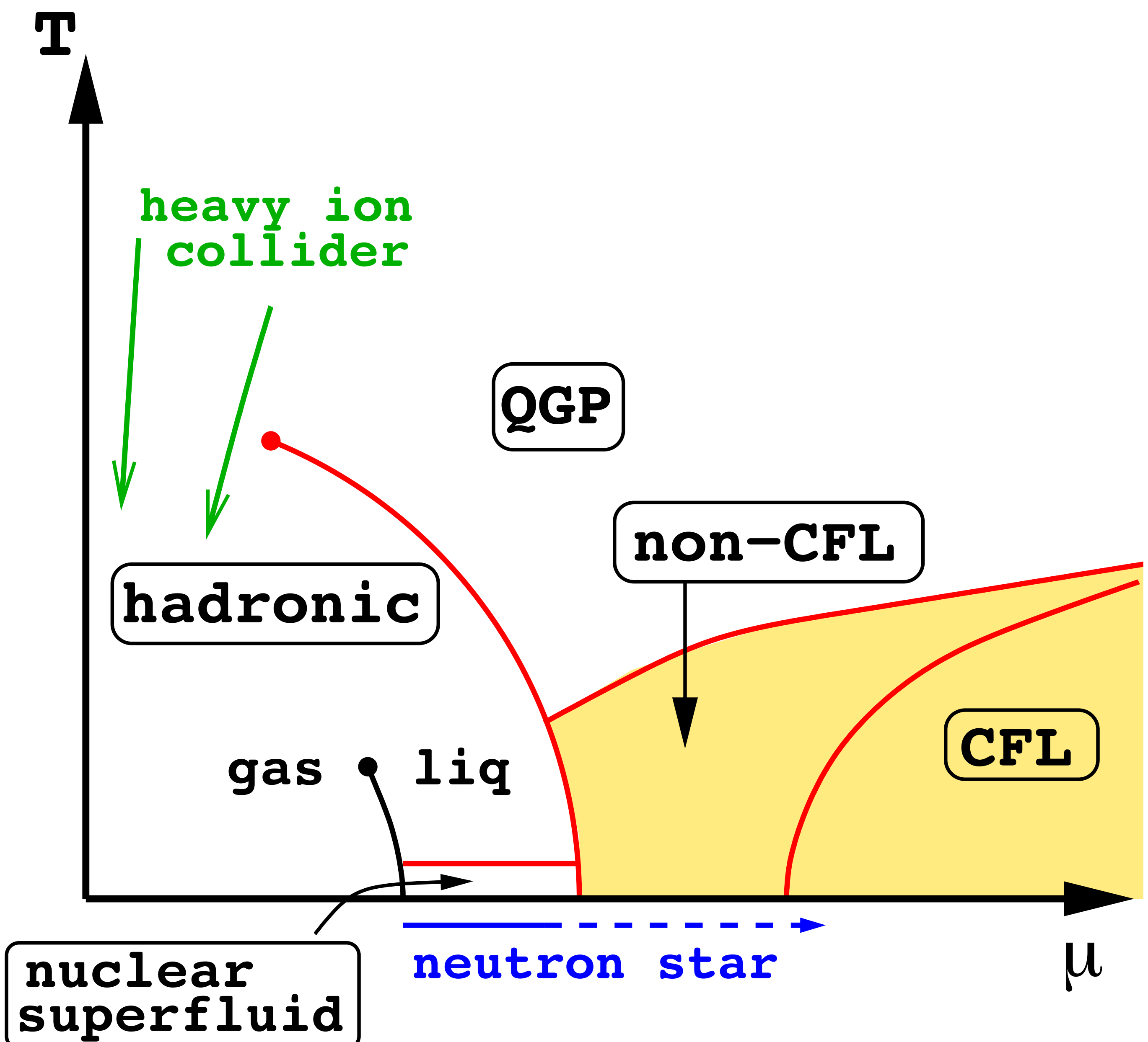


Epochs where vacuum energy is comparable to radiation

primordial gravitational waves
passing through
Cosmic Phase Transitions



Neutron Stars



Alford, Schmitt, Rajagopal, Schaefer
hep-ph/0709.4635

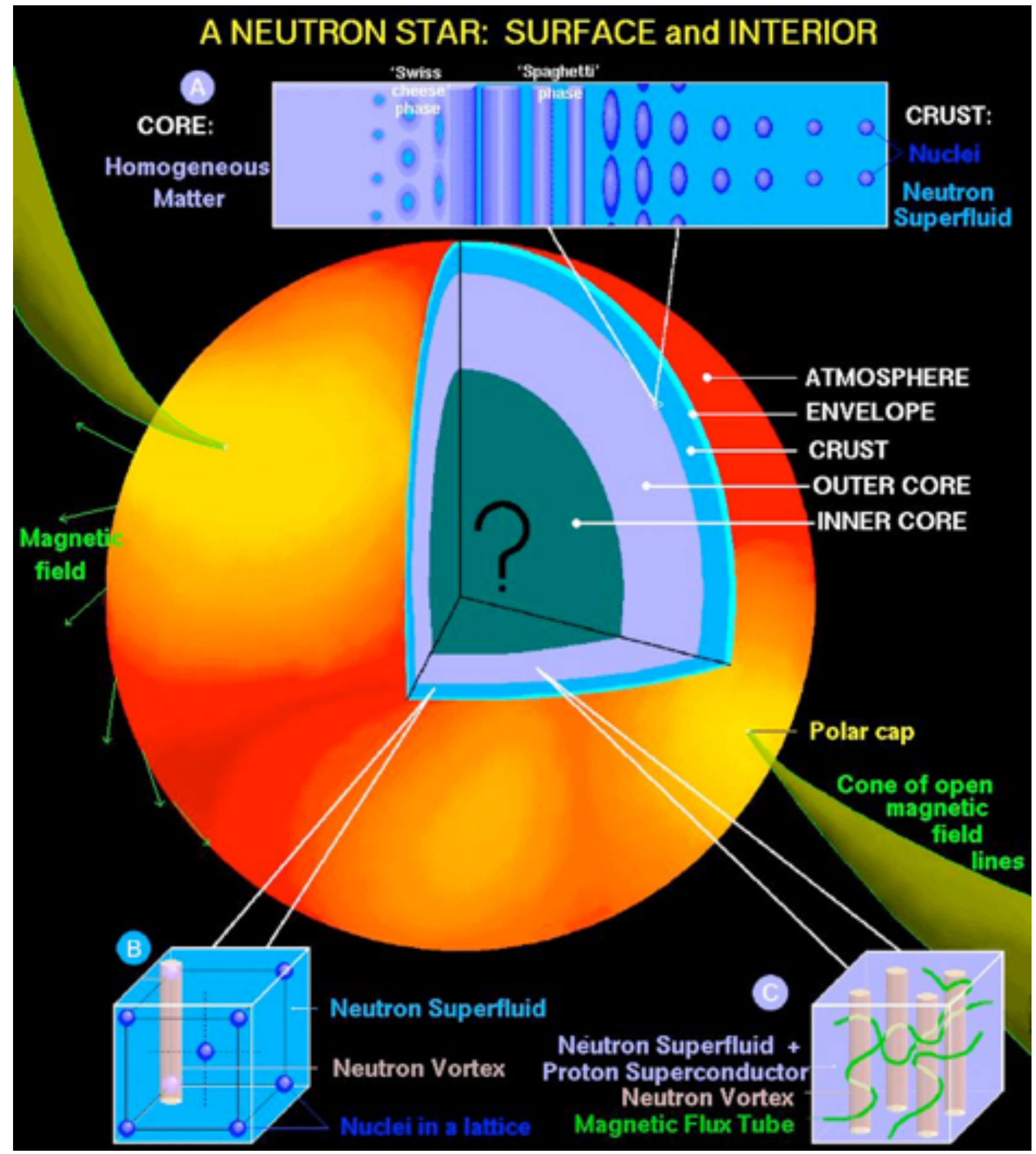
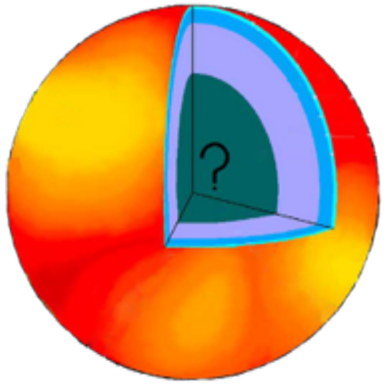


image by Dany P. Page
http://bit.ly/nscross

Model for neutron stars



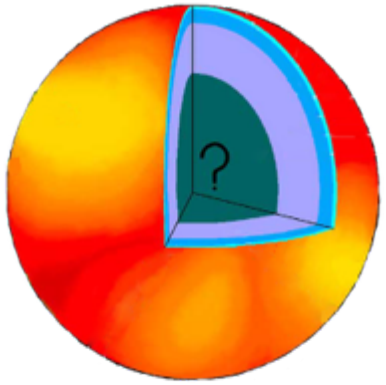
At zero temperature, gravitational pressure balanced by pressure of fluid

$$ds^2 = e^{\nu(r)}dt^2 - (1 - 2GM(r)/r)^{-1}\, dr^2 - r^2 d\Omega^2$$

Einstein eqs (aka Tolman-Oppenheimer-Volkoff):

$$\begin{aligned}
M'(r) &= 4\pi r^2 \rho(r) \\
p'(r) &= -\frac{p(r) + \rho(r)}{r\,(r - 2GM(r))} G \left[M(r) + 4\pi r^3 p(r)\right] \\
\nu'(r) &= -\frac{2p'(r)}{p(r) + \rho(r)}
\end{aligned}$$

Toy model for neutron stars



inner core

$$p_{(-)}(\rho) = p_f(\rho) - \Lambda = K_- \rho_f^{\gamma_-} - \Lambda$$
$$\rho_{(-)} = \rho_f + \Lambda$$

