THERMODYNAMICS IN QUANTUM OPTICAL SETTINGS: WHAT IS TRULY QUANTUM HERE?

D. Gelbwaser-Klimovsky,, M. Kolar. N. Erez, W. Niedenzu, R. Alicki & G.K.

Thermodynamic laws and bounds are not well understood for quantum-system manipulations . We challenge:

- Thermodynamic equilibrium (First law?)
- Carnot efficiency bound of quantum heat engines/refrigerators (Second law)
- Unattainability of absolute zero (Third law)

מכון ויצמן למדע
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Crete-Talk 16

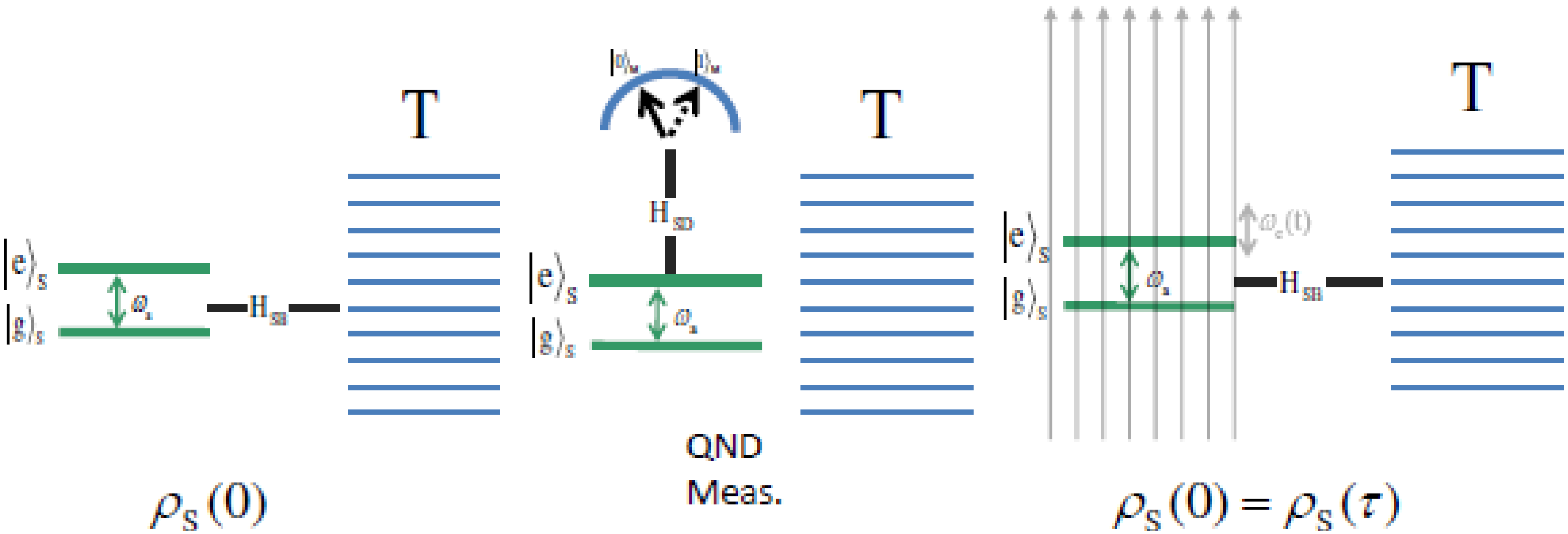
Non-selective measurements disturb thermodynamic equilibrium

'Holy entropy! It's boiling!'

Extracting work via (non-selective) measurements: Demented Maxwell's demon, but bath has memory!

Can we bypass Kelvin: extract work from a single-bath engine?

D. Gelbwaser, N. Erez, R. Alicki & G.K. Phys. Rev. A 88, 022112 (2013)

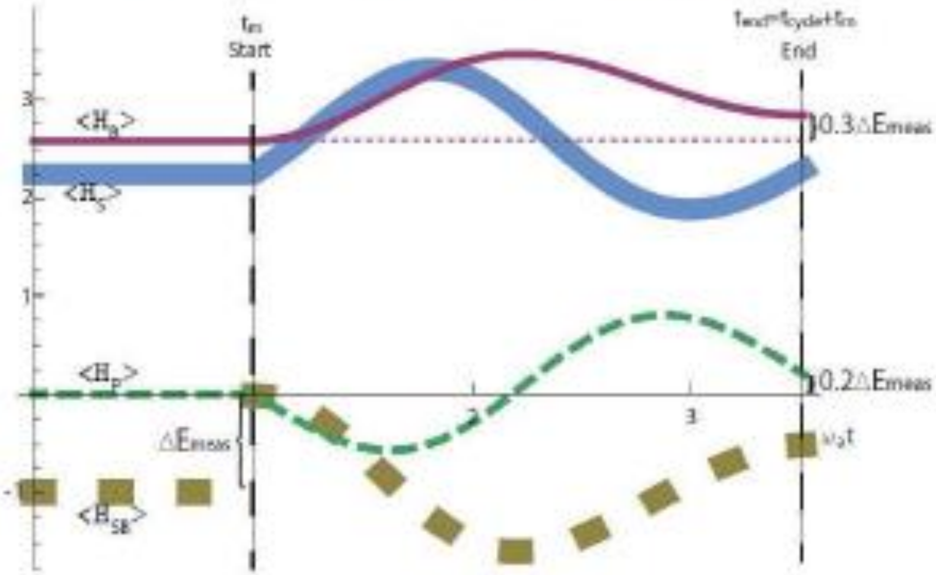


$$\rho_{SB} \xrightarrow{\text{Measurement}} \rho_S \otimes \rho_B \xrightarrow{\text{Modulation}} \ldots$$

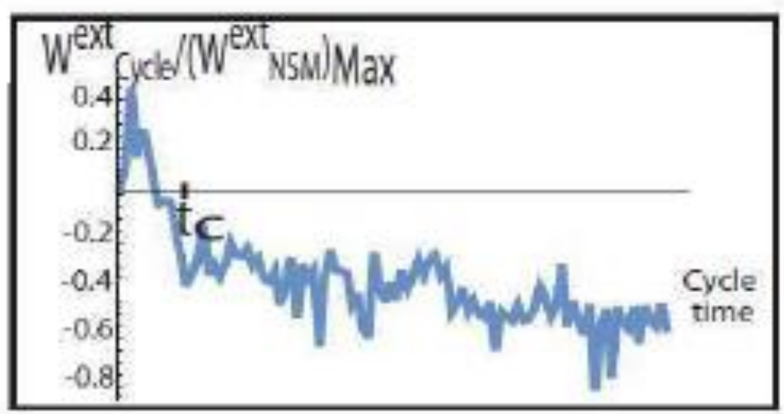
QND non-selective measurement of the system energy (Measure system energy and do not read the results)

The measurement does not change the state of the system or the bath, but changes the total state.

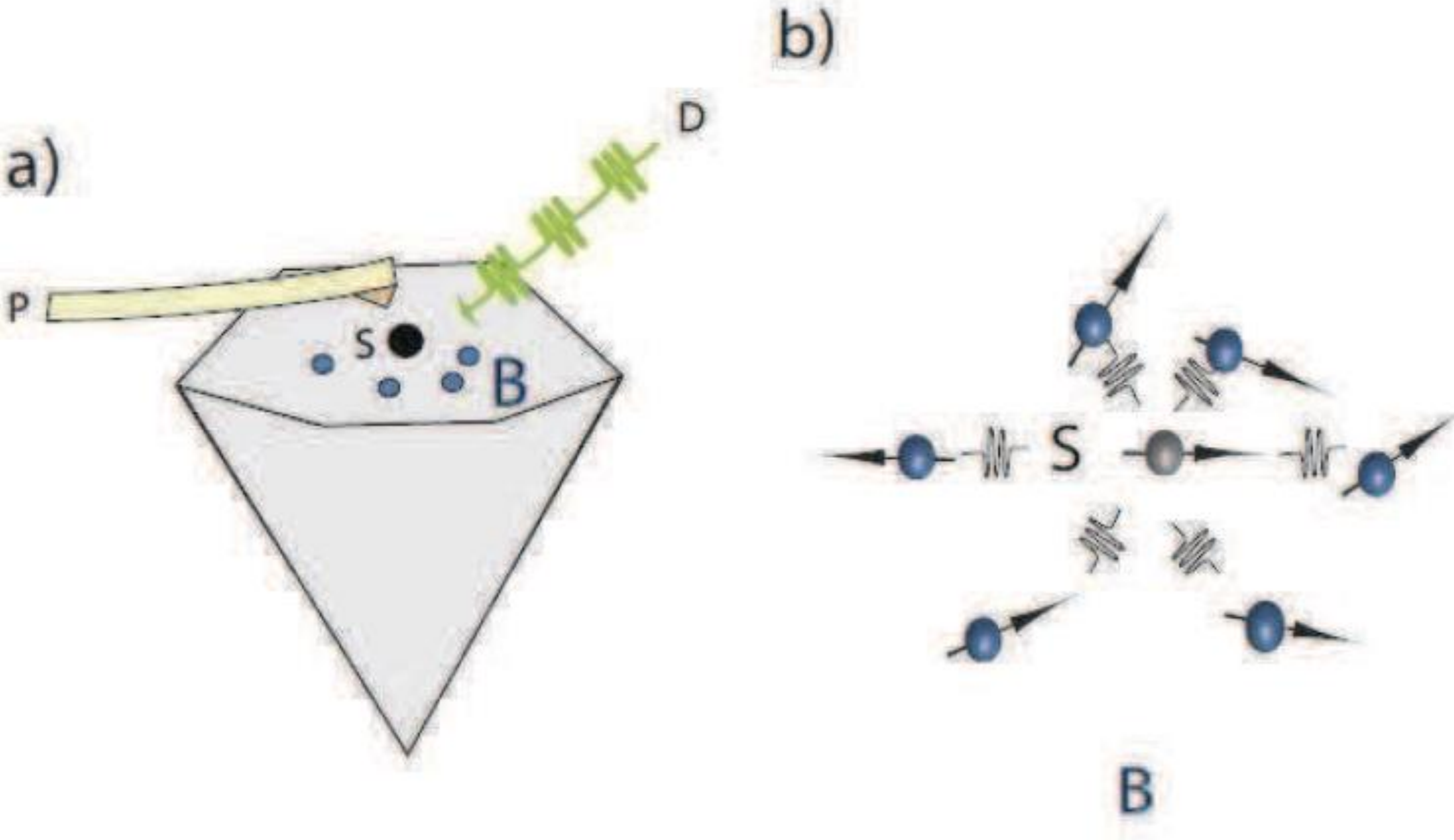
Where does the work come from?



