CERN percolation course: Tentative layout

Jesper Lykke Jacobsen

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1 Introduction and overview

- Potts model and FK expansion
- Percolation: a trivial theory? $Z = 1$ and $\langle \epsilon\epsilon \cdots \rangle = 0$.
- Interesting observables are non-local: wrapping probabilities, connectivity properties, etc.
- Fractal dimensions of cluster, hull, external perimeter, watermelons, shortest path, backbone, k-connected clusters.
- Upper critical dimension $d_{\rm uc} = 6$. Relation with φ^3 theory.
- Lower critical dimension $d_{\rm lc} = 2$. Relation with CFT: $c = 0$ minimal model with trivial Kac table. Interesting operators in extended Kac table, but some are not accounted for ("too non-local"?).
- Layout of the following lectures.

Informal blackboard presentation.

2 Geometrical Coulomb gas ($d = 2$)

- Basically equivalent to Dotsenko-Fateev's Coulomb gas construction. But geometrical framework offers some advantages.
- Equivalences between FK clusters, loops and the six-vertex model. Lattice algebras.

- Geometrical construction of Liouville field theory
- Marginality requirement and the Loop Ansatz
- Derivation of geometrical two-point functions. Kac table interpretation. CG duality.

Follows M2 lecture notes.

3 Boundary CFT ($d = 2$)

- Cardy's CG construction on the annulus
- Crossing formulae
- The $\varphi_{1,2}$ operator, correlation functions and more crossing formulae
- Remarks on the link with Stochastic Loewner Evolution (SLE)
- Conformal Loop Model with one and two boundaries

Follows M2 lecture notes, Cardy (Les Houches), and Potts-CLM papers (1BTL and 2BTL).

4 Observables in general dimension

- Introduction to indecomposability in non-unitary field theories
- Classification of local spinless operators from elementary representation theory of S_Q
- Extension to arbitrary N-point (bulk) operators
- Consequences: Jordan cells, indecomposability parameters β, and logarithmic correlation functions
- Chiral and non-chiral β in $d = 2$

Follows GGI talk (first part), the SCGP review (2d) and puzzle paper.

5 Timelike Liouville theory ($d = 2$)

- Structure constants are beyond the CG framework
- Electric and magnetic three-point functions
- Probability of having three points in the same FK cluster
- DOZZ formula and analytical continuation
- Fun and subtleties at $c = 1$

Follows `arXiv:1509.xxxxx` (due very shortly), Viti-Delfino and unpublished notes.