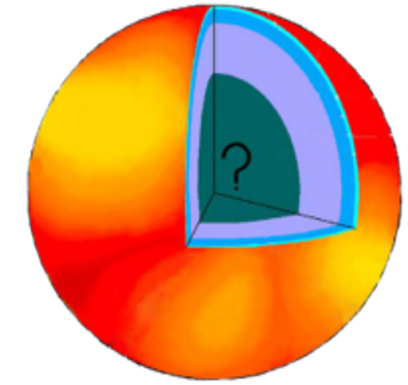
outer core

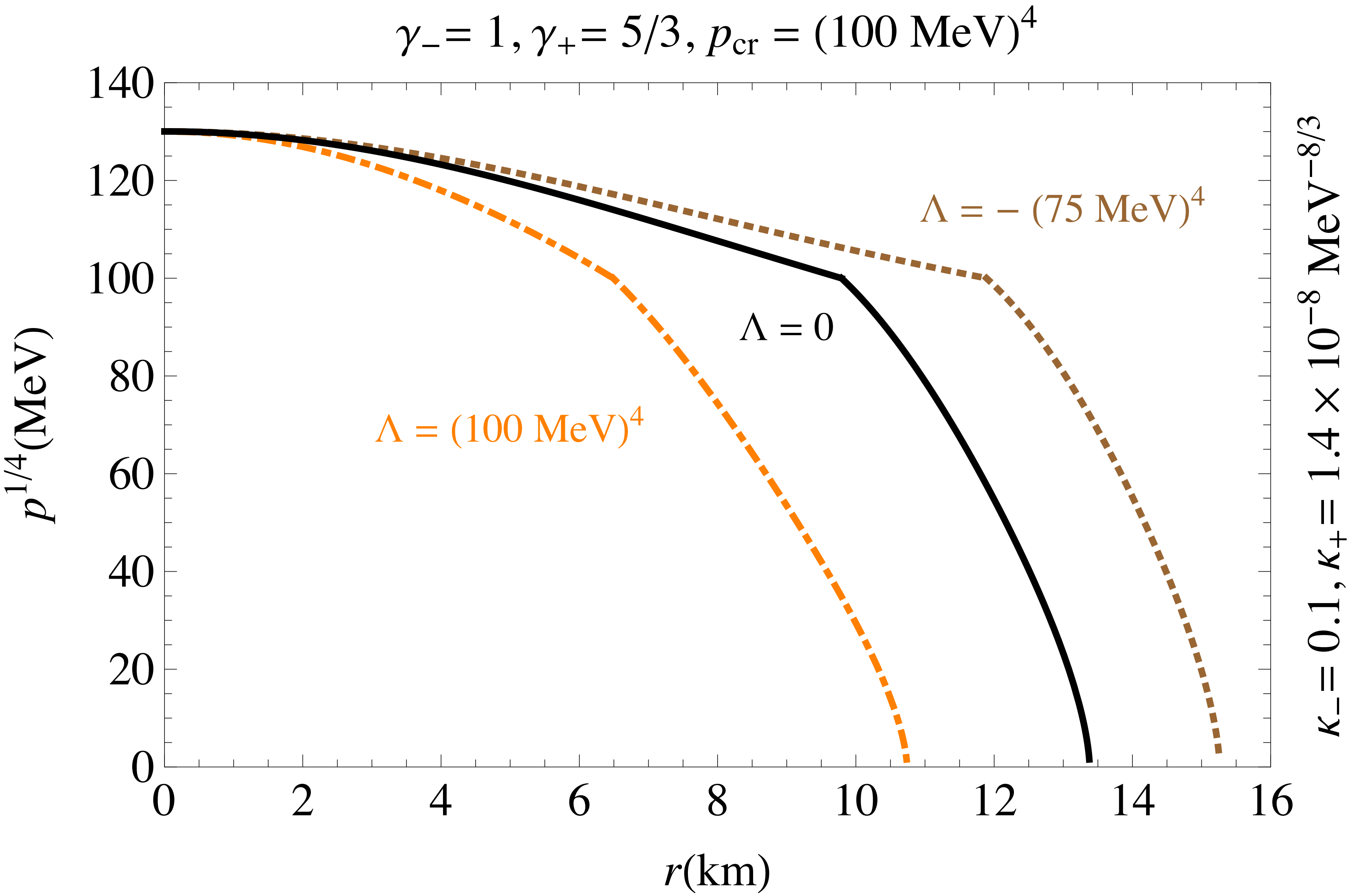
$$p_{(+)}(\rho) = p_f(\rho) = K_+ \rho_f^{\gamma_+}$$
$$\rho_{(+)} = \rho_f \,.$$

Match critical pressure at boundary

Israel Junction condition: $\nu'(r), M(r)$ continuous, thus $p(r)$ also continuous

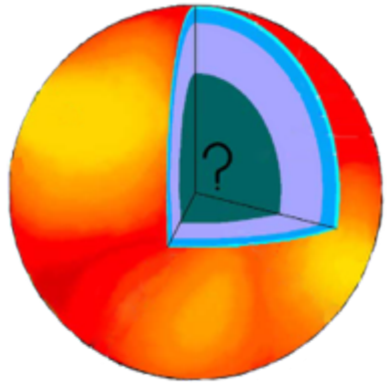
Larger Λ leads to a smaller star



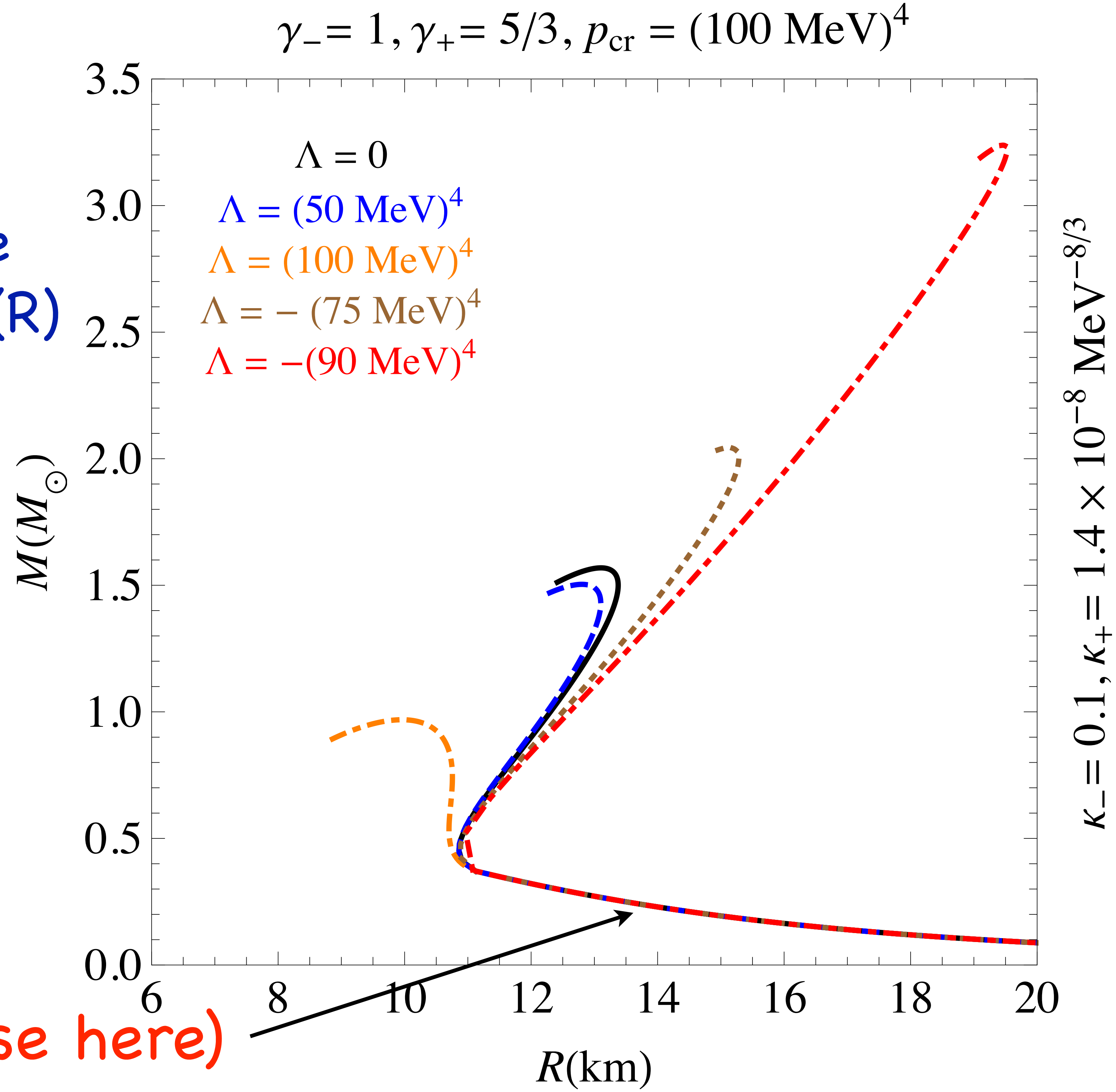


$$p'(r) = -\frac{p(r) + \rho(r)}{r\,(r - 2GM(r))} G\left[M(r) + 4\pi r^3 p(r)\right]$$

Toy model for neutron stars

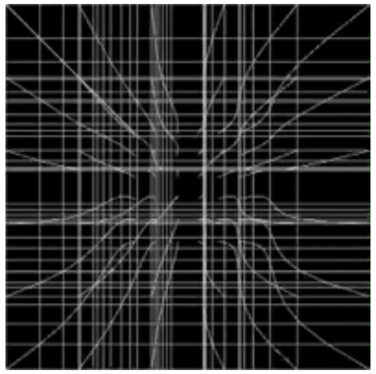


reproduces the characteristic M(R) curve



(No new phase here)

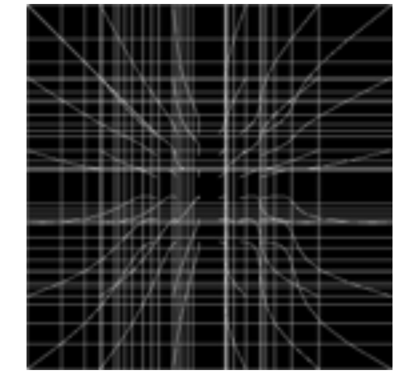
Vacuum energy of the Universe



Cosmological phase transitions

Case study: look at effect of PT's on primordial gravitational waves

Energy density in gravitational waves



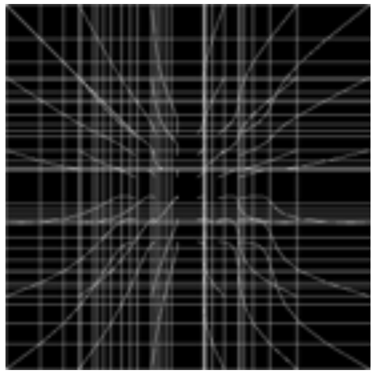
energy density per log scale
normalized to critical density

$$\Omega_h(\tau, k) \equiv \frac{\tilde{\rho}_h(\tau, k)}{\rho_c(\tau)}$$

Approximate expression:

$$\Omega_h(\tau, k) \simeq \frac{(\Delta_h^P)^2}{12 H^2(\tau) a^4(\tau)} k^2 a^2(\tau_{hc})$$

Modes entering during Radiation Domination



Condition for entering:

$$k = (aH)|_{\tau_{hc}}$$

During RD

$$H^2 \propto 1/a^4$$

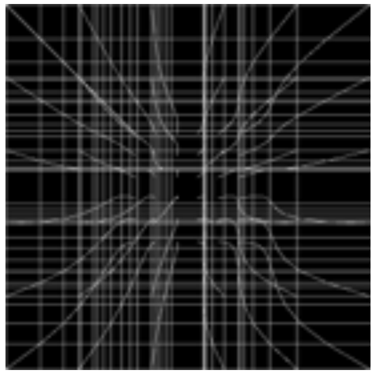
Thus

$$k^2 a^2(\tau_{hc}) \propto const.$$

Spectrum for modes entering during RD

is constant!