Work extraction only for short cycles



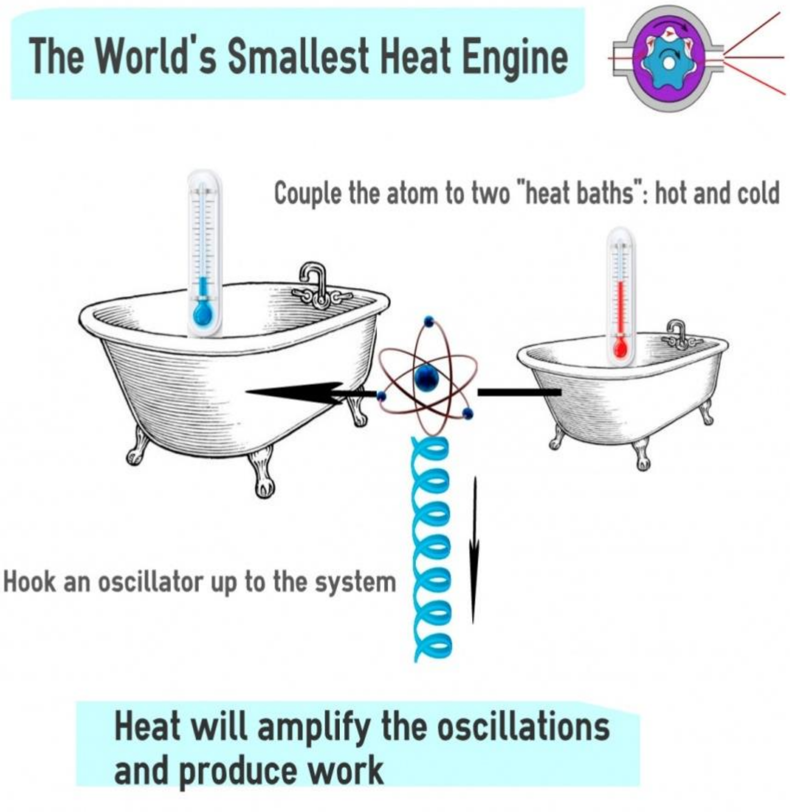
Experimental scenarios



$$t_{cycle} \leq \mu\,\mathrm{sec} \leq t_{correl}$$

Rapport between thermodynamics and quantum mechanics for quantum heat machines?
The basic model: a two-level system (TLS) whose energy is periodically modulated while the system is coupled to two distinct thermal baths.

1)



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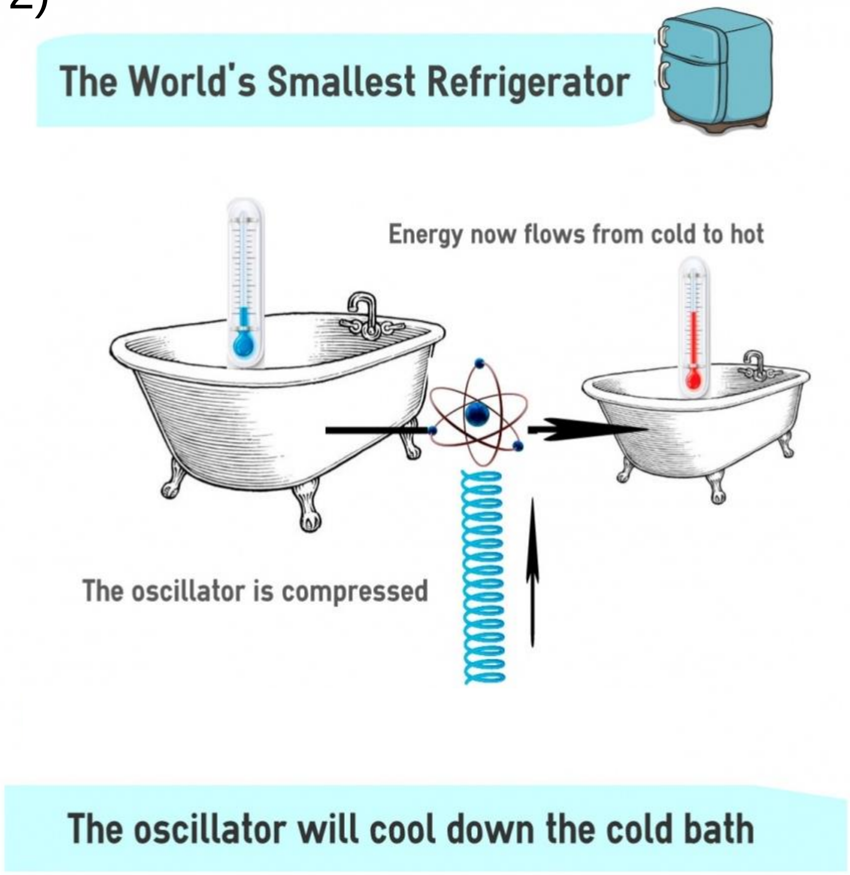


M. Kolar, et al. PRL **109** 090601 (2012));D. Gelbwaser-Klimovsky, et al. PRE **87,** 012140 (2013);
EPL **103** 60005 (2013); PRE (2014; Sci. Repts. (2015); Adv. Atom. Mol Opt. Phys., **64** (2015)

Quantum bath Refrigeration: Towards Absolute Zero?

M. Kolar, D. Gelbwaser-Klimovsky, R. Alicki & G.K. PRL **109,** 90601 (2012)

$$\mathrm{d}T_C/\mathrm{d}t = -AT_C^{\gamma}.$$

γ : Determined by bath dispersion

$$d\omega/dk \sim \omega^{\frac{1-\gamma}{2}}(k)$$

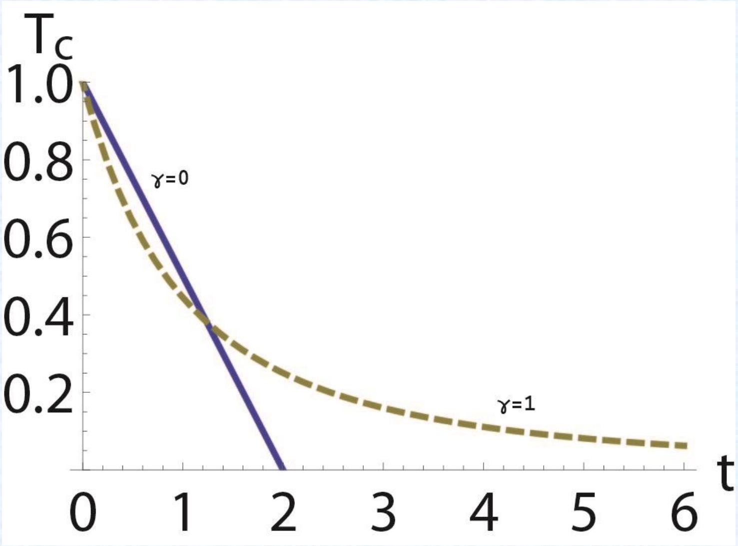
$T_C \to 0$:

i) *Acoustic phonons* $\quad \omega(\vec{k}) \sim v|\vec{k}|; \quad d\omega/dk \to const. \quad (\gamma = 1)$

$\omega(\vec{k}) \sim k^2$

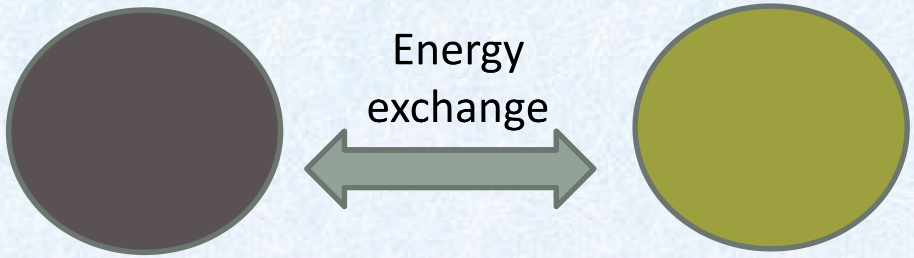
ii) *Magnons (spin-wave)/dipole-coupled atom chain:* $\quad d\omega/dk \to 0$ (freezing)

$(\gamma = 0)$

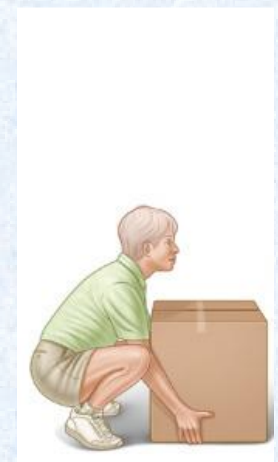


Nernst's 3rd law unattainability principle challenged

ENERGY EXCHANGE



Energy exchange is divided into work and heat….

