# CP violation in Kaons ($\varepsilon'/\varepsilon$) and an emerging anomaly

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Based largely on:
AJ Buras, M Gorbahn, SJ, M Jamin arXiv:1507.06345 (JHEP)
M Cerda Sevilla, M Gorbahn, SJ, A Kokulu (w.i.p.)

# Outline

Kaons are good!

Direct CP violation ( $\varepsilon'$ )

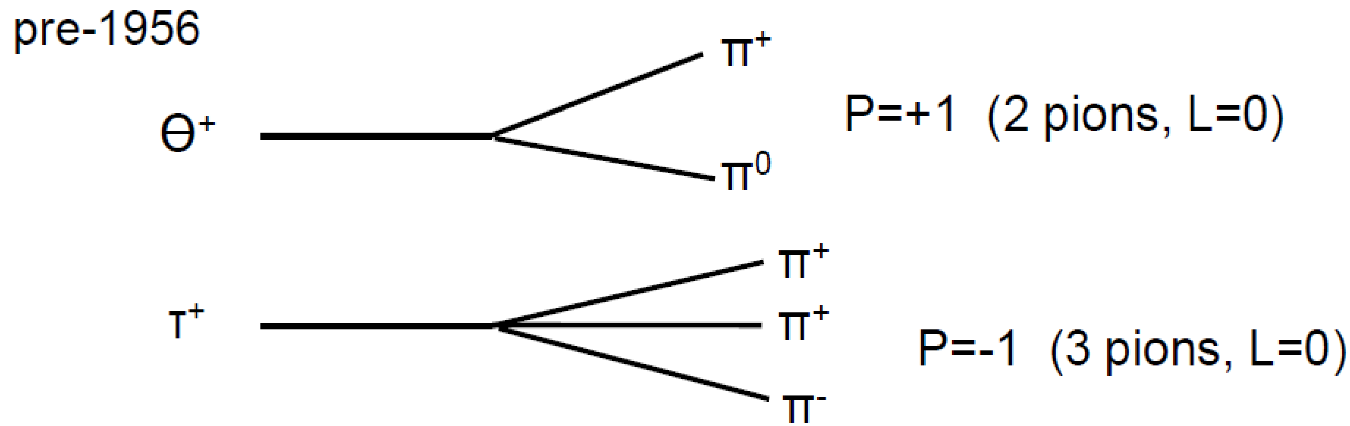
New developments in hadronic matrix elements

A new anomaly

Controlling subleading effects (NNLO, QED, LD charm)

Outlook

# Kaons: track record

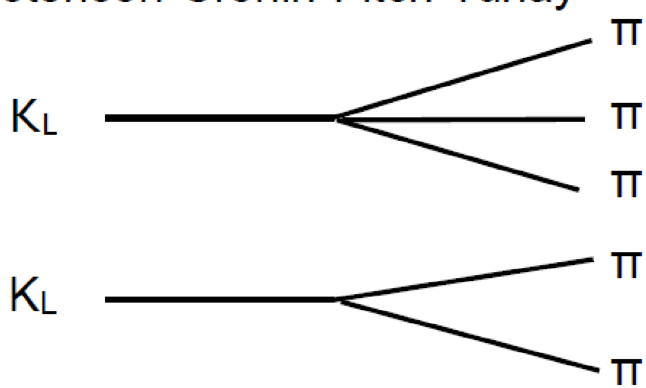



1956 proposal of parity violation (Lee & Yang)
1957 confirmed in nuclear beta decay (Wu et al)
particle renamed to K

CP still (assumed to be) conserved

# CP violation

1964 Christenson-Cronin-Fitch-Turlay




CP=-1 (3 pions, L=0)

CP=+1 (2 pions, L=0)

About one in 350 KL decay to (CP-even) two-pion states
cannot assign a conserved CP number to KL

