Thermodynamic laws and bounds are not well understood for quantum-system manipulations. We challenge:
- Thermodynamic equilibrium (First law?)
- Carnot efficiency bound of quantum heat engines/refrigerators (Second law)
- Unattainability of absolute zero (Third law)
Extracting work via (non-selective) measurements: Demented Maxwell’s demon, but bath has memory!
Can we bypass Kelvin: extract work from a single-bath engine?


\[ \rho_S(0) \text{ Measurement} \rightarrow \rho_S \otimes \rho_B \text{ Modulation} \rightarrow \ldots \]

**QND** non-selective measurement of the system energy (Measure system energy and do not read the results)

The measurement does not change the state of the system or the bath, but changes the total state.
Where does the work come from?

Work extraction only for short cycles
Experimental scenarios

\[ t_{cycle} \leq \mu \text{ sec} \leq t_{correl} \]
Rapport between thermodynamics and quantum mechanics for quantum heat machines?
The basic model: a two-level system (TLS) whose energy is periodically modulated while the system is coupled to two distinct thermal baths.

1) The World’s Smallest Heat Engine

Couple the atom to two “heat baths”: hot and cold

Hook an oscillator up to the system

Heat will amplify the oscillations and produce work

2) The World’s Smallest Refrigerator

Energy now flows from cold to hot

The oscillator is compressed

The oscillator will cool down the cold bath

Quantum bath Refrigeration: Towards Absolute Zero?


\[
dT_C/dt = -AT_C^\gamma.
\]

\(\gamma\) : Determined by bath dispersion

\[
d\omega/dk \sim \omega^{1-\gamma}(k)
\]

\(T_C \rightarrow 0\) :

\(\omega(k) \sim v|k|\); \(d\omega/dk \rightarrow \text{const.}\) \((\gamma = 1)\)

i) Acoustic phonons

ii) Magnons (spin-wave)/dipole-coupled atom chain:

\(d\omega/dk \rightarrow 0\) (freezing) \((\gamma = 0)\)

Nernst’s 3\textsuperscript{rd} law unattainability principle challenged
Energy exchange is divided into work and heat....

... and they are really different.
Work-power? → No! 2\textsuperscript{nd} law violated

\[ P = J_C + J_H = \langle \dot{H}_P \rangle \]

1\textsuperscript{st} law: heat-input = energy-output
CORRECT WORK DEFINITION
D. Gelbwaser-Klimovsky et al. EPL (2013); PRE (2014)

In a thermally adiabatic process $\Delta E = W$ and $\dot{S} = 0 \quad (P = \dot{W})$

Take a state of the piston $\rho_P$. If $\tilde{\rho}_P = U \rho_P U^\dagger$, $(S(\rho_P) = S(\tilde{\rho}_P))$ then

$$W = \langle H_P \rangle_{\rho_P} - \langle H_P \rangle_{\tilde{\rho}_P}$$

$$W^{Max}(\rho_P) = \langle H_P \rangle_{\rho_P} - \langle H_P \rangle_{\tilde{\rho}_P^{Min}}$$

potentially extractable work (“stored” in $\rho_P$)

WORK CAPACITY EVOLUTION (PISTON CHARGING)

\[ T_H \]

\[ J_H \]
\[ \rho_P(t) \]

\[ J_C \]

\[ T_C \]

\[ \rho_P(0) \]
\[ W^{Max}(\rho_P(0)) \]

\[ W^{Max}(\rho_P(t)) \]

Work extracted from engine:
\[ \Delta W(t) \equiv W^{Max}(\rho_P(t)) - W^{Max}(\rho_P(0)) \]

The engine is a charger, the piston is a battery

D. Gelbwaser-Klimovsky, R. Alicki, G. Kurizki  
*EPL* 103 60005 (2013)
Work capacity is non-passivity

D. Gelbwaser-Klimovsky et al. EPL (2013); PRE (2014)

Min. energy (same entropy)

\[
(W_P)_{Max} \leq \langle H_P(\rho_P) \rangle - \langle H_P(\rho'_P) \rangle_{\text{Gibbs}};
\]

\[
(\rho'_P)_{\text{Gibbs}} = Z^{-1} e^{-\frac{H_P}{T_P}}
\]