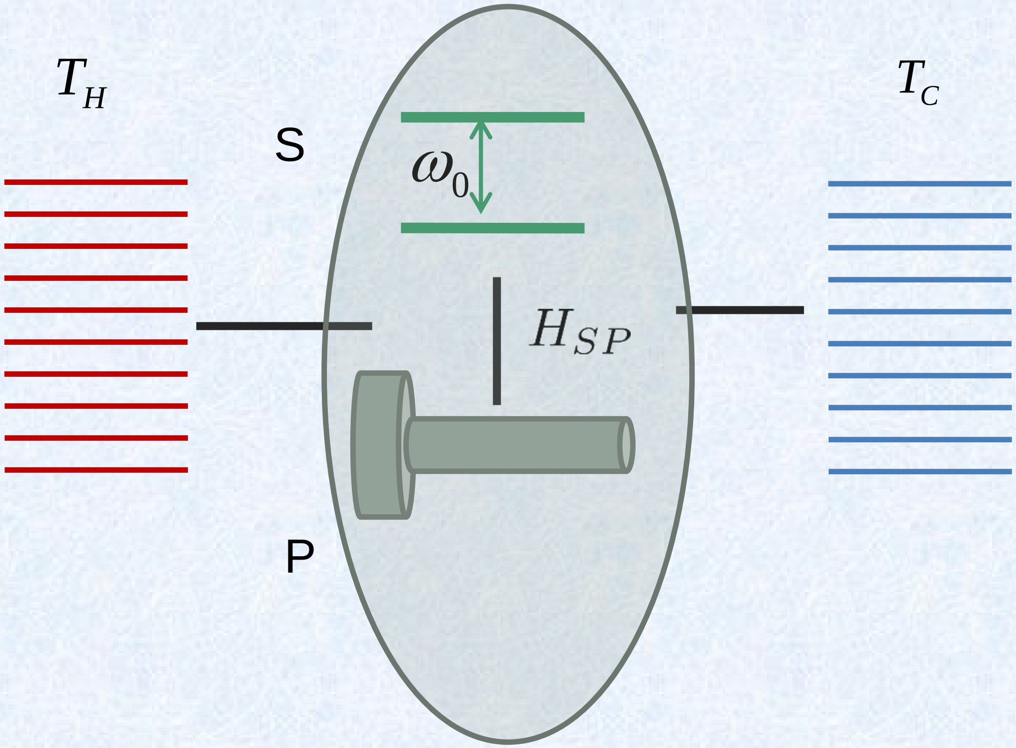


… and they are really different.

Work-power? $\longrightarrow$ $P = J_C + J_H = \langle \dot{H}_P \rangle$

No! 2nd law violated

1st law: heat-input = energy-output



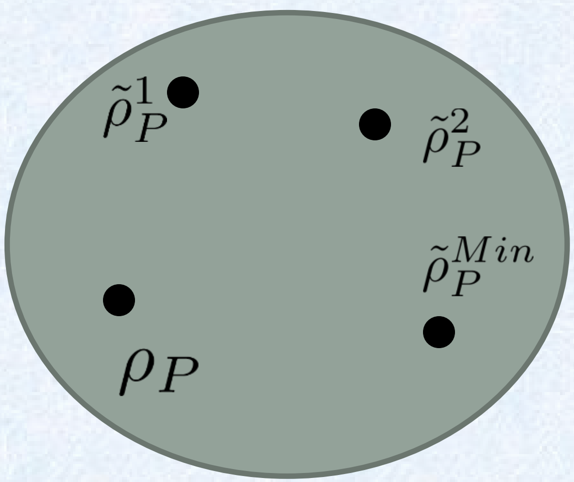
CORRECT WORK DEFINITION

D. Gelbwaser-Klimovsky et al. EPL (2013); PRE (2014)

In a thermally adiabatic process $\Delta E = W$ and $\dot{S} = 0$ $\qquad (P = \dot{W})$

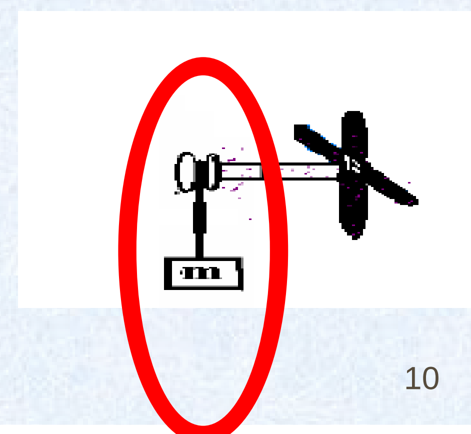
Take a state of the piston ρ_P. If $\tilde{\rho}_P = U \rho_P U^\dagger$, $(S(\rho_P) = S(\tilde{\rho}_P))$ then

$$W = \langle H_P \rangle_{\rho_P} - \langle H_P \rangle_{\tilde{\rho}_P}$$



$$W^{Max}(\rho_P) = \langle H_P \rangle_{\rho_P} - \langle H_P \rangle_{\tilde{\rho}_P^{Min}}$$

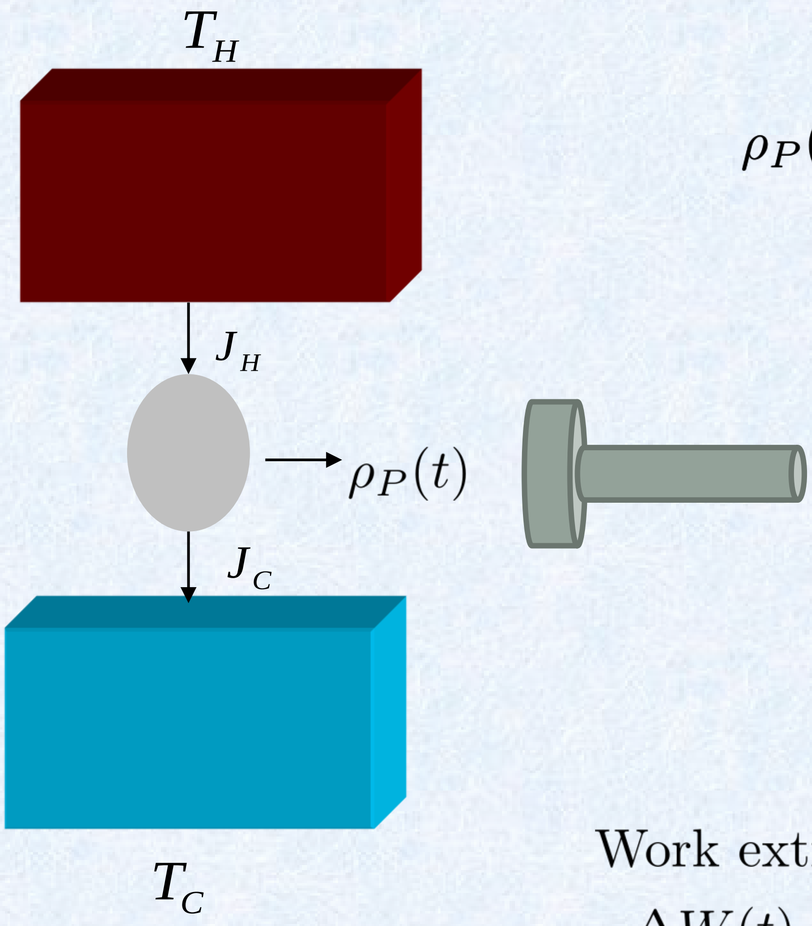
potentially extractable work ("stored" in ρ_P)



Lenard, J. Stat. Phys. **19, 575 (1978).**

W. Pusz, S.L. Woronowicz, Comm. Math. Phys. **58, 273 (1978).**

WORK CAPACITY EVOLUTION (PISTON CHARGING)



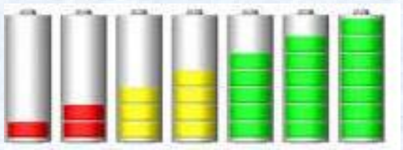
$\rho_P(0)$ $\qquad W^{Max}(\rho_P(0))$

$W^{Max}(\rho_P(t))$

Work extracted from engine:

$$\Delta W(t) \equiv W^{Max}(\rho_P(t)) - W^{Max}(\rho_P(0))$$

The engine is a charger, the piston is a battery



D. Gelbwaser-Klimovsky, R. Alicki, G. Kurizki *EPL* **103** 60005 (2013)

Work capacity is non-passivity

D. Gelbwaser-Klimovsky et al. EPL (2013); PRE (2014)

Min. energy (same entropy)

bound: $$(W_P)_{Max} \leq \langle H_P(\rho_P)\rangle - \langle H_P(\rho'_P)\rangle_{Gibbs};$$

$$(\rho'_P)_{Gibbs} = Z^{-1} e^{-\frac{H_P}{T_P}}$$

Effective temperature

Real work-power: $$\frac{d(W_P)_{Max}}{dt} = \frac{d\langle H_P\rangle}{dt} - T_P(t)\dot{\mathcal{S}}_P,$$