Effect of Phase Transition



Entropy conservation:

$$a \propto T^{-1} g_*^{-1/3}$$

Hubble:

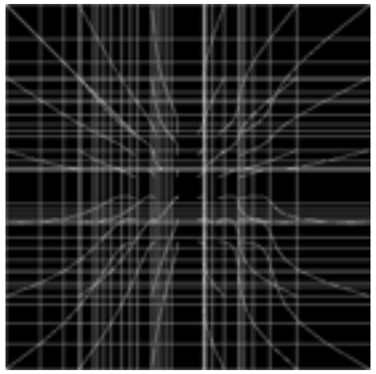
$$H^2 \propto \rho \propto \frac{1}{a^4} g_*^{-1/3}$$

Energy density:

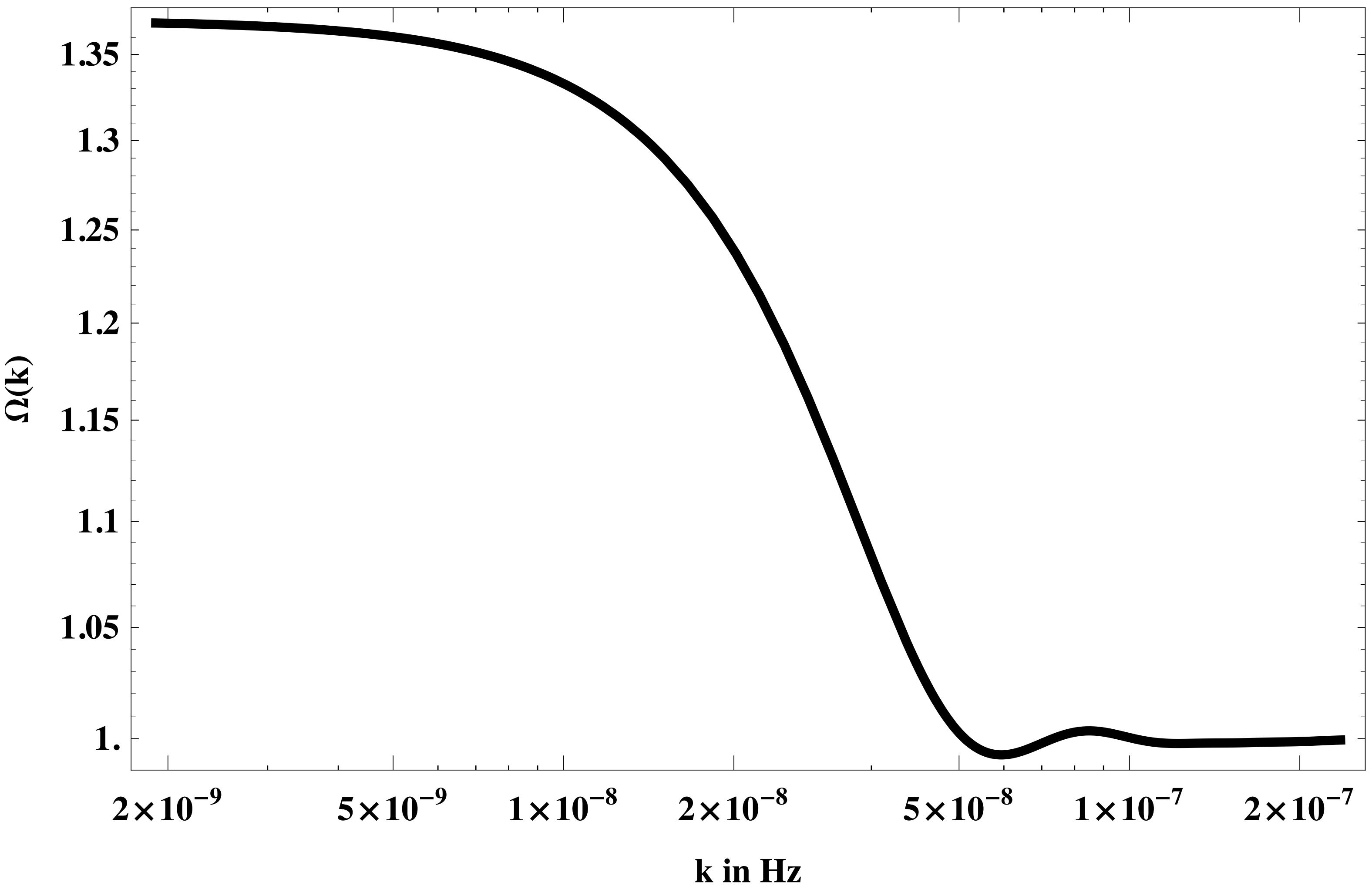
$$\Omega_h \propto k^2 a^2(\tau_{hc}) \propto a^4(\tau_{hc}) H_{hc}^2 \propto g_*^{-\frac{1}{3}}$$

Expect to see a step in spectrum

QCD Phase Transition



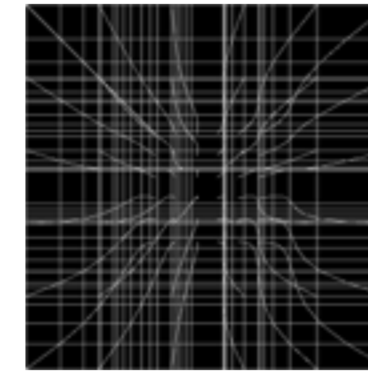
A typical result:



Size of step given by change in DOF

$60 \rightarrow 20$ for QCD

Effect of vacuum energy



include VE

$$\xi = \frac{\rho_\Lambda}{\rho_R} = \frac{\rho_\Lambda}{\bar{\rho}_R} a^4(\tau)$$

modes re-enter when

$$k^2 = a^2 H^2 = (1+\xi) a^2 \rho_R = (1+\xi) a^{-2} \bar{\rho}_R$$

$$\Omega \propto a^2(\tau_{hc}) k^2 = (1+\xi)\bar{\rho}_R = (1+\xi) g_* T^4 a^4 \propto (1+\xi) g_*^{-1/3}$$

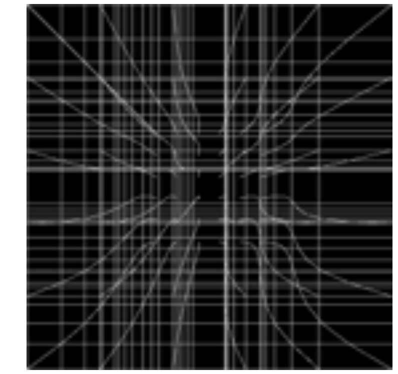
Peaks versus Steps

$$\Omega(k) \propto (1+\xi) g_*^{-1/3}$$

ξ peaks near phase transition

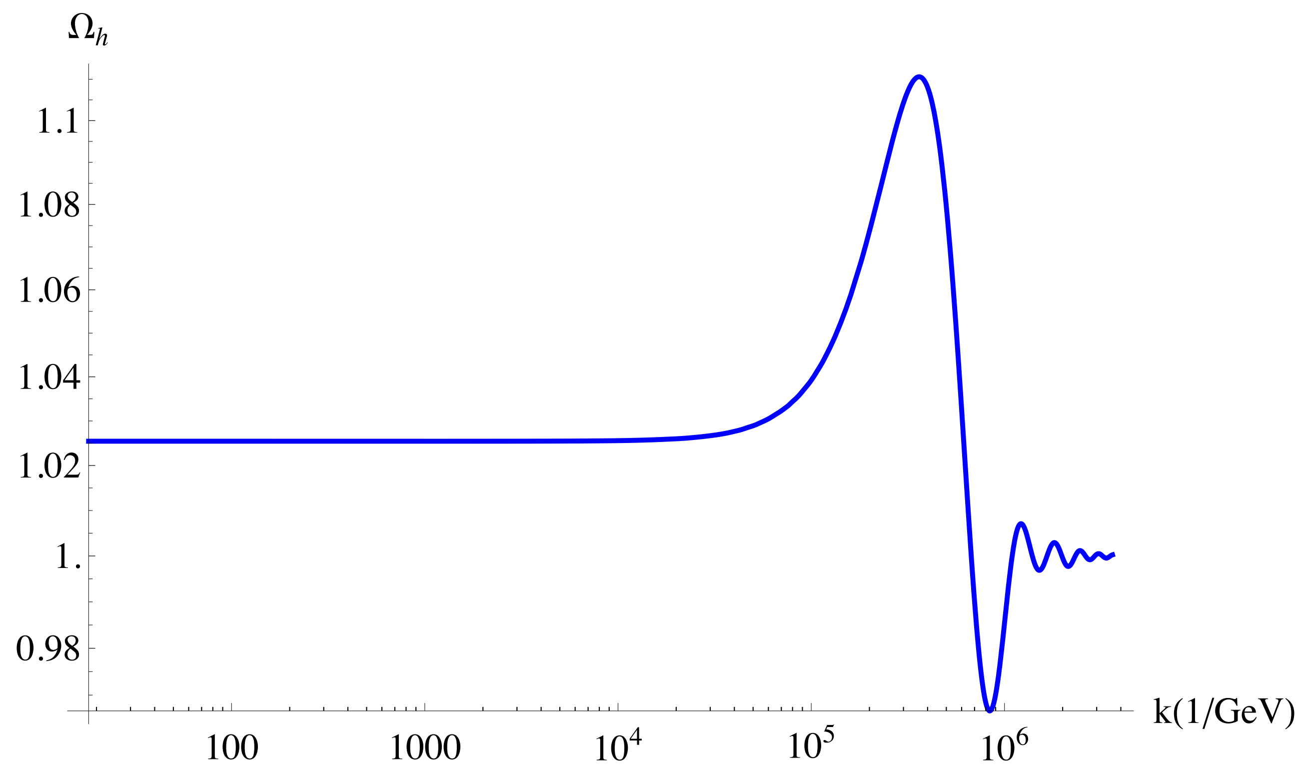
Magnitude of peak set by the maximal ratio of vacuum energy to radiation

large step washes out peak
case for QCD: large change in DOF



A peak in the GW spectrum

example with peak:



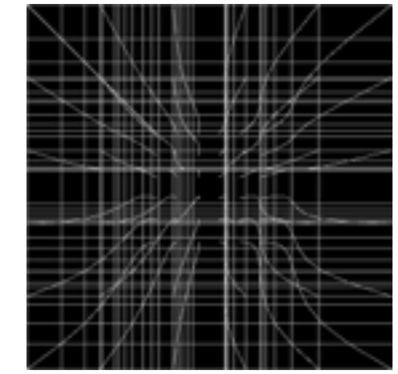
Hypothetical PT with N scalars

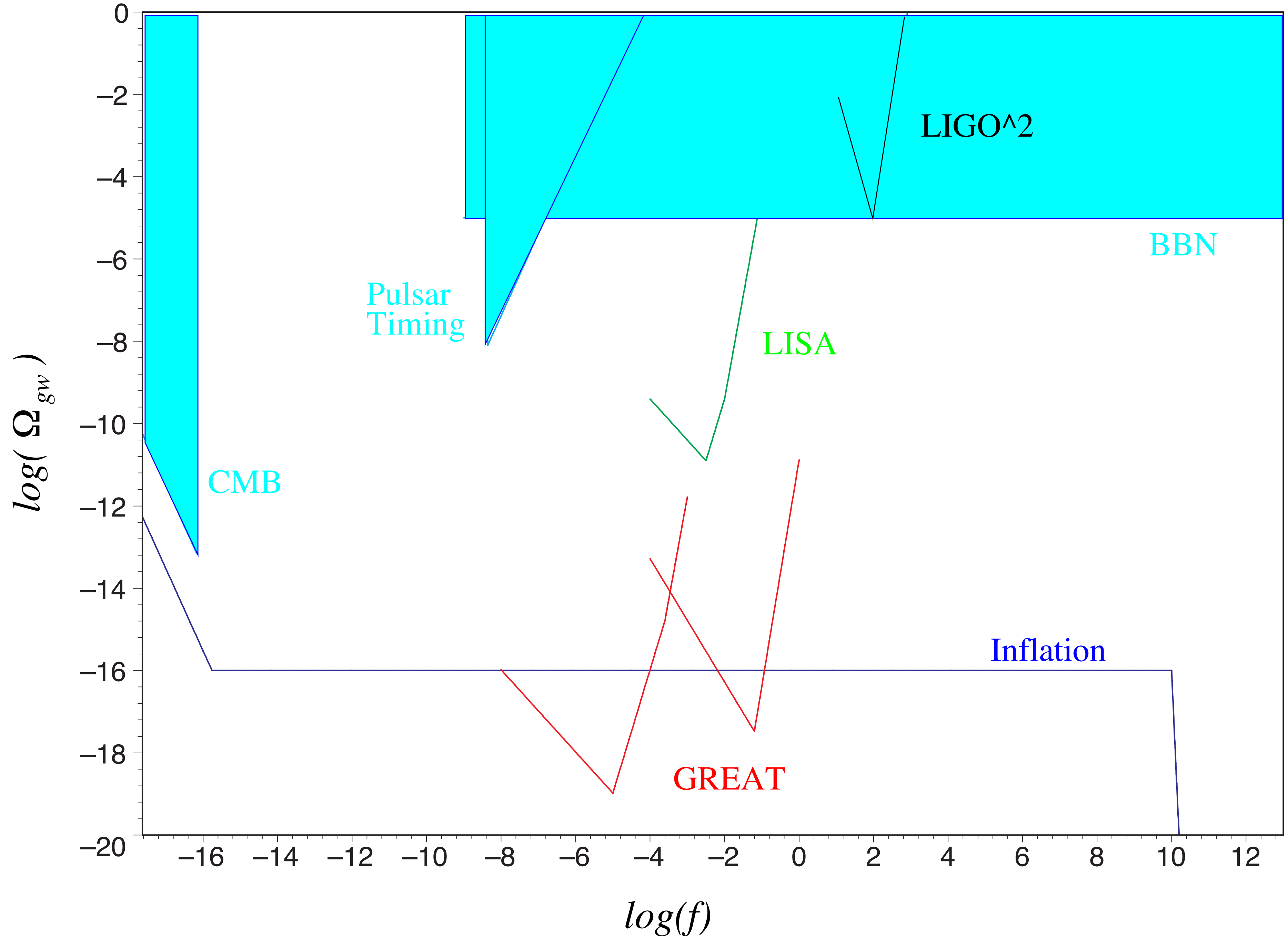
SU(N)->SU(N-1)

at 10^{11} GeV, N=5, DOF 119$\rightarrow$118

$\Lambda\approx$1/3 of radiation

Sensitivity of future experiments





Cornish, Spergel, Bennett

astro-ph/0202001

Summary

vacuum energy should change during
phase transitions

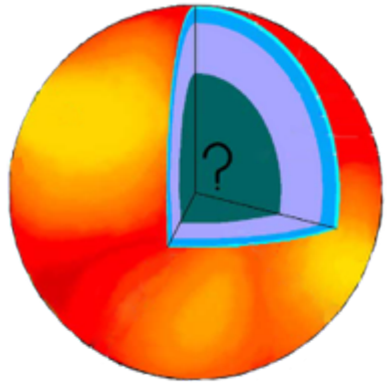
Neutron stars:

VE can cause measurable deviation in
maximal mass and M(R)

Primordial gravitational waves:

hard to see in SM phase transitions,
possible peaks in BSM or large steps for adjustment

Toy model for neutron stars



inner core use polytropic with vacuum energy:

$$p_{(-)}(\rho) = p_f(\rho) - \Lambda = K_- \rho_f^{\gamma_-} - \Lambda$$
$$\rho_{(-)} = \rho_f + \Lambda$$

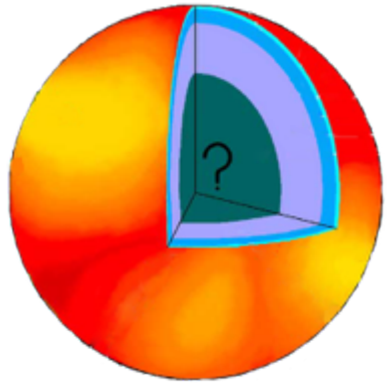
outer core just polytropic

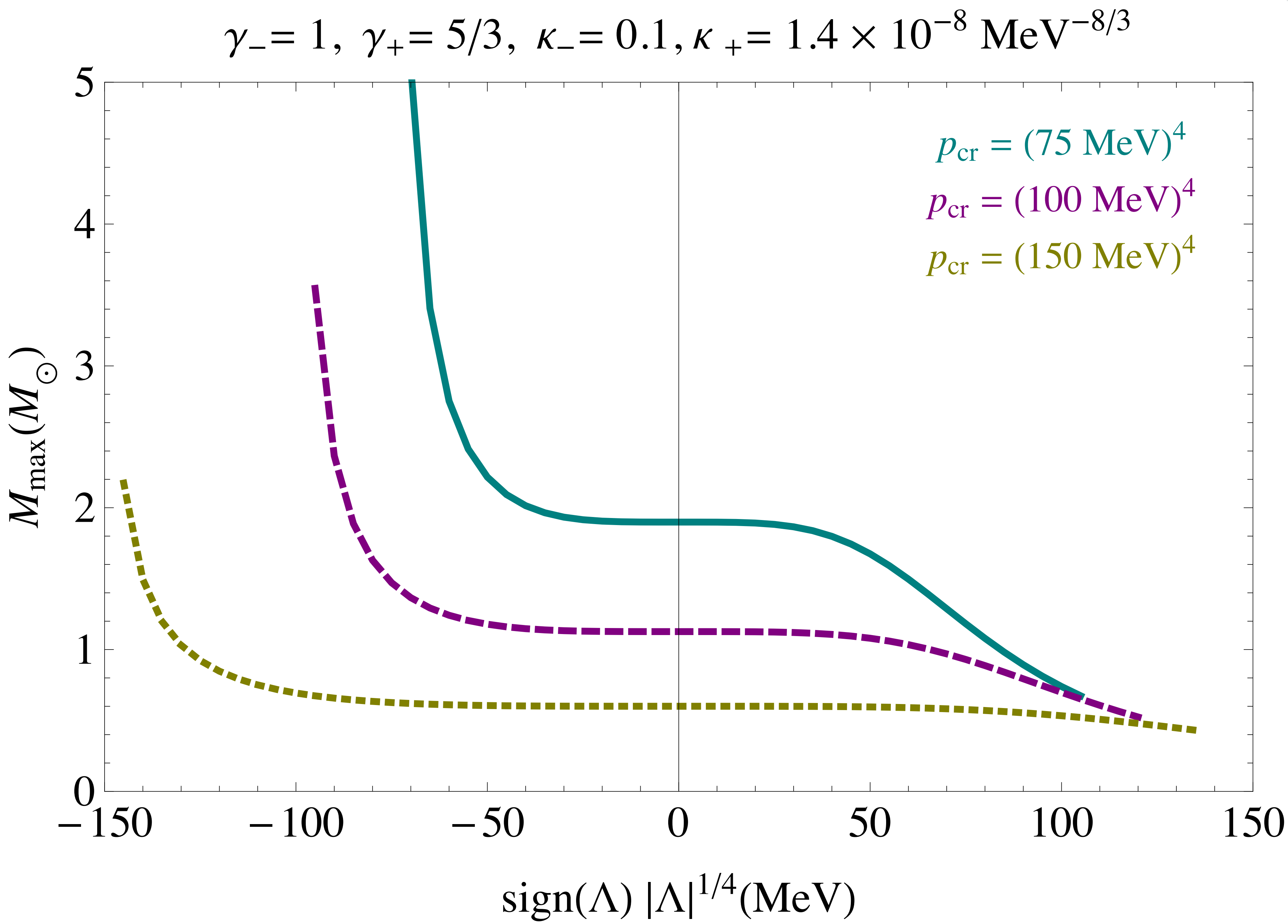
$$p_{(+)}(\rho) = p_f(\rho) = K_+ \rho_f^{\gamma_+}$$
$$\rho_{(+)} = \rho_f \, .$$

$\gamma_+ = 5/3$ for a Fermi fluid

vacuum energy can not be too negative: $\Lambda > -p_{cr}$
otherwise partial pressure of QCD fluid negative

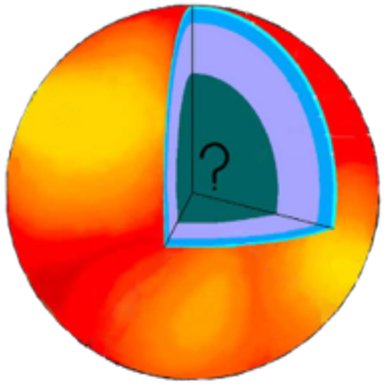
Sensitivities to vacuum energy





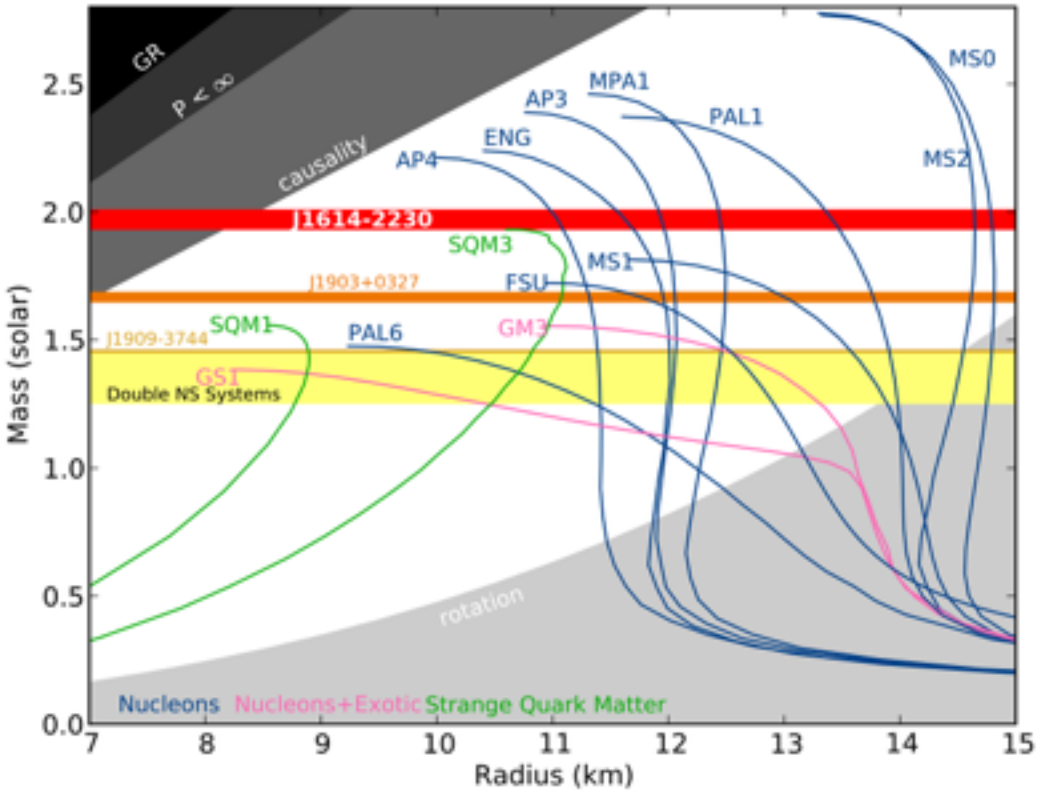
effect on maximum mass varying Λ for fixed p_{cr}