Real work-power:

\[
\frac{d(W_P)_{Max}}{dt} = \frac{d\langle H_P \rangle}{dt} - T_P(t)\dot{S}_P,
\]

Enforced by QM
Under second law constraint:

\[ S_{\text{tot}} \approx \dot{S}_p > \frac{J_C}{T_C} + \frac{J_H}{T_H} \]

\[ \eta(T_P \leq T_C) = \left( \frac{d((W_P)_{\text{Max}})}{dt} \right)/J_H < 1 - \frac{T_P}{T_H} \]

\[ T_C/T_H = 1/50, \alpha^2(0) = 0.98 \]

\( \eta(W_P) \) bound can transgress 2-bath Carnot! Classical analog: 3 baths \( T_P < T_C < T_H \)

QM forces piston to be an effective bath!
Conclusions

- Are there quantum advantages in machines based on quantum resources? Yes

- Do they have classical analogs in different settings? Probably

Quantum mechanics endows us with resources that may boost thermodynamic performance:
heat engine power, refrigeration/cooling speed or work.

**Non-classical effects adhere to thermodynamic laws.**
Measurement-driven control of quantum bits in a spin-bath


Interaction

\[ H_{SB} = J_{CH} \sum_k \hat{S}_k \cdot \hat{I}_k \text{ (CR+RW)} \]

\[
\begin{aligned}
P_C(0) &= 0.05 \quad \text{non-equil} \\
P_H(0) &= 0.2
\end{aligned}
\]

Experimental parameters

13C-methyl iodide (Iodomethane)

\[ J_{CH} = 150\text{Hz}; \quad \frac{\gamma_H}{\gamma_C} \frac{\omega_H}{\omega_C} = 2 \quad \text{(off-resonant)} \]

Induced Dephasings amplify the polarization transfer

No Born: bath changes till

\[ [\rho_{eq}, H_{\text{tot}}^{RW}] \approx 0 \]
I. Purification: Heating or cooling?

Total energy

\[ \langle H_S \rangle + \langle H_B \rangle + \langle H_{SB} \rangle = \langle H_{tot} \rangle \]
\[ \delta \langle H_S \rangle = -\delta \langle H_B \rangle - \delta \langle H_{SB} \rangle \]

B(S) heats & S(B) cools at \( H_{SB} \)'s expense

Non-selective (unread) measurement = induced dephasing:
\[ \langle H_{SB} \rangle \to 0 \to t \langle H_{SB} \rangle < 0 \]

Unfamiliar non-Markov dynamics: Key to purity / decoherence control

Qubit and bath exchange as 2 coupled quantum oscillators, if qubit monitored frequently enough!

Ultrashort times

(bath oscillators)

Long times

(qubit)

(bath oscillators)
Qubit Evolution

\[ H_{SB} = \sum_k \kappa_k \left( \langle b_k \sigma_+ + b_k^\dagger \sigma_- \rangle + \langle b_k \sigma_- + b_k^\dagger \sigma_+ \rangle \right) \]

\[ \text{(RW (Freq. difference))} \quad \text{CR (Freq. sum)} \]

AZE

Long times
Resolved energy levels
relaxation
\( \dot{\rho}_{ee} = -R_e(t) \rho_{ee} + R_g(t) \rho_{gg} \)
< 0
May yield cooling

Nature 405, 546

QZE

Ultrashort times
Unresolved energy levels
relaxation
\( \dot{\rho}_{ee} \xrightarrow{t \to 0} R(t)(\rho_{gg} - \rho_{ee}) > 0 \)
Always yields heating

Nature 452, 724 (2008)
Universal cooling bound


Master Eq. \[ \dot{\rho}_{ee} = R_G(t)\rho_{gg} - R_e(t)\rho_{ee} \]