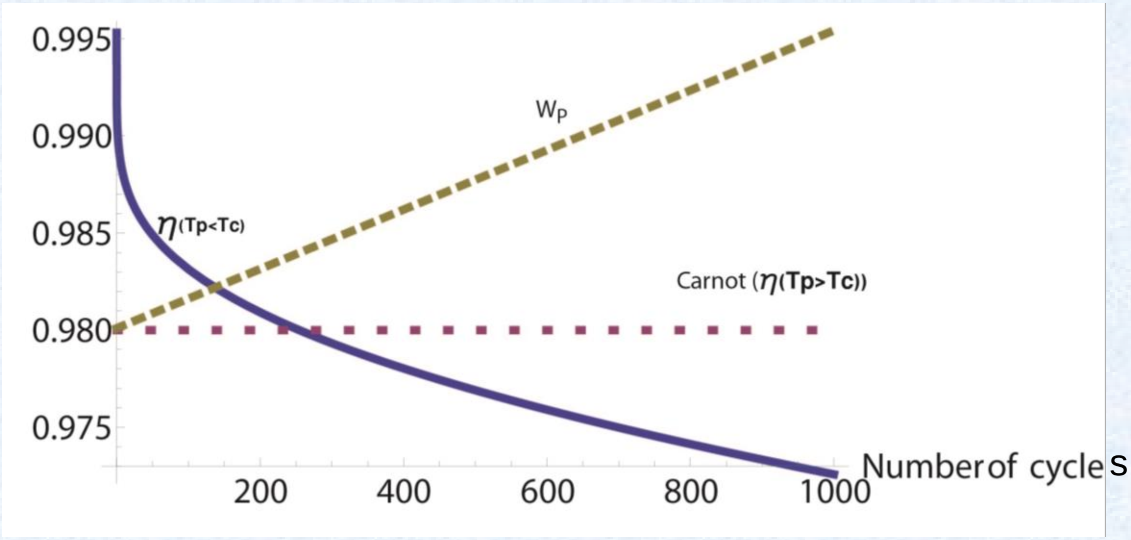
standard

Enforced by QM

Under second law constraint :

$$\dot{\mathcal{S}_{tot}} \approx \dot{\mathcal{S}}_p > \frac{J_C}{T_C} + \frac{J_H}{T_H}$$

$$\eta(T_P \leq T_C) = (\frac{d((W_P)_{Max}}{dt})/J_H < 1 - \frac{T_P}{T_H}$$



$$T_C/T_H = 1/50, \alpha^2(0) = 0.98$$

$\eta(W_P)$ bound can transgress 2-bath Carnot!
Classical analog: **3 baths** $T_P < T_C < T_H$
QM forces piston to be an effective bath!

Conclusions

- ❖ Are there quantum advantages in machines based on quantum resources? Yes

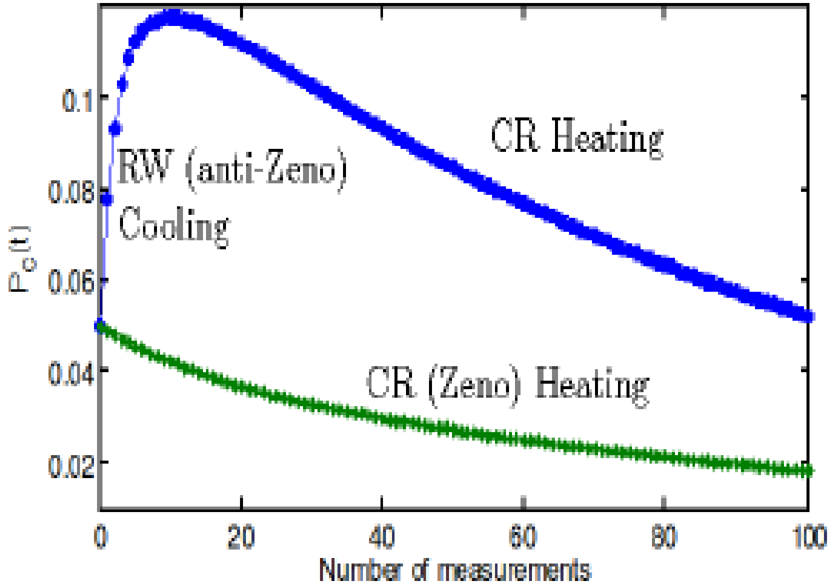
- ❖ Do they have classical analogs in different settings? Probably

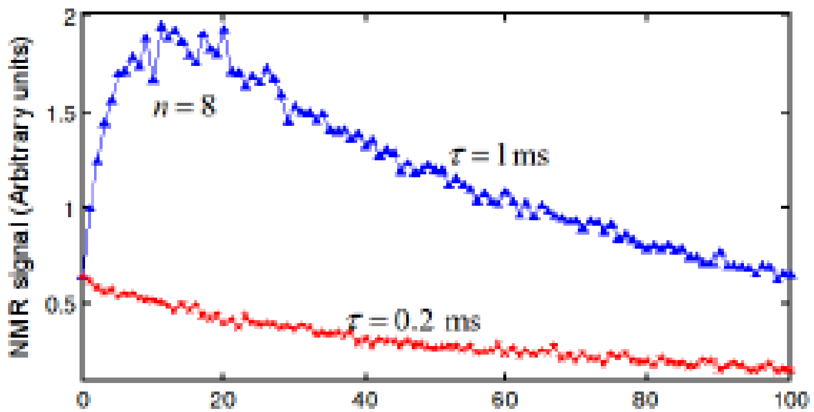
Quantum mechanics endows us with resources that may boost thermodynamic performance:
heat engine power, refrigeration/cooling speed or work.

Non-classical effects adhere to thermodynamic laws.

Measurement-driven control of quantum bits in a spin-bath

G. Alvarez, D. Dasari, L. Frydman and G. K., *PRL* **105**, 160401 (2010)



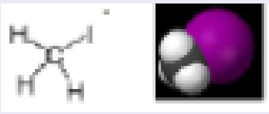


Interaction

$H_{SB} = J_{CH} \sum_k \hat{S}^x \hat{I}_k^x$ (CR+RW)

$\left.\begin{array}{l} P_C(0) = 0.05 \\ P_H(0) = 0.2 \end{array}\right\}$ non-equil

Experimental parameters



13C-methyl iodide (Iodomethane)

$J_{CH} = 150\text{Hz}; \quad \frac{\gamma_H \omega_H}{\gamma_C \omega_C} = 2$ (off-resonant)

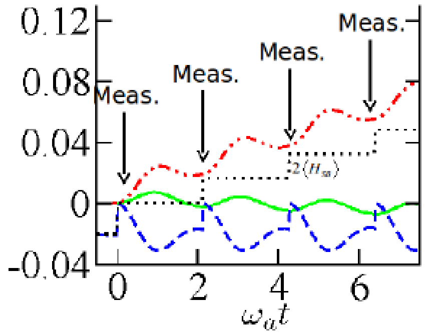
Induced Dephasings amplify the polarization transfer

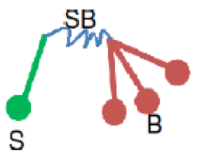
No Born: bath changes till $[\rho_{eq}, H_{tot}^{RW}] \approx 0$

I. Purification: Heating or cooling?

Total energy

$\langle H_S \rangle + \langle H_B \rangle + \langle H_{SB} \rangle = \langle H_{\rm tot} \rangle$
$\delta\langle H_S \rangle = -\delta\langle H_B \rangle - \delta\langle H_{SB} \rangle$
$B(S)$ heats & $S(B)$ cools at H_{SB}'s expense



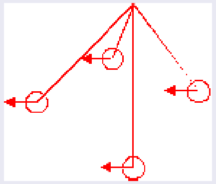


Non-selective (unread) measurement = induced dephasing:
$\langle H_{SB} \rangle \to 0 \to_t \langle H_{SB} \rangle < 0$

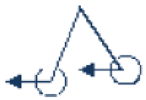
Unfamiliar non-Markov dynamics: Key to purity / decoherence control

Qubit and bath exchange as 2 coupled quantum oscillators, if qubit monitored frequently enough!

Ultrashort times

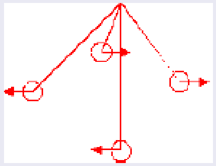


(bath oscillators)



(qubit)

Long times



(bath oscillators)

Qubit Evolution

$$H_{SB} = \sum_k \kappa_k \left(\underbrace{(b_k\sigma_+ + b_k^\dagger\sigma_-)}_{RW\text{(Freq. difference)}} + \underbrace{(b_k\sigma_- + b_k^\dagger\sigma_+)}_{CR\text{(Freq. sum)}} \right)$$

AZE

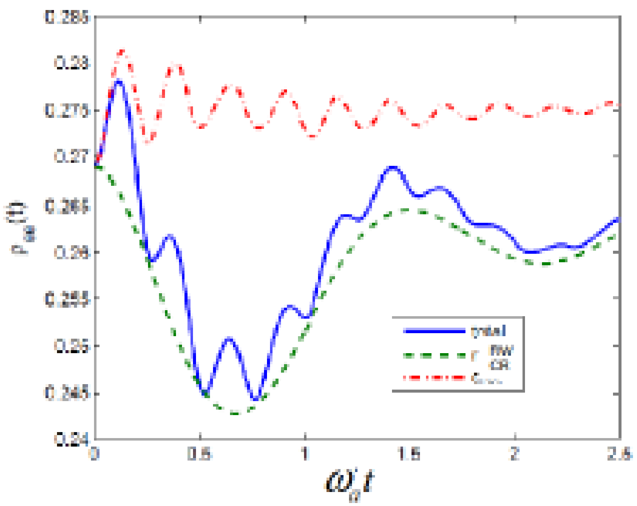
Long times

Resolved energy levels

Nature **405** 546 (2000)

$\dot\rho_{ee} = -R_e(t)\rho_{ee} + R_g(t)\rho_{gg} < 0$

May yield cooling



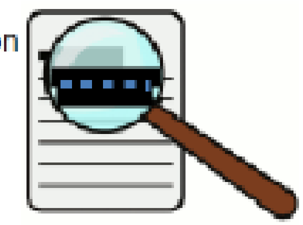
Nature **452**, 724 (2008)