Sensitivity to vacuum energy



maximal mass can change significantly
depends very strongly on EoS parameters

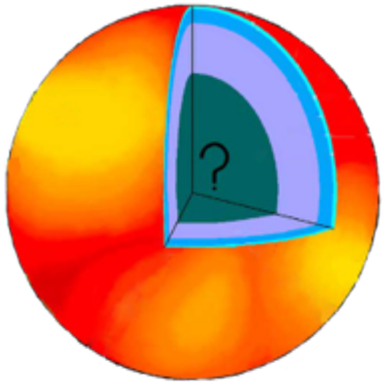
Demorest et. al.
astro-ph/1010.5788

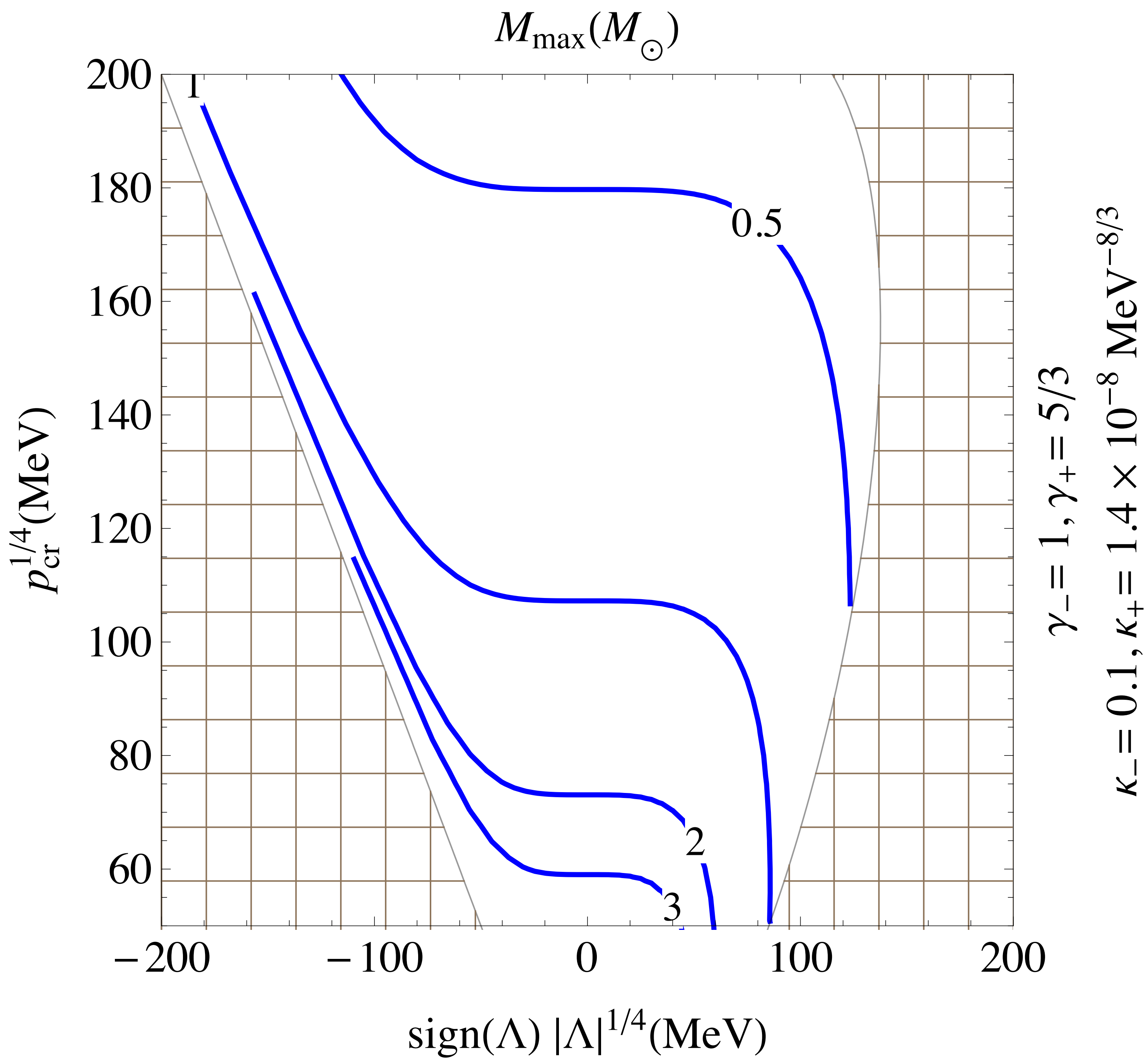


a few radii known from X-ray measurements

Promising: GW from inspiraling
neutron star binaries

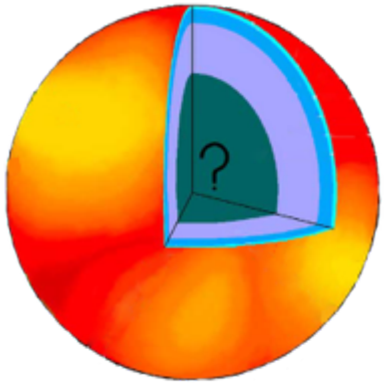
Sensitivities to vacuum energy





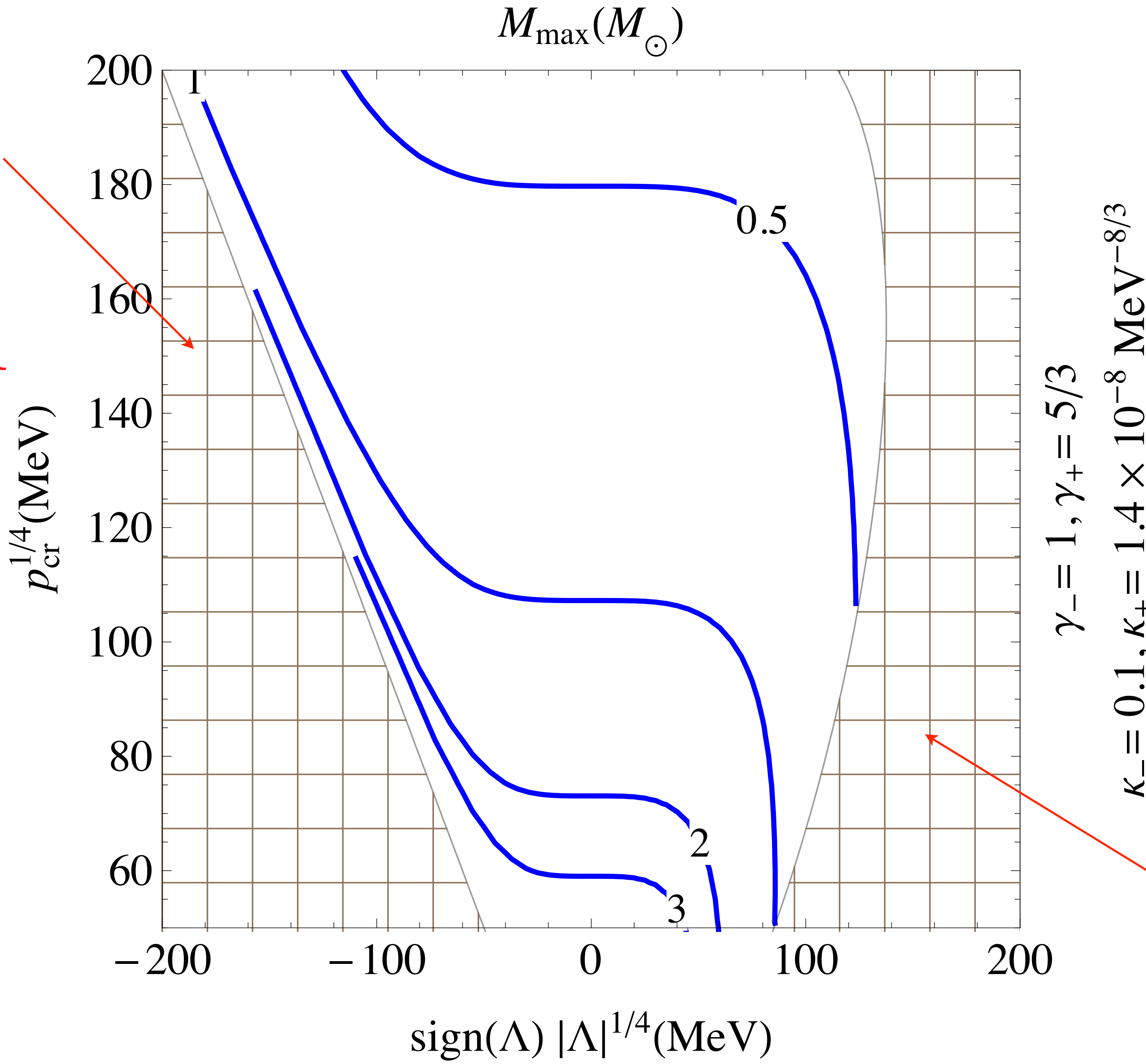
Maximum mass varying Λ and p_{cr}

Sensitivities to vacuum energy



Instability of fluid at core

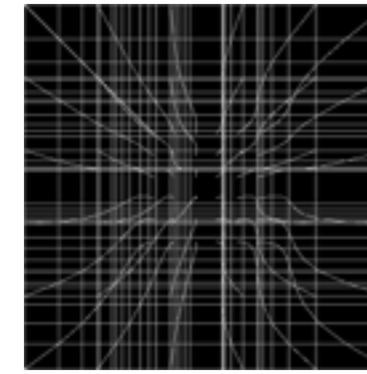
$p_- = p_f - \Lambda$



Instability of neutron star

Maximum mass varying Λ and p_{cr}

Energy density in gw's



energy:

$$\rho_h(\tau) = \frac{1}{16\pi G a^2(\tau)} \int \frac{d^3k}{(2\pi)^3} |h'_{\sigma,k}|^2$$

The power spectrum:

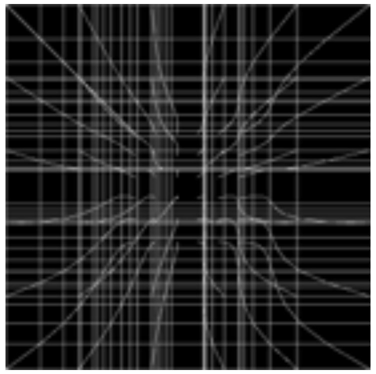
$$\Delta_h^2 = \frac{4k^3}{2\pi^2} |h_k|^2 \ , \quad |h_k|^2 = |h_{\sigma,k}|^2 \ .$$