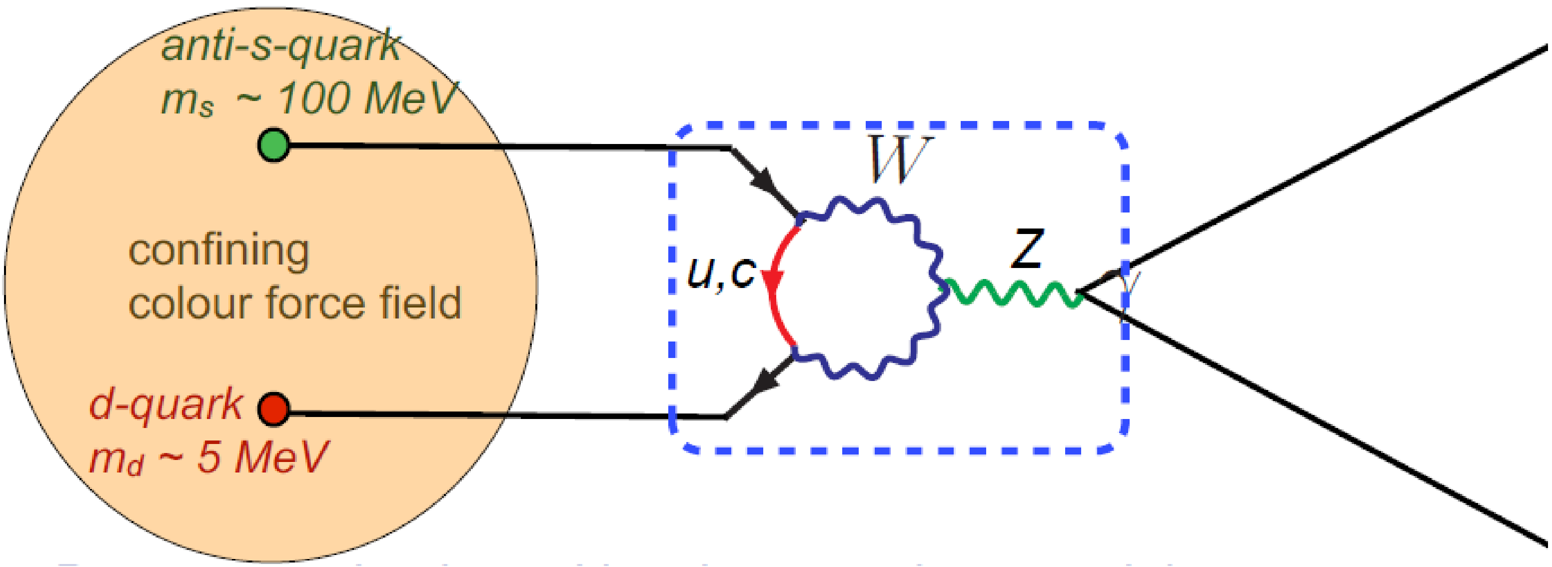
Two possibilities:

1) $K_L$ is superposition of CP states:
   indirect CP violation: parameter $\varepsilon_K = \varepsilon$ -> Zoltan Ligeti's talk

2) CP is violated in the decay of $K_L$:
   direct CP violation: parameter ε' -> this talk

1) accounts for the bulk of the observed effect.

# Charm, GIM, etc




Charm quark invented to forbid K->mu mu at tree level (GIM) (also: Gaillard-Lee, J/psi, …)

The GIM (doublet) structure kills not only K->mu mu, but also CP violation: Any CP violation in Kaons requires top (Kobayashi-Maskawa) in a loop.

# Direct CP violation in $K_L \to \pi\pi$

Precisely known experimentally for a decade

$$(\varepsilon'/\varepsilon)_{\rm exp} = (16.6 \pm 2.3) \times 10^{-4}$$

average of NA48 (CERN) and KTeV

$$\left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 \simeq 1 - 6\,\mathrm{Re}(\frac{\varepsilon'}{\varepsilon})$$

$$\eta_{00} = \frac{A(K_{\rm L} \to \pi^0\pi^0)}{A(K_{\rm S} \to \pi^0\pi^0)}, \qquad \eta_{+-} = \frac{A(K_{\rm L} \to \pi^+\pi^-)}{A(K_{\rm S} \to \pi^+\pi^-)}$$

(magnitudes directly measurable from decay rates)

Even more precise measurement possible in principle at NA62/CERN

Theoretically very complicated multi-scale problem (weak scale, charm, QCD scale)