New J. Phys. **11** 123025 (2009)

QZE

Ultrashort times

Unresolved energy levels



$\dot\rho_{ee} \xrightarrow[t\to 0]{} R(t)(\rho_{gg} - \rho_{ee}) > 0$

Always yields heating

Universal cooling bound

New J. Phys. **11** 123025 (2009)
New J. Phys. **12** 053033 (2010)

Master Eq. $\dot{\rho}_{ee} = R_G(t)\rho_{gg} - R_e(t)\rho_{ee}$

Solution for n measurements (QND disturbances)

$$\rho_{ee}(n\tau) = e^{-nJ(\tau)}\rho_{ee}(0) + \left(1 - e^{-nJ(\tau)}\right)\chi(\tau)$$

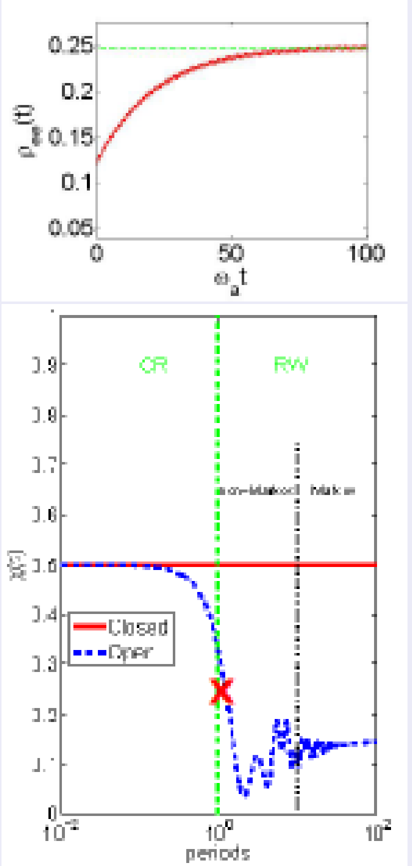
fixed point

$$\chi(\tau) = \frac{\int_0^\tau dt e^{J(t)} R_g(t)}{\int_0^\tau dt e^{J(t)}(R_g(t)+R_e(t))}$$

Relax. integral

$$J(t) = \int_0^t dt'(R_g(t') + R_e(t'))$$

Heating



After $n > t_c/\tau^2\kappa$,
$\tau \ll \omega_a^{-1} \Rightarrow \rho_{ee} \approx \chi \approx 1/2$ — fully mixed
Zeno= "heating"

Universal cooling bound

New J. Phys. 11 123025 (2009)
New J. Phys. 12 053033 (2010)

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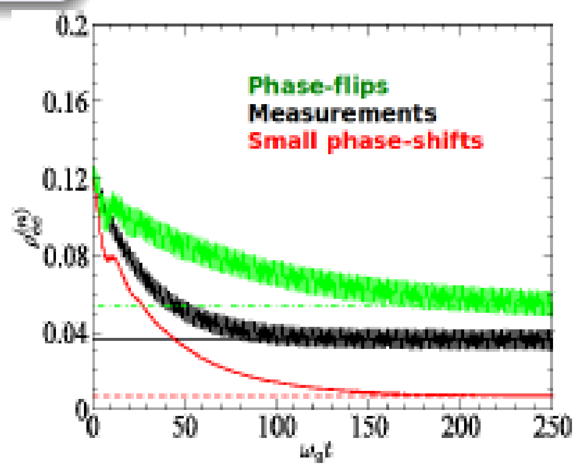
Relax. integral

$$J(t) = \int_0^t dt' (R_g(t') + R_e(t'))$$

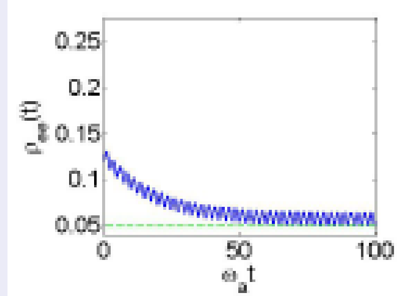
$$\tau \gtrsim \omega_a^{-1} \Rightarrow$$

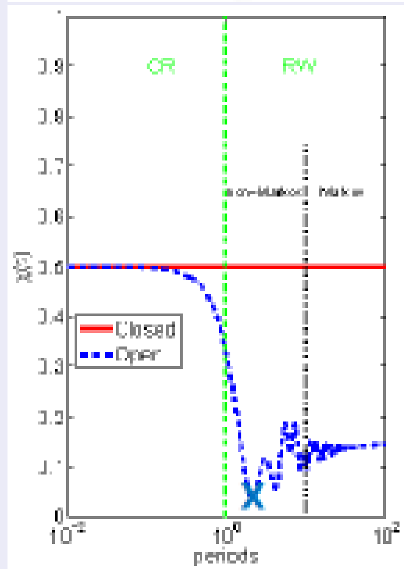
$$\rho_{ee} \approx \chi \ll \rho_{ee}(0):$$

AZE cooling



Cooling





Work-Information relation

Szilard-Landauer bound:
(Maxwell's demon)

$$(W_{Sel})_{Max} = T\mathcal{H}(\rho_S)$$

→ Shannon Entropy

- No correlations between system and bath $\rho_S \otimes \rho_B$
- Zero work at zero temperature

By contrast, our bound

$$(W_{Sel})_{Max} = T\mathcal{H}(\rho_S) + (W_{non-Sel})_{Max}$$

- Correlations between the system and the bath $\rho_{tot}^{eq} \neq \rho_S \otimes \rho_B$
- More work is obtained but higher price is paid for performing the measurement
- Work can be extracted even at zero temperature

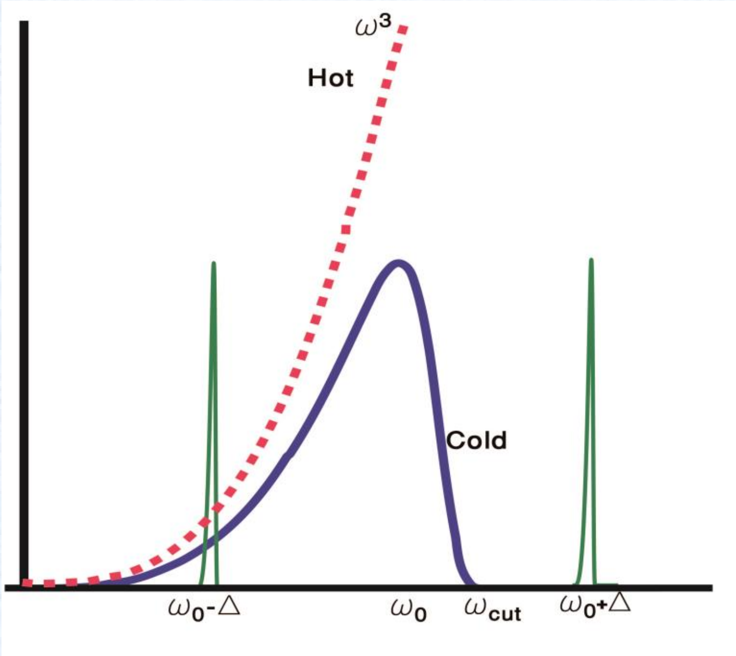
$$W_{non-sel} \neq 0 \Rightarrow W_{sel} \neq 0$$

Spectral separation of C&H baths

(M. Kolar, D. Gelbwaser-Klimovsky, R. Alicki & GK PRL **109** 090601 (2012))

qubit-phase π-flips at $\tau = \frac{2\pi}{\Delta}$:harmonics $m = \pm 1$

cause shifts of $G^{C(H)}(\omega_0) : \omega_0 \to \omega_0 \pm \Delta.$



Example:
Hot bath: black body (broad) spectrum
Cold bath: Lorentzian spectrum
(e.g., cavity mode)

$$J_H = (\omega_0 + \Delta)\mathcal{N}\left(e^{-(\frac{\omega_0+\Delta}{T_H})} - e^{-(\frac{\omega_0-\Delta}{T_C})}\right),$$

$$J_C = -(\omega_0 - \Delta)\mathcal{N}\left(e^{-(\frac{\omega_0+\Delta}{T_H})} - e^{-(\frac{\omega_0-\Delta}{T_C})}\right),$$

$$\mathcal{P} = -2\Delta\mathcal{N}\left(e^{-(\frac{\omega_0+\Delta}{T_H})} - e^{-(\frac{\omega_0-\Delta}{T_C})}\right),$$

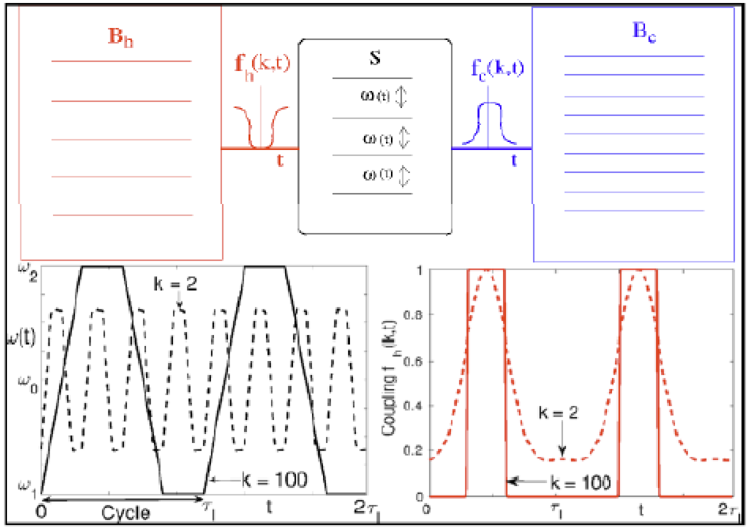
Heat pump (QR) condition: $J_C > 0 : n^C(\omega_0 - \Delta) > n^H(\omega_0 + \Delta)$