Solution for \( n \) measurements (QND disturbances)
\[ \rho_{ee}(n\tau) = e^{-nJ(\tau)}\rho_{ee}(0) + (1 - e^{-nJ(\tau)})\chi(\tau) \]

### fixed point
\[ \chi(\tau) = \frac{\int_0^\tau dt e^{J(t)}R_g(t)}{\int_0^\tau dt e^{J(t)}(R_g(t) + R_e(t))} \]

### Relax. integral
\[ J(t) = \int_0^t dt'(R_g(t') + R_e(t')) \]

After \( n > t_c/\tau^2\kappa \),
\[ \tau \ll \omega_a^{-1} \Rightarrow \rho_{ee} \approx \chi \approx 1/2 — \text{fully mixed} \]

-Zeno= “heating”
Universal cooling bound


**Master Eq.**
\[
\dot{\rho}_{ee} = R_G(t)\rho_{gg} - R_e(t)\rho_{ee}
\]

Solution for *n* measurements (QND disturbances)
\[
\rho_{ee}(n\tau) = e^{-nJ(\tau)}\rho_{ee}(0) + \left(1 - e^{-nJ(\tau)}\right)\chi(\tau)
\]

**Fixed point**
\[
\chi(\tau) = \frac{\int_0^\tau dt e^{-J(t)}R_g(t)}{\int_0^\tau dt e^{-J(t)}(R_g(t)+R_e(t))}
\]

**Relax. integral**
\[
J(t) = \int_0^t dt' (R_g(t') + R_e(t'))
\]

\[\tau \gg \omega_a^{-1} \Rightarrow \rho_{ee} \approx \chi \ll \rho_{ee}(0) : \text{AZE cooling}\]
Work-Information relation

Szilard-Landauer bound: \((W_{\text{sel}})_{\text{Max}} = T \mathcal{H}(\rho_S)\)  
(Maxwell's demon)

- No correlations between system and bath \(\rho_S \otimes \rho_B\)
- Zero work at zero temperature

By contrast, our bound \((W_{\text{Sel}})_{\text{Max}} = T \mathcal{H}(\rho_S) + (W_{\text{non-Sel}})_{\text{Max}}\)

- Correlations between the system and the bath \(\rho^{\text{eq}}_{\text{tot}} \neq \rho_S \otimes \rho_B\)
- More work is obtained but higher price is paid for performing the measurement
- Work can be extracted even at zero temperature

\(W_{\text{non-sel}} \neq 0 \Rightarrow W_{\text{sel}} \neq 0\)
Spectral separation of C&H baths

qubit-phase $\pi$-flips at $\tau = \frac{2\pi}{\Delta}$ : harmonics $m = \pm 1$

cause shifts of $G^{C(H)}(\omega_0) : \omega_0 \rightarrow \omega_0 \pm \Delta$.

Example:
Hot bath: black body (broad) spectrum
Cold bath: Lorentzian spectrum
(e.g., cavity mode)

$$J_H = (\omega_0 + \Delta)N\left(e^{-\frac{(\omega_0+\Delta)}{T_H}} - e^{-\frac{(\omega_0-\Delta)}{T_C}}\right),$$
$$J_C = -(\omega_0 - \Delta)N\left(e^{-\frac{(\omega_0+\Delta)}{T_H}} - e^{-\frac{(\omega_0-\Delta)}{T_C}}\right),$$
$$\mathcal{P} = -2\Delta N\left(e^{-\frac{(\omega_0+\Delta)}{T_H}} - e^{-\frac{(\omega_0-\Delta)}{T_C}}\right),$$

Heat pump (QR) condition: $J_c > 0$: $n^C(\omega_0 - \Delta) > n^H(\omega_0 + \Delta)$
QHE: $J_c < 0$: $n^C(\omega_0 - \Delta) < n^H(\omega_0 + \Delta)$
How fast can QHE operate? What is the best design?

Victor Mukherjee, Wolfgang Niedenzu and GK
How fast can QHE operate? What is the best design?