# Weak Hamiltonian

Weak scale physics, and short-distance QCD/QED (and BSM) corrections to it, well described by a weak effective Hamiltonian. For Kaon decays, it is convenient to write

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{ud} V^*_{us} \sum_{i=1}^{10} \big( z_i(\mu) + \tau \; y_i(\mu) \big) \, Q_i(\mu) \,, \quad \tau \equiv - \frac{V_{td} V^*_{ts}}{V_{ud} V^*_{us}}$$

where z and y are real functions of the strong coupling, quark and W/Z masses (mainly top), a renormalisation scale $\mu$, and $|\tau| \sim 10^{-3}$ with a large weak phase.

$Q_i(\mu)$ are local 4-fermion operators. (Neglect dipoles.)

Decay amplitudes require $\langle \pi\pi | Q_i(\mu) | K \rangle$: nonperturbative

Current–Current:

$$Q_1 = (\bar{s}_\alpha u_\beta)_{V-A}\,(\bar{u}_\beta d_\alpha)_{V-A} \qquad Q_2 = (\bar{s}u)_{V-A}\,(\bar{u}d)_{V-A}$$

**Large coefficients, but CP-conserving (y=0). Account for K->pi pi decay rates.**

QCD–Penguins:

$$Q_3 = (\bar{s}d)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}q)_{V-A} \qquad Q_4 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V-A}$$

$$Q_5 = (\bar{s}d)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}q)_{V+A} \qquad Q_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V+A}$$

$\mathcal{O}(\alpha_s)$ **but CP-violating (y=1). However, isospin-0 final state only**

Electroweak Penguins:

$$Q_7 = \frac{3}{2}\,(\bar{s}d)_{V-A} \sum_{q=u,d,s,c,b} e_q\,(\bar{q}q)_{V+A} \qquad Q_8 = \frac{3}{2}\,(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} e_q\,(\bar{q}_\beta q_\alpha)_{V+A}$$

$$Q_9 = \frac{3}{2}\,(\bar{s}d)_{V-A} \sum_{q=u,d,s,c,b} e_q\,(\bar{q}q)_{V-A} \qquad Q_{10} = \frac{3}{2}\,(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} e_q\,(\bar{q}_\beta q_\alpha)_{V-A}$$

$\mathcal{O}(\alpha_{\rm em})$ **but can create isospin-2 state. Needed for direct CPV!**

# Isospin limit

It is useful to formulate the problem in terms of isospin (as opposed to charge) final states.

Defining $A_I \equiv \langle (\pi\pi)_I | \mathcal{H}_{\rm eff} | K \rangle$

and

$$\langle Q_i \rangle_I \equiv \langle (\pi\pi)_I \, | Q_i | \, K \rangle \,, \qquad\qquad I = 0, 2$$

One has

$$\frac{\varepsilon'}{\varepsilon} = -\frac{\omega_+}{\sqrt{2}\,|\varepsilon_K|} \left[ \frac{{\rm Im}A_0}{{\rm Re}A_0} \left(1 - \hat{\Omega}_{\rm eff}\right) - \frac{1}{a} \frac{{\rm Im}A_2}{{\rm Re}A_2} \right]$$

A small imaginary part on the l.h.s. has been neglected.
In the isospin limit, $A_2$ is pure electroweak penguin.

Moreover, the strong (rescattering) phases for a given isospin all coincide with the pi pi scattering phase shift (Watson's theorem).

Broken by QED and $m_u \neq m_d$ : parameters $\Omega_{\rm eff}, a, \omega_+$

# A trick

An important further simplification (in the 3-flavour theory, with the charm quark integrated out) is that Fierz identities and isospin imply for the purely left-handed operators:

$$\langle Q_9 \rangle_2 = \langle Q_{10} \rangle_2 = \frac{3}{2} \langle Q_1 \rangle_2$$

Hence the isospin-2 ratio depends essentially on a single ratio of hadronic matrix elements

$$\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right)_{V-A} = \mathrm{Im}\tau \, \frac{3(y_9 + y_{10})}{2z_+} \qquad \left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right)_8 = -\frac{G_F}{\sqrt{2}} \mathrm{Im}\lambda_t \, y_8^{\mathrm{eff}} \, \frac{\langle Q_8 \rangle_2}{\mathrm{Re}A_2}$$

**no** nonperturbative input! Can take Re A2 from CP-avg rates

A similar argument shows that the isospin-0 ratio mainly depends on $\langle Q_6 \rangle_0$

# Matrix element calculations

Historically: models of varying sophistication.