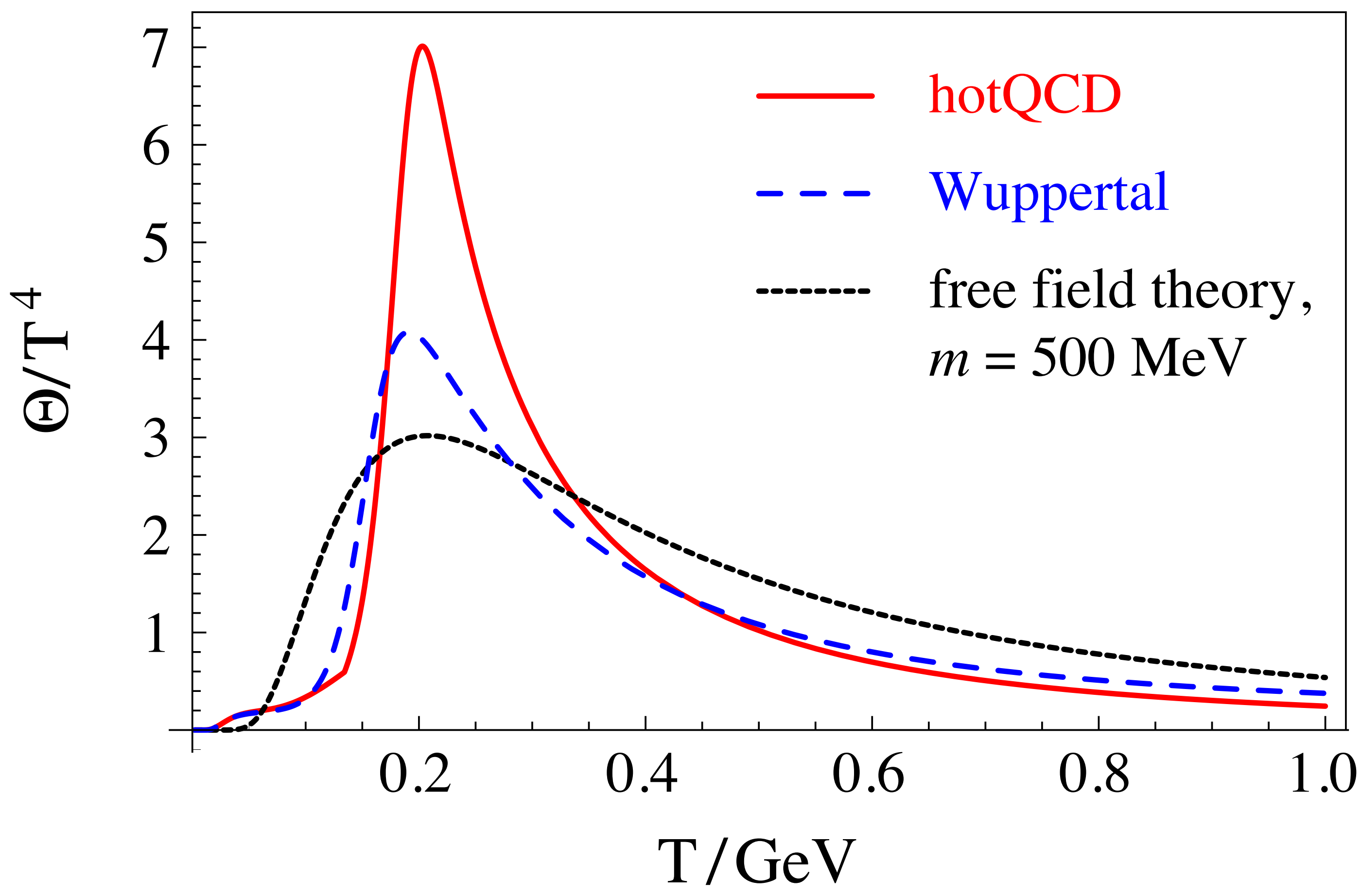
Transfer function: $h_k(\tau) \equiv h_k^P \mathcal{T}(\tau, k)$

the primordial amplitude has approx. constant power

$$(\Delta_h^P)^2 = \frac{4k^3}{2\pi^2} |h_k^P|^2 \simeq \frac{2}{\pi^2} \frac{H_\star^2}{M_P^2}$$

QCD PT from lattice

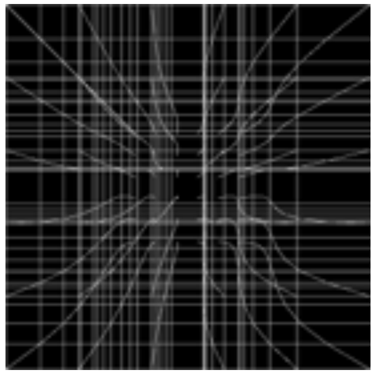




Deviation from radiation domination only during short period during PT...

Caldwell & Gubser astro-ph.CO/1302.1201

QCD Phase Transition



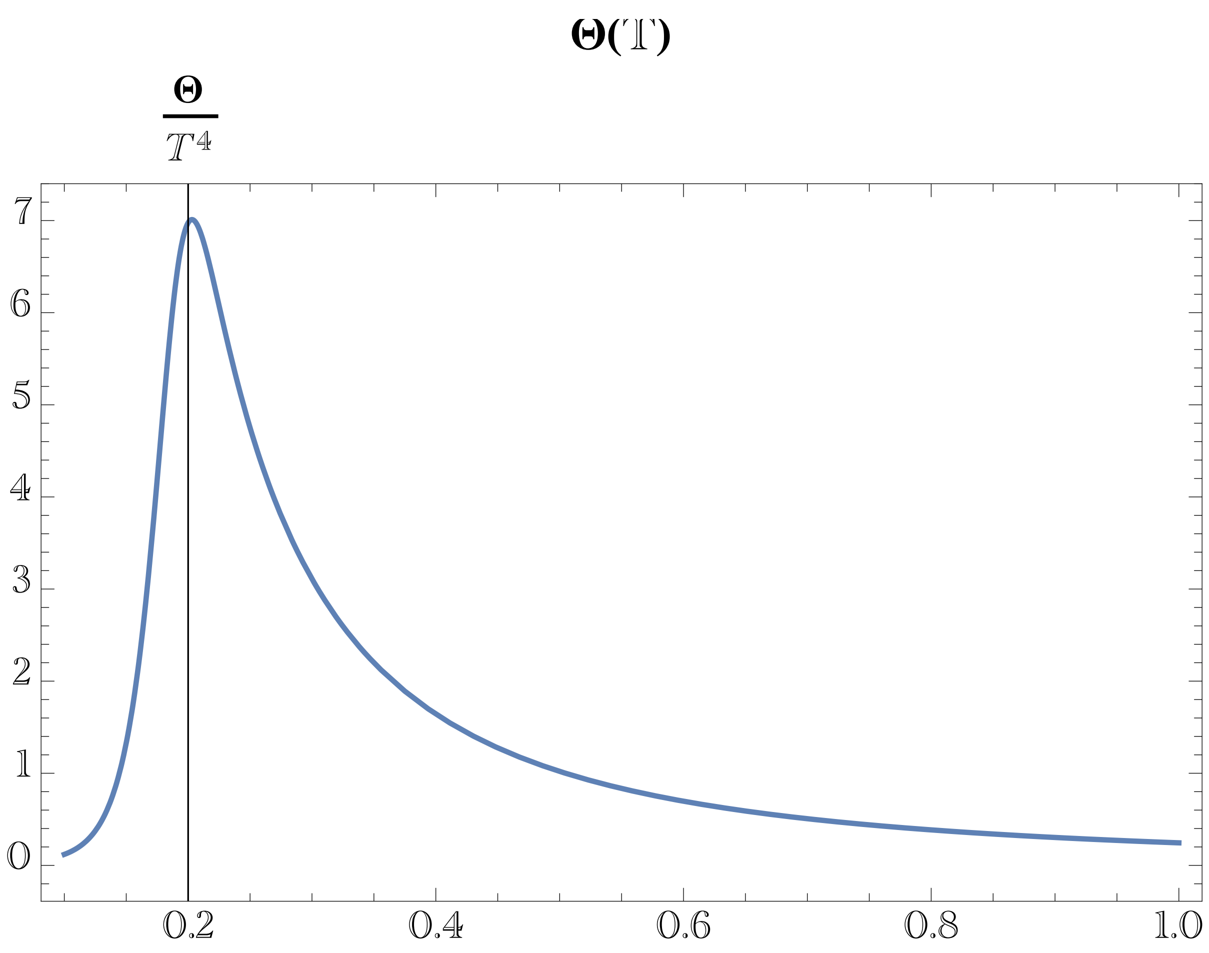
Lattice simulations:

$$\Theta = T^\mu_\mu = T^4 \left(1 - \frac{1}{(1+e^{(T-c_1)/c_2})^2}\right)\left(\frac{d_2}{T^2} + \frac{d_4}{T^4}\right)$$