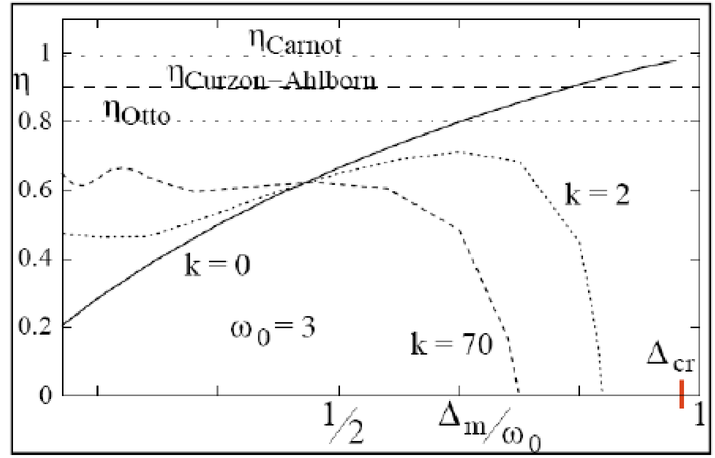
QHE: $J_C < 0 : n^C(\omega_0 - \Delta) < n^H(\omega_0 + \Delta)$

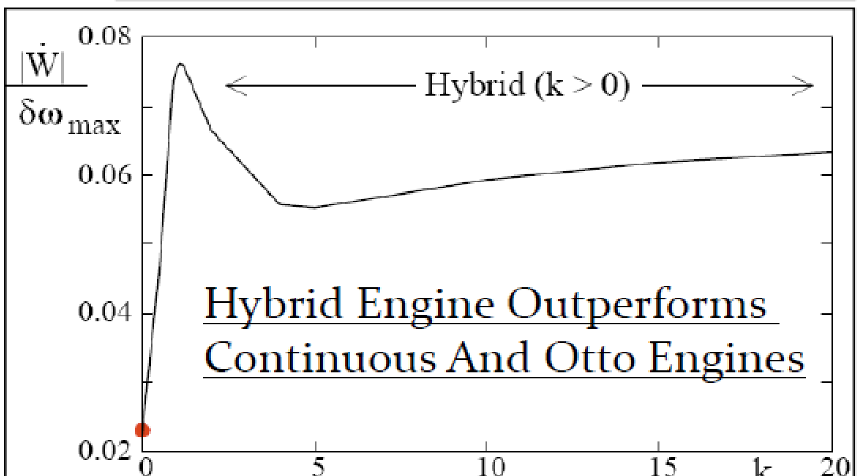
How fast can QHE operate?
What is the best design?

Victor Mukherjee, Wolfgang Niedenzu and GK

Heat Engine And Modulations





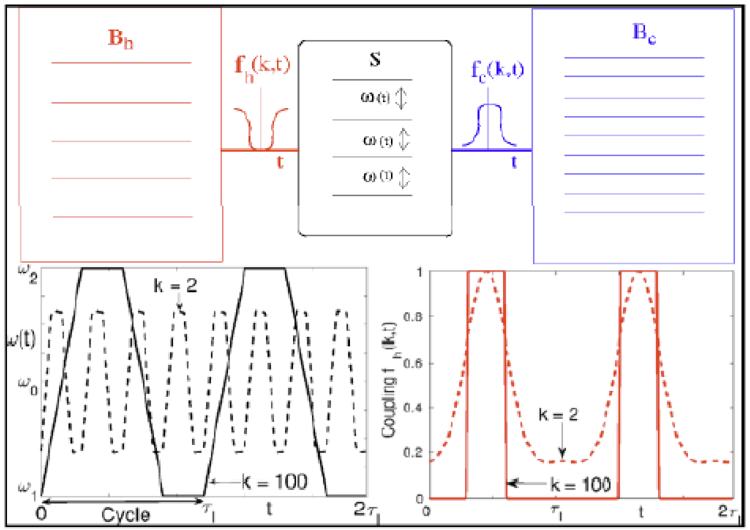


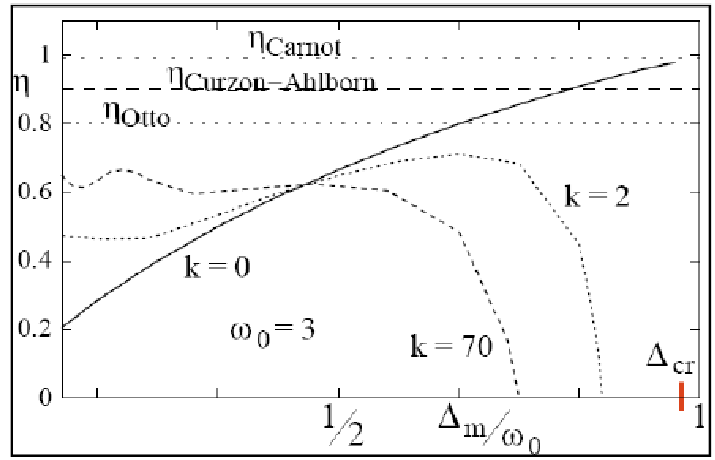
Power

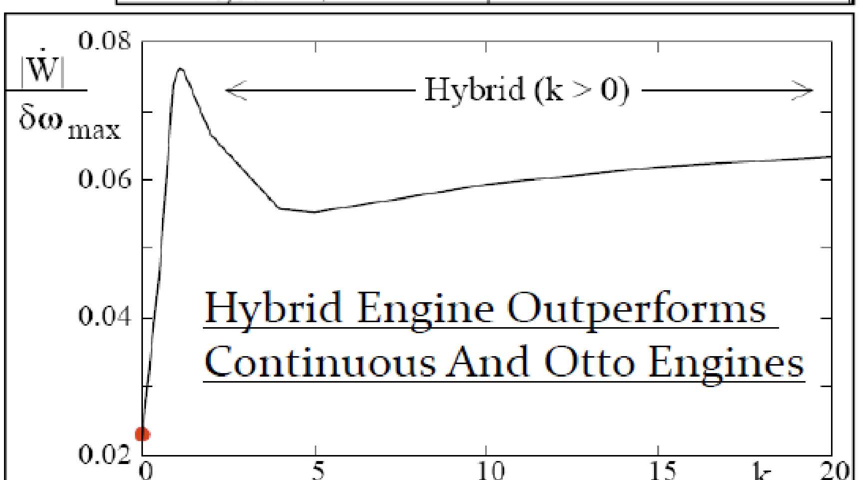
How fast can QHE operate? What is the best design?

Victor Mukherjee, Wolfgang Niedenzu and GK

Heat Engine And Modulations

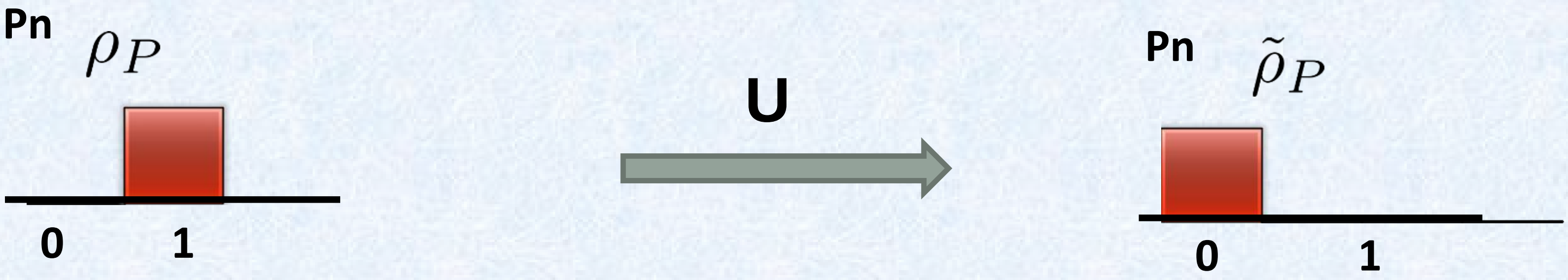






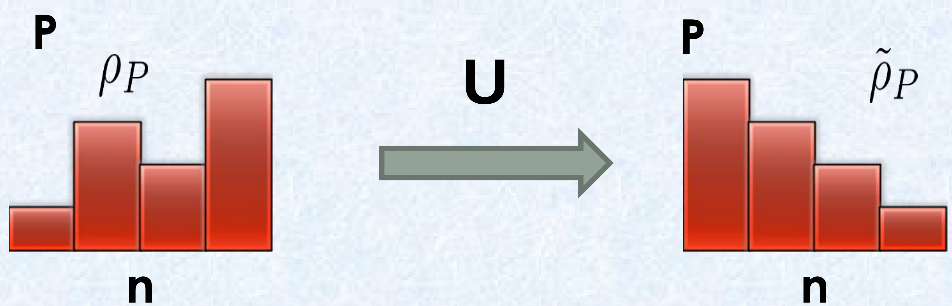
Power

$$W^{Max}(\rho_P) = \langle H_P \rangle_{\rho_P} - \langle H_P \rangle_{\tilde{\rho}_P^{Min}}$$



If $W^{Max}(\rho_P) = 0$: Passive state.
(lowest energy = Gibbs state)