Define "bag parameters" through

$$\langle Q_6(\mu)\rangle_0 = -\,4h\left[\frac{m_{\rm K}^2}{m_s(\mu)+m_d(\mu)}\right]^2 (F_K - F_\pi)\, B_6^{(1/2)}$$

$$\langle Q_8(\mu)\rangle_2 = \sqrt{2}h\left[\frac{m_{\rm K}^2}{m_s(\mu)+m_d(\mu)}\right]^2 F_\pi\, B_8^{(3/2)}\,,$$

Bardeen, Buras, Gerard 1987

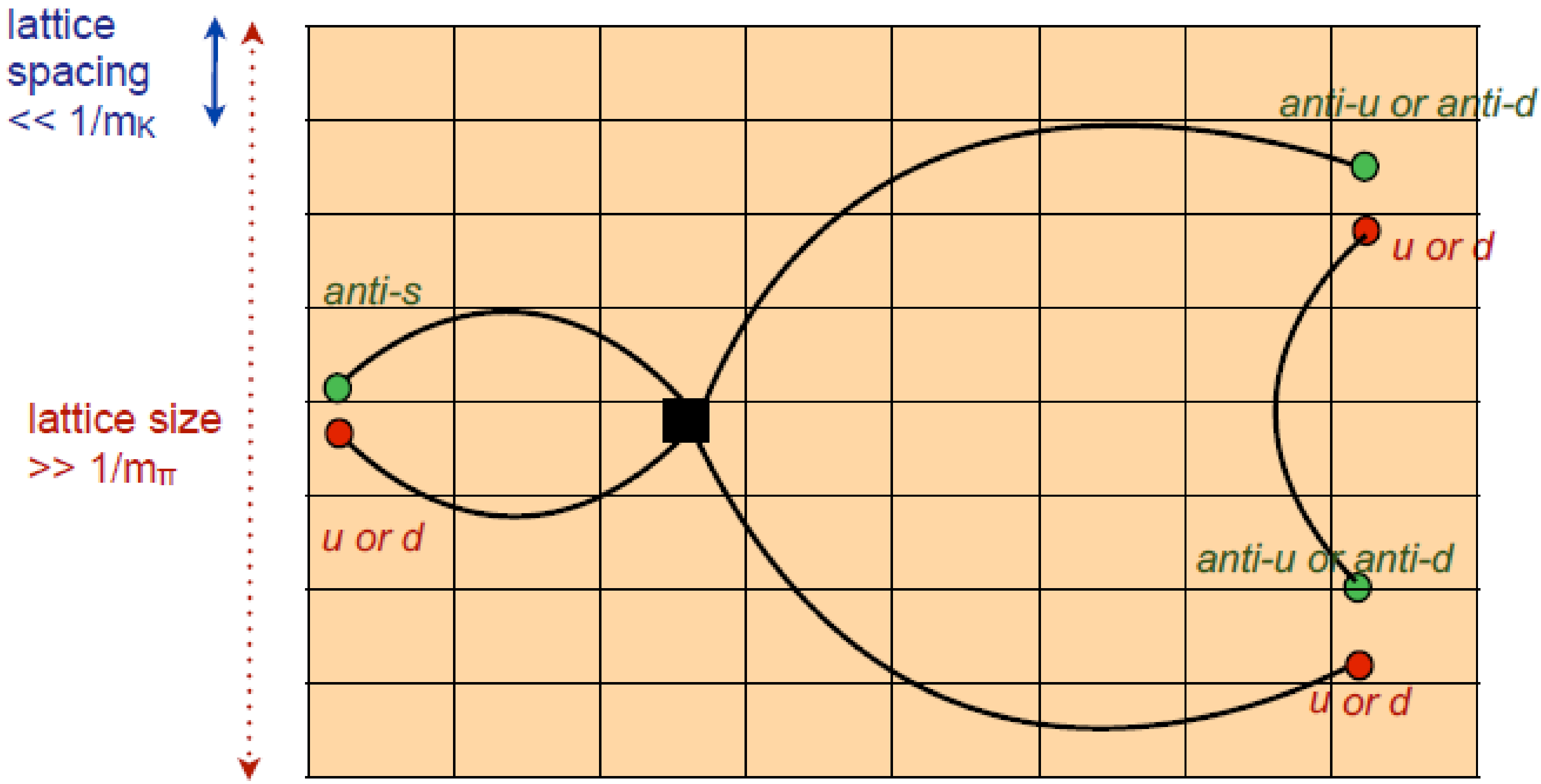
Bardeen-Buras-Gerard: chiral+vector Lagrangian with cutoff, match cutoff-dependence to perturbative RG dependence around 1 GeV to constrain parameters.