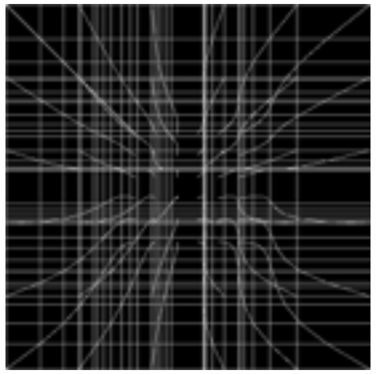
d_4 is vacuum energy term

Valid between 100 MeV and 1 GeV

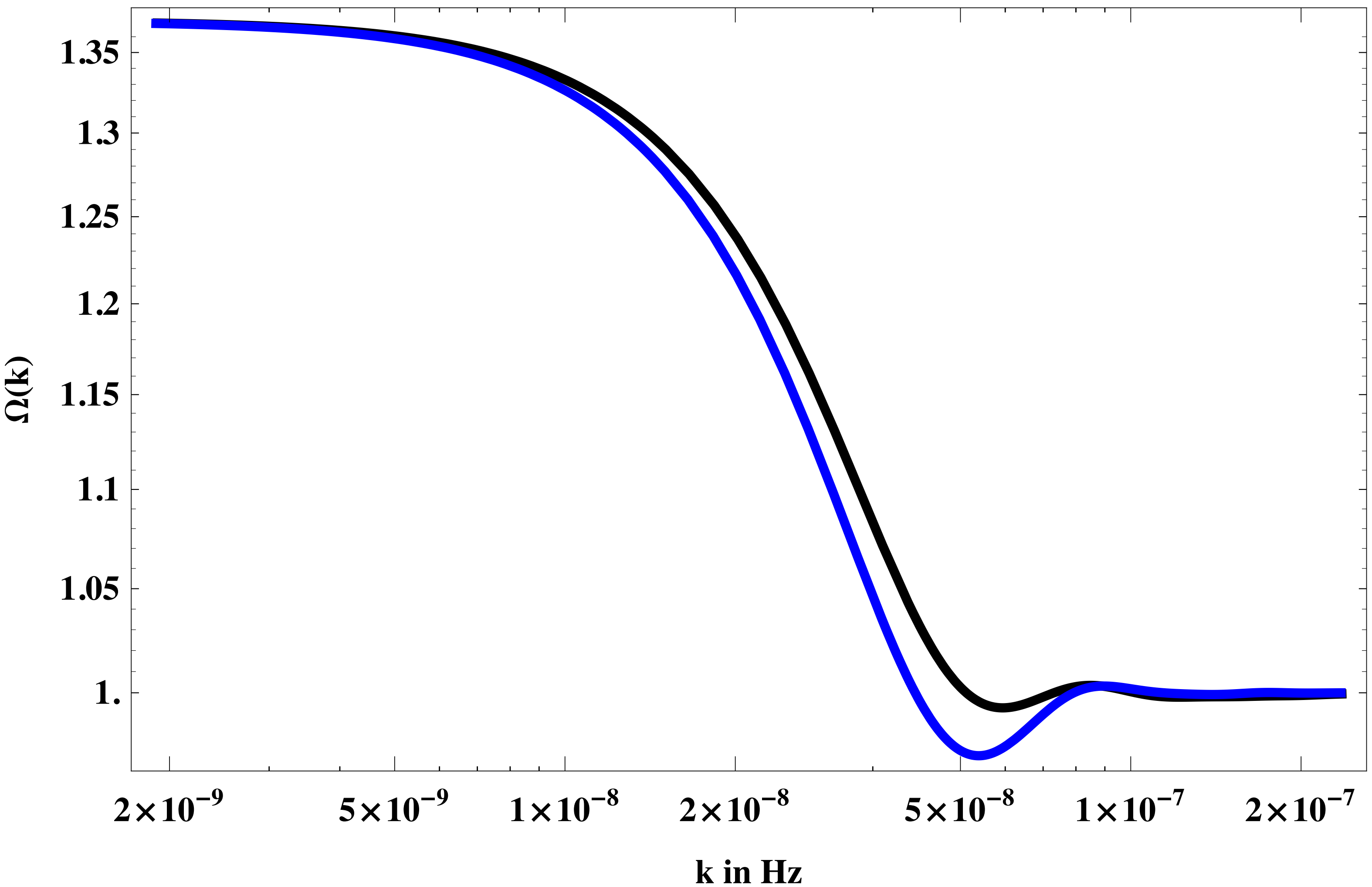
HotQCD Collaboration
hep-lat/0903.4379



QCD Phase Transition

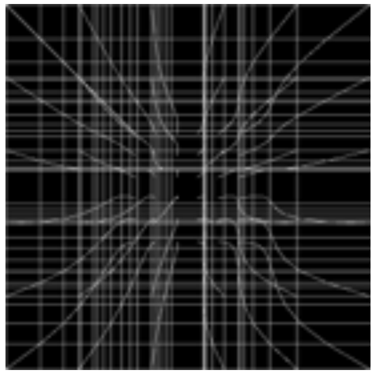


most optimistic vacuum energy shift



Almost no difference

Propagation of primordial gravitational waves



Tensor perturbations h_{ij} transverse, traceless

$$h_i^i = 0, \text{ and } \partial_k h_i^k = 0$$

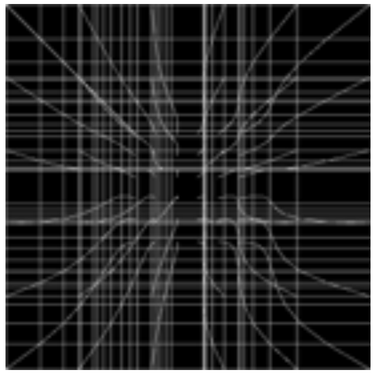
Perturbation of metric in expanding Universe

$$ds^2 = a(\tau)^2 \left(d\tau^2 - (\delta_{ij} + h_{ij}) dx^i dx^j \right)$$

use conformal time τ $\quad a(\tau) d\tau = dt$

$$a' = a\dot{a} = a^2 H \,, \qquad \frac{a''}{a} = a^2 \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right) = \frac{4\pi G}{3} a^2 T_\mu^\mu$$

Propagation of primordial gw's



Einstein equation:

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 0$$

$$h_{ij} = \sum_{\sigma=+,-} \int \frac{d^3k}{(2\pi)^3} \epsilon_{ij}^{(\sigma)} h_k^{(\sigma)}(\tau) e^{ikx}$$

$$\chi_k \equiv a h_k$$

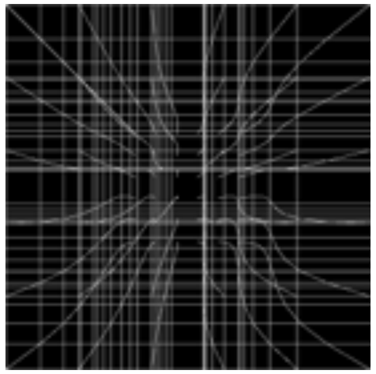
satisfies a very simple equation:

$$\chi''_k + (k^2 - \frac{a''}{a})\chi_k = \chi''_k + \left[k^2 - \frac{4\pi G}{3} a^2 T^\mu_\mu\right] \chi_k = 0$$

Exciting: equation depends on trace of stress tensor!

Might think (we did for a while) that vacuum energy will have big effect...

Energy density in gravitational waves



The energy density is then

$$\rho_h(\tau) = \frac{1}{32\pi G a^2(\tau)} \int d\ln k (\Delta_h^P)^2 \mathcal{T}'^2(\tau, k)$$

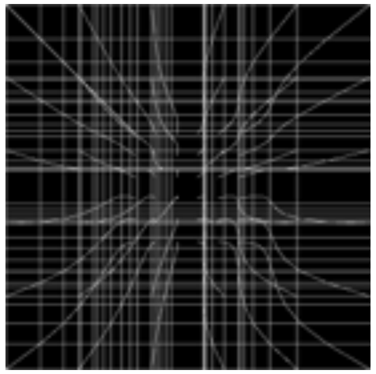
energy density per log scale
normalized to critical density

$$\Omega_h(\tau, k) \equiv \frac{\tilde{\rho}_h(\tau, k)}{\rho_c(\tau)}$$

Approximate expression:

$$\Omega_h(\tau, k) \simeq \frac{(\Delta_h^P)^2}{12 H^2(\tau) a^4(\tau)} k^2 a^2(\tau_{hc})$$

Propagation of primordial gw's



$$\chi_k'' + (k^2 - \frac{a''}{a})\chi_k = \chi_k'' + \left[k^2 - \frac{4\pi G}{3}a^2 T^\mu_\mu\right]\chi_k = 0$$

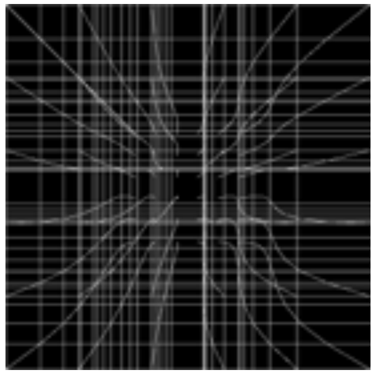
if $k^2 > \frac{a''}{a}$ just free plane wave for χ

inside horizon: actual mode χ/a is damped by 1/a

if $k^2 < \frac{a''}{a}$ then equation is $\frac{\chi''}{\chi} = \frac{a''}{a}$

outside: solution χ~a and actual mode χ/a is frozen

Propagation of primordial gw's



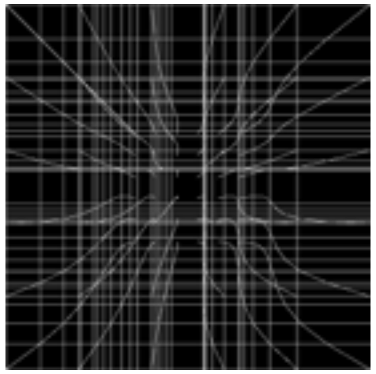
Naive horizon $\frac{a''}{a} = \frac{4\pi G}{3} a^2 T^\mu_\mu$

larger than Hubble horizon

when entering this "naive horizon"
velocity of solution still very large
expands until reaches actual
Hubble horizon

need rate of entering actual horizon

Effect of Phase Transition



Traditional description: changing number of rel. degrees of freedom in equilibrium

$$g_{\star,a} \equiv g_\star(\tau > \tau_t) \neq g_\star(\tau < \tau_t) \equiv g_{\star,b}$$

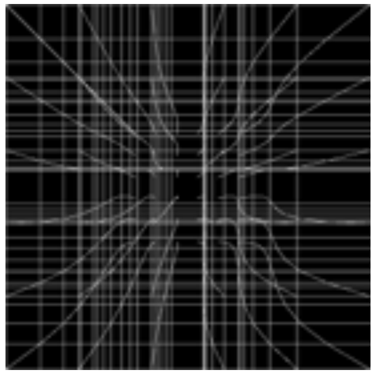
Assuming entropy is conserved:

$$S = \frac{\rho + p}{T} a^3 = const.$$

$$\rho + p \propto g_* T^4$$

$$a \propto T^{-1} g_*^{-1/3}$$

Effect of adjustment mechanism



Depends on adjustment time scale

If very quick: VE set to zero always
hard to make any distinction in QCD & EW

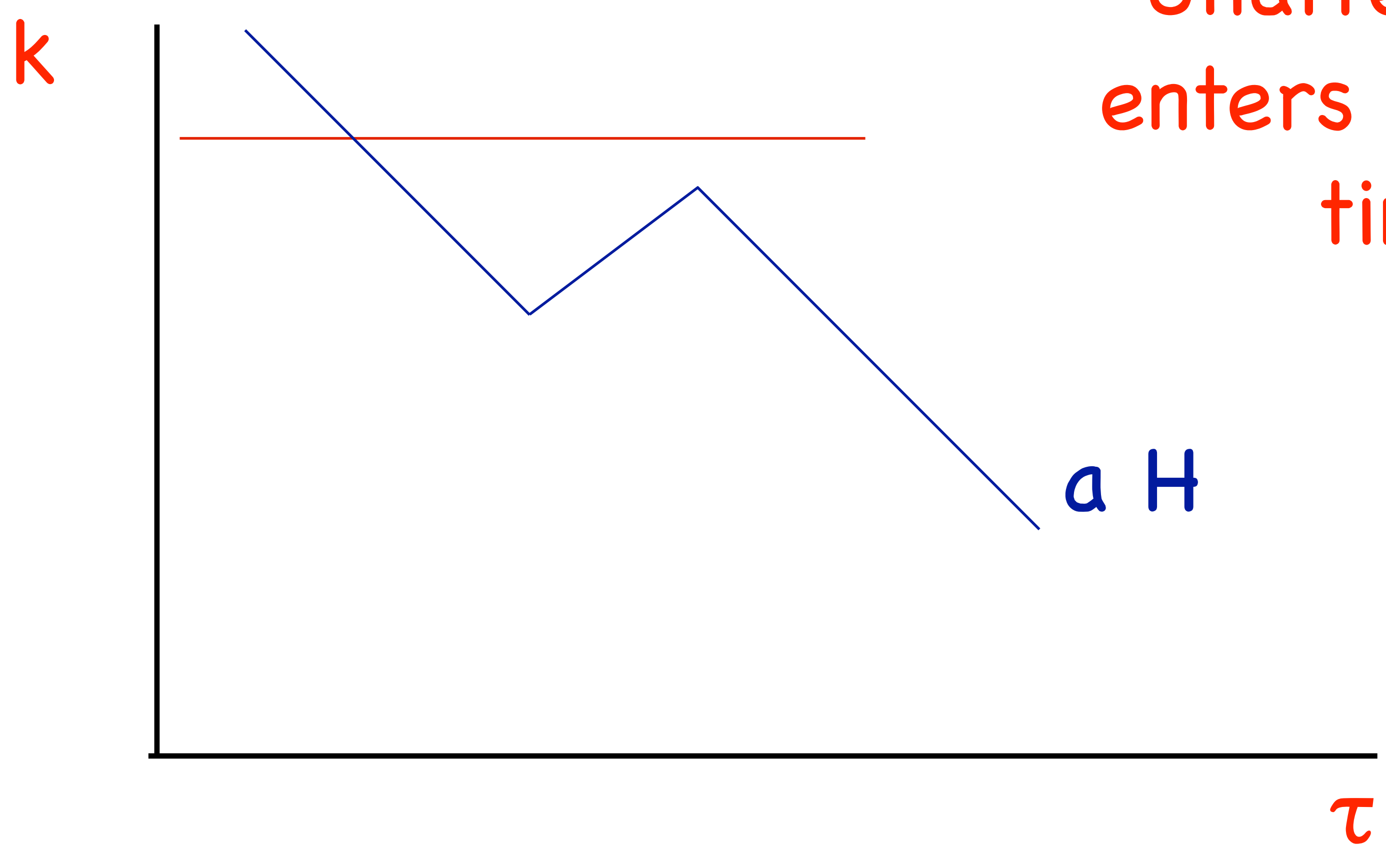
alternative: adjustment time scale
somewhat larger than that of PT