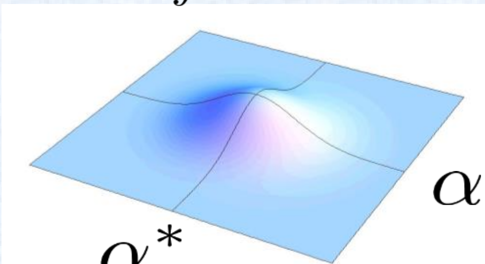
Unitary reshuffle of eigenvalues



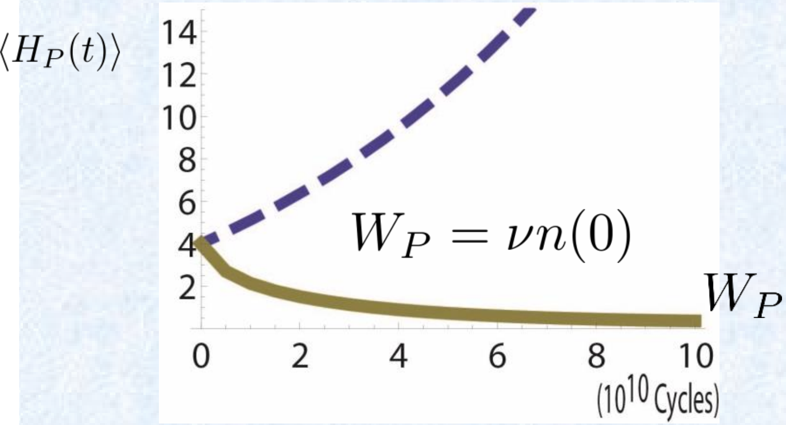
Monotonic
= passive

Passive

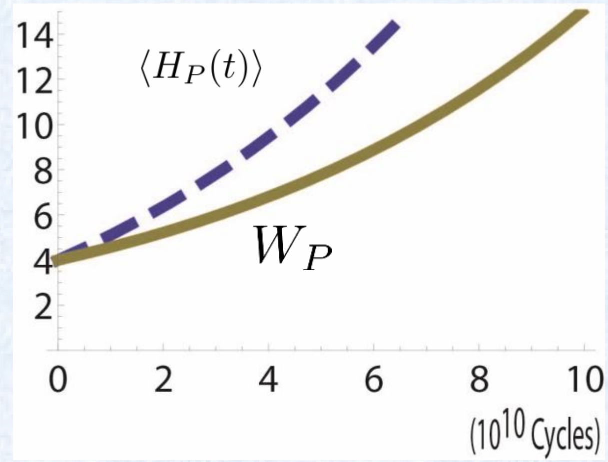
$$\rho_P = \frac{1}{2\pi} \int d^2\alpha \mathbf{P}(\alpha) |\alpha\rangle\langle\alpha|$$



Self-contained QHE: Only nonpassive P yields work. EPL **103** 60005 (2013)

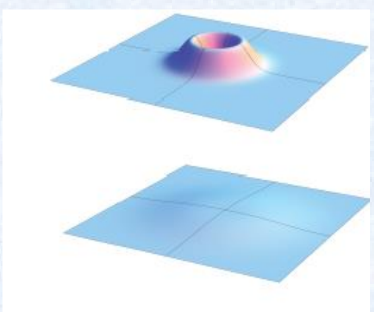


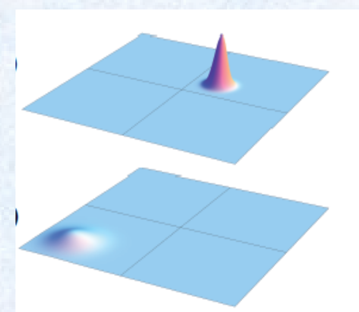
$$W_P = \nu \alpha^2(0) e^{-\gamma t}$$



Fock

Coherent

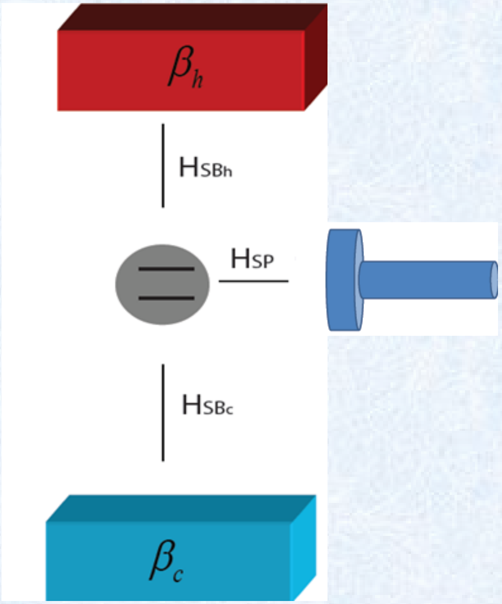




Nonpassive state is low-entropy resource.

Super-Carnot efficiency

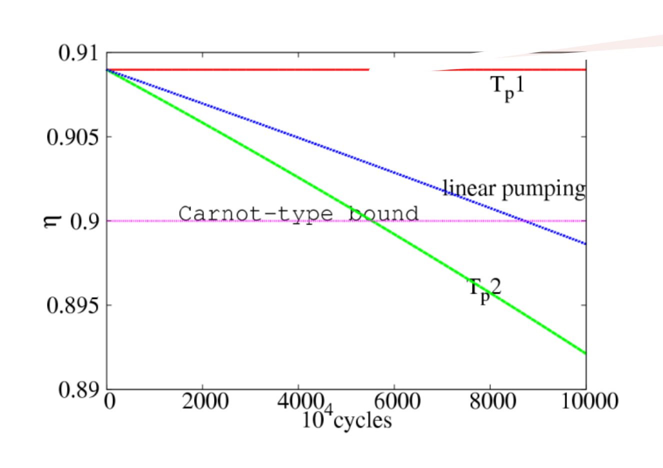
Ghosh et al, (in prep)



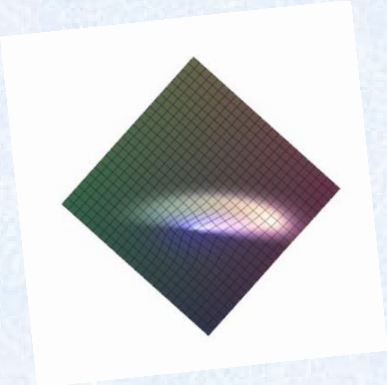
Efficiency:

$$\eta \le \begin{cases} 1-\dfrac{T_C}{T_H}; & T_P > T_C \\ 1-\dfrac{T_P}{T_H}; & T_C > T_P \end{cases}$$

- Nonlinear driven



Stationary super-Carnot efficiency

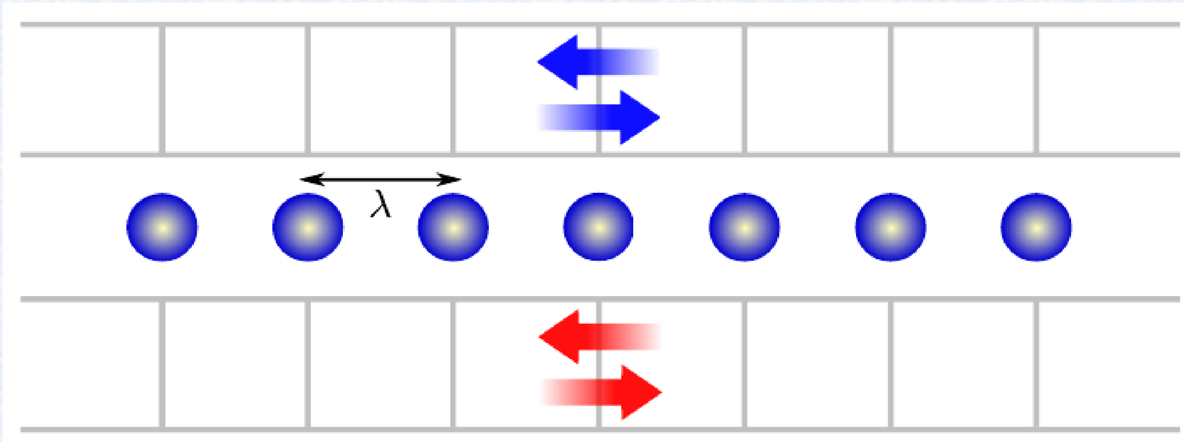


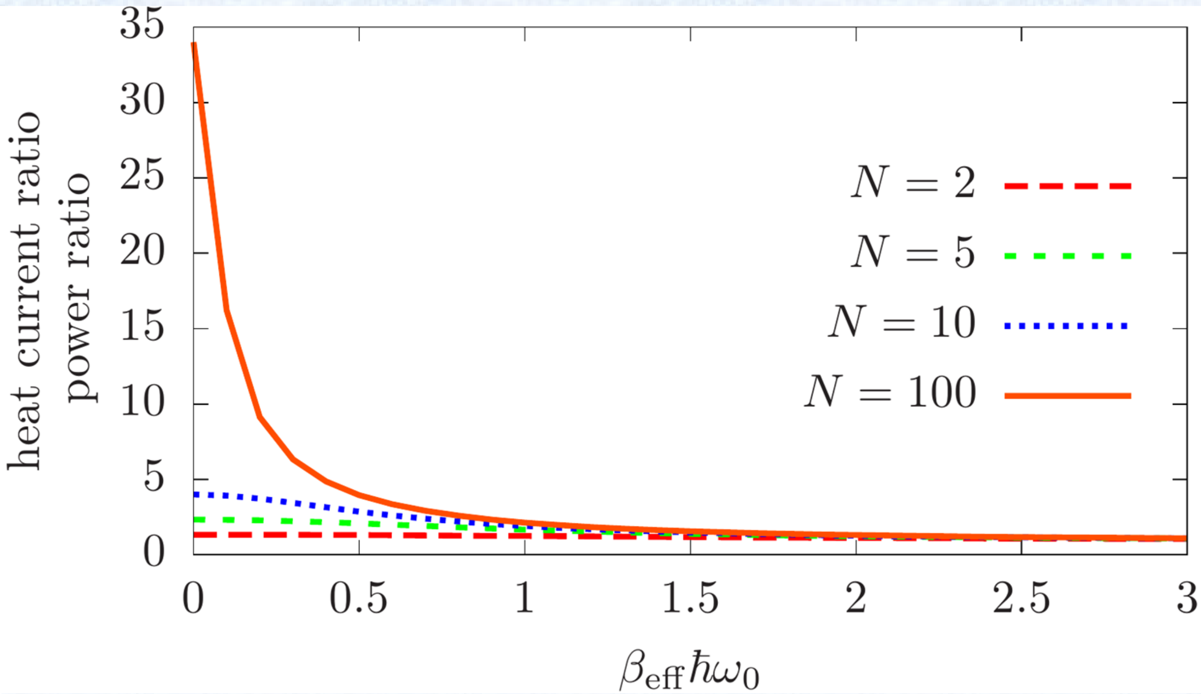
Multiatom quantum heat engine (QHE)