Gives values $B_8^{3/2} \sim 0.7 - 0.9$ , $B_6^{(1/2)} < B_8^{3/2}$ Buras, Gerard 2015

A major but elusive target for lattice QCD for several decades. First results with controlled errors in 2015!

RBC-UKQCD collab 1502.00263 (I=2), 1507.07863 (I=0)

# Lattice QCD computation




Need sufficiently large and fine lattice.
Need to confront several no-go theorems

# Chiral quarks

Nielsen-Ninomya:
Cannot have chiral symmetry in d=4 lattice (no lattice Dirac operator that anticommutes with gamma_5)

various workarounds include:

- staggered fermions and “rooting” (4d, fast, but various issues with continuum limit)

- 5d domain-wall fermions
  gives exact 4d chiral symmetry, zero modes localised at boundary

  employed by RBC-UKQCD

# Strong phases

Maiani-Testa

no access to rescattering phases on Euclidean lattice

Luescher

pi pi phase shifts can be determined from the volume dependence of the pi pi energy spectrum

(lattice calculation anyway done for several finite volumes)

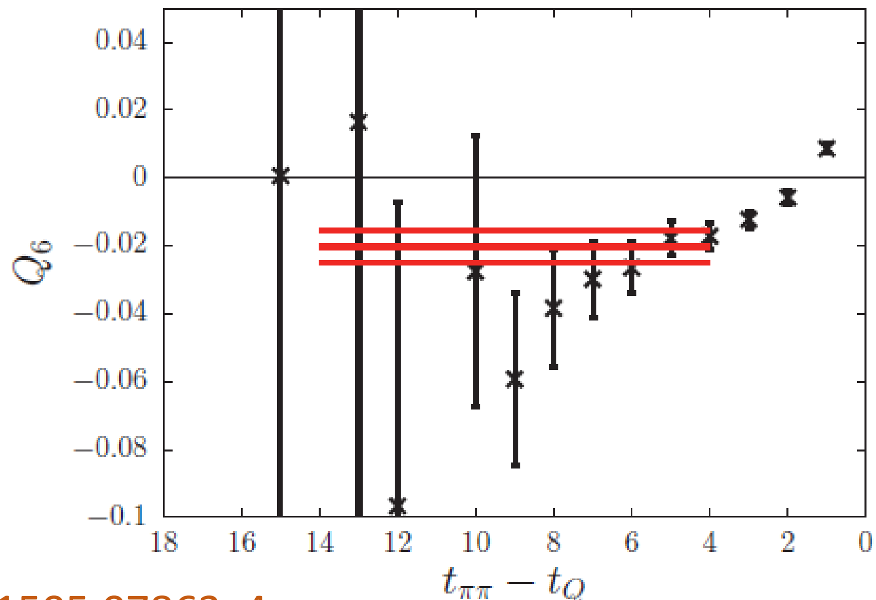
Then used to restore the phases of the matrix elements (cf Watson’s theorem)

# Vacuum subtraction

Lattice calculations are done in position space (and real time), fitting Green's functions to sums of exponentials.

The isospin-2 state, by a clever choice of Green's functions and boundary conditions, can be made the state of smallest energy, so $\langle Q_i \rangle_2$ can be extracted.