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Heat Engine And Modulations

Hybrid Engine Outperforms Continuous And Otto Engines
If $W^{Max}(\rho_P) = 0$: Passive state.
(lowest energy = Gibbs state)

Unitary reshuffle of eigenvalues

$\rho_P = \frac{1}{2\pi} \int d^2\alpha P(\alpha)|\alpha\rangle\langle\alpha|$

Monotonic = passive
Self-contained QHE: Only nonpassive P yields work.  
EPL 103 60005 (2013)

\[ W_P = \nu n(0) \]

\[ W_P = \nu \alpha^2(0) e^{-\gamma t} \]

Nonpassive state is low-entropy resource.
Super-Carnot efficiency
Ghosh et al, (in prep)

Efficiency:

$$\eta \leq \begin{cases} 
1 - \frac{T_C}{T_H} ; & T_P > T_C \\
1 - \frac{T_P}{T_H} ; & T_C > T_P 
\end{cases}$$

- Nonlinear driven

Stationary super-Carnot efficiency
Multiatom quantum heat engine (QHE)

Collective spin operator:

\[ J_- := \sum_{i=1}^{N} \sigma_-^i \]

Efficiency unchanged

\[ \lim_{\beta_{\text{eff}} \to \infty} \frac{\dot{W}}{N\dot{W}_{\text{TLS}}} = 1 \]

\[ \lim_{\beta_{\text{eff}} \to 0} \frac{\dot{W}}{N\dot{W}_{\text{TLS}}} = \frac{N+2}{3} \]
Simplest (Minimal) Model of Quantum Heat Machine

Quasi steady-state
Weak coupling

Coarse graining
\[ J_{C(H)} = \frac{\dot{Q}_{C(H)}}{G_{C(H)}} = \sum_{m} (\omega_0 + m\Delta) \quad G^{C(H)}(\omega_0 + m\Delta) \]

Multi-harmonic Lindblad
\[ \mathcal{L} = \sum \mathcal{L}_m^j, \quad J=H,C, \text{ and } m \text{ is the Floquet harmonic} \]

Transition rates for qubit (TLS) coupled to 2 baths

\[ \mathcal{L}_m^j \rho = \frac{P_m}{2} (G^j(\omega_0 + m\Delta)([\sigma^- \rho, \sigma^+] + [\sigma^-, \rho \sigma^+]) \]
\[ + G^j(-\omega_0 - m\Delta)([\sigma^+ \rho, \sigma^-] + [\sigma^+, \rho \sigma^-])). \]
Quantum heat engine (QHE) \((\mathcal{P} > 0)\):

\[
\Delta < \Delta_{cr} = \omega_0 \frac{T_H - T_C}{T_H + T_C}
\]

\[
\eta = \frac{\mathcal{P}}{J_H} = 1 - \frac{T_C}{T_H} \text{ Carnot bound}
\]

Quantum refrigerator (QR) \((J_C > 0)\):

\[
\Delta > \Delta_{cr} = \omega_0 \frac{T_H - T_C}{T_H + T_C}
\]

\[
\text{COP} : \frac{J_C}{\mathcal{P}} = \frac{\omega_0 - \Delta}{2\Delta} \quad \Delta_{cr} \quad \text{Carnot bound}
\]
**QHE powered by non-thermal baths**

Quantum Otto cycle

J. Roßnagel et al., PRL 112, 030602 (2014)
O. Abah and E. Lutz, EPL 106, 20001 (2014)

Non-thermal bath only provides heat?
No, WF becomes **non-passive**

Work from bath:

\[ W_2 = E_{\text{hot}} - E_{\text{therm}} = \hbar \omega_2 \Delta \bar{n} > 0 \]

Non-thermal engine

\[ \eta = 1 - \frac{\omega_1}{\omega_2} \leq 1 - \frac{T_1}{\Theta_{\text{fic}}(\Delta \bar{n})} \]

\[ T_1 = T_2 = 0 \quad W_2 > 0 \]

No heat