D. Gelbwaser-Klimovsky, W. Niedenzu & G.K., AAMOP **64** (2015)

Collective spin operator:

$$J_- := \sum_{i=1}^{N} \sigma_-^i$$





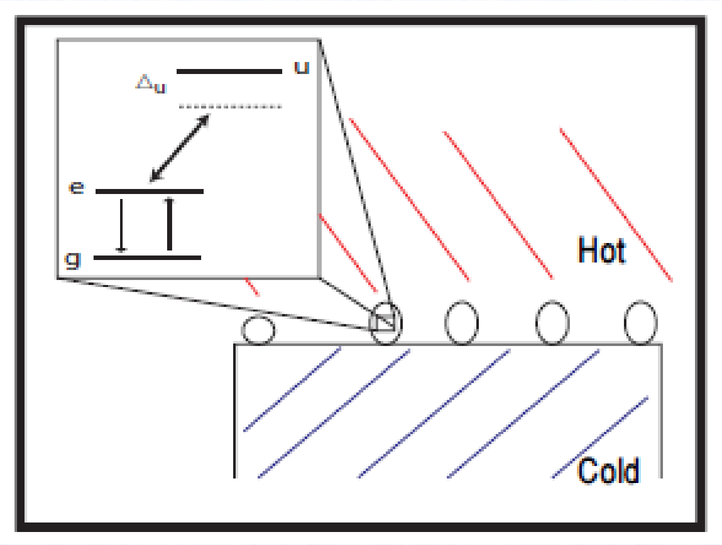
Efficiency unchanged

$$\lim_{\beta_{\mathrm{eff}}\to\infty} \frac{\dot{W}}{N\dot{W}^{\mathrm{TLS}}} = 1$$

$$\lim_{\beta_{\mathrm{eff}}\to 0} \frac{\dot{W}}{N\dot{W}^{\mathrm{TLS}}} = \frac{N+2}{3}$$

Simplest (Minimal) Model of Quantum Heat Machine

(M. Kolar, D. Gelbwaser-Klimovsky, R. Alicki & GK PRL **109**, 090601 (2012))



Transition rates for qubit (TLS) coupled to 2 baths

Quasi steady-state

Weak coupling

Coarse graining $J_{C(H)} = \dot{\bar{Q}}_{C(H)} = \sum_m (\omega_0 + m\Delta) \quad G^{C(H)}(\omega_0 + m\Delta)$

Multi-harmonic Lindblad $\mathcal{L} = \sum \mathcal{L}^j_m,$ *J*=*H*,*C*, and *m* is the Floquet harmonic

$$\mathcal{L}^j_m \rho = \frac{P_m}{2}(G^j(\omega_0 + m\Delta)([\sigma^- \rho, \sigma^+] + [\sigma^-, \rho\sigma^+]) + G^j(-\omega_0 - m\Delta)([\sigma^+ \rho, \sigma^-] + [\sigma^+, \rho\sigma^-])).$$

Minimal (Simplest) Quantum Heat Machine (Under Spectral Separation of Baths)

D. Gelbwaser-Klimovsky, R. Alicki & G.K., PRE **87,** 012140 (2013)

Quantum heat engine (QHE) $(\mathcal{P} > 0)$:

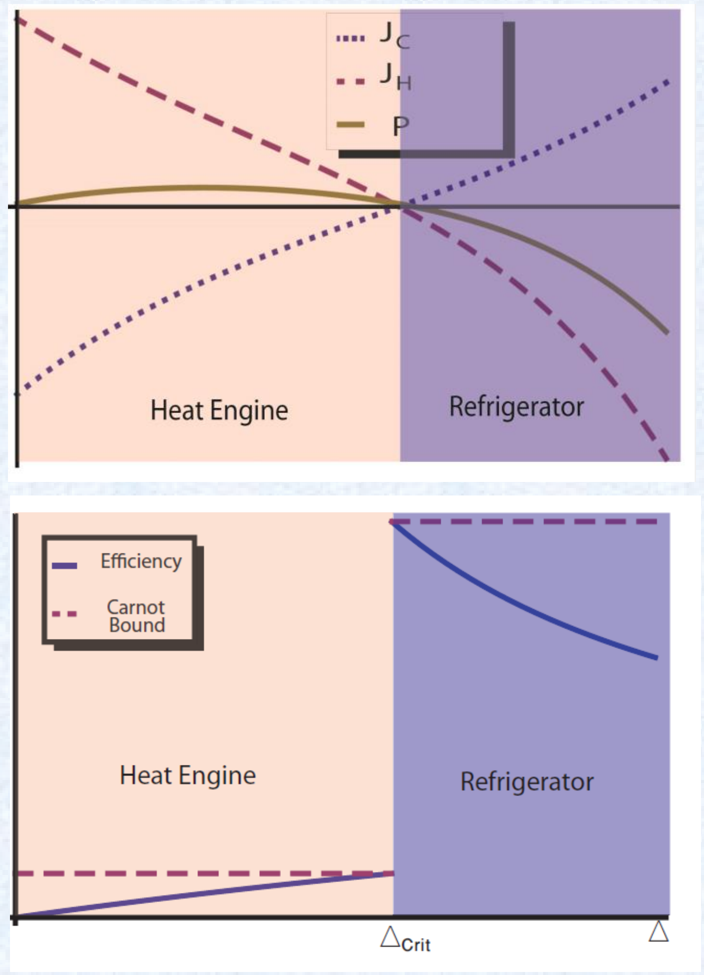
$$\Delta < \Delta_{cr} = \omega_0 \frac{T_H - T_C}{T_H + T_C}$$

$$\eta = \frac{\mathcal{P}}{J_H} = 1 - \frac{T_C}{T_H} \text{ Carnot bound}$$

Quantum refrigerator (QR $(J_C > 0)$:

$$\Delta > \Delta_{cr} = \omega_0 \frac{T_H - T_C}{T_H + T_C}$$

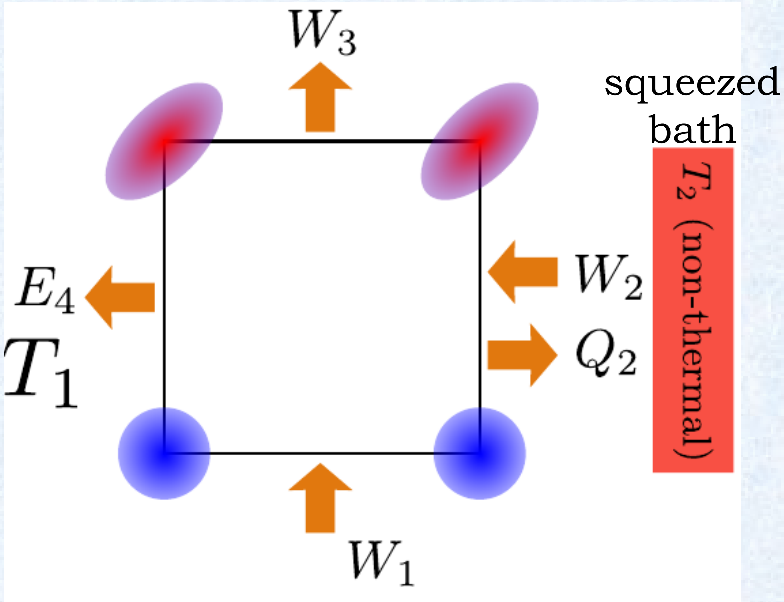
$$COP : \frac{J_C}{\mathcal{P}} = \frac{\omega_0 - \Delta}{2\Delta} \xrightarrow{\Delta_{cr}} \text{Carnot bound}$$



QHE powered by non-thermal baths

Quantum Otto cycle

J. Roßnagel et al., PRL **112**, 030602 (2014)
O. Abah and E. Lutz, EPL **106**, 20001 (2014)



Non-thermal bath only provides heat?
No, WF becomes **non-passive**

Work from bath:

$$W_2 = E_{\rm hot} - E_{\rm hot}^{\rm therm} = \hbar\omega_2 \Delta\bar{n} > 0$$

Non-thermal engine

$$\eta = 1 - \frac{\omega_1}{\omega_2} \leq 1 - \frac{T_1}{\Theta_{\rm fic}(\Delta\bar{n})}$$

$$T_1 = T_2 = 0 \quad W_2 > 0$$

No heat