For isospin-0, the vacuum is always lowest. Use a variety of tricks to suppress the vacuum contributions; sizable statistical uncertainty remains




RBC-UKQCD, 1505.07863v4

# Phenomenology and a new anomaly

# $\varepsilon'/\varepsilon$ master formula

Including estimates of isospin breaking,

Buras, Buchalla, ... 1990; Buras, Jamin 1993;1996; Bosch et al 1999;
Buras, Gorbahn, SJ, Jamin arXiv:1507.06345

$$\omega_+ = a\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} = (4.53 \pm 0.02) \times 10^{-2}$$

from experiment
Cirigliano et al 2003

leading isospin breaking
Cirigliano et al 2003

neglect small imaginary part (for simplicity; could easily be restored)

$$\frac{\varepsilon'}{\varepsilon} = -\frac{\omega_+}{\sqrt{2}|\varepsilon_K|}\left[\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}(1-\Omega_{\mathrm{eff}}) - \frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right]$$

from experiment

QCD isospin amplitudes
calculate in terms of weak Hamiltonian
perturbative NLO Wilson oefficients
& numerous nonperturbative hadronic matrix elements

where the ratios of amplitudes are evaluated making use of operator and isospin relation as discussed before

Buras et al 1990; Buras, Gorbahn, SJ, Jamin arXiv:1507.06345

# Inputs

| | value range |
|---|---|
| $B_6^{(1/2)}$ | $0.57 \pm 0.19$ |
| $B_8^{(3/2)}$ | $0.76 \pm 0.05$ |
| $q$ | $0.05 \pm 0.05$ |
| $B_8^{(1/2)}$ | $1.0 \pm 0.2$ |
| $p_{72}$ | $0.222 \pm 0.033$ |
| $p_3$ | $0 \pm 0.5$ |
| $p_5$ | $0 \pm 0.5$ |
| $p_{70}$ | $0 \pm 1/3$ |
| $\mathrm{Im}\lambda_t$ | $(1.4 \pm 0.1) \times 10^{-4}$ |
| $m_t(m_t)$ | $(163 \pm 3)$ GeV |
| $m_s(m_c)$ | $(109.1 \pm 2.8)$ GeV |
| $m_d(m_c)$ | $(5.4 \pm 1.9)$ GeV |
| $\alpha_s(M_Z)$ | $0.1185 \pm 0.0006$ |
| $s_W^2$ | $0.23126$ |
| $\hat{\Omega}_{\mathrm{eff}}$ | $(14.8 \pm 8.0) \times 10^{-2}$ |

parameterisation of hadronic matrix elements

CKM input

isospin breaking

# Result

Combining all errors in quadrature,

$$(\varepsilon'/\varepsilon)_{\mathrm{SM}} = (1.9 \pm 4.5) \times 10^{-4}$$

Buras, Gorbahn, SJ, Jamin, arXiv:1507.06345

$$(\varepsilon'/\varepsilon)_{\mathrm{exp}} = (16.6 \pm 2.3) \times 10^{-4}$$

average of NA48 and KTeV

**2.9 sigma discrepancy**

New physics or underestimated error? (Cf B-anomalies)

Central values differ by order of magnitude, so a reduced theory error potentially greatly enhance significance

The new theory result is quite consistent with old model estimates; the lattice corroborates these and gives for the first time a meaningful error estimates

# Error budget

Buras, Gorbahn, SJ, Jamin, arXiv:1507.06345

RBC-UKQCD 2015

| quantity | error on $\varepsilon'/\varepsilon$ | quantity | error on $\varepsilon'/\varepsilon$ |
|---|---|---|---|
| $B_6^{(1/2)}$ | 4.1 | $m_d(m_c)$ | 0.2 |
| NNLO | 1.6 | $q$ | 0.2 |
| $\hat{\Omega}_{\rm eff}$ | 0.7 | $B_8^{(1/2)}$ | 0.1 |
| $p_3$ | 0.6 | $\mathrm{Im}\lambda_t$ | 0.1 |
| $B_8^{(3/2)}$ | 0.5 | $p_{72}$ | 0.1 |
| $p_5$ | 0.4 | $p_{70}$ | 0.1 |
| $m_s(m_c)$ | 0.3 | $\alpha_s(M_Z)$ | 0.1 |
| $m_t(m_t)$ | 0.3 | | |

all in units of 10^-4

(still) completely dominated by $\langle Q_6 \rangle_0 \propto B_6^{1/2}$

next is NNLO

isospin-breaking: current treatment relies on chiral perturbation theory and 1/N approximations

# Prospects and goals

With good prospects for more precise lattice results, subleading effects will become leading.