period where VE dominates
brief inflation after PT

Effect of short inflation

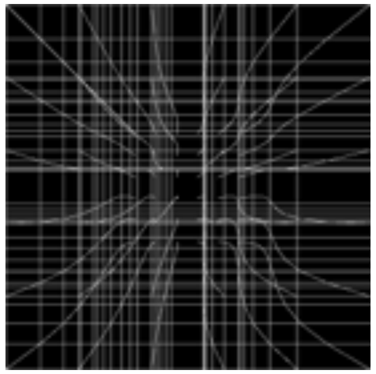
Some of the modes that entered will leave again

Some modes will only enter later



Unaffected - enters at same time

Effect of short inflation



Some of the modes that entered will leave again

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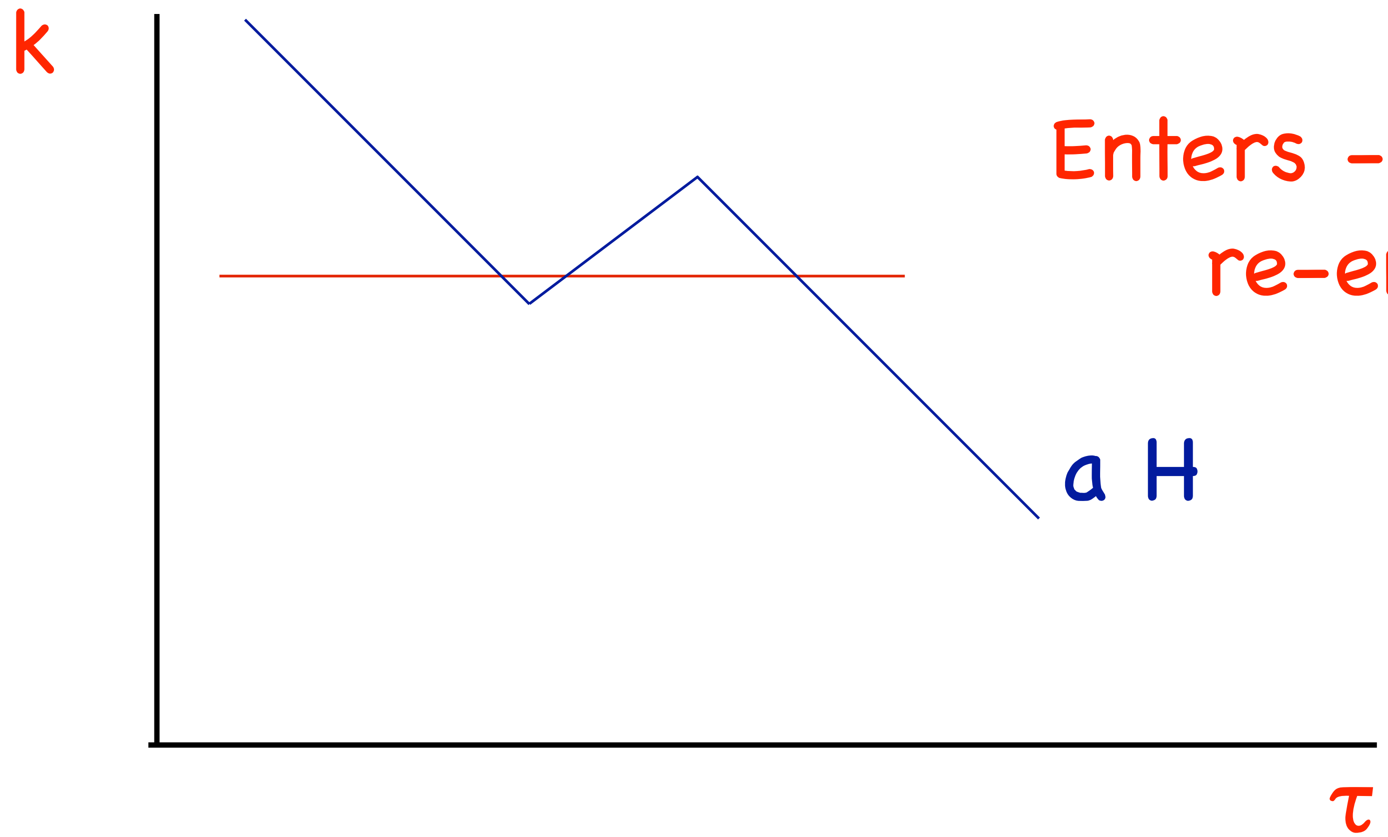
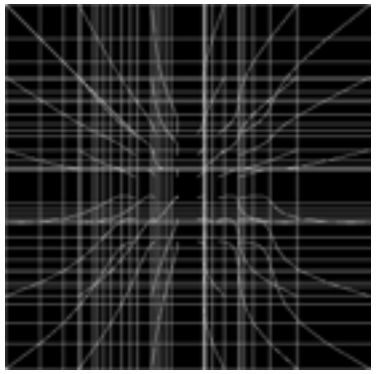


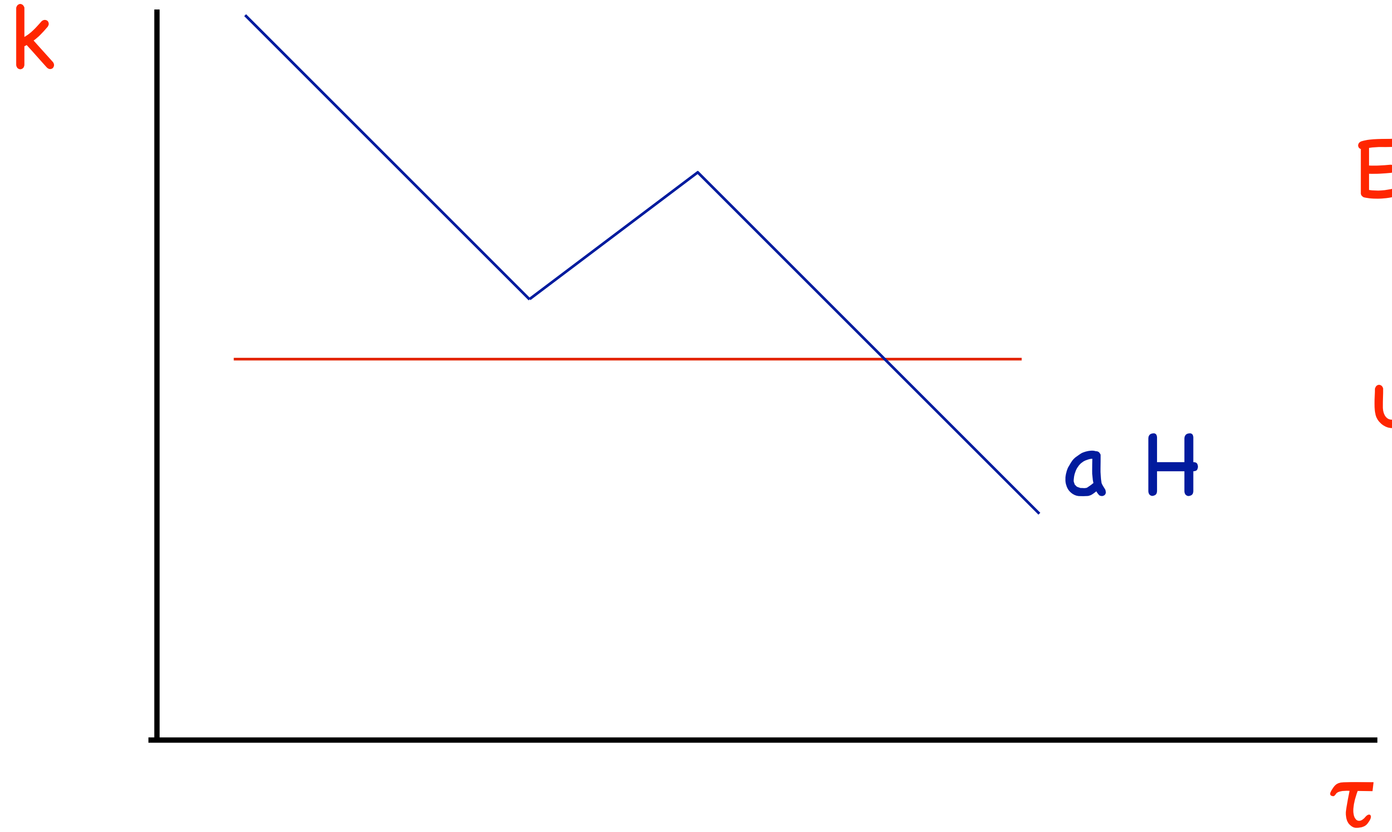
Enters - leaves - re-enters

Effect of short inflation



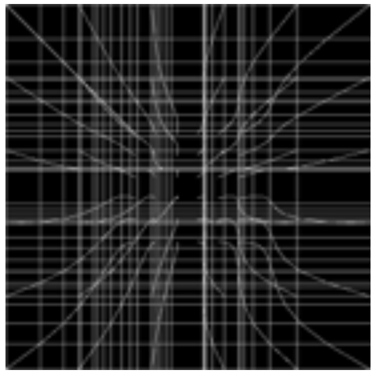
Some of the modes that entered will leave again

Some modes will only enter later



Enters later than w/o inflation unsuppressed by inflation

Effect of short inflation



large changes if the relaxation is slow compared to phase transition