- Current results rely on 3-flavour weak Hamiltonian at NLO (charm integrated out); second largest error.

  Hence NNLO important

  more generally perturbation theory at charm scale doubtful: need to reformulate problem with dynamical charm (4-flavour theory)

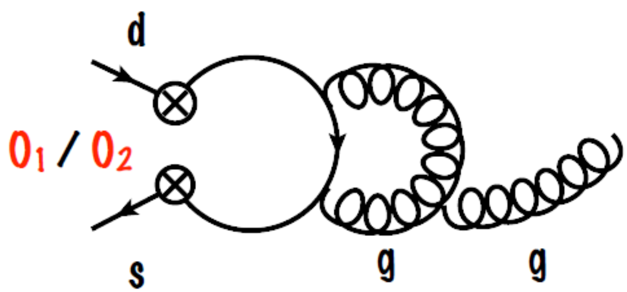
- Current calculations of isospin-breaking quark mass and QED effects rely on chiral perturbation theory and a matching of the 3-flavour weak Hamiltonian on it. Large-N limit only, no systematic improvement

# NNLO calculation

Cerda Sevilla, Gorbahn, SJ, Kokulu, w.i.p

NNLO weak Hamiltonian only known above b mass (from B->Xs gamma)

require bottom and charm threshold matching and (less importantly) NNLO mixing of QCD into EW penguins



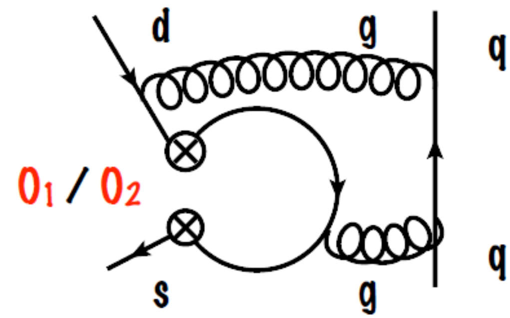




Fig. M Cerda Sevilla

extend the formalism (including operator relations, hadronic matrix element ratios, etc) to 4-flavour theory.
Will eventually obviate the need for perturbation theory at the charm scale.
P reliminary results show small perturbative corrections

# Isospin breaking

complicated, particularly QED effects (IR subtractions, real emission, lattice matching, …)
- don’t respect the two-amplitude structure
- violate Watson’s theorem

Now conceptually understood on the lattice in QED perturbation theory. In practice need to

- define QED expansion of matrix element ratios
- carefully define&express observable at O(alpha).
- disentangle QED RG evolution from matrix element expansion, for matching short-distance and lattice

No more $\Omega_{\rm eff}$ !

Cerda Sevilla, Gorbahn, SJ, Kokulu, w.i.p

# Scheme issues

Wilson coefficients depends on renormalisation scale (and scheme)
Must be cancelled by a proper matrix element calculation.

Wilson coefficients calculated in dim. reg. – not on lattice!

Currently use of momentum-space schemes on lattice.
**Conversion to MSbar more demanding than calculating the Wilson coefficient.**
(The only existing NNLO calculation of this sort is for the light quark masses!)

Separating lattice and continuum parts of calculation is subtle in presence of operator mixing and QED corrections!

Cerda Sevilla, Gorbahn, SJ, Kokulu, w.i.p

# Outlook

Progress on the lattice is removing a long-standing bottleneck for precise calculations of $\varepsilon'/\varepsilon$
(and some other Kaon observables)

A 3 sigma discrepancy has emerged.

The significance is dominated by the theoretical error.

There are systematic ways to improve the three most important error sources: lattice, NNLO, and isospin breaking. I have briefly discussed work on the latter two.